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A TNT DSGE Model for Chile: Explaining the ERPT *

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Abstract

We present a fully-edged dynamic stochastic general equilibrium (DSGE) model for the Chilean economy to explain the economy's adjustments to external shocks, explicitly separating between tradable and non-tradable sectors (TNT). The model was built to explain Chile's linkages with the external sector, to recognize that the sectors of the economy have particular price dynamics that are affected differently by shocks that move the nominal exchange rate, and to study different measures of exchange rate pass through (ERPT). We show unconditional and conditional ERPT measures. The former measures are comparable with the empirical literature, while the latter are defined after a particular shock hit the economy. We highlight important differences in their magnitudes and in their effect on different prices. While a shock to international prices has a transitory and low ERPT, one that affects the uncovered interest rate parity condition has a very high and persistent ERPT for all price indexes. In addition, the prices that are more rapidly affected are those of tradable sectors, while non-tradable prices are affected with a lag, but for longer. We use the model to show that the conditional ERPT measures could have helped to anticipate a great part of the inflationary effects of the depreciation following the tapering announcements of the US in 2013-2015, which was not possible using unconditional ERPT measures of the empirical literature.

*The views and conclusions presented in this paper are exclusively those of the author and do not necessarily reflect the position of the Central Bank of Chile or its Board members. García-Schmidt: mcgarcia@bcentral.cl; García-Cicco: jgarciac@ucema.edu.ar

1 Introduction

Chile is a small and open economy with important linkages to the rest of the world. It buys and sells products in international markets and it also borrows and lends money in foreign currency. The Chilean peso floats since September 1999 and acts as a buffer to external shocks. Because of that it can fluctuate strongly at times, affecting internal prices and the cost of external borrowing, thus generating nominal and real effects in the Chilean economy.

Not all sectors are affected equally by movements in the nominal exchange rate (NER). The general belief is that prices of goods should be more affected than prices of services, because, in general, the former are tradable and so face external competition and the latter are non-tradable and depend only on internal conditions. In addition, other sector-specific characteristics may affect the magnitude and propagation of nominal depreciations, among which are the share of imported inputs used in production and the price setting mechanism in each sector. A higher share of imported inputs makes the marginal cost more sensitive to NER movements, making the price also more sensitive for a given price setting mechanism. More flexibility in setting prices implies that any change in marginal costs can be more rapidly transmitted to final prices. In contrast, if the price setting mechanism implies a higher indexation, the effects will be slower and more persistent, providing a propagation mechanism that takes a while to be felt completely.

Figure 1 shows a graphic representation of the first two paragraphs. The graph on the left shows the very big scale of the changes in the NER (right axis) and the high correlation between variations in the NER and in the consumer price index (CPI) in its headline or core versions (excluding food and energy). The graph on the right shows that while inflation in goods and services seem correlated with nominal exchange rate movements, their behavior is different, being the average of inflation in services higher than in goods (4.3% versus 0.1%), but its standard deviation lower (1.5% versus 2.4%).

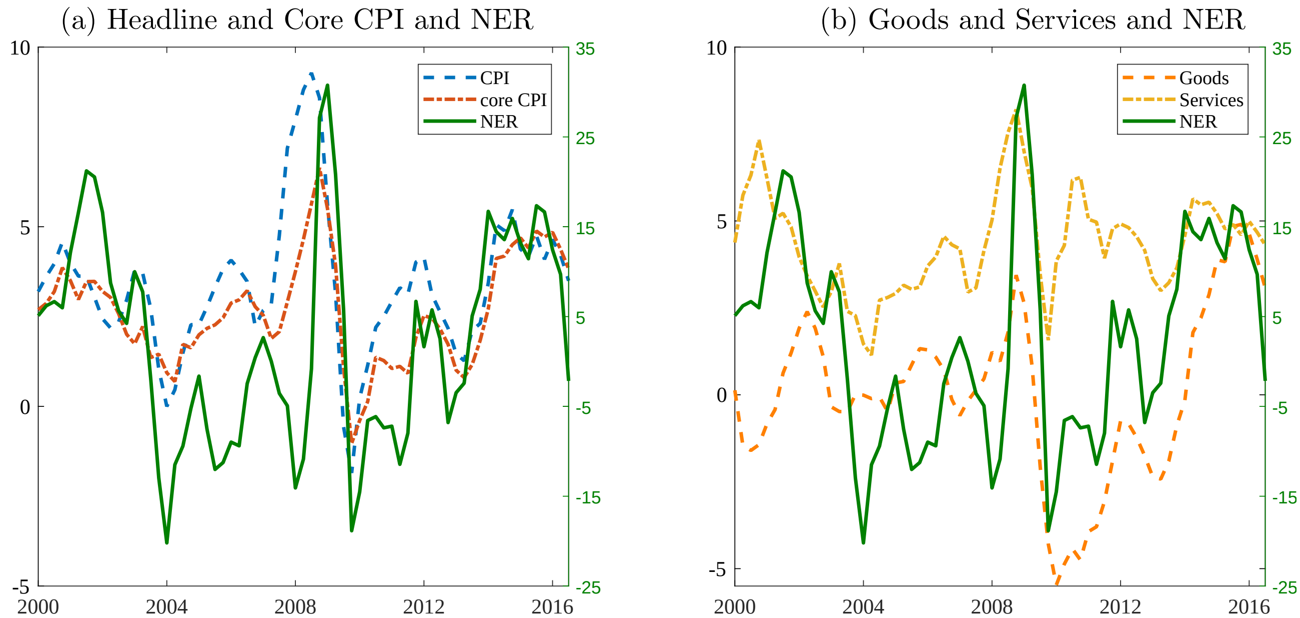
This paper presents a DSGE model for the Chilean economy designed to study how shocks that affect the nominal exchange rate are transmitted into the local economy, while allowing for different reactions of the different sectors. The model includes, among other features, external borrowing, different shares in the use of imported inputs, incomplete short-run ERPT, different pricing mechanisms –including indexation to own-sector price index as well as to the consumer price index (CPI)– and indexation in wages. The parameters of the model are partly calibrated and partly estimated using specific data to highlight the sectoral differences.

Not all shocks that affect the NER affect prices the same way. As discussed in García-Cicco and García-Schmidt (2018), there are important differences in ERPT when conditioning on the shock that hits the economy. Because of this, and particularly when predicting inflationary consequences of a given NER movement, it is important not only to differentiate between different prices, but also to identify which shock or combination of shocks is behind the changes.

In order to see the importance of differentiating between prices and shocks when analyzing NER movements, we first identify the most important shocks in explaining the NER depreciation and then calculate their conditional ERPTs for different price indexes. We find that the main determinants of the NER depreciation are a shock to international prices and a shock that affects the uncovered interest rate parity (UIP) condition. Their conditional ERPTs are very different, the ones conditional to the shock in international prices being low and short-lived for all price indexes and the ones produced by a shock to the UIP very high and persistent. The differences between price indexes is also significant, being the responses of tradable prices higher and shorter lived than the corresponding responses of non-tradables conditional on each shock.

Finally, in order to discuss and show the type of analysis that can be done with the model and

Figure 1: Inflation and nominal exchange rate depreciation in Chile



Note: Each graph displays the annual variation of the variable in the legend. Graph (a) shows the annual variation of headline CPI and core CPI in the left axis and the nominal exchange rate (NER) in the right axis. Graph (b) separates core CPI to goods and services in the left axis and repeats the graph of the NER in the right axis.

concepts discussed here, we study the surge in inflation that occurred in Chile in 2013-2015, after the tapering announcements by the Federal Reserve. We show that the depreciation experienced by the peso during that time was driven by the two main shocks already mentioned and that, because of the timing of the shocks, the major part of the inflation surge that happened afterwards was predictable since the beginning of the period. This contrasts greatly with the analysis that can be done using traditional ERPT measures, which are the ones obtained with the empirical literature and, as will be discussed later, are an average of the conditional ones identified here.

The contribution of this paper to the literature is twofold. The first is to present a fully fledged DSGE model for the Chilean economy that includes the tradable and non-tradable sectors in order to have a platform to study real life questions that need that separation. There are several important contributions of DSGEs for the Chilean economy, but none of them combining differentiation between tradable and non-tradable sectors and being big enough to include a rich set of shocks and data. The only exception is the model behind the last application in García-Cicco and García-Schmidt (2018), but is not explained nor analyzed. The literature does have large size DSGE Chilean models, such as Medina and Soto (2007), García-Cicco et al. (2015), García et al. (2019), but none of these separate between tradables and non-tradables. In contrast, there are models that include traded and non-traded goods for the Chilean economy (e.g. Soto, 2003), but are small. Relatively similar models applied to other countries include Matheson (2010) applied to Australia, Canada and New Zealand, Rees et al. (2016) applied to Australia, Martín-Moreno et al. (2014) applied to Spain, among many others.

The second contribution of this paper is to compute and use the concepts of conditional ERPTs applied to Chile and to a specific episode to highlight the benefits of computing these conditional measures instead of the empirical one. For that purpose we first present the conditional and unconditional ERPTs defined in García-Cicco and García-Schmidt (2018). The unconditional measures are comparable with the results obtained in the empirical literature, such as Justel and Sansone (2015), Contreras and Pinto (2016) and Albagli et al. (2015). All of them find that the ERPT to CPI is between 0.1 and 0.2 in the medium term. In addition, Contreras and Pinto (2016), based on a methodology of Vector autoregressive (VAR) model, calculate ERPTs for different groups getting that the ERPT for goods is much higher than for services (0.15 and 0.08 at 1 year respectively), which we also find. We then use the conditional measures to show the benefits of using those concept instead of the aggregate measures in a specific episode for the Chilean economy.

The paper continues as follows. The next section describes the model in detail, where the problems of each agent and each sector are presented and the driving forces are listed. The optimality conditions, standardization of the variables and the computation of the steady state are left for the appendix. Then, section 3 presents the quantitative analysis, describing the calibration and estimation and presents the main differences between the sectors that have potential effects in the local responses to external shocks. Section 4 presents the computation of the conditional and unconditional ERPTs, showing which shocks are the most important ones that affect the nominal exchange rate. Section 5 applies the model and conditional ERPTs to a specific case, highlighting the benefits of using conditional ERPT measures, but also describing issues of real time instead of ex-post computations. Finally, section 6 concludes.

2 The TNT DSGE Model

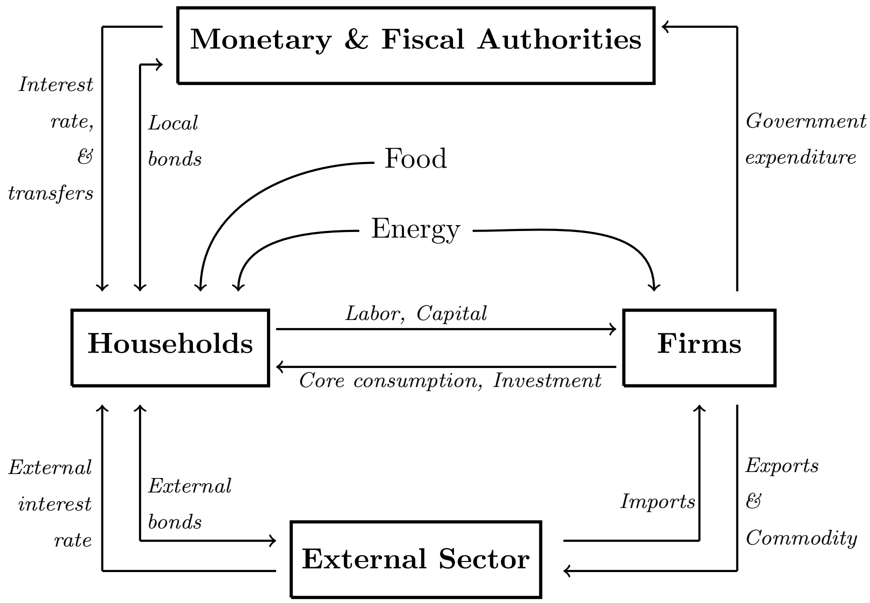
As explained in the introduction, the purpose of the DSGE model is to study the dynamics that external shocks cause to local prices and quantities differentiating between tradable (exportable and importable) and non-tradable sectors. It is based partly on Lombardo and Ravenna (2014), but extended to include specific features of the Chilean economy and of external dynamics[1]. To have a rich setup of inflationary dynamics and of the link between external variables and local characteristics, the model includes, among other features, imported inputs (oil and others) in the production of the tradable and non-tradable outputs; incomplete short run ERPT of importable goods; different pricing dynamics, including indexation to the own-sector price index as well as to the CPI; indexation in wages; consumption of exportable, importable and non-tradable goods apart from energy and food, and energy and food.

The general diagram of the model can be seen in figure 2, to get a big picture of what the model includes. There are four big players in the economy: households, firms, authorities (monetary and fiscal) and an external sector. As described in the figure, households buy energy, food and goods from firms to consume and invest; and supply labor and rent capital for the production of firms. They have access to local and external bonds, for which they take the interest rate as given. The monetary authority sets the interest rate and the fiscal authority supplies local bonds, has an exogenous expenditure and gives transfer to the households.

There are four sectors of firms defined by the type of good that they sell: the commodity, the importable, the exportable and the non-tradable. The commodity sector is assumed to get an exogenous endowment that is fully exported, and the importable is made entirely using imports and is sold domestically to the other two types of firms and households. Finally, exportables and non-tradables are produced with energy, labor, capital and importable goods, and while the first is sold to households and the foreign sector, the second is sold only domestically to households and the government.

[1] This model is very similar to the model used in the real-life application of García-Cicco and García-Schmidt (2018). There are only differences in the external shocks and in the estimation procedure.

Figure 2: General diagram



This section describes the problem of each agent with their main equations and equilibrium conditions. For details of the optimality conditions, normalizations and steady-state calculations, please refer to the appendix.

2.1 Households

We define a representative household that consumes, works, saves, invests and rents capital to the producing sectors. Its goal is to maximize,

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^{\beta} \left\{ \frac{(C_t - \phi_C \tilde{C}_{t-1})^{1-\sigma}}{1-\sigma} - \kappa_t \left(\xi_t^{h,X} \frac{{h_t^X}^{1+\varphi}}{1+\varphi} + \xi_t^{h,N} \frac{{h_t^N}^{1+\varphi}}{1+\varphi} \right) \right\}$$

where C_t is consumption and h_t^J for $J = \{X, N\}$ are hours worked in sector J. $\tilde{C}_t$ denotes aggregate consumption (i.e. the utility exhibits external habits, in equilibrium $\tilde{C}_t = C_t$), and $\kappa_t \equiv (\tilde{C}_t - \phi_C \tilde{C}_{t-1})^{-\sigma}$. This utility specification follows Galí et al. (2012) and is designed to eliminate the wealth effect on the supply of labor while keeping separability between consumption and labor. There are three preference shocks, ξ_t^{β} and $\xi_t^{h,J}$ for $J = \{X, N\}$: the former affects inter-temporal decisions, while the latter are labor supply shifters in sectors $J = \{X, N\}$. The parameters are given by β which is the discount factor, ϕ_C which governs external habits, σ which is risk aversion and φ which is the inverse of the Frisch elasticity of labor supply.

The budget constraint is

$$P_t C_t + S_t B_t^* + B_t + P_t^I I_t^N + P_t^I I_t^X = S_t R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + h_t^{X,d} \int_0^1 W_t^X(i) \left(\frac{W_t^X(i)}{W_t^X} \right)^{-\epsilon_W} di +$$
$$h_t^{N,d} \int_0^1 W_t^N(i) \left(\frac{W_t^N(i)}{W_t^N} \right)^{-\epsilon_W} di + P_t^N R_t^N K_{t-1}^N + P_t^X R_t^X K_{t-1}^X + T_t + \Pi_t.$$

Here P_t denotes the price of the consumption good, S_t the exchange rate, B_t^* the amount of external bonds bought by the household in period t, B_t the amount of local bonds bought by the household in t, P_t^I stands for the price of the investment good, I_t^J for investment in capital of the sector J, $h_t^{J,d}$ is labor demand in sector J, W_t^J is the wage index in sector J, $W_t^J(i)$ is the wage of variety i in sector J (explained below), R_t^* is the external interest rate, R_t the internal interest rate, R_t^J the real rate from renting capital to firms in sector J, K_t^J is capital specific for sector J, P_t^J is the price of good J, T_t are transfers made by the government and finally Π_t encompasses all profits of the firms in all sectors. The parameter ϵ_W is the elasticity of substitution among varieties of labor.

The formulation of the wage-setting problem follows Schmitt-Grohé and Uribe (2006). In this setup, households supply a homogeneous labor input that is transformed by monopolistically competitive labor unions into variety i of a differentiated labor input. The union takes aggregate variables as given and decides the nominal wage, while supplying enough labor to meet the demand in each market. The wage of each variety i is chosen optimally each period with a constant probability $1-\theta_{WJ}$ for $J = \{X, N\}$. When wages cannot be freely chosen they are updated by $(\pi_{t-1})^{\zeta_{WJ}}\bar{\pi}^{1-\zeta_{WJ}}$, with $\zeta_{WJ} \in [0, 1]$, π_t denoting CPI inflation and $\bar{\pi}$ the inflation target set by the Central Bank.

2.1.1 Consumption Goods

We distinguish total and core consumption in order to separate the effects of very volatile and externally given prices from the prices that are determined by local conditions. Consumption C_t is composed by three elements: core consumption (C_t^{NFE}), food (C_t^F) and energy (C_t^E). For simplicity, food and energy consumption are assumed exogenous and normalized to one (so total and core consumption are equal). In contrast, the price of the consumption good will be a composite of the price of the core good, energy and food the following way:

$$P_t = (P_t^{NFE})^{1-\gamma_{FC}-\gamma_{EC}}(P_t^F)^{\gamma_{FC}}(P_t^E)^{\gamma_{EC}}$$

where P_t^{NFE} denotes the price of core consumption, P_t^F the price of food and P_t^E the price of energy. The parameters γ_{FC} and γ_{EC} represent the weights of food and energy in consumption, with $\gamma_{FC}, \gamma_{EC} \geq 0$, $(\gamma_{FC} + \gamma_{EC} \leq 1)$. The goal of this simplified specification is to separate the dynamics of core and headline inflation without complicating significantly the supply side of the model. We further assume that the prices of both F and E relative to that of the tradable composite (T, defined below) follow exogenous processes (p_t^F and p_t^E respectively).

Core consumption is a composite of non-tradable consumption C_t^N and tradable consumption C_t^T, while the latter is composed by exportable C_t^X and importable C_t^M goods,

$$\begin{aligned} C_t^{NFE} &= \left[\gamma^{1/\varrho}(C_t^N)^{\frac{\varrho-1}{\varrho}} + (1-\gamma)^{1/\varrho}(C_t^T)^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}} \\ C_t^T &= \frac{(C_t^X)^{\gamma_T}(C_t^M)^{(1-\gamma_T)}}{(1-\gamma_T)^{(1-\gamma_T)}\gamma_T^{\gamma_T}} \\ C_t^J &= \left[\int_0^1 (C_t^J(i))^{\frac{\epsilon_J-1}{\epsilon_J}} di\right]^{\frac{\epsilon_J}{\epsilon_J-1}} \end{aligned}$$

where ϱ is the elasticity of substitution between non-tradables and tradables, γ and γ_T are the weights of non-tradables in core consumption and of exportables in tradable consumptions respectively. The last equation holds for $J = \{X, M, N\}$, and specifies that exportable, importable and non-tradable consumption are made of a continuum of differentiated goods in each sector, with a constant elasticity of substitution $\epsilon_J > 1$.

2.1.2 Capital and Investment Goods

The investment good is produced by a mixture of tradable and non-tradable goods, similar to the consumption good, but with different weights, γ_I and γ_{TI}, and elasticity of substitution ϱ_I:

$$
\begin{aligned}
I_t &= \left[\gamma_I^{1/\varrho_I}(\tilde{I}_t^N)^{\frac{\varrho_I-1}{\varrho_I}} + (1-\gamma_I)^{1/\varrho_I}(\tilde{I}_t^T)^{\frac{\varrho_I-1}{\varrho_I}}\right]^{\frac{\varrho_I}{\varrho_I-1}} \\
\tilde{I}_t^T &= \frac{(\tilde{I}_t^X)^{\gamma_{TI}}(\tilde{I}_t^M)^{1-\gamma_{TI}}}{(1-\gamma_{TI})^{(1-\gamma_{TI})}\gamma_{TI}^{\gamma_{TI}}}
\end{aligned}
$$

where $I_t = I_t^X + I_t^N$. Similar to consumption, each investment $\tilde{I}_t^J$ is a continuum of the differentiated goods in each sector with the same aggregator as C_t^J, for $J = \{X, M, N\}$.

Households choose how much to invest in each type of capital. The evolution of the capital stock in sector J is given by

$$
K_t^J = \left[1 - \Gamma\left(\frac{I_t^J}{I_{t-1}^J}\right)\right] u_t I_t^J + (1-\delta)K_{t-1}^J,
$$

for $J = \{X, N\}$. It is assumed that installed capital is sector-specific, there are adjustment costs to capital accumulation with $\Gamma'(.) > 0$ and $\Gamma''(.) > 0$ and there is a shock u_t to the marginal efficiency of investment. We assume that u_t is the same for both sectors. The parameter $\delta \in (0, 1)$ denotes the depreciation rate.

Alternatively, we could have modeled the investment good to be produced by a different firm, and then that firm would have sold it to the households. What is implicitly assumed in this version is that the sale from that investment firm to the household happens in a perfectly competitive environment.

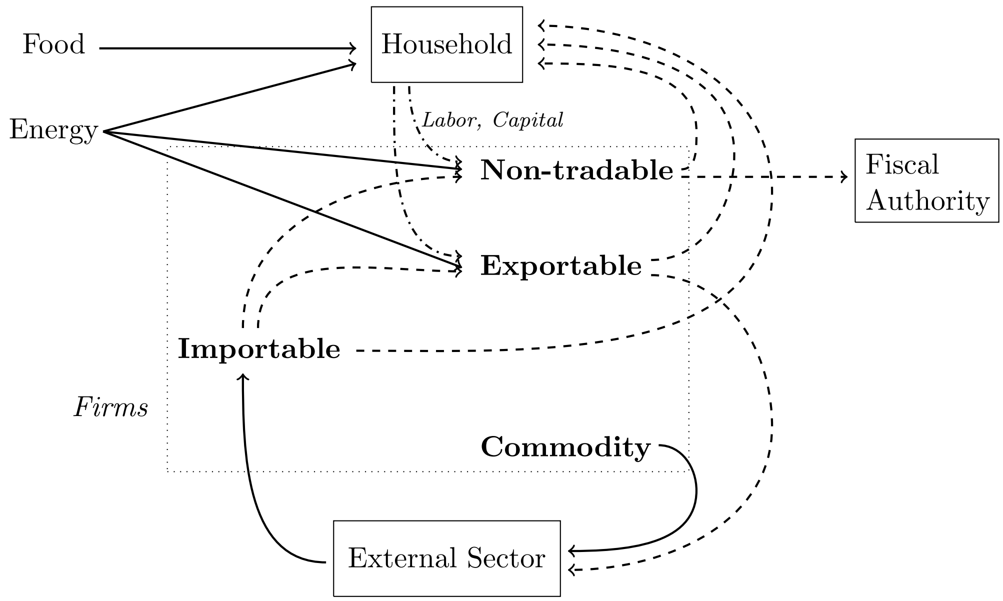
2.2 Firms

As said before, there are four sectors of firms: commodity, importable, exportable and non-tradable. Figure 3 highlights in detail the connections of each type of firm with the rest of the economy, including the pricing mechanisms of each good that is sold and input that is used by each sector. The solid lines represent exogenous prices, the dashed lines represent prices set under monopolistic competition and the dotted lines represent perfect competition. The lines from the household to the firms are dashed-dotted because labor is priced under monopolistic competition and capital competitively.

2.2.1 Commodity

The commodity is assumed to be an exogenous and stochastic endowment, Y_t^{Co}, which has its own trend A_t^{Co} that cointegrates with the other sectors, i.e. $A_t^{Co} = (A_{t-1}^{Co})^{1-\Gamma_{Co}}(A_t^N)^{\Gamma_{Co}}$, where A_t^N is the trend of the non-tradable sector, explained below and $\Gamma_{Co} \in [0, 1]$. We assume $y_t^{Co} \equiv \frac{Y_t^{Co}}{A_{t-1}^{Co}}$ follows an exogenous process. The endowment is exported at the international price P_t^{Co*}, in foreign currency, which is exogenously given for the local economy, as expressed by the solid line that goes from the commodity sector to the external sector of figure 3. To get the price in local currency, one needs to multiply that foreign price by the nominal exchange rate, S_t. This sector represents the mining sector in Chile, which is not totally owned by the country. Because of that, it is assumed that a fraction $\vartheta \in (0, 1)$ of commodity production is owned by the government and the rest, $(1-\vartheta)$, is owned by foreigners.

Figure 3: Production, Sales and Pricing



Note: The format of the lines represent the pricing of the goods: solid is exogenously given, dashed is monopolistic competition and dotted is perfect competition.

2.2.2 Importable Sector

Each firm j in this sector produces a differentiated product, $Y_t^M(j)$, from an homogeneous foreign input, $M_t(j)$, with the technology $Y_t^M(j) = M_t(j)$. The price of their input in local currency is given by $P_{m,t} = S_t P_t^{M*}$, where P_t^{M*} is the price in foreign currency and is exogenously given, as expressed by the solid line in figure 3 that goes from the external sector to the importable sector.

Because each firm supplies a differentiated product, they set their price $P_t^M(j)$ in a monopolistically competitive manner, as shown with the dashed line that goes from the importable firms to the non-tradable and exportable firms and to the households in figure 3. In addition, they face the probability θ_M of not being able to choose their price. In that case, the price is updated according to: $\left[(\pi_{t-1}^M)^{\varrho_M}(\pi_{t-1})^{1-\varrho_M}\right]^{\zeta_M}\bar{\pi}^{1-\zeta_M}$, with parameters $\{\varrho_M, \zeta_M\} \in [0,1]$, and π_t^M the inflation of sector M.

2.2.3 Exportable and Non-Tradable Sectors

The firms in the exportable and non-tradable sectors have the same format. Each firm j of sector $J = \{X, N\}$ produces a differentiated product that is a combination of value added $V_t^J(j)$ and an importable input $M_t^J(j)$, which is by itself a combination of a continuum of the goods sold by the M sector and energy. The firm j in sector J has the technology:

$$Y_t^J(j) = (V_t^J(j))^{\gamma_J}(M_t^J(j))^{1-\gamma_J},$$

with $\gamma_J \in [0,1]$. Value added is produced by:

$$V_t^J(j) = z_t^J\left[K_{t-1}^J(j)\right]^{\alpha_J}\left[A_t^J h_t^{J,d}(j)\right]^{1-\alpha_J}.$$

with $\alpha_J \in [0,1]$, z_t^J is a stationary technology shock and A_t^J is a non-stationary stochastic trend in technology. To maintain a balance-growth path, we assume that both trends co-integrate in the long run. In particular, we assume that $a_t \equiv A_t^N / A_{t-1}^N$ is an exogenous process and A_t^X evolves according to

$$A_t^X = (A_{t-1}^X)^{1-\Gamma_X} (A_t^N)^{\Gamma_X}$$

with $\Gamma_X \in [0,1]$. The problem of these firms can be solved in two stages, the optimal production of the value added, and the optimal production of the final good:

1. Optimal production of $V_t^J(j)$: firms are price takers, so they choose the optimal combination of capital and labor to minimize their cost:

$$\min_{K_{t-1}^J(j), h_t^d(j)} P_t^J R_t^J K_{t-1}^J(j) + W_t^J h_t^J(j) + \mu \left\{ V_t^J(j) - z_t^J \left[K_{t-1}^J(j)\right]^{\alpha_J} \left[A_t^J h_t^{J,d}(j)\right]^{1-\alpha_J} \right\}$$

   From this problem, we define MC_t^{VJ} as the marginal cost of producing $V_t^J(j)$ and is the same for all firms.

2. Optimal production of $Y_t^J(j)$: firms choose the optimal combination of value added and imported inputs to minimize their cost:

$$\min_{M_t^J(j), V_t^J(j)} MC_t^{VJ} V_t^J(j) + P_t^{ME} M_t^J(j) + \mu \left\{ Y_t^J(j) - [V_t^J(j)]^{\gamma_J} [M_t^J(j)]^{1-\gamma_J} \right\}$$

   where P_t^{ME} is the price of a composite between a continuum of the importable goods sold by the M sector and the price of energy, i.e.

$$P_t^{ME} = (P_t^M)^{1-\gamma_{EM}} (P_t^E)^{\gamma_{EM}}$$

   with $\gamma_{EM} \in [0,1]$ and P_t^M being the price of the composite of importables, which is the same as the composite for households. As in the case of the household with energy and food, $M_t^J(j)$ can be interpreted as only the continuum of importable goods or the composite between energy and the importable goods, since firm take the quantity of energy as exogenous and so it has been normalized to one.

The price setting mechanism of the firms in the J sector, for $J = \{X, N\}$, is the same as for the firms in the M sector and is done in a monopolistically competitive manner, as shown by the dashed lines that go from the exportable to the external sector and to the household, and the lines that go from the non-tradable sector to the household and the fiscal authority. In addition, and similar to the M sector, firms in sector J for $J = \{X, N\}$ face Calvo problems with a probability θ_J of not being able to choose their price optimally. When that happens, the price is updated with the rule: $\left[(\pi_{t-1}^J)^{\varrho_J} (\pi_{t-1})^{1-\varrho_J}\right]^{\zeta_J} \bar{\pi}^{1-\zeta_J}$, with parameters $\{\varrho_J, \zeta_J\} \in [0,1]$ and π_t^J inflation in sector J.

As shown in figure 3, the difference between the firms in the exportable and non-tradable sectors, in addition to the specific values of their parameters, is the buyers of their product. While the firms in the exportable sector sell to households and to the external sector, the ones in the non-tradable sector sell only internally (to households and the fiscal authority).

2.3 Fiscal and Monetary Authorities

The fiscal and monetary policies are assumed as simplified as possible to maintain focus on the other more complex features of the model. As shown in the budget constraint below, the fiscal authority receives a part ϑ of the profits of the commodity sector, receives the interest from the bonds it bought last period B^G_{t-1}, spends an amount G_t in the non-tradable goods, can buy new local bonds and gives transfers to households, T_t.

$$\vartheta S_t P_t^{Co*} Y_t^{Co} + R_{t-1} B^G_{t-1} = P_t^N G_t + T_t + B_t^G.$$

Similarly to the household, government expenditure is the same composite of non-tradable varieties. We assume $g_t \equiv \frac{G_t}{A^N_{t-1}}$ follows an exogenous process.

Monetary policy follows a Taylor-type rule of the form

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\varrho_R} \left[\left(\frac{(\pi_t^{NFE})^{\alpha_\pi^{NFE}} \pi_t^{1-\alpha_\pi^{NFE}}}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{GDP_t/GDP_{t-1}}{a} \right)^{\alpha_Y} \right]^{1-\varrho_R} e_t^m$$

where the variables without a time subscript stand for steady-state values, π_t^{NFE} denotes core inflation, GDP_t is gross domestic product and e_t^m a monetary shock, which is assumed exogenous. The parameters that govern the rule are $\varrho_R \in (0,1)$ for the autoregressive component, $\alpha_\pi, \alpha_y > 0$ for the reactions to the total inflation measure and output growth respectively, and $\alpha_\pi^{NFE} \in [0,1]$ for the weight of core inflation in the total inflation measure.

2.4 External Sector

The rest of the world sells the imported inputs, $M_t = \int_0^1 M_t(j)dj$, at price P_t^{M*}, buys the commodity, Y_t^{Co}, at price $P_t^{Co,*}$ and buys the varieties j of exported products, $Y_t^X(j)$, at the price set by local producers, $P_t^X(j)$. For these last goods, the aggregator of the varieties is the same as for the households. In contrast, the demand for the composite exportable is,

$$C_t^{X,*} = \left(\frac{P_t^X}{S_t P_t^*}\right)^{-\epsilon^*} Y_t^* \xi_t^{X*},$$

where P_t^* is the external CPI index, Y_t^* is external demand, ξ_t^{X*} is a disturbance to external demand and ϵ^* is the elasticity of external demand. It is assumed that P_t^*, $y_t^* = Y_t^*/A^N_{t-1}$ and ξ_t^{X*} follow exogenous processes.

The closing device of the model is given by the equation for the international interest rate

$$R_t^* = R_t^W \exp\left\{\phi_B \left(\bar{b} - \frac{S_t B_t^*}{P_t^Y GDP_t}\right)\right\} \xi_t^{R1} \xi_t^{R2}. \tag{1}$$

which can be decomposed and interpreted in three parts. The first part is R_t^W and represents the world interest rate. The second part is the term $\exp\left\{\phi_B \left(\bar{b} - \frac{S_t B_t^*}{P_t^Y GDP_t}\right)\right\} \xi_t^{R1}$ and represents the country premium, where ξ_t^{R1} is an exogenous shock, $\bar{b}$ represents a threshold for the external debt and P_t^Y is the GDP deflator. Finally, the third part, ξ_t^{R2}, is a risk-premium shock that captures deviations from the EMBI-adjusted uncovered interest parity (UIP). As we will later see, this last shock is very important for the behavior of the nominal exchange rate in Chile. What it actually represents is everything that affects the relevant external interest rate for Chile without coming from the rest of the variables described in the equation above. In particular, it can be affected by different factors in different periods depending on what is affecting the exchange rate that is not explicitly accounted for in the model.

2.5 Equilibrium Conditions

To close the model all the relevant markets have to clear: local bonds, investment, labor and goods[2].

2.6 Driving Forces

The model features a total of 20 exogenous state variables. Those of domestic origin are consumption preferences (ξ_t^{β}), labor supply ($\xi_t^{h,N}$ and $\xi_t^{h,X}$), stationary productivity (z_t^H and z_t^X), the growth rate of the long-run trend (a_t), the commodity endowment (y_t^{Co}), the relative prices of food and energy (p_t^F and p_t^E), efficiency of investment (u_t), government consumption (g_t), and monetary policy (e_t^m). In turn foreign driving forces are the world interest rate (R_t^W), foreign risk premium (ξ_t^{R1} and ξ_t^{R2}), international prices of commodities (P_t^{Co*}), imported goods (P_t^{M*}) and CPI of trade partners (P_t^*), demand for exports of X (ξ_t^{X*}), and GDP of trade partners (y_t^*). All these processes are assumed to be Gaussian in logs. The monetary policy shock is *i.i.d.* while the rest, with the exception of international prices, are independent AR(1) processes.

As the model features a balanced growth path and preferences are such that relative prices are stationary, foreign prices should co-integrate, all growing at the same long-run rate. Defining inflation of foreign CPI as $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, with steady state value of π^*, we propose the following model for international prices:

$$P_t^j = (\pi^* P_{t-1}^j)^{\Gamma_j} (F_t^*)^{1-\Gamma_j} u_t^j, \qquad \text{with } \Gamma_j \in [0,1), \quad \text{for } j = \{Co*, M*, *\}, \tag{2}$$

$$\Delta F_t^* \equiv \frac{F_t^*}{F_{t-1}^*}, \qquad \frac{\Delta F_t^*}{\pi^*} = \left(\frac{\Delta F_{t-1}^*}{\pi^*}\right)^{\rho_{F*}} \exp(\epsilon_t^{F*}), \qquad \text{with } \rho_{F*} \in (-1,1) \tag{3}$$

$$u_t^j = \left(u_{t-1}^j\right)^{\rho_j} \exp(\epsilon_t^j), \qquad \text{with } \rho_j \in (-1,1), \quad \text{for } j = \{Co*, M*, *\}, \tag{4}$$

where ϵ_t^i are *i.i.d.* $\mathcal{N}(0, \sigma_i^2)$ for $i = \{Co*, M*, *, F*\}$.

Under this specification, each price is driven by two factors: a common trend (F_t^*) and a price-specific shock (u_t^j). The parameter Γ_j determines how changes in the trend affect each price. The presence of a common trend generates co-integration among prices (as long as $\Gamma_j < 1$), and the fact that the power in the trend and in the lagged price in (2) add-up to one forces relative prices to remain constant in the long run.[3] The usual assumption for these prices in DSGE models with nominal rigidities is obtained as a restricted version of this setup, imposing $\Gamma_j = 0$ for $j = \{Co*, M*\}$ and $\sigma_*^2 = 0$. In other words, the relative prices of both commodities and imports, and the inflation of trading partners are driven by stationary AR(1) processes. The specification in (2)-(4) generalizes this usual assumption in several dimensions. First, in the usual setup, the common trend of all prices is exactly equal to the CPI of trading partners. This might lead to the wrong interpretation that inflation of trading partners is an important driver of domestic variables, while actually this happens because it represents a common trend in all prices. Second, the usual specification imposes that every change in the common trend has a contemporaneous one-to-one impact in all prices, while in reality different

[2]For details, please refer to the appendix.

[3]If $\Gamma_j = 1$, each price is a random walk with a common drift π^*. Although this implies that in the long run all prices will grow at the same rate, they will not be co-integrated and relative prices may be non-stationary.

prices may adjust to changes in this common trend at different speeds. Finally, for our specific sample, the data favors the general specification (2)-(4) relative to the restricted model. Overall, the model features 21 exogenous disturbances, related to the 20 exogenous state variables previously listed plus the common trend in international prices.

3 Quantitative Analysis

This section presents the quantitative analysis performed with the model previously described. It firsts explains how the parameters were by combining calibration and estimation and it then shows the fit of the model to main Chilean statistics highlighting the differences of the pricing mechanisms of each sector, thus showing the importance of differentiating them.

3.1 Calibration and Estimation

The values of the parameters in the model are determined using calibration and estimation. Parameters that are calibrated, shown in table A.1 of the appendix, include preference parameters such as the risk aversion coefficient and the Frisch elasticity, the sectorial parameters such as shares of consumption, investment and inputs in production and some statistics measured as fractions of GDP. Table 1 reproduces the sectoral calibration, which was made using averages of input-output tables for Chile, to highlight important differences across sectors. From the consumption side, there are three main expenditures: household consumption, investment and government expenditure. The table shows that core consumption and investment use the same proportion of non-tradables, which is 62 percent, and that among the tradables, both use a higher proportion of importables than exportables, being the share of importables in consumption lower than that in investment. In addition, government expenditure (not shown) is fully consumed in non-tradables, which was constructed that way, because the input-output tables showed that 99% is completely spent in those goods.

The bottom part of table 1 shows the shares in the production side. By assumption, the importable sector uses only the importable input in its production. The other two sectors produce using labor, capital and imported inputs and, as shown in the table, the exportable production uses more capital, much less labor and more imported inputs in its production than the non-tradable. Note that since capital is partly made with importable products, and the exportable sector uses more capital than the non-traded sector, the exposure of the production of exportables to the external sector is not only the 20% used in imported inputs but more. This exposure is in addition to the exposure on the demand side, because as the definition implies, the non-tradable is only sold locally while the exportable is partly sold abroad.

The rest of the calibration, which is not directly related to sectoral distinctions was set by targeting several steady-state ratios to sample averages of their observable counterparts and by relying on previous studies estimating DSGE models for Chile. Parameters characterizing the dynamics of some of the external driving forces were calibrated by estimating separate AR(1) processes.

The parameters that were not calibrated, were estimated using a Bayesian approach. These, shown in tables A.2-A.4 in the appendix, relate to preferences, wage and price setting mechanisms, growth in the different sectors, monetary policy and exogenous processes. The series used at quarterly frequency and from 2001.Q3 to 2016.Q3 were:[4]

[4]The source for all data used is the Central Bank of Chile. Variables are seasonally adjusted using the X-11 filter, expressed in logs, multiplied by 100, and demeaned. All growth rates are changes from two consecutive quarters.

Table 1: Sectoral Calibration

	Non-tradable (N)	Exportable (X)	Importable (M)
Consumption (C^{NFE})	62	8,7	29,3
Investment (I)	62	0,8	37,2
	Capital (K)	Labor (L)	Importable (M)
Exportable production (Y^X)	52,8	27,2	20
Importable production (Y^M)	-	-	100
Non-tradable production (Y^N)	45,1	46,9	8

Note: Numbers correspond to percentages. Each row sums up to 100.

- Real growth rate of: GDP, exportable GDP (agriculture, fishing, industry, utilities, transportation), non-tradable GDP (construction, retail, services), mining GDP, private consumption (C), total investment (I), and government consumption (G).
- The ratio of nominal trade balance to GDP.
- Quarterly CPI-based inflation of non-tradables (π^N, services, excluding food and energy), tradables (π^T, goods excluding food and energy), importables (π^M, excluding food and energy), food (π^F) and energy (π^E).
- The growth of nominal wages (π^{WX} and π^{WN}) measured as the cost per unit of labor (the CMO index), using sectors consistent with the GDP definition.
- The nominal dollar exchange-rate depreciation (π^S) and the monetary policy rate (R).
- External: world interest rate (R^W, LIBOR), country premium (EMBI Chile), foreign inflation (π^*, inflation index for trading partners, the index IPE), inflation of commodity prices (π^{Co*}, copper price) and imports (π^{M*}, price index for imported goods, the IVUM index), external GDP (Y^*, GDP of trading partners).

All domestic observables were assumed to have a measurement error, with calibrated variance equal to 10% of the observable variance, except for the interest rate which had no measurement error. Priors and posteriors are also shown in tables A.2 to A.4. When possible, priors were set centering the distributions around previous results in the literature. To get percentiles for the graphs shown below, we got 500,000 draws from the posterior using the Metropolis Hastings algorithm.

3.2 Fit of the Model and Price Dynamics

The model achieves a very good match of the main volatilities and autocorrelation coefficients of the Chilean economy, as shown in table 2. The orders of magnitude are very similar between data and model and also their ordering. The autocorrelations of the inflation variables are in general higher in the model than in the data, but of similar magnitudes. The autocorrelation that is most different is the one of investment, which is higher in the model.

One of the beliefs behind the motivation to build this model was that price dynamics were very different in tradable and non-tradable sectors. To check if that is actually true, table 3 shows how

Table 2: Second Moments in the Data and in the Model

	St. Dev (%)			AC(1)		
	Data		Model	Data		Model
ΔGDP	0,9	0,1	1,1	0,5	0,2	0,5
$\Delta CONS$	1	0,1	0,8	0,7	0,2	0,7
ΔINV	3,9	0,4	4,5	0,3	0,1	0,7
ΔGDP^X	1,5	0,1	1,4	0,2	0,1	0
ΔGDP^N	0,8	0,1	1,6	0,7	0,1	0,6
TB/GDP	5,5	0,5	5,4	0,8	0,1	0,9
π	0,6	0,1	0,5	0,6	0,2	0,6
π^T	0,7	0,1	0,6	0,6	0,1	0,8
π^M	0,8	0,1	0,6	0,7	0,1	0,9
π^N	0,4	0	0,4	0,7	0,2	0,9
π^{WX}	0,6	0	0,7	0,6	0,1	0,8
π^{WN}	0,4	0	0,4	0,8	0,2	0,9
R	0,4	0	0,5	0,9	0,2	0,9
π^S	5,2	0,7	5,2	0,2	0,2	0

Note: The variables are: the growth rates of GDP, private consumption, investment, and GDP in the X and N sectors, the trade-balance-to-output ratio, inflation for headline CPI, tradables, non-tradables and imported, the growth rate of nominal wages in sectors X and N, the monetary policy rate, and the nominal depreciation. Columns two to four correspond to standard deviations, while five to seven are first-order autocorrelations. For each of these moments, the three columns shown are: point estimates in the data, GMM standard-errors in the data, and unconditional moments in the model evaluated at the posterior mode.

prices and wages are chosen or changed each period. Let's recall from the model that all prices are revised each period to either an optimal choice or indexed to a mix of past inflations and the inflation target. Since firms and households are otherwise equal, one can interpret the coefficients (that sum up to one) as a percentage of firms (unions) that change their price to each option. The table shows the percentage of each firm (union) that: (*i*) changes the price optimally, (*ii*) indexes its previous price to the inflation target (*iii*) indexes its previous price to past CPI inflation and (*iv*) indexes its previous price to the own-sector inflation (in the case of firms).

As seen in the table, there are important differences between the sectors in the case of the prices set by firms. The non-tradable sector is an extreme, in which only 2% of the prices are chosen optimally each quarter and a 51% is indexed to own-sector inflation. The rest is indexed to CPI inflation and the target. This produces a high degree of inertia which implies that if there is a movement in CPI inflation, for example because of a movement in the exchange rate, there will be inflationary consequences in this sector for a long period. The other two sectors, exportables and importables, show lower total dynamic indexation to CPI and own-sector past inflation, being 42% in each. In the exportable sector the great majority of the dynamic indexation is to CPI inflation, while in the importable it is more evenly distributed. In addition, the exportable sector shows more flexibility, in the sense that there is a higher percentage of prices (45%) chosen optimally, while for the importable sector that value is 15%. This last value implies that the ERPT for imports is incomplete and takes a while for the price to completely reflect changes in either the international price of the importable good or the exchange rate. The average duration of prices in the importable sector chosen optimally (and indexed thereafter) is 6,6 quarters.

Table 3: Price dynamics in Chile

	Change optimally	Indexed to		
		Inflation target	CPI inflation	Own-sector inflation
Non-tradable prices (P^N)	2	17	30	51
Exportable prices (P^X)	46	13	39	3
Importable prices (P^M)	15	43	19	23
Non-tradable wages (W^N)	3	83	14	-
Exportable wages (W^X)	6	79	15	-

Note: Numbers correspond to percentages. Each row sums up to 100.

In contrast, wages show a similar behavior in both sectors. Only a very small percentage is chosen optimally, around 80% of the wages are indexed to the inflation target and around 15% are indexed to past inflation. These characteristics make wages show a lot of inertia in both sectors, which implies longer persistence of the effects after a given shock, but no additional sectoral differences.

In sum, there are important sectoral differences that make the non-tradable sector very different from its tradable counterparts and that need to be accounted for. First, on the demand side, because it is only demanded by local agents. Second, on the supply side because it uses relatively more labor and less capital and importable input. And third, on the pricing mechanism, because it shows the highest inertia, because a very small fraction of firms chooses its price optimally each period. In the next section, we show that, aside from this distinction, and in particular when explaining the local effects of external shocks, it is important to also identify which external shock is the one affecting the nominal exchange rate.

4 Inflationary Consequences of the Different External Shocks

There is no doubt that changes in the nominal exchange rate affect prices in the Chilean economy, but the intensity and duration of their effects need to be studied to be able to do proper predictions and policy recommendations. The effect depends not only on the sectors, but also on the type of shock, as highlighted by García-Cicco and García-Schmidt (2018).

To see which shock is cause for more concern because of its effect on the exchange rate and subsequent inflationary dynamics, we first need to identify which shocks are more important in explaining exchange-rate movements. Table 4 lists the shocks that explain the great majority (96%) of the changes in nominal exchange rate depreciation and also how those shocks explain the different inflation measures. As shown in García-Cicco and García-Schmidt (2018), the shock that affects the exchange rate the most is the shock that is common to all international prices (ΔF^*) which explains almost 70%, and second is the shock to the uncovered interest parity condition (UIP, ξ^{R2}), which is a shock that includes all that is not modeled. With a much lower value, the following shock is to the external interest rate, which enters the model the same way as the UIP shock[5].

As shown in the same table, these shocks are also important in explaining inflation in the economy.

[5]The differences between the shocks are their estimated autoregressive coefficient and standard deviation, which depend on the data used to identify and estimate each one.

Table 4: Shocks explaining the Exchange Rate

Var.	Shocks M.P.	Ext. I.R.	C.P.	UIP	Int. Prices	Sum
π^S	2	6	1	18	69	96
π	2	17	3	15	6	43
π^T	2	19	3	23	9	56
π^M	3	21	4	27	11	66
π^N	1	26	3	13	6	49
R	14	28	4	16	6	68
rer	2	8	2	17	9	38

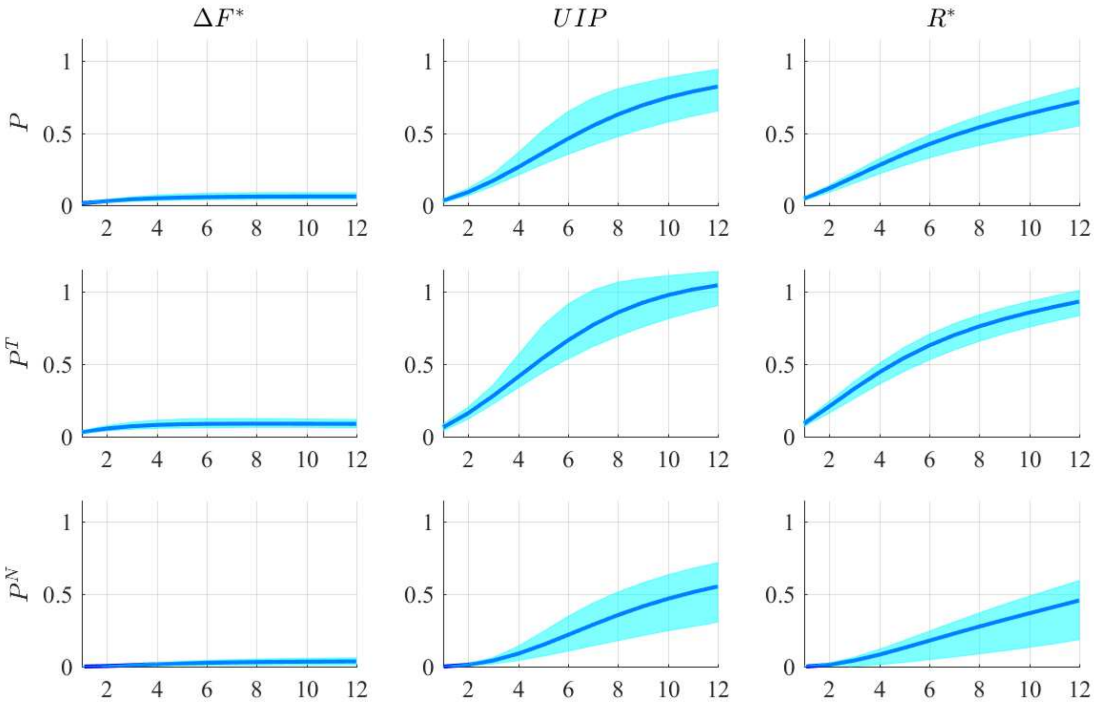
Note: The shocks are from left to right: monetary policy (ϵ^m), external interest rate (R^W), country premium (ξ^{R1}), uncovered interest parity condition (ξ^{R2}), and external prices (ΔF^*).

They explain 56% of the variance of the tradable sector and 49% of the non-tradable. To get intuition we will first describe the evolution of the endogenous variables after the most important shocks and then we will show the computation of the conditional ERPTs. For the impulse response functions, please refer to the Appendix. After a negative shock to the international trend in prices, aggregate demand falls. As the market for non-tradable goods has to clear domestically, the shock generates a fall in the relative price of non-tradables, a real exchange rate depreciation, a drop in the production of non-tradables, an increase in the output of exportables, and an overall drop in GDP. Moreover, given the real depreciation and the presence of price rigidities, the nominal exchange rate depreciates as well. To explain the dynamics of inflation, first note that without indexation the required fall in the relative price of non-tradables would lead to an increase in the price of tradables (due to the nominal depreciation) and a drop in the price of non-tradables, which can actually be observed in the very short run. But with indexation to aggregate inflation (in both wages and prices), inflation of non-tradables starts to rise after a few periods. Therefore, the indexation channel affects significantly the dynamics of inflation (and the ERPT) in the non-tradable sector. Finally, given the monetary policy rule, the domestic interest rate increases to smooth-out the increase in inflation.

The shock to the UIP and the shock to the external interest rate work in the same way, because both affect the relevant interest rate at which Chile can borrow in international markets. A positive realization of these shocks increases the cost of foreign borrowing, which triggers both income and substitution effects, leading to a contraction in aggregate demand. This leads to both real and nominal depreciations, and a reduction in all measures of activity, except for production of exportables, which is favored by the reallocation of resources from the non-tradable sector. All measures of inflation increase, and the role of indexation in explaining non-tradable inflation is similar to what we described before. Accordingly, the policy rate rises after this shock.

Even though both shocks have an impact through aggregate demand, the shock to external prices has also a direct impact on inflation that dampens the effect generated by nominal exchange rate changes. This happens through two different channels. First, a drop in international prices puts downward pressure to the domestic price of imports. Second, given the presence of imported inputs in the production of both exportable and non-tradable output, a reduction in world prices will, ceteris paribus, reduce the marginal cost in these sectors, dampening also the response of inflation in the exportable and non-tradable inflation.

Figure 4: Conditional ERPTs



Note: Each graph displays the ERPT of the shock in each row to the price in each column. The shaded are show the 2.5th and 97.5th percentiles.

Conditional ERPTs after a shock to international prices is much lower than after shocks to UIP and the international interest rate as can be seen in figure 4. This can be explained by the dampening effect of lower international prices. Note that by construction the ERPTs of all shocks with the exception of international prices, must have a long-term value of 1, since the real exchange rate is assumed stationary.[6] In contrast, a negative shock to international prices generates a depreciation in the NER and so the change in local prices can be much lower to get a stationary real exchange rate in the long run.

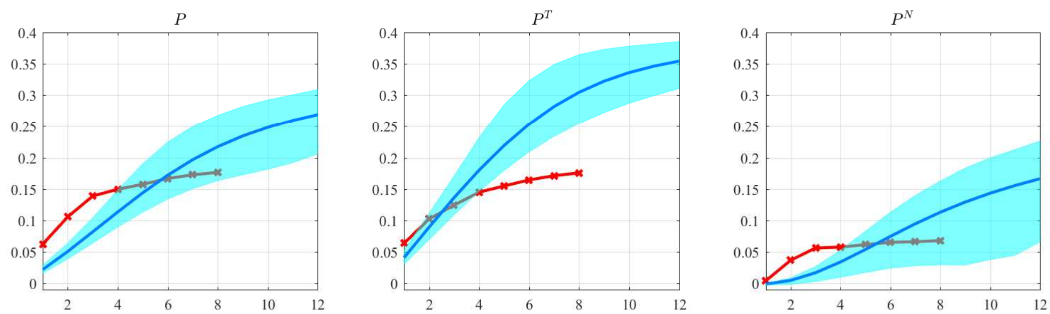
The conditional ERPTs to international prices, is also shorter lived. As seen in the second row of figure 4, in the first column, the great majority of the effect in tradable prices happens within 4 quarters. In contrast, and probably because of indexation, the effect in non-tradable prices happens later, but is also very small. In great contrast, the effects of the other two shocks are very high and take a while to completely affect the prices. After a UIP shock, the effect of the first 4 quarters is almost the same as the marginal effect of the next 4, as can be seen in the second and third rows and middle column of figure 4.

It is important to highlight that the differences in magnitude and timing of the conditional ERPTs is very important for policy analysis. Monetary policy acts with a lag, which in general is thought

[6]Recall that the real exchange rate is defined as: $rer_t = P_t^* S_t / P_t$, and because the external price index, P_t^* is exogenously given, any shock that affects the nominal exchange rate S_t must translate one-to-one to internal prices P_t in the long run.

to be around 4-8 quarters, so if the inflationary effects of a particular shock happen in the short run, as is the case of the shock to external prices, monetary policy cannot react to counteract it, however desirable. In addition, since the inflationary effect of the shock to external prices is very low, the desirability is also questionable. A very different picture is drawn after a shock to either UIP or the international interest rate. Since that effect is high and, at least, a great part of it takes a while to unfold, monetary policy has time to react and can actually counteract some of the inflationary consequences of such shocks.

Figure 5: Unconditional ERPTs



Note: Each graph displays the unconditional ERPT to the price in each column. In blue is the estimation based on this paper and in red the estimation in Contreras and Pinto (2016). The shaded are show the 2.5th and 97.5th percentiles.

To get a sense of the importance of identifying the shock that produces the nominal depreciation, we compare the conditional measures computed previously with the unconditional ones defined in García-Cicco and García-Schmidt (2018). As discussed in that paper, with the conditional ERPTs, one can build measures of the unconditional ERPT, that resemble the ERPT that one gets in the empirical literature. Figure 5 shows the unconditional ERPT calculated as a weighted average of the conditional ones, with the weights equal to the importance of that shock in explaining movements in the nominal exchange rate depreciation, shown in table 4. When comparing the conditional ERPTs in figure 4 with the unconditional ERPTs in figure 5 one can see that the unconditional measures are in between the conditional ones (by construction). Since the differences in the conditional ERPTs are important, so are the differences between each conditional ERPT and the unconditional ones.

It is important to note that in all cases the behavior of the tradable prices is very different from the one of non-tradable prices (and CPI is an average of both). When comparing per shock (and the unconditional measures), and as expected after the discussion in the previous section, the effect in the price of tradables is higher and more immediate than the effect in the price of non-tradables. This is explained by the larger amount of imported goods and the higher price flexibility in the tradable sectors compared to non-tradables. This implies that, while the initial impact is felt specially through tradable prices, the effects are then amplified and prolonged through the non-tradable sector.

The differences in the conditional and unconditional measures, in addition to differentiating between tradables and non-tradables, become very important when trying to predict the inflationary consequences of a particular movement in the nominal exchange rate. Policy decision makers in general, and monetary authorities in particular, are very interested in how different external shocks affect inflation, and in general, use unconditional measures to make such a prediction. As seen in this section, using the unconditional measures is very imprecise since the mixture of shocks that happen in the economy at different points in time is almost surely not the same as the one that would make that prediction correct. To illustrate this point, consider a nominal depreciation of 10%. If we use the unconditional measures to predict its inflationary effect, we would predict a 1.1% increase in CPI inflation in the first year and 1.1% in the second year (2.2% accumulated). The corresponding numbers for tradable inflation would be 1.8% and 1.3% (3.1% accumulated) and for non-tradables 0.3% and 0.6% (0.9% accumulated). In great contrast, if we think (or identify) that the shock was to international prices, we would predict 0.5% and 0.1% (0.6% accumulated) for headline CPI, 0.8% and 0.1% (0.9% accumulated) for tradables, and 0.2% and 0.1% (0.3% accumulated) for non-tradables. Finally and in the other direction, if we think that the depreciation was a consequence of a shock to the UIP condition, the respective numbers are 2.6% and 3.7% (6.3% accumulated) for headline CPI, 4.1% and 4.5% (8.6% accumulated) for tradables, and 0.9% and 2.7% (3.6% accumulated) for non-tradables.

It is important to recognize that the previous analysis assumes that one can identify the shocks that hit the economy accurately and in time to make the predictions, which is not always an easy task. There are several considerations to take into account that can make the prediction inaccurate or change in time. First, there is some instability in the models estimated, because some of the parameters can be re-estimated as new data becomes available. If the model is well identified and there is enough initial data, this should not imply a major parameter change, but it can anyways affect the exact numbers. Second, and more importantly, the reading of past shocks is affected by new data. The shocks are normally recovered through a smoothing algorithm, which includes past and future data for its predictions. Because of this, when an additional quarter is added to the information set, even holding the parameters fixed, one gets new readings of the shocks that were already in the dataset. An example of this will be seen in the next section for a particular case.

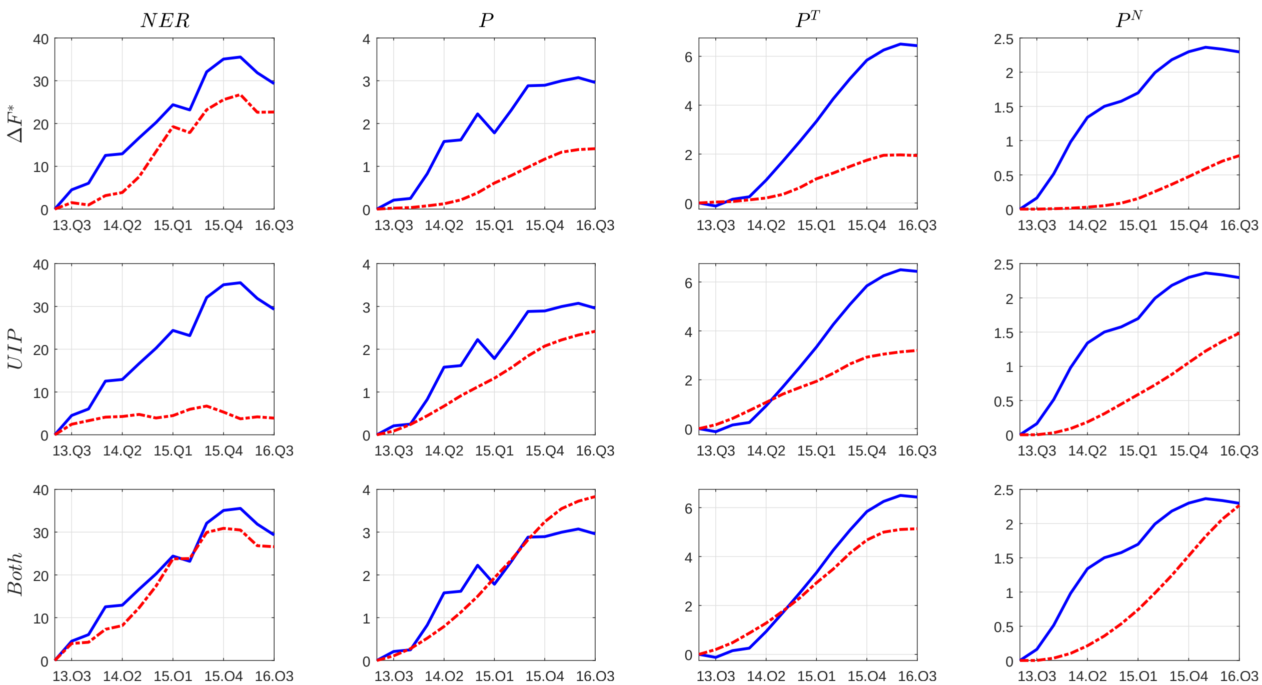
Another limitation with the analysis done here, related to the Lucas critique, is that ERPTs also depend on the reaction of monetary policy. The previous analysis assumes that the monetary policy follows the Taylor rule stated in the model but if the monetary authority were to deviate, as shown by García-Cicco and García-Schmidt (2018), ERPTs would also be altered. In particular, the direction of the change of any given monetary policy is not ex-ante evident, since a relatively more expansionary policy increases inflation and nominal depreciation, and so, it is not clear which effect dominates.

To sum up, in order to get the whole picture of the inflationary effects of external shocks, it is important not only to separate between the sectors, but also to identify and calculate separately the effects of the different shocks. Different shocks have different inflationary consequences. In the case of Chile, while the shock that most frequently affects the nominal depreciation (shock to international prices) has low and short lived ERPTs, the second most important shock (shock to the UIP condition) has very strong and persistent effects.

5 Application: Explaining Inflation after the Large Depreciation of 2013-2015

This section presents the application of the distinctions discussed in previous sections to a particular episode. Beginning in June 2013, following conversations about the tapering of the US Federal Reserve, most currencies in the world depreciated strongly against the US dollar, including the Chilean peso. After that, there was an important surge in inflation that was thought to be caused (at least partly) by the depreciation. In this section we will use the previous conceptual discussion and the estimated model to try to discern the role that NER movements played in explaining the observed dynamics and we will question whether the inflationary consequences were predictable or not, and with what anticipation.

Figure 6: Inflationary consequences of the nominal depreciation during the tapering



Note: The blue line shows the accumulated percentage deviation (with respect to the sample trend) of the variable indicated in each column, compared with its value in the second quarter of 2013. In each column, the variables are: nominal exchange rate, CPI, tradables and non-tradables. The red dotted lines indicate the counterfactual path of the same variable that would have occurred if only the shock indicated in each row had been present (computed with the Kalman smoother). In each row, the shocks are: international price trend (ΔF^*), deviations from the parity of rates (UIP) and both at the same time.

During this period the nominal exchange rate accumulated an increase of nearly 40%, which can be seen in the blue line of the first graph in figure 6, which shows the cumulative percentage change in the exchange rate with respect to its value in the second quarter of 2013. The red lines show the path that this variable would have followed if only one of the shocks (or both in the third row) had been present. Looking at the first row, we can observe that the shock to the trend in international prices (ΔF^*) was the main determinant of the depreciation observed, particularly from the first quarter of 2014 onwards. On the contrary, in the second row it can be seen that the shock to the parity of interest rates (UIP) affected the exchange rate in the first quarters after the tapering, but as of 2014 its effect on the exchange rate was marginal. Thus, as observed in the last row, the effect of both shocks combined can account for the evolution of the exchange rate observed in that period.

In the first row, columns two to four, we can see how the shock to international prices affected the different price indices. The blue lines show the accumulated percentage deviations (with respect to the sample trend) of the index, compared to its value in the second quarter of 2013, while the red ones are the corresponding counterfactuals. Even though these shocks account for most of the exchange rate movements up to 2014, their effect on prices was moderate. By the end of 2015, these shocks explain about a third of the deviations from the CPI and tradables, and around a fifth for non-tradables. These results reflect the discussion in the previous section of the low ERPTs of this particular shock.

Turning to the effect on prices of shocks to the UIP, in the second row, columns two to four, we can observe that these can explain about two thirds of the deviations accumulated in the CPI until the end of 2015, and around half of the accumulated inflation for the other two indices. This contrasts greatly with the low fraction of the NER variation that is explained by this shock. So, it was the shocks at the beginning of the period, which affected mostly the prices because of their very high and persistent ERPTs.

Finally, the last row shows that the shocks that determined the exchange rate movements in the analyzed period can largely explain the movements observed in the CPI and the prices of tradables. On the contrary, inflation of non-tradables was not so related to exchange rate movements, particularly during 2013 and 2014.

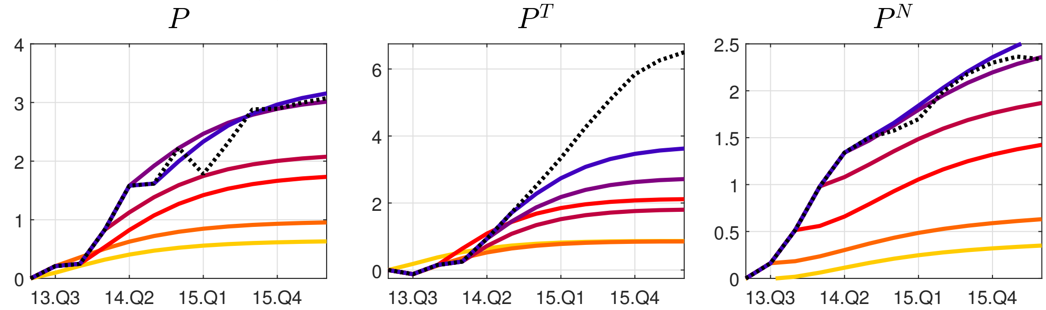
The main message to take from this analysis is that not every movement in the nominal exchange rate has the same inflationary consequences. While the main driver of the movements in the nominal exchange rate was the shock to external prices, the one that caused most inflation was the shock to the UIP that was more active at the beginning of the sample. Because of this, a very important part of the inflation of 2015 could have been anticipated by the end of 2013/early 2014 using conditional ERPTs applied to the identified shocks. Figure 7 shows the inflationary projections using unconditional ERPTs in the first graph and the main conditional ones in the second graph.

Graph (a) uses the unconditional ERPTs shown previously and applies it to the nominal depreciation seen since 2013.Q3 until 2014.Q4, doing each time the projection conditional on the new information seen. As seen in this graph, unconditional ERPTs recognize that the depreciation will cause inflation in the future periods, but the projection falls very short of what actually happened, which is shown by the black dotted line. In order to get a good prediction, you need to wait until at least the end of 2014.

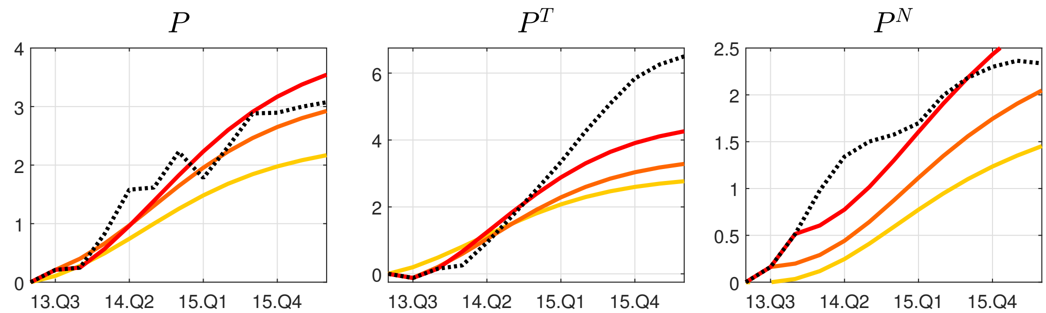
Graph (b) uses the conditional ERPTs to do the same exercise. You can see that already in 2013.Q3, using the conditional measures, you can predict an important increase in inflation and then you get a similar prediction using the conditional measures and nominal depreciation data until 2014.Q1 as the unconditional prediction using data until 2014.Q4, which is because a great part of the increase is experienced during 2014. Note that the conditional exercise done here uses only information of the main two shocks. To make the best possible prediction, one would use information of all the shocks,

Figure 7: Inflationary Predictions using Unconditional vs Conditional ERPTs

(a) Projections using Unconditional ERPTs



(b) Projections using Conditional ERPTs



Note: Both figures show the projection of inflation using ERPTs measures discussed previously. The clearer color (yellow) is the projection made with the nominal depreciation until 2013.Q3 (but inflation measures until 2013.Q2) and then each darker line presents one additional quarter of data, being the red one 2014.Q1 and the blue one in the top graph 2014.Q4. Graph (b) sums the contribution of shocks to international prices (ΔF^*) and to the UIP condition.

since actually what happens starting 2014 is that other shocks decrease inflation and counteract the effects of the shock to international prices and the shock to the UIP condition[7].

5.1 Pseudo-Real Time vs Ex-Post Considerations

The analysis done in this section suggests that, only with the shocks inferred until the end of 2013 and using the concepts of conditional ERPT, at least half of the inflation deviations from the target experienced in 2015 could have been anticipated. However, it is important to note, as stated in the previous section, that the inference of real-time shocks may differ from that previously presented. This is mainly because the shocks predicted by the model are affected not only by past data, but also by future data.

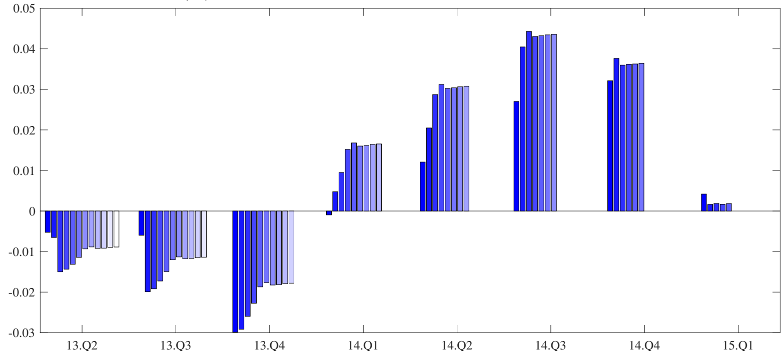
Figure 8 shows the revision in the projection of 1-year inflation given by the review of shocks to international prices -panel (a)- and to the interest rate parity condition -panel (b)- between the second quarter of 2013 and the first quarter of 2015. As seen in the first graph, the effects of revising the shock to international prices is very low, not exceeding 0.05% of inflation and on average it is around 0.02% in absolute value. In contrast, the inflationary effect of the review of the UIP shock is significantly higher, even standing above 0.2%, averaging 0.08%.

Note also that the first revisions are the most important ones, and after one to four revisions

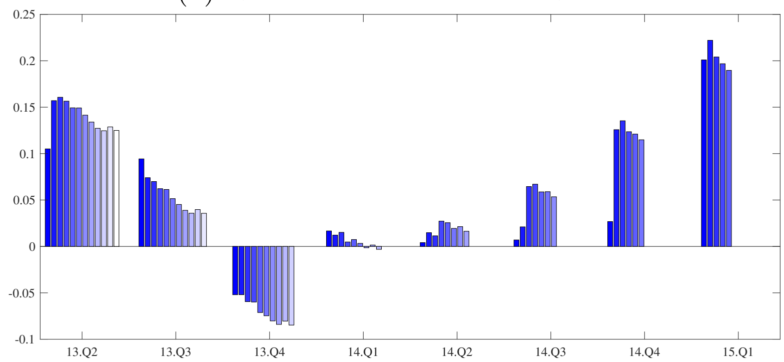
[7]To get the part of the nominal depreciation that was due to the main shocks, we used the shock decomposition of the nominal depreciation and then with the conditional ERPTs we calculated the inflationary predictions.

Figure 8: Revision of Effect on Inflation

(a) Revision after a shock to ΔF^*



(b) Revision after a shock to UIP



Note: The graphs show the effects of the revisions to shocks to ΔF^* in the first graph and UIP in the second graph, in 1 year projected inflation, measured in percentage points. The darkest blue shows the review between the first quarter in which the data is known and the next. Each additional column represents the revised effect projected initially and one additional quarter, being the white column (when available) the revision of the effect 12 quarters ahead. Each set of columns corresponds to the review of the quarter defined in the abscissa axis.

(depending on which shock and period), the subsequent revisions are very minor. This implies that even though there are uncertainties, the most important part of them are corrected after one quarter of data and then there is only a very small part left after a year.

6 Conclusions

This paper presents a DSGE model built for the Chilean economy with the objective of explaining ERPTs and the inflationary effects of external shocks. It first presented the construction of the model with the focus in explaining interactions between external shocks and different sectors of the economy. It then showed the fit of the model to the main statistics of the Chilean economy and some important features of the estimation of the model's parameters. It also showed that for the Chilean economy the main shocks that explain changes in the NER are a shock to international prices and a shock to the uncovered interest rate parity condition. These shocks have very different inflationary consequences, since the conditional ERPTs of the first are very low and their main effect are felt within the first year, while the conditional ERPTs for the second are very high and persistent, making the effect during the second year to be as important as the one during the first.

In addition, we analyzed, in the light of the model, the inflationary episode that occurred in Chile between 2013 and 2015 after the tapering announcements. In particular, we emphasized that, the UIP shock was particularly important at the beginning of the period and the shock to international prices during the rest. The first, because of its very high and persistent ERPTs, can explain about half of the inflation that occurred in that period, which means, in turn, that this could have been anticipated at the end of 2013. Finally, we highlight the importance of revisions in the reading of shocks in real time and how these revisions can alter the initial analysis.

There are several alternative assumptions excluded from the analysis presented here, which can be important for the effects of NER movements on other variables and for policy analysis. On the one hand, the analysis abstracts from any imperfection of information and any departure from complete rationality. These can be particularly important for the identification of shocks and of the responses of prices to new information. On the other hand, the model used in this paper omits financial frictions, which can be potentially important in an emerging economy with a tradable/non-tradable distinction. These considerations would probably imply an additional amplification mechanism (as in Mendoza, 2002) and could yield a different reading of the shocks that hit the economy (as in García-Cicco et al., 2015). We leave all of this for future research.

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Appendix

A.1 Calibration

Table A.1: Calibrated Parameters

Param.	Description	Value	Source
σ	Risk Aversion	1	García et al. (2019)
φ	Inv. Frish elast.	1	Medina and Soto (2007)
γ	Share C^N in C^{NFE}	0.62	I-O Matrix, average 08-13
γ_T	Share C^X in C^T	0.23	I-O Matrix, average 08-13
γ_I	Share I^N in I	0.62	I-O Matrix, average 08-13
γ_{TI}	Share I^X in I^T	0.02	I-O Matrix, average 08-13
γ_{EC}	Share C^E in C	0.09	Share E in CPI 2013 base
γ_{FC}	Share C^F in C	0.19	Share E in CPI 2013 base
α_X	Capital in V.A. X	0.66	I-O Matrix, average 08-13
α_N	Capital in V.A N	0.49	I-O Matrix, average 08-13
$1-\gamma_X$	Imports in Prod. X	0.2	I-O Matrix, average 08-13
$1-\gamma_N$	Imports in Prod. M	0.08	I-O Matrix, average 08-13
γ_{EM}	Share E in Interm. Imports	0.09	Same as γ_{EC}
δ	Capital depreciation	0.01	García et al. (2019)
ϵ^j	Elast. of Subst. Varieties	11	García et al. (2019)
ϑ	Fraction sector Co owned by Gov.	0.56	Average 01-15[a]
s^{TB}	Ratio of TB to PIB	0.05	Average 01-15
s^{PIB_N}	Ratio of PIB^N to PIB	0.6	Average 01-15
s^{Co}	Ratio of Co to GDP	0.1	Average 01-15
s^G	Ratio of G to GDP	0.12	Average 01-15
π	Inflation (annual)	1.03	Target Central Bank of Chile
R	Monetary Policy Rate (annual)	1.058	Average 01-15
a	Long-run growth (annual)	1.016	Average 01-15
R^W	World Interest Rate (annual)	1.045	Average 01-15
ξ^{R1}	EMBI Chile (annual)	1.015	Average 01-15
ρ_g	AR(1) coef. gov. expenditure	0.849	Estimated separately
σ_g	St. dev. gov. expenditure	0.0105	Estimated separately
ρ_{R^W}	AR(1) coef. LIBOR	0.966	Estimated separately
σ_{R^W}	St. dev. LIBOR	0.0010	Estimated separately
ρ_{y^*}	AR(1) coef. GDP comm. partners	0.884	Estimated separately
σ_{y^*}	St. dev. GDP comm. partners	0.0055	Estimated separately
p^X/p^I	Relative price of X to I	1	Normalization
p^M/p^I	Relative price of M to I	1	Normalization

Notes: [a] This includes the public production and the taxes received by the government of the rest of the production.

A.2 Estimation

Table A.2: Estimated Parameters

			Prior		Posterior	
Param.	Description	Dist.	Mean	St.D.	Mode	St. D.
ϕ_C	Habits C	β	0.65	0.2	0.892	0.03
ϕ_I	Inv. Adj. Costs	$\mathcal{N}^+$	4	1	4.248	0.802
θ_{WX}	Calvo W^X	β	0.65	0.2	0.94	0.013
ζ_{WX}	Din. Index. W^X	β	0.5	0.27	0.156	0.183
θ_{WN}	Calvo W^N	β	0.65	0.2	0.974	0.005
ζ_{WN}	Din. Index. W^N	β	0.5	0.27	0.15	0.088
ϱ	Sust. C^T,C^N	$\mathcal{N}^+$	1.1	0.6	0.902	0.535
ϱ_I	Sust. I^T,I^N	$\mathcal{N}^+$	1.1	1.5	1.112	0.891
θ_X	Calvo X	β	0.5	0.27	0.545	0.085
θ_M	Calvo M	β	0.5	0.27	0.853	0.013
θ_N	Calvo N	β	0.5	0.27	0.979	0.004
ϱ_X	Index. Own X	β	0.5	0.27	0.058	0.132
ϱ_M	Index. Own M	β	0.5	0.27	0.548	0.31
ϱ_N	Index. Own N	β	0.5	0.27	0.633	0.052
ζ_X	Din. Index. X	β	0.5	0.27	0.764	0.469
ζ_M	Din. Index. M	β	0.5	0.27	0.498	0.177
ζ_N	Din. Index. N	β	0.5	0.27	0.827	0.065
Γ_X	Adj. Trend X	β	0.65	0.2	0.772	0.247
Γ_{Co}	Adj. Trend Co	β	0.65	0.2	0.773	0.241
ϵ^*	Elast. Ext. Dem.	$\mathcal{N}^+$	0.3	0.15	0.028	0.082
ϕ_B	Elast. External i.r.	IG	0.001	∞	0.002	0.001
ρ_R	Smoothing	β	0.8	0.05	0.772	0.03
v α_π	Reaction to π	$\mathcal{N}^+$	1.7	0.1	1.58	0.098
α_π^{NFE}	Reaction to π^{NFE}	β	0.5	0.2	0.384	0.183
α_y	Reaction to y	$\mathcal{N}^+$	0.125	0.05	0.175	0.047

Note: Prior distributions: β Beta, $\mathcal{N}^+$ Normal truncated for positive values, IG Inverse Gamma, $\mathcal{U}$ Uniform. The standard deviation of the posterior is approximated by the inverse Hessian evaluated at the posterior mode.

Table A.3: Estimated Parameters, Coefficients of Exogenous Processes

		Prior		Posterior	
Para.	Dist.	Mean	St.D.	Mode	St.D.
		Dynamics of Driving Forces			
$\rho_{\xi^{\beta}}$	β	0.65	0.2	0.792	0.079
ρ_{a}	β	0.35	0.15	0.279	0.155
$\rho_{\xi^{hX}}$	β	0.65	0.2	0.811	0.076
$\rho_{\xi^{hN}}$	β	0.65	0.2	0.897	0.051
ρ_{z^X}	β	0.5	0.23	0.847	0.084
ρ_{z^N}	β	0.5	0.23	0.944	0.028
ρ_{u}	β	0.65	0.2	0.62	0.117
$\rho_{\xi^{X*}}$	β	0.65	0.2	0.86	0.061
$\rho_{\xi^{R1}}$	β	0.65	0.2	0.936	0.051
$\rho_{\xi^{R2}}$	β	0.65	0.2	0.851	0.065
ρ_{yCo}	β	0.55	0.2	0.873	0.068
ρ_{p^E}	β	0.65	0.2	0.903	0.054
ρ_{p^F}	β	0.65	0.2	0.974	0.016
Γ_{*}	$\mathcal{U}$	0.5	0.29	0.324	0.115
Γ_{M*}	$\mathcal{U}$	0.5	0.29	0.505	0.082
Γ_{Co*}	$\mathcal{U}$	0.5	0.29	0.117	0.12
ρ_{F*}	$\mathcal{U}$	0	0.58	0.239	0.121
ρ_{*}	$\mathcal{U}$	0	0.58	0.768	0.153
ρ_{M*}	$\mathcal{U}$	0	0.58	0.478	0.123
ρ_{Co*}	$\mathcal{U}$	0	0.58	0.918	0.045

Note: Prior distributions: β Beta, $\mathcal{U}$ Uniform. The standard deviation of the posterior is approximated by the inverse Hessian evaluated at the posterior mode.

Table A.4: Estimated Parameters, Standard Deviations of Exogenous shocks

		Prior		Posterior	
Para.	Dist.	Mean	St.D.	Mode	St.D.
$\sigma^{\xi^{\beta}}$	$\mathcal{N}^+$	0.03	0.03	0.047	0.013
σ^{a}	IG	0.01	∞	0.004	0.001
$\sigma^{\xi^{hX}}$	$\mathcal{N}^+$	0.1	0.15	0.252	0.102
$\sigma^{\xi^{hN}}$	$\mathcal{N}^+$	0.1	0.15	0.239	0.093
σ^{z^X}	$\mathcal{N}^+$	0.01	0.03	0.014	0.002
σ^{z^N}	$\mathcal{N}^+$	0.005	0.03	0.047	0.006
σ^{u}	$\mathcal{N}^+$	0.03	0.03	0.062	0.017
σ^{e^m}	$\mathcal{N}^+$	0.01	0.01	0.002	0
$\sigma^{\xi^{X*}}$	$\mathcal{N}^+$	0.01	0.01	0.018	0.002
$\sigma^{\xi^{R1}}$	$\mathcal{N}^+$	0.01	0.01	0.001	0
$\sigma^{\xi^{R2}}$	$\mathcal{N}^+$	0.01	0.01	0.004	0.002
σ^{yCo}	$\mathcal{N}^+$	0.02	0.02	0.021	0.002
σ^{p^E}	$\mathcal{N}^+$	0.04	0.04	0.024	0.002
σ^{p^F}	$\mathcal{N}^+$	0.02	0.02	0.012	0.001
σ^{F*}	$\mathcal{U}$	0.25	0.14	0.031	0.006
σ^{*}	$\mathcal{U}$	0.25	0.14	0.013	0.002
σ^{M*}	$\mathcal{U}$	0.25	0.14	0.014	0.002
σ^{Co*}	$\mathcal{U}$	0.25	0.14	0.117	0.011

Note: Prior distributions: $\mathcal{N}^+$ Normal truncated for positive values, IG Inverse Gamma, $\mathcal{U}$ Uniform. The standard deviation of the posterior is approximated by the inverse Hessian evaluated at the posterior mode.

A.3 Optimality Conditions

A.3.1 Household

From the decision of final consumption, labor, bonds and capital and defining as λ_t the multiplier of the budget constraint, $\mu_t^J \lambda_t$ the multiplier of the capital accumulation equation for $J = \{X, N\}$ and as $\mu_t^{WJ} W_t^J \lambda_t$ the multiplier of the equalization of labor demand and supply, we have the first order conditions:

$$\xi_t^{\beta}(C_t - \phi_c \tilde{C}_{t-1})^{-\sigma} - P_t \lambda_t = 0$$
$$-\xi_t^{\beta} \kappa_t \xi_t^{h,J} (h_t^J)^{\varphi} + \mu_t^{WJ} W_t^J \lambda_t = 0$$
$$-\lambda_t + \beta E_t \lambda_{t+1} R_t = 0$$
$$-\lambda_t S_t + \beta E_t \lambda_{t+1} S_{t+1} R_t^* = 0$$
$$-\mu_t^J \lambda_t + \beta E_t \left\{ \lambda_{t+1} P_{t+1}^J R_{t+1}^J + \mu_{t+1}^J \lambda_{t+1}(1-\delta) \right\} = 0$$
$$-\lambda_t P_t^I + \mu_t^J \lambda_t \left\{ \left[1 - \Gamma\left(\frac{I_t^J}{I_{t-1}^J} \right) \right] u_t + \left(-\Gamma'\left(\frac{I_t^J}{I_{t-1}^J} \right) \frac{1}{I_{t-1}^J} \right) u_t I_t^J \right\} +$$
$$\beta E_t \left\{ \mu_{t+1}^J \lambda_{t+1} \left(-\Gamma'\left(\frac{I_{t+1}^J}{I_t^J} \right) \right) \left(-\frac{I_{t+1}^J}{(I_t^J)^2} \right) u_{t+1} I_{t+1}^J \right\} = 0$$

where $J = \{X, N\}$ for the second and last two equations. The functional form for $\Gamma(x)$ is:

$$\Gamma(x) = 1 - \frac{\phi_I}{2}(x-a)^2$$

where a is the steady-state value of the trend growth. From the optimality conditions of choosing wages, we can write the first order conditions as:

$$\frac{\epsilon_W - 1}{\epsilon_W} W_t^{J,*} E_t \sum_{\tau=0}^{\infty} (\theta_{WJ}\beta)^{\tau} \lambda_{t+\tau} \left\{ \frac{h_{t+\tau}^{J,d}}{(W_{t+\tau}^J)^{-\epsilon_W}} (W_t^{J,*})^{-\epsilon_W} \left[a^{\tau} \prod_{s=1}^{\tau} \pi_{t+s-1}^{\zeta_{WJ}} \bar{\pi}_{t+s}^{1-\zeta_{WJ}} \right]^{1-\epsilon_W} \right\} =$$
$$E_t \sum_{\tau=0}^{\infty} (\theta_{WJ}\beta)^{\tau} \mu_{t+\tau}^{WJ} \lambda_{t+\tau} W_{t+\tau}^J \left\{ \frac{h_{t+\tau}^{J,d}}{(W_{t+\tau}^J)^{-\epsilon_W}} (W_t^{J,*})^{-\epsilon_W} \left[a^{\tau} \prod_{s=1}^{\tau} \pi_{t+s-1}^{\zeta_{WJ}} \bar{\pi}_{t+s}^{1-\zeta_{WJ}} \right]^{-\epsilon_W} \right\}$$

where $W_t^{J,*}$ is the optimal wage chosen and this equation holds for $J = \{X, N\}$.

In addition, the optimality conditions for the decision between tradable and non-tradable consumption are:

$$C_t^N = \gamma \left(\frac{P_t^N}{P_t} \right)^{-\varrho} C_t$$
$$C_t^T = (1-\gamma) \left(\frac{P_t^T}{P_t} \right)^{-\varrho} C_t$$

where it was used the fact that $C_t^{NFE} = C_t$.

And between the exportable and importable:

$$C_t^X = \gamma_T \left(\frac{P_t^T C_t^T}{P_t^X} \right)$$
$$C_t^M = (1-\gamma_T) \left(\frac{P_t^T C_t^T}{P_t^M} \right)$$

A.3.2 Investment Good Production

The first order conditions between tradable and non-tradable investment can be written as:

$$\begin{aligned}\tilde{I}_t^N &= \gamma_I \left(\frac{P_t^N}{P_t^I}\right)^{-\varrho_I} I_t \\ \tilde{I}_t^T &= (1-\gamma_I)\left(\frac{P_t^{TI}}{P_t^I}\right)^{-\varrho_I} I_t\end{aligned}$$

where P_t^{TI} is the price index defined for the tradable investment. The FOC between exportable and importable investment is given by:

$$\begin{aligned}\tilde{I}_t^X &= \gamma_{TI}\left(\frac{P_t^{TI}\tilde{I}_t^T}{P_t^X}\right) \\ \tilde{I}_t^M &= (1-\gamma_{TI})\left(\frac{P_t^{TI}\tilde{I}_t^T}{P_t^M}\right)\end{aligned}$$

A.3.3 Firms

The first order conditions are the same for each firm i in each sector and so the subscript will be omitted. First, given the marginal costs, the first order condition of the price setting can be written as:

$$\frac{\epsilon_J - 1}{\epsilon_J}(P_t^{J,*})^{-\epsilon_J}\sum_{\tau=0}^{\infty}(\beta\theta_J)^{\tau}\Lambda_{t,t+\tau}\frac{1}{(P_{t+\tau}^J)^{-\epsilon_J}}Y_{t+\tau}^J\left[\prod_{s=1}^{\tau}\left((\pi_{t+s-1}^J)^{\varrho_J}\pi_{t+s-1}^{1-\varrho_J}\right)^{\zeta_J}\bar{\pi}_{t+s}^{1-\zeta_J}\right]^{1-\epsilon_J} =$$

$$(P_t^{J,*})^{-\epsilon_J-1}\sum_{\tau=1}^{\infty}(\beta\theta_J)^{\tau}\Lambda_{t,t+\tau}MC_{t+\tau}^J\frac{1}{(P_{t+\tau}^J)^{-\epsilon_J}}Y_{t+\tau}^J\left[\prod_{s=1}^{\tau}\left((\pi_{t+s-1}^J)^{\varrho_J}\pi_{t+s-1}^{1-\varrho_J}\right)^{\zeta_J}\bar{\pi}_{t+s}^{1-\zeta_J}\right]^{-\epsilon_J}$$

where $P_t^{J,*}$ is the optimal price chosen at t. To get the marginal cost of each sector, we distinguish between the importable and the other sectors

- Sector M: Cost minimization implies that their marginal cost is the same for all firms and is given by the price in local currency of the imported input:

$$MC_t^M = P_{m,t}$$

  Note the difference between the price set by the M sector, P_t^M, and the price of its input, $P_{m,t}$.

- Sector X and N:

  1. Optimal production of V_t^J: The optimality conditions and the marginal cost are:

$$
\begin{aligned}
h_t^{J,d} &= \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J}} \left[\frac{1-\alpha_J}{\alpha_J} \frac{P_t^J R_t^J}{W_t^J} \right]^{\alpha_J} \\
K_{t-1}^J &= \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J}} \left[\frac{\alpha_J}{1-\alpha_J} \frac{W_t^J}{P_t^J R_t^J} \right]^{1-\alpha_J} \\
MC_t^{V,J} &= \frac{1}{z_t^J (A_t^J)^{1-\alpha_J}} (P_t^J R_t^J)^{\alpha_J} (W_t^J)^{1-\alpha_J} \left[\frac{1}{(1-\alpha_J)^{1-\alpha_J} \alpha_J^{\alpha_J}} \right]
\end{aligned}
$$

2. Optimal production Y_t^J:

$$
\begin{aligned}
M_t^J &= Y_t^J(i) \left[\frac{1-\gamma_J}{\gamma_J} \frac{MC_t^{V,J}}{P_t^{ME}} \right]^{\gamma_J} \\
V_t^J &= Y_t^J(i) \left[\frac{\gamma_J}{1-\gamma_J} \frac{P_t^{ME}}{MC_t^{V,J}} \right]^{1-\gamma_J} \\
MC_t^J &= (MC_t^{V,J})^{\gamma_J} (P_t^{ME})^{1-\gamma_J} \left[\frac{1}{(1-\gamma_J)^{1-\gamma_J} \gamma_J^{\gamma_J}} \right]
\end{aligned}
$$

where MC_t^J is the marginal cost of producing Y_t^J.

A.3.4 Market Clearing

All markets clear:

$$
\begin{aligned}
B_t &= B_t^G \\
I_t &= I_t^X + I_t^N \\
h_t^X &= \Delta_t^{WX} h_t^{X,d} \\
h_t^N &= \Delta_t^{WN} h_t^{N,d} \\
Y_t^X &= \Delta_t^X \left(C_t^X + \tilde{I}_t^X + C_t^{X,*} \right) \\
Y_t^M &= \Delta_t^M \left(C_t^M + \tilde{I}_t^M + M_t^X + M_t^N \right) \\
Y_t^N &= \Delta_t^N \left(C_t^N + \tilde{I}_t^N + G_t^N \right)
\end{aligned}
$$

Which correspond to the local bonds market, the investment market, labor markets and goods market. The Δ variables are measures of dispersion in prices in the different markets, given by:

$$
\begin{aligned}
\Delta_t^{WJ} &= \int_0^1 \left(\frac{W_t^J(i)}{W_t^J} \right)^{-\epsilon_W} dj \\
\Delta_t^J &= \int_0^1 \left(\frac{P_t^J(i)}{P_t^J} \right)^{-\epsilon_J} dj
\end{aligned}
$$

the first equation for $J = \{X, N\}$ and the second for $J = \{X, M, N\}$. The rest of the equations correspond to the policy and foreign equations described in the previous section.

A.4 Equilibrium Conditions

This sections describes the equilibrium conditions after the variables were redefined to make them stationary. The transformations made to the variables were: all lower case prices are the corresponding capital price divided by the CPI Index with the exception of $p_t^{Co,*}$ and p_t^{M*} which are divided by the foreign CPI price index and $p_t^{J,*} = P_t^{J,*}/P_t^J$ for $J = \{X, M, N\}$. All lower case real variables (consumption, investment, capital, government expenditure, production, imports, productivity, output, foreign demand) are the upper case divided by A_{t-1} with the exception of $y_t^{Co} = Y_t^{Co}/A_{t-1}^{Co}$. All inflation definitions are the corresponding price index divided by the price index in the previous period. And particular definitions are: $\tilde{\xi}_t^{h,J} = \xi_t^{h,J}/A_{t-1}$, $\tilde{\mu}_t^J = \mu_t^J/P_t$, $b_t^* = B_t^*/(A_{t-1}P_t^*)$, $\tilde{f}_t^J = f_t^{1,J}/(A_{t-1}P_t^\sigma)$, $\tilde{f}_t^{WJ} = f_t^{WJ}/A_{t-1}^{1-\sigma}$, $\tilde{\lambda}_t = P_t\lambda_t/A_{t-1}^{-\sigma}$, $w_t^J = W_t^J/(A_{t-1}P_t)$, $w_t^{J,*} = W_t^{J,^*}/W_t^J$, $mc_t^J = MC_t^J/P_t^J$ and $mc_t^{V,J} = MC_t^{V,J}/P_t^J$ for $J = \{X, M, N\}$ or $J = \{X, N\}$ depending on the variable. In addition, new variables were defined as the real exhange rate, the trade balance, the gdp deflactor among others.

There are 84 endogenous variables,

$$\{c_t, \tilde{\lambda}_t, h_t^X, \mu_t^{WX}, w_t^X, h_t^N, \mu_t^{WN}, w_t^N, R_t, \pi_t, R_t^*, \pi_t^S, \tilde{\mu}_t^X, p_t^X, R_t^X, \tilde{\mu}_t^N, p_t^N, R_t^N, p_t^I, i_t^X, i_t^N, k_t^X, k_t^N, \tilde{f}_t^{WX}, w_t^{X,*}, h_t^{X,d}, \pi_t^X, \tilde{f}_t^{WN}, w_t^{N,*}, h_t^{N,d}, \pi_t^N, c_t^N, p_t^{SAE}, c_t^T, p_t^T, c_t^X, p_t^M, c_t^M, p_t^{TI}, \tilde{i}_t^N, \tilde{i}_t^T, \tilde{i}_t^X, \tilde{i}_t^M, i_t, mc_t^M, y_t^M, m_t, p_{m,t}, v_t^X, a_t^X, v_t^N, mc_t^{V,X}, mc_t^{V,N}, y_t^X, p_t^{ME}, m_t^X, y_t^N, m_t^N, mc_t^X, mc_t^N, \tilde{f}_t^X, p_t^{X,*}, \tilde{f}_t^M, p_t^{M,*}, \pi_t^M, \tilde{f}_t^N, p_t^{N,*}, gdp_t, \pi_t^{SAE}, c_t^{X,*}, rer_t, \Delta_t^{WX}, \Delta_t^{WN}, \Delta_t^N, \Delta_t^X, \Delta_t^M, tb_t, a_t^{Co}, b_t^*, p_t^Y, f_t^*, p_t^{Co,*}, p_t^{M*}, \pi_t^*\}$$

and 21 shocks:

$$\{\xi_t^\beta, a_t, \tilde{\xi}_t^{h,X}, \tilde{\xi}_t^{h,N}, p_t^A/p_t^T, p_t^E/p_t^T, u_t, z_t^X, z_t^N, g_t, e_t^m, y_t^*, \xi_t^{X,*}, R_t^W, \xi_t^{R1}, \xi_t^{R2}, y_t^{Co}, \Delta F_t^*, u_t^*, u_t^{Co*}, u_t^{M*}\}.$$

$$\xi_t^\beta \left(c_t - \phi_C \frac{c_{t-1}}{a_{t-1}}\right)^{-\sigma} = \tilde{\lambda}_t \tag{A.1}$$

$$\tilde{\xi}_t^{h,X}(h_t^X)^\varphi = \mu_t^{WX} w_t^X \tag{A.2}$$

$$\tilde{\xi}_t^{h,N}(h_t^N)^\varphi = \mu_t^{WN} w_t^N \tag{A.3}$$

$$\tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \frac{\tilde{\lambda}_{t+1} R_t}{\pi_{t+1}} \tag{A.4}$$

$$\tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \frac{\tilde{\lambda}_{t+1} R_t^* \pi_{t+1}^S}{\pi_{t+1}} \tag{A.5}$$

$$\tilde{\mu}_t^X \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{\tilde{\lambda}_{t+1} p_{t+1}^X R_{t+1}^X + \tilde{\mu}_{t+1}^X \tilde{\lambda}_{t+1}(1-\delta)\right\} \tag{A.6}$$

$$\tilde{\mu}_t^N \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{\tilde{\lambda}_{t+1} p_{t+1}^N R_{t+1}^N + \tilde{\mu}_{t+1}^N \tilde{\lambda}_{t+1}(1-\delta)\right\} \tag{A.7}$$

$$\tilde{\lambda}_t p_t^I = \tilde{\mu}_t^X \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 - \phi_I \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right) \frac{i_t^X}{i_{t-1}^X} a_{t-1} \right\} u_t + \beta a_t^{-\sigma} E_t \tilde{\mu}_{t+1}^X \tilde{\lambda}_{t+1} \phi_I \left(\frac{i_{t+1}^X}{i_t^X} a_t - a \right) \left(\frac{i_{t+1}^X}{i_t^X} a_t \right)^2 u_{t+1} \tag{A.8}$$

$$\tilde{\lambda}_t p_t^I = \tilde{\mu}_t^N \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 - \phi_I \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right) \frac{i_t^N}{i_{t-1}^N} a_{t-1} \right\} u_t + \beta a_t^{-\sigma} E_t \tilde{\mu}_{t+1}^N \tilde{\lambda}_{t+1} \phi_I \left(\frac{i_{t+1}^N}{i_t^N} a_t - a \right) \left(\frac{i_{t+1}^N}{i_t^N} a_t \right)^2 u_{t+1} \tag{A.9}$$

$$k_t^X = \left[1 - \frac{\phi_I}{2} \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 \right] u_t i_t^X + (1-\delta) \frac{k_{t-1}^X}{a_{t-1}} \tag{A.10}$$

$$k_t^N = \left[1 - \frac{\phi_I}{2} \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 \right] u_t i_t^N + (1-\delta) \frac{k_{t-1}^N}{a_{t-1}} \tag{A.11}$$

$$\tilde{f}_t^{WX} = \frac{\epsilon_W - 1}{\epsilon_W} (w_t^{X,*})^{1-\epsilon_W} \tilde{\lambda}_t h_t^{X,d} + \theta_{WX} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{X,*}}{w_{t+1}^{X,*}} \frac{w_t^X}{w_{t+1}^X} \right)^{1-\epsilon_W} \left[\frac{a}{a_t} \frac{\pi_t^{\zeta_{WX}} \bar{\pi}^{1-\zeta_{WX}}}{\pi_{t+1}} \right]^{1-\epsilon_W} \frac{w_{t+1}^X}{w_t^X} \tilde{f}_{t+1}^{WX} \tag{A.12}$$

$$\tilde{f}_t^{WN} = \frac{\epsilon_W - 1}{\epsilon_W} (w_t^{N,*})^{1-\epsilon_W} \tilde{\lambda}_t h_t^{N,d} + \theta_{WN} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{N,*}}{w_{t+1}^{N,*}} \frac{w_t^N}{w_{t+1}^N} \right)^{1-\epsilon_W} \left[\frac{a}{a_t} \frac{\pi_t^{\zeta_{WN}} \bar{\pi}^{1-\zeta_{WN}}}{\pi_{t+1}} \right]^{1-\epsilon_W} \frac{w_{t+1}^N}{w_t^N} \tilde{f}_{t+1}^{WN} \tag{A.13}$$

$$\tilde{f}_t^{WX} = \left(w_t^{X,*} \right)^{-\epsilon_W} \mu_t^{WX} \tilde{\lambda}_t h_t^{X,d} + \theta_{WX} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{X,*}}{w_{t+1}^{X,*}} \frac{w_t^X}{w_{t+1}^X} \right)^{-\epsilon_W} \left[\frac{a}{a_t} \frac{\pi_t^{\zeta_{WX}} \bar{\pi}^{1-\zeta_{WX}}}{\pi_{t+1}} \right]^{-\epsilon_W} \frac{w_{t+1}^X}{w_t^X} \tilde{f}_{t+1}^{WX} \tag{A.14}$$

$$\tilde{f}_t^{WN} = \left(w_t^{N,*} \right)^{-\epsilon_W} \mu_t^{WN} \tilde{\lambda}_t h_t^{N,d} + \theta_{WN} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{N,*}}{w_{t+1}^{N,*}} \frac{w_t^N}{w_{t+1}^N} \right)^{-\epsilon_W} \left[\frac{a}{a_t} \frac{\pi_t^{\zeta_{WN}} \bar{\pi}^{1-\zeta_{WN}}}{\pi_{t+1}} \right]^{-\epsilon_W} \frac{w_{t+1}^N}{w_t^N} \tilde{f}_{t+1}^{WN} \tag{A.15}$$

$$1 = \theta_{WX} \left(\frac{w_{t-1}^X}{w_t^X} \frac{a}{a_{t-1}} \frac{\pi_{t-1}^{\zeta_{WX}} \bar{\pi}^{1-\zeta_{WX}}}{\pi_t} \right)^{1-\epsilon_W} + (1-\theta_{WX}) \left(w_t^{X,*} \right)^{1-\epsilon_W} \tag{A.16}$$

$$1 = \theta_{WN} \left(\frac{w_{t-1}^N}{w_t^N} \frac{a}{a_{t-1}} \frac{\pi_{t-1}^{\zeta_{WN}} \bar{\pi}^{1-\zeta_{WN}}}{\pi_t} \right)^{1-\epsilon_W} + (1-\theta_{WN}) \left(w_t^{N,*} \right)^{1-\epsilon_W} \tag{A.17}$$

$$c_t^N = \gamma \left(\frac{p_t^N}{p_t^{SAE}} \right)^{-\varrho} c_t \tag{A.18}$$

$$c_t^T = (1-\gamma)\left(\frac{p_t^T}{p_t^{SAE}}\right)^{-\varrho} c_t \tag{A.19}$$

$$c_t^X = \gamma_T \left(\frac{p_t^T c_t^T}{p_t^X}\right) \tag{A.20}$$

$$c_t^M = (1-\gamma_T)\left(\frac{p_t^T c_t^T}{p_t^M}\right) \tag{A.21}$$

$$1 = \left(p_t^{SAE}\right)^{1-\gamma_{AC}-\gamma_{EC}} \left(p_t^A\right)^{\gamma_{AC}} \left(p_t^E\right)^{\gamma_{EC}} \tag{A.22}$$

$$1 = (1-\gamma)\left(\frac{p_t^T}{p_t^{SAE}}\right)^{1-\varrho} + \gamma\left(\frac{p_t^N}{p_t^{SAE}}\right)^{1-\varrho} \tag{A.23}$$

$$p_t^T = (p_t^X)^{\gamma_T}(p_t^M)^{1-\gamma_T} \tag{A.24}$$

$$p_t^I = \left(\gamma_I (p_t^N)^{1-\varrho_I} + (1-\gamma_I)(p_t^{TI})^{1-\varrho_I}\right)^{\frac{1}{1-\varrho_I}} \tag{A.25}$$

$$p_t^{TI} = (p_t^X)^{\gamma_{TI}}(p_t^M)^{1-\gamma_{TI}} \tag{A.26}$$

$$\tilde{i}_t^N = \gamma_I \left(\frac{p_t^N}{p_t^I}\right)^{-\varrho_I} i_t \tag{A.27}$$

$$\tilde{i}_t^T = (1-\gamma_I)\left(\frac{p_t^{TI}}{p_t^I}\right)^{-\varrho_I} i_t \tag{A.28}$$

$$\tilde{i}_t^X = \gamma_{TI}\left(\frac{p_t^{TI}\tilde{i}_t^T}{p_t^X}\right) \tag{A.29}$$

$$\tilde{i}_t^M = (1-\gamma_{TI})\left(\frac{p_t^{TI}\tilde{i}_t^T}{p_t^M}\right) \tag{A.30}$$

$$mc_t^M = \frac{p_{m,t}}{p_t^M} \tag{A.31}$$

$$y_t^M = m_t \tag{A.32}$$

$$h_t^{X,d} = \frac{v_t^X}{z_t^X (a_t^X)^{1-\alpha_X}}\left[\frac{1-\alpha_X}{\alpha_X}\frac{p_t^X}{w_t^X} R_t^X\right]^{\alpha_X} \tag{A.33}$$

$$k_{t-1}^X = a_{t-1}\frac{v_t^X}{z_t^X (a_t^X)^{1-\alpha_X}}\left[\frac{\alpha_X}{1-\alpha_X}\frac{w_t^X}{p_t^X R_t^X}\right]^{1-\alpha_X} \tag{A.34}$$

$$h_t^{N,d} = \frac{v_t^N}{z_t^N a_t^{1-\alpha_N}}\left[\frac{1-\alpha_N}{\alpha_N}\frac{p_t^N}{w_t^N} R_t^N\right]^{\alpha_N} \tag{A.35}$$

$$k_{t-1}^{N} = a_{t-1}\frac{v_t^N}{z_t^N a_t^{1-\alpha_N}}\left[\frac{\alpha_N}{1-\alpha_N}\frac{w_t^N}{p_t^N R_t^N}\right]^{1-\alpha_N} \tag{A.36}$$

$$mc_t^{V,X} = \frac{1}{z_t^X (a_t^X)^{1-\alpha_X}}\frac{(p_t^X R_t^X)^{\alpha_X}(w_t^X)^{1-\alpha_X}}{p_t^X}\left[\frac{1}{(1-\alpha_X)^{1-\alpha_X}\alpha_X^{\alpha_X}}\right] \tag{A.37}$$

$$mc_t^{V,N} = \frac{1}{z_t^N a_t^{1-\alpha_N}}\frac{(p_t^N R_t^N)^{\alpha_N}(w_t^N)^{1-\alpha_N}}{p_t^N}\left[\frac{1}{(1-\alpha_N)^{1-\alpha_N}\alpha_N^{\alpha_N}}\right] \tag{A.38}$$

$$v_t^X = y_t^X\left[\frac{\gamma_X}{1-\gamma_X}\frac{p_t^{ME}}{mc_t^{V,X}}\frac{1}{p_t^X}\right]^{1-\gamma_X} \tag{A.39}$$

$$m_t^X = y_t^X\left[\frac{1-\gamma_X}{\gamma_X}\frac{mc_t^{V,X}}{p_t^{ME}}p_t^X\right]^{\gamma_X} \tag{A.40}$$

$$v_t^N = y_t^N\left[\frac{\gamma_N}{1-\gamma_N}\frac{p_t^{ME}}{mc_t^{V,N}}\frac{1}{p_t^N}\right]^{1-\gamma_N} \tag{A.41}$$

$$m_t^N = y_t^N\left[\frac{1-\gamma_N}{\gamma_N}\frac{mc_t^{V,N}}{p_t^{ME}}p_t^N\right]^{\gamma_N} \tag{A.42}$$

$$mc_t^X = (mc_t^{V,X})^{\gamma_X}\left(\frac{p_t^{ME}}{p_t^X}\right)^{1-\gamma_X}\frac{1}{(1-\gamma_X)^{1-\gamma_X}\gamma_X^{\gamma_X}} \tag{A.43}$$

$$mc_t^N = (mc_t^{V,N})^{\gamma_N}\left(\frac{p_t^{ME}}{p_t^N}\right)^{1-\gamma_N}\frac{1}{(1-\gamma_N)^{1-\gamma_N}\gamma_N^{\gamma_N}} \tag{A.44}$$

$$a_t^X = \left(\frac{a_{t-1}^X}{a_{t-1}}\right)^{1-\Gamma_X}(a_t)^{\Gamma_X} \tag{A.45}$$

$$p_t^{ME} = \left(p_t^M\right)^{1-\gamma_{EF}}\left(p_t^E\right)^{\gamma_{EF}} \tag{A.46}$$

$$\begin{aligned}\tilde{f}_t^X =& \frac{\epsilon_X-1}{\epsilon_X}\left(p_t^{X,*}\right)^{1-\epsilon_X} y_t^X + \\ & \beta a_t^{1-\sigma}\theta_X E_t\left(\frac{p_t^{X,*}}{p_{t+1}^{X,*}}\frac{p_t^X}{p_{t+1}^X}\right)^{1-\epsilon_X}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{\left[\left((\pi_t^X)^{\varrho_X}\pi_t^{1-\varrho_X}\right)^{\zeta_X}\bar{\pi}^{1-\zeta_X}\right]^{1-\epsilon_X}}{\pi_{t+1}^{1-\epsilon_X}}\frac{\pi_{t+1}^X}{\pi_{t+1}}\tilde{f}_{t+1}^X\end{aligned} \tag{A.47}$$

$$\begin{aligned}\tilde{f}_t^M =& \frac{\epsilon_M-1}{\epsilon_M}\left(p_t^{M,*}\right)^{1-\epsilon_M} y_t^M + \\ & \beta a_t^{1-\sigma}\theta_M E_t\left(\frac{p_t^{M,*}}{p_{t+1}^{M,*}}\frac{p_t^M}{p_{t+1}^M}\right)^{1-\epsilon_M}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{\left[\left((\pi_t^M)^{\varrho_M}\pi_t^{1-\varrho_M}\right)^{\zeta_M}\bar{\pi}^{1-\zeta_M}\right]^{1-\epsilon_M}}{\pi_{t+1}^{1-\epsilon_M}}\frac{\pi_{t+1}^M}{\pi_{t+1}}\tilde{f}_{t+1}^M\end{aligned} \tag{A.48}$$

$$\tilde{f}_t^N = \frac{\epsilon_N - 1}{\epsilon_N}\left(p_t^{N,*}\right)^{1-\epsilon_N} y_t^N + \\ \beta a_t^{1-\sigma}\theta_N E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}}\frac{p_t^N}{p_{t+1}^N}\right)^{1-\epsilon_N} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^N)^{\varrho_N}\pi_t^{1-\varrho_N}\right)^{\zeta_N}\bar{\pi}^{1-\zeta_N}\right]^{1-\epsilon_N}}{\pi_{t+1}^{1-\epsilon_N}} \frac{\pi_{t+1}^N}{\pi_{t+1}}\tilde{f}_{t+1}^N \tag{A.49}$$

$$\tilde{f}_t^X = \left(p_t^{X,*}\right)^{-\epsilon_X} mc_t^X y_t^X + \\ \beta a_t^{1-\sigma}\theta_X E_t \left(\frac{p_t^{X,*}}{p_{t+1}^{X,*}}\frac{p_t^X}{p_{t+1}^X}\right)^{-\epsilon_X} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^X)^{\varrho_X}\pi_t^{1-\varrho_X}\right)^{\zeta_X}\bar{\pi}^{1-\zeta_X}\right]^{-\epsilon_X}}{\pi_{t+1}^{-\epsilon_X}} \frac{\pi_{t+1}^X}{\pi_{t+1}}\tilde{f}_{t+1}^X \tag{A.50}$$

$$\tilde{f}_t^M = \left(p_t^{M,*}\right)^{-\epsilon_M} mc_t^M y_t^M + \\ \beta a_t^{1-\sigma}\theta_M E_t \left(\frac{p_t^{M,*}}{p_{t+1}^{M,*}}\frac{p_t^M}{p_{t+1}^M}\right)^{-\epsilon_M} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^M)^{\varrho_M}\pi_t^{1-\varrho_M}\right)^{\zeta_M}\bar{\pi}^{1-\zeta_M}\right]^{-\epsilon_M}}{\pi_{t+1}^{-\epsilon_M}} \frac{\pi_{t+1}^M}{\pi_{t+1}}\tilde{f}_{t+1}^M \tag{A.51}$$

$$\tilde{f}_t^N = \left(p_t^{N,*}\right)^{-\epsilon_N} mc_t^N y_t^N + \\ \beta a_t^{1-\sigma}\theta_N E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}}\frac{p_t^N}{p_{t+1}^N}\right)^{-\epsilon_N} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^N)^{\varrho_N}\pi_t^{1-\varrho_N}\right)^{\zeta_N}\bar{\pi}^{1-\zeta_N}\right]^{-\epsilon_N}}{\pi_{t+1}^{-\epsilon_N}} \frac{\pi_{t+1}^N}{\pi_{t+1}}\tilde{f}_{t+1}^N \tag{A.52}$$

$$\pi_t^X = \frac{p_t^X}{p_{t-1}^X}\pi_t \tag{A.53}$$

$$\pi_t^M = \frac{p_t^M}{p_{t-1}^M}\pi_t \tag{A.54}$$

$$\pi_t^N = \frac{p_t^N}{p_{t-1}^N}\pi_t \tag{A.55}$$

$$\pi_t^{SAE} = \frac{p_t^{SAE}}{p_{t-1}^{SAE}}\pi_t \tag{A.56}$$

$$1 = (1-\theta_X)\left(p_t^{*,X}\right)^{1-\epsilon_X} + \theta_X\left[\left((\pi_{t-1}^X)^{\varrho_X}\pi_{t-1}^{1-\varrho_X}\right)^{\zeta_X}\bar{\pi}^{1-\zeta_X}\right]^{1-\epsilon_X}\left(\frac{1}{\pi_t^X}\right)^{1-\epsilon_X} \tag{A.57}$$

$$1 = (1-\theta_M)\left(p_t^{*,M}\right)^{1-\epsilon_M} + \theta_M\left[\left((\pi_{t-1}^M)^{\varrho_M}\pi_{t-1}^{1-\varrho_M}\right)^{\zeta_M}\bar{\pi}^{1-\zeta_M}\right]^{1-\epsilon_M}\left(\frac{1}{\pi_t^M}\right)^{1-\epsilon_M} \tag{A.58}$$

$$1 = (1-\theta_N)\left(p_t^{*,N}\right)^{1-\epsilon_N} + \theta_N\left[\left((\pi_{t-1}^N)^{\varrho_N}\pi_{t-1}^{1-\varrho_N}\right)^{\zeta_N}\bar{\pi}^{1-\zeta_N}\right]^{1-\epsilon_N}\left(\frac{1}{\pi_t^N}\right)^{1-\epsilon_N} \tag{A.59}$$

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\varrho_R} \left[\left(\frac{(\pi_t^{SAE})^{\alpha_\pi^{SAE}} \pi_t^{1-\alpha_\pi^{SAE}}}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{gdp_t a_{t-1}/gdp_{t-1}}{a}\right)^{\alpha_Y}\right]^{1-\varrho_R} e_t^m \tag{A.60}$$

$$c_t^{X,*} = \left(\frac{p_t^X}{rer_t}\right)^{-\epsilon^*} y_t^* \xi_t^{X,*} \tag{A.61}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t} \tag{A.62}$$

$$p_{m,t} = rer_t p_{m,t}^* \tag{A.63}$$

$$R_t^* = R_t^W \exp\left\{\phi_B \left(\bar{b} - \frac{b_t^* rer_t}{p_t^Y gdp_t}\right)\right\} \xi_t^{R1} \xi_t^{R2} \tag{A.64}$$

$$i_t = i_t^X + i_t^N \tag{A.65}$$

$$h_t^X = \Delta_t^{WX} h_t^{X,d} \tag{A.66}$$

$$h_t^N = \Delta_t^{WN} h_t^{N,d} \tag{A.67}$$

$$y_t^N = \Delta_t^N (c_t^N + g_t + \tilde{i}_t^N) \tag{A.68}$$

$$y_t^X = \Delta_t^X (c_t^X + \tilde{i}_t^X + c_t^{X,*}) \tag{A.69}$$

$$y_t^M = \Delta_t^M (c_t^M + \tilde{i}_t^M + m_t^X + m_t^N) \tag{A.70}$$

$$\Delta_t^{WX} = (1-\theta_{WX}) \left(w_t^{X,*}\right)^{-\epsilon_W} + \theta_{WX} \left(\frac{w_{t-1}^X}{w_t^X} \frac{a}{a_{t-1}} \frac{\pi_{t-1}^{\zeta_{WX}} \bar{\pi}^{1-\zeta_{WX}}}{\pi_t}\right)^{-\epsilon_W} \Delta_{t-1}^{WX} \tag{A.71}$$

$$\Delta_t^{WN} = (1-\theta_{WN}) \left(w_t^{N,*}\right)^{-\epsilon_W} + \theta_{WN} \left(\frac{w_{t-1}^N}{w_t^N} \frac{a}{a_{t-1}} \frac{\pi_{t-1}^{\zeta_{WN}} \bar{\pi}^{1-\zeta_{WN}}}{\pi_t}\right)^{-\epsilon_W} \Delta_{t-1}^{WX} \tag{A.72}$$

$$\Delta_t^X = (1-\theta_X) \left(p_t^{*,X}\right)^{-\epsilon_X} + \theta_X \left(\frac{\left((\pi_{t-1}^X)^{\varrho_X} \pi_{t-1}^{1-\varrho_X}\right)^{\zeta_X} \bar{\pi}^{1-\zeta_X}}{\pi_t^X}\right)^{-\epsilon_X} \Delta_{t-1}^X \tag{A.73}$$

$$\Delta_t^M = (1-\theta_M) \left(p_t^{*,M}\right)^{-\epsilon_M} + \theta_M \left(\frac{\left((\pi_{t-1}^M)^{\varrho_M} \pi_{t-1}^{1-\varrho_M}\right)^{\zeta_M} \bar{\pi}^{1-\zeta_M}}{\pi_t^M}\right)^{-\epsilon_M} \Delta_{t-1}^M \tag{A.74}$$

$$\Delta_t^N = (1-\theta_N) \left(p_t^{*,N}\right)^{-\epsilon_N} + \theta_N \left(\frac{\left((\pi_{t-1}^N)^{\varrho_N} \pi_{t-1}^{1-\varrho_N}\right)^{\zeta_N} \bar{\pi}^{1-\zeta_N}}{\pi_t^N}\right)^{-\epsilon_N} \Delta_{t-1}^N \tag{A.75}$$

$$tb_t = rer_t p_t^{Co,*} y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} + p_t^X c_t^{X,*} - p_{m,t} m_t \tag{A.76}$$

$$a_t^{Co} = \left(\frac{a_{t-1}^{Co}}{a_{t-1}}\right)^{1-\Gamma_{Co}} a_t^{\Gamma_{Co}} \tag{A.77}$$

$$rer_t b_t^* = tb_t + \frac{rer_t}{\pi_t^* a_{t-1}} R_{t-1}^* b_{t-1}^* - (1-\vartheta) rer_t p_t^{Co,*} y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} \tag{A.78}$$

$$gdp_t = c_t + g_t + i_t + c_t^{X,*} + y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} - m_t \tag{A.79}$$

$$p_t^Y gdp_t = c_t + p_t^G g_t + p_t^I i_t + tb_t \tag{A.80}$$

The equations for the external prices are:

$$p_t^{Co*} = \left(\frac{\pi_{ss}^* p^{Co*}}{\pi_t^*}\right)^{\Gamma_{Co*}} (f_t^*)^{1-\Gamma_{Co*}} u_t^{Co*} \tag{A.81}$$

$$p_t^{M*} = \left(\frac{\pi_{ss}^* p^{M*}}{\pi_t^*}\right)^{\Gamma_{M*}} (f_t^*)^{1-\Gamma_{M*}} u_t^{M*} \tag{A.82}$$

$$\pi_t^* = (\pi_{ss}^*)^{\Gamma_*} (f_t^* \pi_t^*)^{1-\Gamma_*} u_t^* \tag{A.83}$$

$$\Delta F_t^* = \frac{f_t^* \pi_t^*}{f_{t-1}^*} \tag{A.84}$$

A.4.1 Exogenous Processes

$$\log(\xi_t^\beta) = \rho_{\xi^\beta} \log(\xi_{t-1}^\beta) + \epsilon_t^{\xi^\beta} \tag{A.85}$$

$$\log(\xi_t^{h,N}/\xi_{ss}^{h,N}) = \rho_{\xi^{h,N}} \log(\xi_{t-1}^{h,N}/\xi_{ss}^{h,N}) + \epsilon_t^{\xi^{h,N}} \tag{A.86}$$

$$\log(\xi_t^{h,X}/\xi_{ss}^{h,X}) = \rho_{\xi^{h,X}} \log(\xi_{t-1}^{h,X}/\xi_{ss}^{h,X}) + \epsilon_t^{\xi^{h,X}} \tag{A.87}$$

$$\log(z_t^X/z_{ss}^X) = \rho_{z^X} \log(z_{t-1}^X/z_{ss}^X) + \epsilon_t^{z^X} \tag{A.88}$$

$$\log(z_t^N) = \rho_{z^N} \log(z_{t-1}^N) + \epsilon_t^{z^N} \tag{A.89}$$

$$\log(a_t/a_{ss}) = \rho_a \log(a_{t-1}/a_{ss}) + \epsilon_t^a \tag{A.90}$$

$$\log(y_t^{Co}/y_{ss}^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y_{ss}^{Co}) + \epsilon_t^{y^{Co}} \tag{A.91}$$

$$\log(p_t^E/p_t^T) = \rho_{p^E} \log(p_{t-1}^E/p_{t-1}^T) + \epsilon_t^{p^E} \tag{A.92}$$

$$\log(p_t^F/p_t^T) = \rho_{p^F} \log(p_{t-1}^F/p_{t-1}^T) + \epsilon_t^{p^F} \tag{A.93}$$

$$\log(u_t) = \rho_u \log(u_{t-1}) + \epsilon_t^u \tag{A.94}$$

$$\log(g_t/g_{ss}) = \rho_g \log(g_{t-1}/g_{ss}) + \epsilon_t^g \tag{A.95}$$

$$\log(R_t^W/R_{ss}^W) = \rho_{R^W} \log(R_{t-1}^W/R_{ss}^W) + \epsilon_t^{R^W} \tag{A.96}$$

$$\log(\xi_t^{R1}/\xi_{ss}^{R1}) = \rho_{\xi^{R1}} \log(\xi_{t-1}^{R1}/\xi_{ss}^{R1}) + \epsilon_t^{\xi^{R1}} \tag{A.97}$$

$$\log(\xi_t^{R2}) = \rho_{\xi^{R2}} \log(\xi_{t-1}^{R2}) + \epsilon_t^{\xi^{R2}} \tag{A.98}$$

$$\log(\xi_t^{X*}) = \rho_{\xi^{X*}} \log(\xi_{t-1}^{X*}) + \epsilon_t^{\xi^{X*}} \tag{A.99}$$

$$\log(y_t^*/y_{ss}^*) = \rho_{y^*}\log(y_{t-1}^*/y_{ss}^*) + \epsilon_t^{y^*} \tag{A.100}$$

$$\log(\Delta F_t^*/\pi_{ss}^*) = \rho_{\Delta F^*}\log(\Delta F_{t-1}^*/\pi_{ss}^*) + \epsilon_t^{\Delta F^*} \tag{A.101}$$

$$\log(u_t^*) = \rho_*\log(u_{t-1}^*) + \epsilon_t^* \tag{A.102}$$

$$\log(u_t^{Co*}) = \rho_{Co*}\log(u_{t-1}^{Co*}) + \epsilon_t^{Co*} \tag{A.103}$$

$$\log(u_t^{M*}) = \rho_{M*}\log(u_{t-1}^{M*}) + \epsilon_t^{M*} \tag{A.104}$$

All ϵ variables are *i.i.d.* with mean zero (including the monetary shock).

A.4.2 Steady State

The given endogenous are: $\{R, h^X, h^N, p^X/p^I, p^M/p^I, s^{Co} = rer\ p^{Co,*}y^{Co}/(p^Y gdp), s^M = p_m y^M/(p^Y gdp), s^g = p^N g/(p^Y gdp)\}$[8] and the exogenous variables or parameters that are calculated endogenously are: $\{\beta, \tilde{\xi}^{h,N}, z^X, g, y^*, \pi^*, y^{Co}, \gamma, \bar{b}\}$. The rest of the steady state values of the exogenous variables, (not endogenously determined nor listed in table A.1) are normalized to one as is implied in the previous equations and $\tilde{\xi}^{h,X}$ is set equal to $\tilde{\xi}^{h,N}$. By (A.64) (assuming that the part inside the bracket is zero):

$$R^* = R^W \xi^{R1}$$

By (A.45)

$$a^X = a^{\frac{2\Gamma_X - 1}{\Gamma_X}}$$

By (A.77)

$$a^{Co} = a^{\frac{2\Gamma_{Co} - 1}{\Gamma_{Co}}}$$

By (A.60) and (A.56) (assuming $\epsilon^m = 1$):

$$\pi^{SAE} = \pi = \bar{\pi}$$

By (A.4):

$$\beta = \frac{a^\sigma \pi}{R}$$

By (A.5):

$$\pi^S = \frac{a^\sigma \pi}{R^* \beta}$$

By (A.62):

$$\pi^* = \frac{\pi}{\pi^S}$$

By (A.83):

$$f^* = 1$$

By (A.81)-(A.82):

$$p^{Co*} = p^{M*} = f^*$$

By (A.84):

$$\Delta F^* = \pi^*$$

[8]The values for h^X, h^N, s^M were set to get as close as possible to the targets for $\gamma, tby, p^N y^N / p^Y gdp$.

By (A.63)-(A.65):

$$\pi^X = \pi^M = \pi^N = \pi$$

By (A.57)-(A.59):

$$p^{X,*} = p^{M,*} = p^{N,*} = 1$$

By (A.16)-(A.17):

$$w^{X,*} = w^{N,*} = 1$$

By (A.71)-(A.75):

$$\Delta^{WX} = \Delta^{WN} = \Delta^X = \Delta^M = \Delta^N = 1$$

By (A.47)-(A.52)

$$mc^X = \frac{\epsilon_X - 1}{\epsilon_X}$$

$$mc^M = \frac{\epsilon_M - 1}{\epsilon_M}$$

$$mc^N = \frac{\epsilon_N - 1}{\epsilon_N}$$

By (A.12)-(A.15)

$$\mu^{WX} = \mu^{WN} = \frac{\epsilon_W - 1}{\epsilon_W}$$

By (A.66)-(A.67):

$$h^{X,d} = h^X$$

$$h^{N,d} = h^N$$

From the relative prices p^X/p^I and p^M/p^I, we get using (A.24)-(A.26) the relative prices:

$$\frac{p^{TI}}{p^I} = \left(\frac{p^X}{p^I}\right)^{\gamma_{TI}} \left(\frac{p^M}{p^I}\right)^{(1-\gamma_{TI})}$$

$$\frac{p^N}{p^I} = \left(\frac{1-(1-\gamma_I)\left(p^{TI}/p^I\right)^{1-\varrho_I}}{\gamma_I}\right)^{\frac{1}{1-\varrho_I}}$$

$$\frac{p^T}{p^I} = \left(\frac{p^X}{p^I}\right)^{\gamma_T} \left(\frac{p^M}{p^I}\right)^{(1-\gamma_T)}$$

From (A.8)-(A.9):

$$\frac{\tilde{\mu}^X}{p^I} = \frac{\tilde{\mu}^N}{p^I} = 1/u$$

By (A.6)-(A.7):

$$R^X = \frac{(\tilde{\mu}^X/p^I)(1-\beta a^{-\sigma}(1-\delta))}{\beta a^{-\sigma}(p^X/p^I)}$$

$$R^N = \frac{(\tilde{\mu}^N/p^I)(1-\beta a^{-\sigma}(1-\delta))}{\beta a^{-\sigma}(p^N/p^I)}$$

By (A.31):

$$\frac{p_m}{p^I} = mc^M(p^M/p^I)$$

By (A.63):

$$\frac{rer}{p^I} = \frac{p_m/p^I}{p_m^*}$$

It is further assumed that $p^A = p^E = p^T$, and so, we also have p^A/p^I and p^E/p^I. By (A.46):

$$\frac{p^{ME}}{p^I} = \left(\frac{p^M}{p^I}\right)^{1-\gamma_{EF}} \left(\frac{p^T}{p^I}\right)^{\gamma_{EF}}$$

By (A.43)-(A.44):

$$mc^{V,X} = \left(\frac{mc^X (p^X/p^I)^{1-\gamma_X} (1-\gamma_X)^{1-\gamma_X} \gamma_X^{\gamma_X}}{(p^{ME}/p^I)^{1-\gamma_X}}\right)^{\frac{1}{\gamma_X}}$$

$$mc^{V,N} = \left(\frac{mc^N (p^N/p^I)^{1-\gamma_N} (1-\gamma_N)^{1-\gamma_N} \gamma_N^{\gamma_N}}{(p^{ME}/p^I)^{1-\gamma_N}}\right)^{\frac{1}{\gamma_N}}$$

By (A.38):

$$\frac{w^N}{p^I} = \left(\frac{mc^{V,N} z^N a^{1-\alpha_N} (p^N/p^I)(1-\alpha_N)^{1-\alpha_N} \alpha_N^{\alpha_N}}{((p^N/p^I)R^N)^{\alpha_N}}\right)^{\frac{1}{1-\alpha_N}}$$

By (A.3):

$$\frac{\tilde{\xi}^{h,N}}{p^I} = \frac{\mu^{WN}}{(h^N)^{\varphi}} \frac{w^N}{p^I}$$

Assuming that $\tilde{\xi}^{h,X} = \tilde{\xi}^{h,N}$, we also have $\tilde{\xi}^{h,X}/p^I$ and with (A.2):

$$\frac{w^X}{p^I} = \frac{(\tilde{\xi}^{h,X}/p^I)(h^X)^{\varphi}}{\mu^{WX}}$$

By (A.37):

$$z^X = \frac{((p^X/p^I)R^X)^{\alpha_X} (w^X/p^I)^{1-\alpha_X}}{mc^{V,X} (a^X)^{1-\alpha_X} (p^X/p^I)(1-\alpha_X)^{1-\alpha_X} \alpha_X^{\alpha_X}}$$

By (A.33) and (A.35):

$$v^X = h^{X,d} z^X (a^X)^{1-\alpha_X} \left[\frac{\alpha_X}{1-\alpha_X} \frac{w^X/p^I}{(p^X/p^I)R^X}\right]^{\alpha_X}$$

$$v^N = h^{N,d} z^N a^{1-\alpha_N} \left[\frac{\alpha_N}{1-\alpha_N} \frac{w^N/p^I}{(p^N/p^I)R^N}\right]^{\alpha_N}$$

By (A.34) and (A.36):

$$k^X = a \frac{v^X}{z^X (a^X)^{1-\alpha_X}} \left[\frac{\alpha_X}{1-\alpha_X} \frac{w^X/p^I}{(p^X/p^I)R^X}\right]^{1-\alpha_X}$$

$$k^N = a \frac{v^N}{z^N a^{1-\alpha_N}} \left[\frac{\alpha_N}{1-\alpha_N} \frac{w^N/p^I}{(p^N/p^I)R^N}\right]^{1-\alpha_N}$$

By (A.39) and (A.41):

$$y^X = v^X \left[\frac{\gamma_X}{1-\gamma_X} \frac{p^{ME}/p^I}{mc^{V,X}} \frac{1}{p^X/p^I} \right]^{-(1-\gamma_X)}$$

$$y^N = v^N \left[\frac{\gamma_N}{1-\gamma_N} \frac{p^{ME}/p^I}{mc^{V,N}} \frac{1}{p^N/p^I} \right]^{-(1-\gamma_N)}$$

By (A.40) and (A.42):

$$m^X = y^X \left[\frac{1-\gamma_X}{\gamma_X} \frac{mc^{V,X}}{p^{ME}/p^I} p^X/p^I \right]^{\gamma_X}$$

$$m^N = y^N \left[\frac{1-\gamma_N}{\gamma_N} \frac{mc^{V,N}}{p^{ME}/p^I} p^N/p^I \right]^{\gamma_N}$$

By (A.47) and (A.49):

$$\tilde{f}^X = \frac{\epsilon_X - 1}{\epsilon_X} \frac{y^X}{(1-\beta a^{1-\sigma}\theta_X)}$$

$$\tilde{f}^N = \frac{\epsilon_N - 1}{\epsilon_N} \frac{y^N}{(1-\beta a^{1-\sigma}\theta_N)}$$

By (A.10)-(A.11):

$$i^X = \frac{k^X}{u} \left(1 - \frac{1-\delta}{a} \right)$$

$$i^N = \frac{k^N}{u} \left(1 - \frac{1-\delta}{a} \right)$$

By (A.65):

$$i = i^X + i^N$$

By (A.27)-(A.30):

$$\tilde{i}^N = \gamma_I \left(\frac{p^N}{p^I} \right)^{-\varrho_I} i$$

$$\tilde{i}^T = (1-\gamma_I) \left(\frac{p^{TI}}{p^I} \right)^{-\varrho_I} i$$

$$\tilde{i}^X = \gamma_{TI} \left(\frac{p^{TI}/p^I}{p^X/p^I} \right) \tilde{i}^T$$

$$\tilde{i}^M = (1-\gamma_{TI}) \left(\frac{p^{TI}/p^I}{p^M/p^I} \right) \tilde{i}^T$$

When replacing equations (A.68)-(A.70) into equation (A.80) (and using the identities of expenditures), one gets an alternative sum for nominal gdp:

$$p^Y gdp = p^X y^X + rer\ p^{Co,*} y^{Co} \frac{a^{Co}}{a} + p^N y^N + p^M y^M - p^M (m^X + m^N) - p_m m$$

which can also be written in terms of prices relative to investment:

$$\frac{p^Y}{p^I} gdp = \frac{p^X}{p^I} y^X + \frac{rer}{p^I} p^{Co,*} y^{Co} \frac{a^{Co}}{a} + \frac{p^N}{p^I} y^N + \frac{p^M}{p^I} y^M - \frac{p^M}{p^I} (m^X + m^N) - \frac{p_m}{p^I} m$$

And using s^{Co}, s^M:

$$\frac{p^Y}{p^I} gdp = \frac{\frac{p^X}{p^I} y^X + \frac{p^N}{p^I} y^N - \frac{p^M}{p^I}(m^X + m^N)}{1 - s^{Co} - s^M \frac{((p^M - p_m)/p^I)}{p_m/p^I}}$$

With this, we can get:

$$y^{Co} = \frac{s^{Co}(p^Y/p^I)gdp}{(rer/p^I)p^{Co,*}} \frac{a}{a^{Co}}$$

$$y^M = \frac{s^M(p^Y/p^I)gdp}{p_m/p^I}$$

$$g = \frac{s^g(p^Y/p^I)gdp}{p^N/p^I}$$

By (A.51):

$$\tilde{f}^M = \frac{\epsilon_M - 1}{\epsilon_M} \frac{y^M}{(1 - \beta a^{1-\sigma}\theta_M)}$$

By (A.32):

$$m = y^M$$

By (A.68):

$$c^N = y^N - g - \tilde{i}^N$$

By (A.70):

$$c^M = y^M - \tilde{i}^M - m^X - m^N$$

By (A.21):

$$c^T = \frac{c^M}{1 - \gamma_T} \left(\frac{p^M/p^I}{p^T/p^I} \right)$$

By (A.20):

$$c^X = \gamma_T \left(\frac{p^T/p^I}{p^X/p^I} \right) c^T$$

By (A.18)-(A.19):

$$\gamma = \frac{(p^N/p^I)^\varrho c^N}{(p^T/p^I)^\varrho c^T + (p^N/p^I)^\varrho c^N}$$

By (A.22)-(A.23):

$$\frac{p^{SAE}}{p^I} = \left[(1-\gamma)\left(\frac{p^T}{p^I}\right)^{1-\varrho} + \gamma \left(\frac{p^N}{p^I}\right)^{1-\varrho} \right]^{\frac{1}{1-\varrho}}$$

$$p^I = \left[\left(\frac{p^{SAE}}{p^I}\right)^{1-\gamma_{AC}-\gamma_{EC}} \left(\frac{p^T}{p^I}\right)^{\gamma_{AC}+\gamma_{EC}} \right]^{-1}$$

Now, we get all prices by multiplying the price relative to investment by p^I: $\{p^X, p^M, p^N, p^T, p^{TI}, p^{SAE}, p^{ME}, rer, w^X, w^N, \tilde{\mu}^X, \tilde{\mu}^N, p_m, \tilde{\xi}^{h,N}\}$

By (A.18):

$$c = \frac{1}{\gamma}(p^N)^\varrho c^N$$

(also check equation $c = c^T(p^T)^\varrho/(1-\gamma)$)

By (A.69):

$$c^{X,*} = y^X - \tilde{i}^X - c^X$$

By (A.61):

$$y^* = \frac{c^{X,*}}{\xi^{X,*}} \left(\frac{p^X}{rer} \right)^{\epsilon^*}$$

By (A.79):

$$gdp = c + g + i + c^{X,*} + y^{Co} \frac{a^{Co}}{a} - m$$

$$p^Y = \frac{p^Y gdp}{gdp}$$

By (A.76):

$$tb = rer\ p^{Co,*} y^{Co} \frac{a^{Co}}{a} + p^X c^{X,*} - p_m m$$

By (A.78):

$$b^* = \frac{tb - (1-\vartheta) rer\ p^{Co,*} y^{Co} \frac{a^{Co}}{a}}{rer \left(1 - \frac{R^*}{\pi^* a}\right)}$$

By (A.64) (part that was assumed zero):

$$\bar{b} = \frac{b^* rer}{p^Y gdp}$$

By (A.1):

$$\tilde{\lambda} = \xi^\beta c^{-\sigma} \left(1 - \frac{\phi_C}{a}\right)^{-\sigma}$$

By (A.14)-(A.15):

$$\tilde{f}^{WX} = \frac{\mu^{WX} \tilde{\lambda} h^{X,d}}{1 - \theta_{WX} a^{1-\sigma} \beta}$$

$$\tilde{f}^{WN} = \frac{\mu^{WN} \tilde{\lambda} h^{N,d}}{1 - \theta_{WN} a^{1-\sigma} \beta}$$

A.4.3 IRFs of selected shocks

Figure 9: IRF after a shock to External Prices

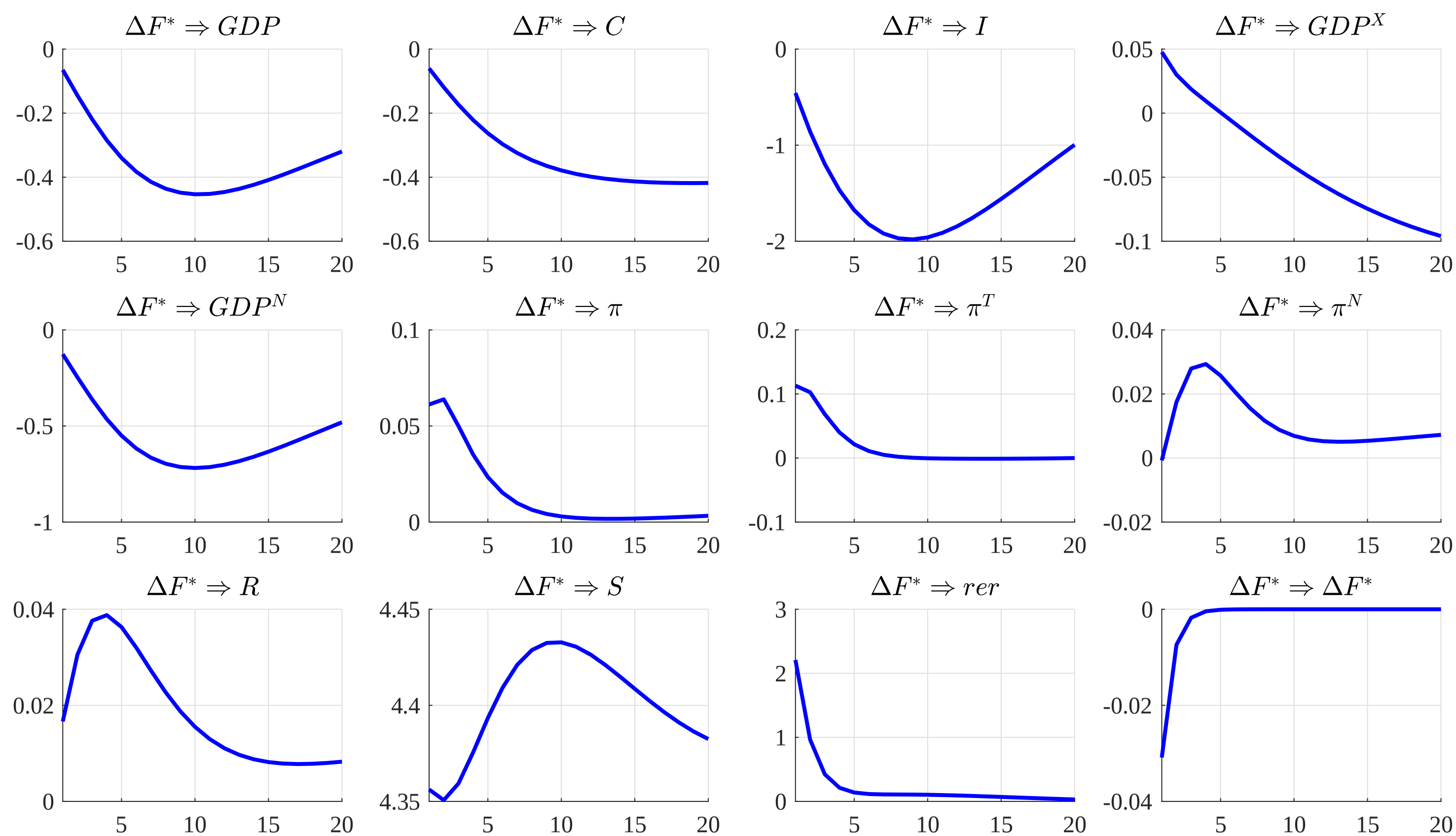

Figure 10: IRF after a shock to UIP