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Carrot *and* Stick: A Role for Benchmark-Adjusted Compensation in Active Fund Management*

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ABSTRACT

Investors delegating their wealth to privately informed managers face not only an intrinsic asymmetric information problem but also a potential misalignment in risk preferences. In this setting, we show that by tying fees symmetrically to the appropriate benchmark investors can tilt a fund portfolio toward their optimal risk exposure and realize nearly all the value of managers' information. They attain these benefits despite an inherent inefficiency in the choice of the benchmark, and at no extra cost of compensating managers for exposure to relative-performance risk. Under certain conditions, benchmark-adjusted performance fees are necessary to prevent passive alternatives from dominating active management. Our results can shed light on a recent debate on the appropriate fee structure of active funds in contexts of high competition from passive funds.

Keywords: Portfolio delegation, benchmarking, fulcrum fees, asymmetric information, passive management.

JEL Classification: D82, G11, G23.

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1 Introduction

In the past decade, trillions of dollars have left active asset management firms, as investors have become drawn to less-expensive passive alternatives like index funds and exchange-traded funds (ETFs). In response to this trend, active managers of prominent fund companies have advocated a radical shift in their funds' fee structure in favor of so-called "fulcrum" performance fees.[1] Like performance fees used by hedge funds and other segments of the industry, fulcrum fees give managers a proportion of the fund's excess return over a prespecified benchmark. Unlike traditional performance fees, fulcrum schemes penalize underperformance with respect to the benchmark by deducting fees *symmetrically* (i.e., "linearly") in proportion to the shortfall. This feature challenges academic views on linear benchmarking practices in asset management as suboptimal. In this paper, we re-examine these practices by assessing the role of benchmark-adjusted compensation in the portfolio delegation decision to *active* versus *passive* funds. Our innovation is to focus on an under-explored tradeoff, intrinsic to this decision, between portfolio efficiency and risk alignment.

The use of benchmarks in practice has puzzled academics over the years.[2] A main practical argument in favor of benchmarking is its ability to address the potential misalignment in risk preferences between the active managers and their clients. However, in an influential article Admati and Pfleiderer (1997) claim that for the purposes of aligning the managers' portfolios with the optimal portfolio of investors or, more generally, for optimal risk sharing between the two parties, linear benchmarking is at best irrelevant, and more likely harmful to fund investors. Their concern has been echoed by subsequent literature, leading to a "benchmark irrelevance" puzzle that has proved hard to tackle. More recent studies have rationalized the use of benchmarks for risk-sharing purposes, mainly through non-linear variants or complex time-varying benchmark portfolios, in

[1] For example, *The Wall Street Journal* (November 5, 2018) reports that "a wave of stock-picking firms are stepping up their fight against cheap exchange-traded and index funds with new offerings that dial back fees if they can't beat the market." The article mentions AllianceBernstein Holding LP, Allianz Global Investors and other managers as offering new funds that charge fulcrum fee arrangements. Similarly, the *Financial Times* (October 26, 2017) quotes Abigail Johnson, chairwoman and chief executive of Fidelity International, as calling for a "fundamental rethink" of the fees charged by asset managers, arguing that fulcrum fees "should be used more widely." This proposal is finding support in other segments of the industry such as active ETFs.

[2] DelGuercio and Tkac (2002) and BIS (2003) describe the use of explicit and implicit benchmarking practices, respectively, in the pension fund industry. Elton et al. (2003) document the use of benchmark-adjusted fees in the U.S mutual fund industry, while Chevalier and Ellison (1997) and, more recently, Spiegel and Zhang (2013), characterize empirically the implicit relation between the benchmark-adjusted performance of mutual funds and their future investor flows. More recently, Pavlova and Sikorskaya (2021) examine the benchmarks specified in the prospectuses of U.S. equity mutual funds and provide estimates of the total assets under management benchmarked against different popular (e.g., Russell) indices.

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contexts of symmetric information between the two parties.[3] The questions of whether benchmarks are optimal under the type of asymmetric information environment more likely to prevail in active asset management, and whether they are implementable through linear schemes, remain open. Using a model of delegated portfolio management, we offer the following answers to these questions: whereas linear benchmarking is in general suboptimal, fulcrum fees associated to simple passive benchmarks improve over linear schemes with no benchmark, achieve near-optimal outcomes, and avoid a dominance of informed active funds by inexpensive passive alternatives.

Extensive evidence suggests that the search for both risk alignment and superior information are core components of investors' portfolio delegation decision.[4] We make this feature a central piece of our analysis. In the model, investors can delegate financial wealth management either to a passive fund (e.g., an ETF) or to an active and privately informed asset manager. The passive fund caters perfectly to the risk preferences of the investors but, by definition, offers no investment skill. The opposite is true with the active fund, which has the potential to outperform the index but also to expose the fund investors to a level of risk different from the desired. The setup approximates a real-life situation in which, absent managers' superior information, there would be no extra benefits from delegation to active funds and, absent differences in risk preferences, active funds run by managers with skill would dominate their passive counterparts.[5]

Following the standard practice in the industry, we assume that passive funds are offered at a fixed fee over assets under management, while active funds can charge additional fees tied to relative performance with respect to a benchmark. We focus on performance fees of the fulcrum type, although we also consider the case of asymmetric (nonlinear) performance fees that reward relative

[3] We expand on these studies in our literature review of Section 2.

[4] Arguably, investors' belief that managers can produce superior returns, or "alpha," is one of the main reasons behind the existence of the active asset management industry. Several authors document that investors across segments of the money management industry chase fund performance (see, e.g., DelGuercio and Tkac, 2002, Sirri and Tufano, 1998 and Getmansky et al., 2015 for evidence in the pension fund, mutual fund and hedge fund segments, respectively), a behavior that Berk and Green (2004) rationalize as resulting from investors' search for managers with superior investment skills. Moreover, recent research backs the belief that at least certain subgroups of fund managers have superior investment skills (e.g., Kosowski et al., 2006), even if the average manager does not (Carhart, 1997). The extensive use of provisions to limit managers' risk-taking behavior such as tracking error and position limits, leverage and short selling constraints, represents direct evidence of the concern of fund investors about inappropriate levels of risk on their portfolios. See, e.g., BIS (2003) and Almazan et al. (2004) for a description of the use of these constraints in practice.

[5] This tension in the relation between portfolio managers and their clients is broadly recognized. The *Financial Times* (26 October, 2017) quotes chief executives of large fund companies as recognizing that "a big part of our ability to add value for investors is the skill of our people," but also that the remuneration packages of their managers, based on total shareholder returns, "is a misalignment of interests and it can create tensions." In the same vein, these managers acknowledge that "one of the advantages of index-tracking strategies is that risks are better controlled and incentives are aligned between the asset manager and the client."

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outperformance without penalizing underperformance. Funds invest in a risky security and a bond over a finite investment period. At the end of this period, the final value of the actively managed portfolio is split between investors and the manager according to the compensation contract.

We first characterize explicitly, for a given contract, the active fund's optimal portfolio and the manager's compensation. Under positive fulcrum fees, the fund portfolio partially mimics the composition of the benchmark to hedge the manager's compensation against relative performance risk. For a given benchmark, this effect increases (respectively, decreases) the allocation to the risky security after low (high) realizations of the manager's signal. Conversely, for a given low (respectively, high) realization of the signal, the tilt toward (away from) the risky security is larger (smaller) for riskier benchmarks. Thus, fulcrum fees affect the portfolio choice, but not the compensation, of the fund manager. This implies that active fund investors can calibrate the fees and benchmark composition to maximize the benefits from delegation without having to compensate the manager for exposure to undesired risk.[6]

Turning to the contract design problem we next show, analytically, the optimality of including a benchmarking component in the linear compensation contract of the manager. In designing this contract, investors seek to take advantage of the superior information of the manager at the lowest possible compensation cost. By the information asymmetry underlying the relationship, the parties cannot contract on the manager's signal. Absent a benchmark, the manager may respond to a given realization of this signal by choosing a smaller (respectively, larger) long or short position in the stock than a relatively risk-tolerant (risk-averse) investor would prefer conditional on the same signal. The inclusion of the benchmark in the compensation contract allows investors to tilt the portfolio allocation of the manager toward their desired positions. Benchmarking comes at a cost, though, as the asymmetry in information implies that investors choose the benchmark conditional on their own, inferior information. Thus, benchmarking distorts the fund investment towards a conditionally inefficient portfolio and hurts (risk-adjusted) performance. Our main analytical result is that investors should be willing to bear the costs associated with this distortion and assign a non-zero weight to the fulcrum performance component whenever a misalignment in risk preferences exists.

Offering quantitative predictions about the optimal fee levels, the benchmark composition and

[6] For simplicity, we interpret the results from the point of view of fund investors that offer the manager (or management company) a fee contract. As we explain in Section 4, an equivalent interpretation holds in the alternative case in which it is fund managers who offers their clients a fee arrangement.

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the benefits of benchmarking requires that we analyze the model numerically. In line with the underlying tradeoff between the risk-alignment and conditional efficiency of the fund portfolio, the optimal fulcrum fee increases with the misalignment in risk preferences between manager and investors but decreases with the information advantage of the manager. The latter feature can help understand, in a context of highly efficient financial markets and widespread information dissemination (i.e., shrinking information advantage by any party), the recent arguments among prominent active managers in favor of fulcrum fees.

More surprisingly, the optimal benchmark has either a higher or a lower risk exposure than the unconditionally efficient portfolio that fund investors would choose under self-management. Intuitively, from the perspective of relatively risk-tolerant investors, the manager underreacts to good news and overreacts to bad news, increasing and decreasing the risky asset allocation of the fund less and more, respectively, than investors would prefer. *Both* under- and overreaction problems are alleviated by including a high-risk benchmark in the managerial contract.[7]

We find that the optimal linear contract allows investors to realize most of the ex-ante value of the manager's private information. Under a linear contract, managers do not have to be compensated for choosing a suboptimal—from their perspective—level of risk because they can perfectly hedge their exposure to the benchmark. Thus, the optimal benchmark allows investors to benefit from the manager's private information at no greater cost than under proportional-only fees. Regardless of the difference in risk aversion between manager and investors, the information advantage of the manager, or the investment horizon, adding a fulcrum fee to the manager's compensation allows investors to enjoy either a substantial fraction or almost all (e.g., 98% or more for some fund investors-manager pairs) of the gains associated with the manager's private information—i.e., the extra risk-adjusted performance that investors could attain if they traded on the manager's signal.

We also provide sufficient conditions under which active funds with purely proportional fees are not only dominated by active counterparts with fulcrum fees but also by passive alternatives. For a given risk-aversion misalignment between manager and investors, these conditions are more likely to be met for strategies or markets in which the manager's information advantage is slimmer or the risky asset's Sharpe ratio is higher, consistent with the value of active strategies falling with these attributes relative to the value offered by passive strategies. In these situations, investors who are restricted to pay the manager no fulcrum fee are better off investing in the passive alternative. This

[7] A similar reasoning explains why relatively risk-averse investors optimally have the manager mimic a low-risk benchmark.

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is especially the case when active management offers little potential for "alpha", a result that helps explain the increasing preference of investors for passive over active mutual funds in the U.S., where purely proportional fees are prevalent.

Lastly, we find that linear contracts dominate nonlinear variants such as the asymmetric benchmark-adjusted performance fees charged by hedge funds. A simple compensation contract consisting of a fee proportional to total assets plus a fulcrum fee attains higher risk-aversion-aligning benefits at lower cost. The symmetric nature of fulcrum fees is consistent with the requirements of the SEC starting in 1971. Our findings contribute to the debate that has emerged since then by alleviating concerns about the welfare losses to investors that the ban on asymmetric performance fees entail.

The rest of the paper is organized as follows. Section 2 summarizes our contributions relative to prior literature. Section 3 introduces the economic environment, the information structure and the class of managerial contracts considered. Section 4 describes the contracting problem and characterizes the manager's optimal investment strategy for an arbitrary contract. We establish the optimality of fulcrum fees, characterize the optimal linear compensation contract and quantify the benefits of benchmarking in Section 5. We discuss extensions and practical implications of our model in Sections 6. Section 7 contains our conclusions.

2 Related Literature

Our paper contributes an analysis of the effect of asymmetric information about asset returns to the literature on portfolio choice and risk sharing under institutional asset management. Under symmetric information between managers and investors, prior studies derive the contract that achieves perfect risk-sharing when the asset manager can exert costly effort to improve the portfolio return (Ou-Yang, 2003; Cadenillas et al., 2007). A different strand of the literature studies the effect of linear and nonlinear benchmarking on the risk-taking of the money manager (Starks, 1987; Basak et al., 2008; Chen and Pennacchi, 2009; Koijen, 2014), on equilibrium prices (Basak and Pavlova, 2013; Buffa and Hodor, 2018), on information acquisition and market efficiency (Palomino, 2005; Breugem and Buss, 2019; Sockin and Xiaolan, 2019), and on firm's corporate decisions (Kashyap et al., 2018). In the context of a centralized decision maker who hires multiple asset managers and whose compensation depends only on relative performance, van Binsbergen et al. (2008) derive an optimal unconditional linear performance benchmark that aligns risk preferences across managers.

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Our results show that a linear compensation contract that includes both absolute and relative performance components, similar to those used in practice, can nearly eliminate the risk-sharing distortions introduced by the *asymmetric* information about asset returns between fund managers and their investors.

Our work is also related to the recent literature on the optimal compensation of asset managers that accounts for general-equilibrium implications on asset prices. When fund investors delegate the equity portion of their portfolios, have the same information as the manager, and take the contract parameters as given, Cuoco and Kaniel (2011) find that asymmetric performance fee contracts can dominate both fulcrum and pure-proportional fee contracts even when risk preferences are aligned. By contrast, we find that when these conditions are not met fulcrum fees arise as the dominant arrangement. Buffa et al. (2019) study the impact of managers' private benefits on the optimal managerial contract in a symmetric-information setup with CARA preferences. They show that including compensation for relative performance with respect to an exogenous benchmark in the optimal contract improves risk sharing between fund managers and investors, while rewarding absolute performance addresses the agency friction created by private benefits. Cvitanić and Xing (2018) extend Buffa et al. (2019) by generalizing the contract and allowing the manager to invest privately in individual risky assets or an index. They find that the optimal contract involves rewarding the manager for returns in excess of a benchmark, and for the deviation's quadratic variation. We demonstrate that risk sharing is at the core of the optimality of linear benchmarking under the alternative agency friction created by the private information of active managers. Complementing these papers, we show how the endogenously designed benchmark portfolio can differ from the market portfolio or other unconditionally efficient passive alternatives.[8]

Lastly, our paper contributes to the broader literature on optimal compensation in the context of delegated portfolio management. Admati and Pfleiderer (1997) argue that linear benchmarking is irrelevant not only for portfolio-alignment or risk-sharing purposes but also for inducing the manager to exert more effort or for screening out bad managers. In a model featuring CARA preferences, Li and Tiwari (2009) devise a nonlinear benchmarking contract that overcomes effort inducement problems. With the precedent of Gómez and Sharma (2006), who showed that benchmarks become relevant in the presence of short-selling constraints, Dybvig et al. (2010) deviate from the previous

[8] In this sense, our paper is also related to Huddart (1999). Using a model of portfolio misalignment between investors and managers due to reputation concerns rather than differences in risk aversion, this author rationalizes linear performance fees relative to an exogenous benchmark as a tool to align the portfolio choices of privately informed managers and their fund investors.

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work by considering logarithmic utility and making the investment opportunity set of the money manager a central piece of their analysis. They show that the optimal contract to address effort inducement problems when the manager's private information is non-contractible must include both nonlinear benchmark-adjusted fees and portfolio constraints. Similarly, He and Xiong (2013) derive the optimality of investment mandates and tracking error constraints in an institutional asset management industry with agency costs, hidden information, and the availability of negatively skewed bets. Accounting for the value of benchmark-adjusted compensation as a signaling device for superior managers, Das and Sundaram (2002) identify equilibria under which option-like performance fees improve investor welfare relative to fulcrum fees. We demonstrate the effectiveness of *linear* benchmarking, before accounting for investment constraints, to improve risk-sharing and to align the portfolio allocation of the manager with the optimal portfolio of investors.

3 Model Setup

We consider an economy in which investors, henceforth also referred to as households, delegate their financial wealth w to an investment company (e.g., a mutual fund) over a certain investment horizon denoted by $[0, T]$. The investment company manages investors' portfolio by allocating their wealth among the available financial assets. During the investment period, no additional funds share purchases or redemptions take place. In return for its services, at $t = T$ households pay the investment company a monetary sum according to a prespecified (at $t = 0$) fee contract. We introduce this contract in Section 3.2.

The rest of the paper focuses on the relationship between the fund manager in one such company and the fund investors.[9] For most of our analysis, we assume that both the fund manager and the households have constant relative risk aversion (CRRA) preferences, with coefficients of intrinsic

[9] Although in practice the manager of a fund advisory company need not be the same person (or team) that manages the fund portfolio, we do not make that distinction and assume that the fund company's compensation accrues entirely to the portfolio manager. See Ibert et al. (2018) for a detailed analysis of the compensation of mutual fund managers. Similarly, we abstract away from the potential for agency conflicts in the relation between the fund advisory company and its portfolio manager, as well as the design of the optimal contract between these two parties. In practice, it is often the case that the compensation of the portfolio manager itself depends on her portfolio's benchmark-adjusted performance; see Farnsworth and Taylor (2006) and Ma et al. (2019).

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relative risk aversion $\gamma, \gamma_h > 1$, respectively:[10]

$$u(w) = \begin{cases} \frac{w^{1-\gamma}}{1-\gamma} & \text{if } w > 0 \\ -\infty & \text{if } w \leq 0 \end{cases}, \quad u_h(w) = \begin{cases} \frac{w^{1-\gamma_h}}{1-\gamma_h} & \text{if } w > 0 \\ -\infty & \text{if } w \leq 0 \end{cases}. \tag{1}$$

The power utility assumption (1) departs from the standard literature on delegated portfolio management (e.g., Admati and Pfleiderer, 1997), which for tractability assumes either constant absolute risk aversion (CARA) or log utility for both agents. At least two reasons justify this departure. First, power utility preferences allow simultaneously for differences in risk aversion and realistic portfolio behavior (e.g., wealth effects on the demand for assets), two key features to assess the portfolio-alignment and risk-sharing roles of linear benchmarking. Second, power utility is the workhorse in the general class of asset management and asset pricing problems to which a portfolio delegation decision belongs.[11] Notwithstanding these reasons, in sections 4 and 5 we discuss the implications of assuming either CARA or log preferences for our analysis.

3.1 Financial Markets, Information Structure and Fund Companies

Financial markets consist of one riskfree asset and one risky asset, with prices β and S, respectively. The riskless asset can be a short-term bond or a bank account. The risky asset can be a stock or any portfolio of risky assets (such as the market portfolio or other traded benchmark). All agents are atomistic participants in the asset markets, so they take asset price dynamics as exogenously given. The bond has initial price $\beta_0 = 1$ and pays a constant interest rate r per unit of time. The bond's price dynamics are given by $d\beta_t = r\beta_t dt$.

We let the mean rate of return on the stock, μ, be the realization at $t = 0$ of a random variable $\tilde{\mu}$. We assume that $\tilde{\mu}$ is normally distributed, $\tilde{\mu} \sim \mathbf{N}(r + \sigma\bar{m}, \sigma^2\bar{v}_0)$, for given constants $\bar{m}$ and $\bar{v}_0 \geq 0$, and given stock volatility σ. Equivalently, the market price of risk $\tilde{\eta} \equiv (\tilde{\mu} - r)/\sigma$ follows a normal distribution $\mathbf{N}(\bar{m}, \bar{v}_0)$ with realized value $\eta \equiv (\mu - r)/\sigma$ at $t = 0$. After η is realized, the stock price dynamics are:

$$dS_t^\eta = S_t^\eta(r + \sigma\eta)dt + S_t^\eta \sigma dB_t, \tag{2}$$

where B is a standard Brownian motion process, independent of $\tilde{\eta}$, defined on a filtered probability

[10] We refer to these coefficients as capturing "intrinsic" relative risk aversion to distinguish them from the *effective* relative risk aversion of each party at their respective after-fee wealth and compensation levels, which in general depends on the managerial compensation contract introduced in Section 3.2.

[11] See, e.g., Campbell (2018)

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space $(\Omega, \mathcal{F}, P, \{\mathcal{F}\}_{t \leq T})$ over the interval $0 \leq t \leq T$, and the superscript η makes explicit the dependency of the stock price path on the realized value η of the market price of risk. We normalize the stock's initial price S_0^η to s for all realizations of $\tilde{\eta}$.

Arguably, the belief that fund managers have superior information or investment skills is a key reason behind the existence of active money management. To account for this possibility, we assume that at $t = 0$ (i.e., before the investment period starts) fund managers have access to η as a private signal. Since the signal allows managers to choose the portfolio that best suits the current market conditions (with low and high realizations of η reflecting, respectively, "bear" and "bull" markets), the manager's private information represents a source of value from delegation to the fund's investors. In Section 6.1 we relax the assumption that the managers' signal is perfectly correlated with η to allow for noisy signals, which reduce but do not eliminate their information advantage relative to households.

Households can delegate their financial wealth either to an *actively* managed fund, or to a *passively* managed fund (e.g., an index fund or an ETF). In case they choose the active fund, households offer the manager the performance fee contract that we introduce in the next subsection. Given this contract, the manager chooses the fund portfolio throughout the investment period to maximize utility of end-of-period compensation.

The passive fund allocates a fixed fraction $\phi^P \in \mathbb{R}$ of the fund's value in the stock and the remaining fraction $1 - \phi^P$ in the riskfree asset, with ϕ^P chosen at the start of the investment period to maximize households' utility of end-of-period wealth. The passive fund charges a fixed fee $k_m > 0$ proportional to total assets under management (AUM),[12] and its manager keeps the commitment to invest in the agreed portfolio ϕ^P throughout the investment period.

Remark 1 (Multiple Risky Assets and Investment Opportunity Set): The assumption of a single risky asset is standard in the literature (e.g., Dybvig et al., 2010). We adopt it for ease of interpretation and comparison with this literature. Notwithstanding, our main results generalize to the existence of multiple risky asset. In particular, the proof of Theorem 1 remains valid if, in addition to the capital allocation between the riskfree asset and the risky portfolio, households are allowed to decide the relative weights $(\phi_1^Y, \cdots, \phi_N^Y)$ of the $N > 1$ risky assets in the risky portfolio ϕ^Y (see

[12] This is the standard fee arrangement in the passive fund and ETF industries. Although we take this compensation structure as given, it is easy to see from our results in Section 5 that it is optimal in the context we study. This is because, by construction, the passive investment entails perfect alignment with the fund investors' risk preferences, in which case a pure-proportional fee schedule is optimal (Theorem 1).

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Appendix A). Likewise, our assumption that households do not diversify their portfolio between the active and passive funds is inessential. In particular, we argue in Section 6.2 that allowing for this possibility cannot fully substitute for the role of benchmarks in aligning risk preferences.

3.2 Managerial Contract

The active manager's compensation is determined by a *linear* fee contract.[13] Specifically, for given constants $k > 0$, $\theta \in [0, 1]$, and α, households pay the manager a fee rate

$$f(R_T^W, R_T^Y; k, \theta, \alpha) = (1-\theta)k\beta_T + \theta k R_T^W + \alpha(R_T^W - R_T^Y) \tag{3}$$

per initial dollar of AUM W_0. $R_T^W \equiv W_T/W_0$ and $R_T^Y \equiv Y_T/Y_0$ denote, respectively, the fund's absolute performance (where the dynamics of total AUM are as specified by Eq. (8) below) and relative performance with respect to a benchmark Y, during the investment period $[0, T]$. We refer to $(1-\theta)k$ as a *load* fee rate, k as a total asset-based or *proportional* fee rate, and to α as a relative or *fulcrum* performance fee rate. We assume that $\theta k + \alpha < 1$, meaning that the manager's share of the funds' returns must be less than one hundred per cent.

The benchmark is the value of a portfolio with constant weight ϕ^Y in the risky asset and the remaining $1-\phi^Y$ in the riskfree asset. In principle, investors can choose ϕ^Y to differ from the passive fund portfolio ϕ^P. Since its portfolio composition is fixed over the investment period, the benchmark approximates a *passive* index, as is typically the case in practice (see references in Section 1). For a realization η, the benchmark value process satisfies the self-financing dynamics:

$$dY_t^{\eta;\phi^Y} = Y_t^{\eta;\phi^Y}\left(r + \phi^Y\sigma\eta\right)dt + Y_t^{\eta;\phi^Y}\phi^Y\sigma dB_t, \tag{4}$$

where without loss of generality we normalize the benchmark's initial value $Y_0^{\eta;\phi^Y} = y$ to equal w.

Given the fee contract (3), the manager's compensation is:

$$\begin{aligned} X_T \equiv f_T W_0 &= (1-\theta)kw\beta_T + \theta k W_T + \alpha(W_T - Y_T) \\ &= f\left(W_T, Y_T; k, \theta, \alpha\right). \end{aligned} \tag{5}$$

A management fee contract $\mathscr{C} \equiv (k, \theta, \alpha, \phi^Y)$ specifies the load, proportional, and symmetric

[13] Strictly, the contract functional form is *affine*. For consistency with prior literature (e.g., Ou-Yang, 2003), however, we refer to it as a linear contract throughout the paper.

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benchmark-adjusted (fulcrum) fees as given by k, θ, α, as well as the benchmark composition ϕ^Y. Since investors do not observe the realization of the manager's signal, the contract parameters are restricted to be invariant to realizations of $\tilde{\eta}$.

Our assumption that managerial compensation is a linear function of the performances of the fund and the benchmark serves two purposes. First, it allows us to assess whether including a benchmarking component in the class of contracts considered by prior literature (e.g., Admati and Pfleiderer, 1997) is optimal in our setup and, if so, to quantify the derived gains relative to the optimal linear contract that excludes it. Second, it is general enough to encompass the fee structures observed on different type of investment companies (e.g., mutual funds and pension funds). In Section 6, we extend our analysis to contracts that include *asymmetric* (nonlinear) benchmark-adjusted performance fees. This type of contracts includes, as a special case, the "two-and-twenty" fee schedule (2% of total AUM plus 20% of profits in excess of a hurdle) prevalent in the U.S. hedge fund industry.

4 Contracting Problem

The households' problem consists in designing the contract $\mathscr{C}$ that induces the manager to use her private information in their best interest at the lowest possible cost. To this aim, they choose the fee parameters, within the class of contracts (5), under which the manager optimally implements the investment policy that maximizes households' expected utility from delegation, the manager agrees to actively manage the fund, and households agree to delegate their wealth to the active fund. We assume that both parties agree on the contract before $\tilde{\eta}$ is realized and that neither party can renege on the contracts afterwards. Households' contracting problem is then:

$$\max_{\{k,\theta,\alpha,\phi^Y\}} E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\mathscr{C}} - X_T^{\tilde{\eta};\mathscr{C}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] \tag{6}$$

$$s.t. = \begin{cases} \forall\eta: \quad \hat{W}_T^{\eta;\mathscr{C}} = \arg\max_{W_T} E\left[\frac{f\left(W_T, Y_T^{\tilde{\eta};\phi^Y}; k,\theta,\alpha\right)^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right], & \text{(M's ICC)} \\ E\left[\frac{(X_T^{\tilde{\eta};\mathscr{C}})^{1-\gamma}}{1-\gamma}\right] = c\bar{U}, & \text{(M's PC)} \\ E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};\mathscr{C}} - X_T^{\tilde{\eta};\mathscr{C}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] \geq U_{h,P}, & \text{(HH's PC)} \end{cases}$$

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where expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$, and $0 < c \leq 1$ is a constant summarizing the relative bargaining power of households.[14]

Condition (M's ICC) reflects the manager's incentive compatibility constraint: given a contract $\mathscr{C} = (k, \theta, \alpha, \phi^Y)$, the terminal fund value is the one that maximizes the manager's expected utility for each realization of $\tilde{\eta}$. To preserve the asymmetry in information about asset returns between the manager and households we assume that, up until $t = T$, households do not observe the manager's portfolio choice and the fund value (relative to the benchmark) simultaneously.[15]

Conditions (M's PC) and (HH's PC) reflect, respectively, the manager's and households' participation constraints given reservation utilities of $\bar{U}$ and $U_{h,P}$. The utility that a manager obtains from actively managing a fund can be no lower than the utility $\bar{U}$ she derives from offering a passively managed fund. An informed manager, in particular, can require a strictly greater utility than $\bar{U}$ to engage in active management (e.g., to cover the costs of active research), a possibility for which we account by allowing c to be strictly less than 1. Likewise, the utility that households expect to obtain from delegating their money to an active fund can be no lower than the expected utility $U_{h,P}$ they derive under passive management.

Problem (6) defines a contracting model similar to the ones considered by Ou-Yang (2003), Cadenillas et al. (2007), and Cuoco and Kaniel (2011), among others. Like in these studies, a solution to (6) aims for the best achievable risk sharing between households and the manager when the parties face a potential misalignment in risk preferences but no effort inducement or adverse selection problems. Our innovation is to introduce information asymmetry about asset returns between the parties, which allows us to examine the role of superior investment ability and risk misalignment, intrinsic to the choice between active and passive management, in explaining recent trends in asset management (see Section 1).

To tackle problem (6) we proceed in several steps. For any given fee contract $\mathscr{C}$ and realized market price of risk η, we first characterize the solution $\hat{W}_T^{\eta;\mathscr{C}}$ to the manager's optimization problem that determines her incentive compatibility constraint (Section 4.1). We next solve for households'

[14] Because for $\gamma_h > 1$ the left- and right-hand sides of (M's PC) are negative, this specification is equivalent to the more standard formulation of the participation constraint requiring that the left-hand side is greater than or equal to the right-hand side.

[15] If households observed both the manager's portfolio choice $\hat{\phi}_t$ and the fund's relative performance at each point in time they could infer the value η from the expression of the optimal portfolio (9) below. Given that professional asset managers disclose their portfolio holdings at a much lower frequency (e.g., only every three months in the case of mutual funds, with lower frequency for other asset management firms like hedge funds) than asset return data is available, the assumption seems realistic.

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optimal passive portfolio and characterize the reservation utilities $\bar{U}$ and $U_{h,P}$ underlying the participation constraints of households and the manager (Section 4.2). We complete the characterization of the optimal contract in Section 5.

Remark 2 (Contract Interpretation): For most of our analysis below we follow the standard convention in the literature (e.g., Dybvig et al., 2010) of referring to the contract as "offered by households to the manager," but our results remain the same under the alternative interpretation in which it is the manager who offers the contract to households. Indeed, it is straightforward to verify that a solution $(\hat{k}, \hat{\theta}, \hat{\alpha}, \hat{\phi}^Y)$ to the contracting problem (6) is also a solution to the problem of maximizing the expected utility of the manager subject to the same constraints. Thus, the total surplus generated is the same under the two formulations and how this surplus is split between the contracting parties depends on c and uniquely determines the optimal proportional fee $\hat{k}$ (see Section 5.2).

Remark 3 (Static versus Dynamic Features): Contracting problem (6) is essentially *static*, in the sense that the compensation contract $\mathscr{C}$ is decided at $t = 0$ and unchanged throughout the investment period. Our assumption of *dynamic* trading in the financial markets follows from its analytical tractability under CRRA preferences and log-normally distributed returns. We believe the assumption is particularly realistic in the context of *active* fund management.

4.1 Incentive Compatibility and Optimal Active Fund Portfolio

After observing the realization of the market price of risk η at $t = 0$, and for a given fee contract $\mathscr{C}$, the manager chooses a dynamic investment policy ϕ_t^η representing the fraction of the active fund's wealth W_t^η allocated in the risky asset over the investment period $t \in [0, T]$. The investment policy seeks to maximize her expected utility conditional on η over end-of-period compensation $X_T^{\eta;\mathscr{C}}$:

$$E\left[\left.\frac{(X_T^{\tilde{\eta};\mathscr{C}})^{1-\gamma}}{1-\gamma}\right|\tilde{\eta} = \eta\right], \tag{7}$$

subject to the self-financing constraint:

$$dW_t^\eta = W_t^\eta\,(r + \phi_t\sigma\eta)\,dt + W_t^\eta\phi_t\sigma dB_t, \tag{8}$$

and initial wealth $W_0^\eta = w$.

Financial markets are complete in our setup (a single risky asset driven by a single Brownian

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motion). Absent arbitrage opportunities, and given the realization η of $\tilde{\eta}$, the manager sees financial markets as driven by a unique state-price deflator $\pi_t^\eta = \exp\{-(r+\eta^2/2)t - \eta B_t\}$. For a given compensation contract $\mathscr{C}$, the active fund's value $\hat{W}^\eta$ and investment strategy $\hat{\phi}^\eta$ are characterized explicitly in the following:

Proposition 1. *Given contract $\mathscr{C} \equiv (k, \theta, \alpha, \phi^Y)$ and realization η of the market price of risk, the active fund's optimal risk exposure at time $t \in [0, T]$ is given by:*

$$\hat{\phi}_t^{\eta;\mathscr{C}} = \frac{\eta}{\gamma\sigma} + \frac{(1-\theta)k\beta_t}{(\theta k + \alpha)R_t^{\hat{W}}}\frac{\eta}{\gamma\sigma} + \frac{\alpha}{\theta k + \alpha}\frac{R_t^Y}{R_t^{\hat{W}}}\left(\phi^Y - \frac{\eta}{\gamma\sigma}\right), \tag{9}$$

where $R_t^{\hat{W}} \equiv \hat{W}_t/W_0$ and $R_t^Y \equiv Y_t/Y_0$.

The time-t ($t \in [0, T]$) optimal fund value and the manager's end-period compensation are given by:

$$\hat{W}_t^{\eta;\mathscr{C}} = \frac{kw}{\theta k + \alpha} g(t;\eta,\gamma)(\pi_t^\eta)^{-\frac{1}{\gamma}} + \frac{\alpha}{\theta k + \alpha} Y_t^{\eta;\phi^Y} - \frac{(1-\theta)kw\beta_t}{\theta k + \alpha}. \tag{10}$$

$$\hat{X}_T^{\eta;k} = kwg(T;\eta,\gamma)(\pi_T^\eta)^{-\frac{1}{\gamma}}, \tag{11}$$

where $g(t;a,b) \equiv \exp((1-1/b)(r + a^2/(2b))t)$.

The manager's investment policy combines a standard mean-variance portfolio $\eta/(\gamma\sigma)$ with an additional component. The mean-variance portfolio reflects the manager's optimal risk exposure absent load fees and benchmarking ($1-\theta = \alpha = 0$). It implies that, if trading for her own account, the manager would choose a constant allocation to the stock over the entire investment period (i.e., the Merton, 1971 portfolio), which reflects *both* her superior information (η) and her own relative risk aversion (γ).

The difference between the manager's optimal policy and the mean-variance portfolio has the usual interpretation as a *hedging* demand. Part of this hedging demand is driven by the effectively lower managerial risk aversion that load fees ($\theta < 1$) induce as a result of guaranteeing a positive minimum compensation, and implies an increase in the fund position in the stock by a factor $(1-\theta)k\beta_t/((\theta k + \alpha)R_t^{\hat{W}}) > 0$. The rest hedges the exposure of the manager's compensation to benchmark risk under positive fulcrum fees ($\alpha > 0$) by partially mimicking the benchmark. Depending on whether the stock allocation of the mean-variance portfolio is greater or less than the allocation ϕ^Y of the benchmark, this demand will either decrease or increase the portfolio weight in the stock relative to the mean-variance portfolio. Each of these hedging components vary inversely with, respectively, the fund's absolute and relative performances $R_t^{\hat{W}}$ and $R_t^{\hat{W}}/R_t^Y$.

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Eq. (11) shows that the optimal policy fully hedges the manager's exposure to the benchmark. By levering up or down the stock in the portfolio, the manager's trading policy can undo any unwanted asset exposure induced by the load-to-proportional fee parameter θ, the fulcrum fee α, or the composition of the benchmark ϕ^Y. The result follows from the symmetric nature of linear benchmarking, and is standard in prior studies (e.g., Stoughton, 1993). In our setup, it implies that households can calibrate these fees and the benchmark composition to maximize utility of end-of-period after-fee wealth $W_T^{\eta;\mathscr{C}} - X_T^{\eta;k}$ without having to compensate the manager for inducing an undesired level of risk in her portfolio. This feature reduces the compensation cost of linear benchmarking relative to more complex, nonlinear contracts that expose the manager to either excessive or insufficient risk.

In the absence of load fees ($\theta = 1$) it is easy to see (e.g., by expressing α as a multiple of k in Eq. (9)) that the proportional fee θk does not affect the optimal portfolio composition $\hat{\phi}_t$. Unlike in a CARA-normal setup, fees tied to total fund returns play no role in potentially distorting the manager's portfolio in the direction of either increasing or decreasing the allocation to the stock. Eq. (9) shows that, in the current setup, the fulcrum fee α and the benchmark's stock allocation ϕ^Y subsume this role. In particular, a higher value of α increases the manager's concern relative to the benchmark, while a higher value of ϕ^Y increases the fraction of the manager's portfolio invested in the stock.

4.2 Reservation Utilities

Instead of delegating their money to the active manager, households can opt to invest in a passively managed fund, or ETF. In this case, households select the ETF with stock allocation $\phi^P \in \mathbb{R}$ that maximizes expected utility of their after-fees wealth, $(1-k_m)W_{P,T}$, where $W_{P,T}$ is the terminal value (total AUM) of the ϕ^P-ETF. In reality, different choices of ϕ^P could correspond, for instance, to different passive investment styles (e.g., "balanced portfolio") available in the market. Importantly, households do not observe the signal $\tilde{\eta}$ when deciding the composition of the ETF portfolio, so the portfolio is *inefficient* conditional on the realization of the signal.

As is standard in the literature, we assume that the manager is endowed with no initial wealth. However, she can set up and offer a passively managed fund at no cost. In particular, she can offer the ϕ^P-ETF that households would select if they decided to delegate their wealth to a passive fund. If so, the manager collects fees $k_m W_{P,T}$ at the end of the period. The expected utility over these

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fees determines her reservation utility $\bar{U}$.[16]

The following result characterizes the ETF optimal composition ϕ^P, households' derived utility from delegation to the passive fund, $U_{h,P}$, and the manager's reservation utility $\bar{U}$:

Proposition 2. *When households delegate their wealth to a passively managed fund, they choose an ETF with fixed weight ϕ^P in the stock as given by:*

$$\phi^P = \frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T} \frac{\bar{m}}{\sigma} \tag{12}$$

The ex-ante utility of households under passive portfolio management (households' reservation utility) is:

$$\begin{aligned} U_{h,P}(w) \equiv E\left[\frac{(W_{P,T})^{1-\gamma_h}}{1-\gamma_h}\right] =& \frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h} \\ & \times \exp\left\{-(\gamma_h - 1)\left(r + \frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T}\frac{\bar{m}^2}{2}\right)T\right\}, \end{aligned} \tag{13}$$

whereas the ex-ante utility of the manager (the manager's reservation utility) is:

$$\begin{aligned} \bar{U}(w) \equiv E\left[\frac{(W_{P,T})^{1-\gamma}}{1-\gamma}\right] =& \frac{(k_m w)^{1-\gamma}}{1-\gamma} \\ & \times \exp\left\{-(\gamma - 1)\left(r + \frac{2\gamma_h - \gamma + (2\gamma_h - \gamma - 1)\bar{v}_0 T}{(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)^2}\frac{\bar{m}^2}{2}\right)T\right\}. \end{aligned} \tag{14}$$

According to Proposition 2, the reservation utility of households depends positively on the expected market conditions $\bar{m}$ and negatively on the level of uncertainty $\bar{v}_0$.[17] On the one hand, the better the expected market conditions the higher the ex ante utility that households attain by investing in a passive portfolio that contains some stock allocation, even if this allocation is (conditionally) inefficient. On the other hand, the greater the prior uncertainty the lower the expected utility that households attain by dispensing with the information of the manager, and the higher the opportunity cost to an informed manager of offering a passively managed rather than an actively managed fund—as the information she forgoes becomes more valuable. As a result, the reservation utilities of both households and the manager fall, enlarging the set of fee contracts that satisfy the participation constraints and make active fund management optimal.

[16] To examine other, exogenously specified reservation utility levels, we consider values of $c < 1$ in our analysis.

[17] Note that $U_{h,P}$ is negative for our assumption that $\gamma_h > 1$, and a smaller argument of the exponential function on the RHS of (13) increases households' reservation utility.

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5 The Optimal Linear Contract

5.1 Value of the Manager's Information to Households

Before establishing the optimality of benchmarking in the contracting problem (6) (Section 5.2) and examining the optimal contract parameters (Section 5.3), we characterize the ex-ante value of the manager's information to households. To this end, we solve for the benchmark case in which, conditional on receiving (state-dependent) payment $k_{CI}(\eta)w$ at $t = 0$, the manager reveals the realized value η of her signal $\tilde{\eta}$ to households. In this *contractible information* (CI) scenario, *both* households and the manager have symmetric information about the stock fundamentals over the investment period $[0, T]$. Conditional on η, the portfolio that maximizes households' utility solves the following problem:

$$\max_{\{\phi_t\}_{t\in[0,T]}} E_0 \left[\frac{(W_T^{\tilde{\eta}})^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\eta} = \eta \right], \tag{15}$$

subject to the self-financing constraint (8) and initial wealth $W_0^\eta = (1 - k_{CI}(\eta))w$. The solution to this problem is the standard Merton (1971) portfolio:

$$\phi_{CI}^\eta = \frac{\eta}{\gamma_h \sigma}, \tag{16}$$

with associated wealth process, for $t \in [0, T]$, equal to:

$$W_{CI,t}^\eta = (1 - k_{CI}(\eta))wg(t; \eta, \gamma_h)(\pi_t^\eta)^{-\frac{1}{\gamma_h}}, \tag{17}$$

for $g(\cdot)$ as defined in Proposition 1. For each realization of $\tilde{\eta}$ at $t = 0$, the CI portfolio (16) is constant over time. Thus, households can implement the CI investment policy by choosing, after observing η, the ETF that allocates the fraction $\phi^P = \phi_{CI}^\eta$ to the stock. For simplicity, we assume that households can invest in ETFs at no cost in this idealized case. Their end-of-period wealth is $W_{CI,T}^\eta$, for W_{CI}^η as characterized by (17) at $t = T$.

In turn, the manager collects the amount $k_{CI}(\eta)w$ at $t = 0$ from households and invests this amount over the investment period $[0, T]$ according to her own preferences. Her end-of-period wealth is then $\hat{X}_T^{\eta;k}$, as characterized by (11), for $k = k_{CI}(\eta)$. Setting $k_{CI}(\eta)$ to equate the manager's utility to $c\bar{U}$ *state-by-state*, we obtain the following:

Proposition 3. *In the contractible-information scenario, households can acquire the manager's*

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private signal at cost

$$k_{CI}(\eta)w = \exp\left\{-\frac{1}{2\gamma}\left(\eta - \frac{\gamma\bar{m}}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T}\right)^2 T\right\}\frac{k_m w}{c^{\frac{1}{\gamma-1}}}. \tag{18}$$

By acquiring the manager's signal and allocating their remaining wealth in a ϕ^{η}_{CI}-ETF at zero cost, households attain an ex-ante utility of

$$U_{h,CI}(w) \equiv E\left[\frac{(W^{\tilde{\eta}}_{CI,T})^{1-\gamma_h}}{1-\gamma_h}\right] = \frac{w^{1-\gamma_h}}{1-\gamma_h}\int_{-\infty}^{+\infty}(1-k_{CI}(x))^{1-\gamma_h} e^{-(\gamma_h-1)\left(r+\frac{x^2}{2\gamma_h}\right)T}\mathbf{n}_\eta(x)dx \tag{19}$$

where $\mathbf{n}_\eta(\cdot)$ is the (normal) probability density function of $\tilde{\eta}$, while the manager attains ex-ante utility of

$$U_{CI}(w) \equiv E\left[\frac{\left(\hat{X}^{\eta;k_{CI}(\tilde{\eta})}_T\right)^{1-\gamma}}{1-\gamma}\right] = c\bar{U}(w), \tag{20}$$

where expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$.

Households certainty equivalent of (19) for $c = 1$ (i.e., the manager receives exactly her reservation utility) in excess of the certainty equivalent of their reservation utility $U_{h,P}$ gives the *maximum ex-ante value that households can extract from the manager's information.*

5.2 Optimality of Benchmark-Adjusted Fees in a Linear Contract

When the manager's signal (the realization of $\tilde{\eta}$) is not observable, it cannot be verified and no contract can be written on it.[18] In this scenario, households choose the contract parameters $\mathscr{C}$ to solve problem (6). We state our main result, namely the optimality of linear benchmarking in the active fund's fee structure, in the following:

Theorem 1. *Let*

$$k^* = \frac{k_m}{c^{\frac{1}{\gamma-1}}\sqrt{\left(1+\left(1-\frac{1}{\gamma}\right)\bar{v}_0 T\right)^{\frac{1}{\gamma-1}}}} \times \exp\left\{\left(\frac{2\gamma_h - \gamma + (2\gamma_h - \gamma - 1)\bar{v}_0 T}{(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)^2} - \frac{1}{\gamma + (\gamma - 1)\bar{v}_0 T}\right)\frac{\bar{m}^2}{2}T\right\}, \tag{21}$$

[18] Households cannot rely on the manager truthfully revealing her signal either. Indeed, her expected utility is increasing in $k_{CI}(\eta)$, so the manager finds it optimal to report $\hat{\eta} = \gamma\bar{m}/(\gamma_h + (\gamma_h - 1)\bar{v}_0 T)$ across all realizations of $\tilde{\eta}$.

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and assume a solution (a^*, b^*) *exists to the following system of equations in* (a, b)*:*

$$\begin{cases} E\left[\left(\hat{W}_T^{\tilde{\eta};(k^*,1,a,b)} - \hat{X}_T^{\tilde{\eta};k^*}\right)^{-\gamma_h} \left(Y_T^{\tilde{\eta};b} - wg(t;\tilde{\eta},\gamma)(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] & = 0 \\ E\left[\left(\hat{W}_T^{\tilde{\eta};(k^*,1,a,b)} - \hat{X}_T^{\tilde{\eta};k^*}\right)^{-\gamma_h} aY_T^{\tilde{\eta};b}\big(\sigma(\tilde{\eta} - b\sigma)T + \sigma B_T\big)\right] & = 0 \end{cases}, \tag{22}$$

where expectations are taken with respect to the joint distribution of $(\tilde{\eta}, B_T)$*, and* $g(\cdot)$ *is as in Proposition 1. Then the contract* $(\hat{k}, \hat{\theta}, \hat{\alpha}, \hat{\phi}^Y) = (k^*, 1, a^*, b^*)$ *solves the linear contracting problem (6). In particular, when the intrinsic relative risk aversion of households and the manager are*

(i) *identical* $(\gamma_h = \gamma)$*, a pure-proportional fee contract* $(a^* = 0, b^* \in \mathbb{R})$ *is optimal;*

(ii) *different* $(\gamma_h \neq \gamma)$*, a fulcrum performance fee contract* $(a^* \neq 0)$ *is optimal.*

The optimal linear contract has no load fees $(1-\hat{\theta} = 0)$. For any positive load fee, and regardless of the other contract parameters, households cannot rule out states of the world in which their end-of-period after-fee wealth $W_T - X_T$ is negative. Because they derive negative infinite utility in these states, the states must have zero probability of occurrence under the optimal contract. With no load fees, we argued in Section 4.1 that the proportional fee θk does not affect the optimal portfolio composition $\hat{\phi}_t$ but only how total wealth is split between households and the manager. Since households' share falls with k, the optimal proportional fee $\hat{k}$ decreases with their bargaining power c. Keeping c constant, $\hat{k}$ also decreases with the manager's utility gains from active relative to passive management, as measured by the reciprocal of the exponential function in (21). Importantly, these utility gains do not change with the fulcrum fee or the benchmark portfolio composition but only with the difference in relative risk aversion between the parties, the manager's information advantage $\bar{v}_0$, and the expected market conditions $\bar{m}_0$.

Implications (i) and (ii) of Theorem 1, hence the relative optimality of fulcrum versus purely proportional fees, follow from the two conditions that the contract must meet to allow households to realize the maximum possible value of the manager's information. The first condition is that the contract aligns the active fund portfolio with households' CI portfolio (*portfolio-alignment* role). Comparing Eqs. (9) and (16) we see that, in general, linear compensation contracts fulfill this role imperfectly:[19]

[19] In our setup, perfect portfolio alignment requires a nonlinear compensation contract $s(W_T) = (\kappa W_T)^{(1-\gamma_h)/(1-\gamma)}$, for some constant $\kappa > 0$. We analyze the value of the manager's information that households can realize under the optimal linear contract versus this nonlinear contract in Section 6.3.

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Lemma 1. *When households and the manager have identical intrinsic relative risk aversion ($\gamma_h = \gamma$), a pure-proportional fee contract $(k, 1, 0, \phi^Y)$, with $k > 0, \phi^Y \in \mathbb{R}$, implements portfolio ϕ^{η}_{CI} for all realizations of $\tilde{\eta}$. When households' and the manager's intrinsic relative risk aversions differ ($\gamma_h \neq \gamma$), no linear fee contract implements portfolio ϕ^{η}_{CI} for all realizations of $\tilde{\eta}$.*

In the knife-edge case in which the intrinsic relative risk aversions of households and the manager coincide, there is no portfolio-alignment role for fulcrum fees. In any other case, a fee proportional to total returns cannot align the manager's allocation with the CI portfolio. An interesting corollary of this result is that, outside of the standard CARA-normal setup (see Appendix B for an analysis of this case), the superiority of purely proportional fees over benchmark-linked fees as a portfolio aligning device (e.g., Admati and Pfleiderer, 1997) is no longer granted.

The second condition is that the contract achieves portfolio alignment at the lowest possible cost of compensating the manager (*risk-sharing* role). In general, inducing the manager to choose a given portfolio can be costly if this portfolio exposes her to inadequate levels of risk, for which she must be compensated. Under the fulcrum structure (3), we showed in Section 4.1 that the benchmark-linked fee and benchmark portfolio can be calibrated to tilt the active fund portfolio in the desired direction without having to compensate the manager for exposure to the benchmark's risk. Thus, regardless of the sensitivity of pay to the fund's benchmark-adjusted performance or the riskiness of the benchmark, the compensation of the manager under the fulcrum arrangement is the same as under proportional-only fees.

These conditions explain why there is no risk sharing role for linear benchmarking when the relative risk aversions of households and the manager are perfectly aligned (implication (i) of Theorem 1). Indeed, purely proportional fees achieve perfect portfolio alignment (Lemma 1) at the same managerial compensation cost as fulcrum fees. The result is standard in delegated portfolio management models in which the principal (households, in our setup) both dictates the terms of the contract and delegates the management of the entire portfolio to the agent (e.g., Cadenillas et al., 2007).[20] It implies, in particular, that fulcrum fees are irrelevant in the limiting case in which both households and the manager exhibit log preferences (i.e., $\gamma_h = \gamma = 1$). A similar reasoning explains why fulcrum fees do not improve risk-sharing over purely proportional fees, and actually worsen it, in the CARA setup with risk preferences misalignment of Appendix B.

Notably, Theorem 1 also implies that the proportional-only fee contract is suboptimal whenever

[20] In a setting where neither of these conditions are met, Cuoco and Kaniel (2011) show that a linear sharing rule can be suboptimal even when the relative risk aversion coefficients of households and the manager are the same.

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the risk preferences of households and the manager differ (implication (ii)). In fact, in this case we can show that active funds with purely proportional fees are not only dominated by active counterparts with fulcrum fees but also, under certain conditions, by passive alternatives:

Proposition 4. *A sufficient condition for households to prefer delegation to the optimal passive fund over* any *pure-proportional fee active fund, regardless of its fee, is that*

$$\ln(1-k_m) + \frac{1}{\gamma_h - 1}\ln\gamma - \frac{1}{2(\gamma_h - 1)}\ln\left(\gamma^2 + (\gamma_h - 1)(2\gamma - \gamma_h)\bar{v}_0 T\right) + \left(\frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T} - \frac{2\gamma - \gamma_h}{\gamma^2 + (\gamma_h - 1)(2\gamma - \gamma_h)\bar{v}_0 T}\right)\frac{\bar{m}^2}{2}T > 0. \tag{23}$$

Although condition (23) does not lend itself to an intuitive interpretation, we verify in Section 5.3.2 that it is satisfied under reasonable model parameterizations. Most notably, it holds for a wide range of relative risk aversion misalignment between households and the manager in contexts of high expected market conditions ($\bar{m}_0$) or low information asymmetry ($\bar{v}_0$).

The result of Theorem 1 that including fulcrum fees in the fee contract increases the ex-ante value of the manager's information realizable by investors (implication (ii)) overturns the benchmark irrelevance of prior studies. Once again, the result can be interpreted in terms of the portfolio alignment at minimum compensation cost conditions that the optimal contract must meet. Absent a benchmark, the manager may respond to a given realization of her private signal by choosing a smaller (respectively, larger) long or short position in the stock than a relatively risk-tolerant (risk-averse) investor would prefer conditional on the same signal. The asymmetry in information between the parties implies that households cannot fully control this response by contracting on the signal. Instead, it is the inclusion of the benchmark in the compensation contract what allows investors to tilt the fund portfolio allocation toward their desired positions. The benchmark-linked component plays this portfolio-alignment role at no extra cost of compensating the manager for exposure to additional risks relative to the pure-proportional fee arrangement.

The flip side of the coin is that contract implementability requires that investors choose the benchmark conditional on their own, inferior information. This imposes an inefficiency cost, as benchmarking distorts the fund investment towards a conditionally inefficient portfolio and hurts (risk-adjusted) performance. Theorem 1 establishes that both relatively risk tolerant and risk averse households are willing to bear the costs associated with this distortion and include a benchmarking component in the fund manager's linear compensation contract.

We highlight that the rationale and nature of linear benchmarking in our framework differ from

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those in Ou-Yang (2003). In that article, the benchmark irrelevance is overturned by enlarging the set of admissible benchmarks to include not only passive portfolios but also "active indexes" whose portfolio composition varies over time. With CARA preferences, the optimal contract is shown to consists of a fixed fee plus a fraction of total assets plus a symmetric benchmark-adjusted fee. Crucially, because households and the manager have symmetric (complete) information about asset returns in this setup, households can choose an efficient portfolio as the benchmark to induce the manager to invest the right amount of money in the assets at each point in time. In contrast, households in our setup do not observe the efficient portfolio but are constrained to optimize risk sharing by trading off the benefits and costs of an inefficient benchmark in the objective of the manager. Theorem 1 states that including fulcrum fees in a linear contract is optimal even when the benchmark associated to these fees is inefficient. In fact, our numerical analysis in the next section indicates that the benchmark portfolio that households optimally select is generally inefficient also with respect to their own, inferior information.

5.3 Numerical Analysis

Theorem 1 establishes the optimality of fulcrum fee arrangements in informed active management but does not offer a full characterization of the optimal contract $(\hat{k}, \hat{\theta}, \hat{\alpha}, \hat{\phi}^Y)$, or a quantitative assessment of the benefits of linear benchmarking relative to the pure-proportional fee and CI cases.[21] To address these issues, in this section we analyze our model numerically using the baseline and alternative parameterizations that we describe in Appendix C.

Figure 1 plots the mapping of the choice variables (α, ϕ^Y), keeping k and θ constant at their optimal values $(\hat{k} = k^*, \hat{\theta} = 1)$, to households' excess certainty equivalent returns (CER) from delegation to an active manager. The resulting surface is smooth and typically has a unique maximum. These features make the numerical search for a solution to Eqs. (22) fast and reliable.

5.3.1 Optimal Linear Contract Parameters

Figure 2 plots the optimal contract as a function of the risk aversion misalignment $|\gamma_h - \gamma|$ between households and the manager. Panels (a) and (b) depict the optimal proportional and fulcrum fees $\hat{k}$ and $\hat{\alpha}$ (top row) and the benchmark allocation to the stock $\hat{\phi}^Y$ (bottom row) of, respectively,

[21] In general, the expectations in (22) that characterize (implicitly) $\hat{\alpha}$ and $\hat{\phi}^Y$ cannot be computed explicitly. However, they are defined with respect to two independent normally distributed variables. Thus, they can be approximated with arbitrary precision using standard numerical integration methods.

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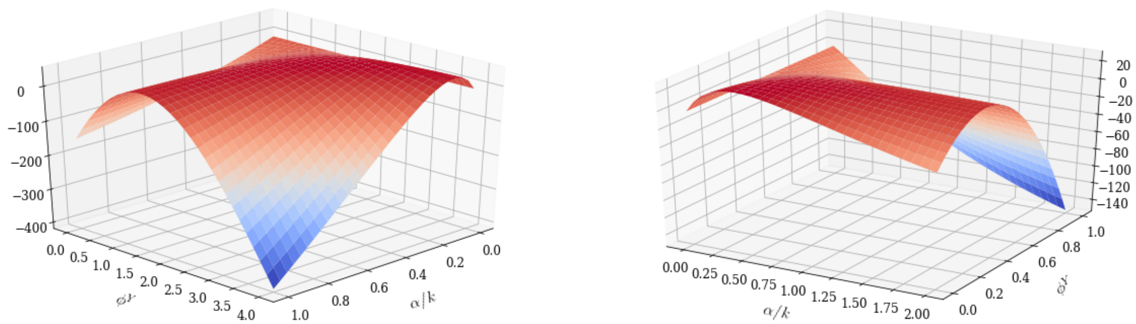

(a) Risk-Tolerant Investors ($\gamma_h = 3$) (b) Risk-Averse Investors ($\gamma_h = 7$)

Figure 1: **Excess CER (in bps) from Delegation to an Active Manager.**
The figure plots households' annualized excess certainty equivalent returns (CER) from delegation to an active manager under a linear contract $(k^*, 1, \alpha, \phi^Y)$. Excess CER are computed with respect to households' CER from delegation to a passively managed fund. The model parameters are: $T = 3, r = 3\%, \sigma = 0.158, \bar{m} = 0.513, \bar{v}_0 = 0.04, \gamma = 5, c = 1$.

relatively risk-tolerant ($\gamma_h < \gamma$) and risk-averse ($\gamma_h > \gamma$) fund investors. For comparison with the benchmark portfolio, the bottom row also displays the optimal allocation ϕ^P of the passive fund to the stock. The figure illustrates our main result in Theorem 1: under relative risk aversion misalignment ($\gamma \neq \gamma_h$), the optimal fulcrum fee is always non-zero (positive in this case) regardless of whether fund investors are relatively more or less risk averse than the fund manager.

Other novel patterns are evident. First, the optimal fulcrum fee $\hat{\alpha}$ increases, while the optimal proportional fee $\hat{k}$ decreases, in the risk aversion misalignment $|\gamma - \gamma_h|$ across investor types. The pattern for the fulcrum fee follows the intuition laid out in Section 5.2, whereby a greater misalignment in risk aversion raises the benefits of tilting the manager's portfolio toward the allocation that households prefer, even if this allocation is conditionally inefficient. The average value of the proportional fee is tied to the assumption that households have all the bargaining power and pay the manager the reservation certainty equivalent compensation she would receive if she offered the optimal passive fund. Because the passive fund portfolio increasingly differs from the manager's preferred portfolio as the risk-aversion misalignment between the parties rises, the utility that the manager derives from offering the passive fund falls with the misalignment and determines the observed decreasing pattern for the proportional fee.

Second, the optimal benchmark portfolio in the fee contract is *unconditionally* inefficient. Households' uninformed optimal portfolio is represented by ϕ^P, the stock allocation of the ETF that households would select under passive management of their wealth. The bottom panels of Fig. 2 show that the optimal stock allocation $\hat{\phi}^Y$ of the benchmark is either substantially higher or sub-

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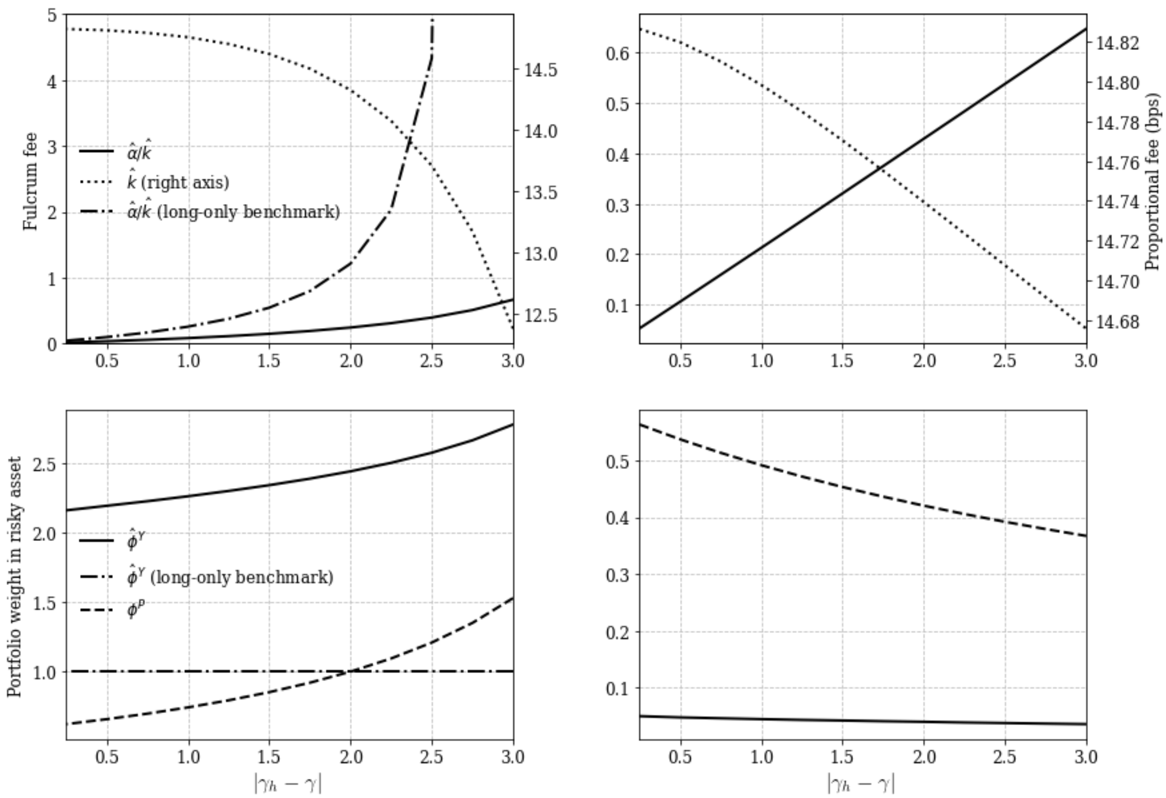

(a) Risk-Tolerant Investors ($\gamma_h = 3$) (b) Risk-Averse Investors ($\gamma_h = 7$)

Figure 2: **Optimal Linear Contract.**

The figure plots the proportional and fulcrum fees $\hat{k}$ and $\hat{\alpha}$ (dotted and solid lines, top row) and benchmark allocation in the stock $\hat{\phi}^Y$ (solid line, bottom row) of the optimal linear contract for relatively risk-tolerant ($\gamma_h < 5$, Panel (a)) and risk-averse ($\gamma_h > 5$, Panel (b)) fund investors. The left panels include the fulcrum fee and benchmark allocation to the stock of the optimal constrained contract whose benchmark portfolio contains only long positions (dash-and-dot lines). The bottom row also displays the optimal allocation ϕ^P of the passive fund to the stock (dashed line). The rest of the model parameters are as in Fig. 1.

stantially lower than ϕ^P depending on whether active fund investors are, respectively, relatively risk tolerant or risk averse. The result departs from the implications of prior models with no information asymmetry about asset returns (e.g., the second-best contract in Dybvig et al., 2010), where the prescribed benchmark portfolio is unconditionally optimal.

The optimal active fund portfolio (Section 4.1) and the CI portfolio of households (Section 5.1) provide the rationale for this result. From the perspective of relatively risk-tolerant fund investors, the manager *underreacts to good news and overreacts to bad news*, in the sense that she increases the stock allocation of the fund too conservatively in response to a high signal ($\eta > \bar{m} > 0$), but decreases it too aggressively in response to a low signal ($0 < \eta < \bar{m}$). Both under- and overreaction problems can be alleviated by setting a high-risk benchmark in the fulcrum arrangement. Conversely, from

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the perspective of relatively risk-averse fund investors, the manager *overreacts to good news and underreacts to bad news*, as she increases the stock allocation of the fund too aggressively in response to a high signal, and decreases it too little in response to a low signal. Both problems can be alleviated by having the manager partially mimic a highly conservative benchmark.

				Panel A: Unconstrained Benchmark			Panel B: Constrained Benchmark		
γ_h	T	$\sqrt{\bar{v}_0}$	ϕ^P	$\hat{k}/T$ (in bps)	$\hat{\alpha}/\hat{k}$	$\hat{\phi}^Y$	$\hat{k}/T$ (in bps)	$\hat{\alpha}/\hat{k}$	$\hat{\phi}^Y$
3	1	0.1	1.07	14.8	0.35	2.25	14.8	264.6	1.00
		0.2	1.05	14.8	0.19	3.08	14.8	1.33	1.00
		0.3	1.02	14.7	0.12	3.83	14.7	0.30	1.00
	3	0.1	1.06	14.4	0.50	1.85	14.4	26.4	1.00
		0.2	1.00	14.3	0.24	2.44	14.3	1.21	1.00
		0.3	0.92	14.2	0.15	2.98	14.2	0.34	1.00
	5	0.1	1.05	14.1	0.59	1.71	14.1	14.0	1.00
		0.2	0.95	13.9	0.27	2.20	13.9	1.10	1.00
		0.3	0.83	13.7	0.15	2.64	13.7	0.35	1.00
7	1	0.1	0.46	15.0	0.44	0.05	15.0	0.44	0.05
		0.2	0.45	14.9	0.41	0.02	14.9	0.41	0.02
		0.3	0.43	14.8	0.41	0.01	14.8	0.41	0.01
	3	0.1	0.45	14.9	0.50	0.10	14.9	0.50	0.10
		0.2	0.42	14.7	0.43	0.04	14.7	0.43	0.04
		0.3	0.38	14.6	0.41	0.02	14.6	0.41	0.02
	5	0.1	0.44	14.8	0.53	0.12	14.8	0.53	0.12
		0.2	0.40	14.6	0.44	0.05	14.6	0.44	0.05
		0.3	0.33	14.3	0.41	0.02	14.3	0.41	0.02

Table 1: **Optimal Managerial Contract.** This table reports the optimal allocation ϕ^P of the passive fund to the stock and the optimal contract parameters $\hat{k}, \hat{\alpha}$ and $\hat{\phi}^Y$ solving problem (6) for different combinations of household's risk aversion, time horizons T, and degrees of managerial information advantage $\bar{v}_0$. In Panel A, we impose no additional constraint on the contract space beyond those introduced in Section 4. In Panel B, we constrain the benchmark to adopt a long-only position in both the riskfree and risky assets ($0 \leq \phi^Y \leq 1$). The rest of the model parameters are as in Fig. 1.

Table 1 reports the optimal contract parameters under the different investment horizons T and degrees of managerial information advantage $\bar{v}_0$ in our baseline and alternative scenarios. Of particular interest is the relation between the manager's information advantage and the optimal contract parameters. From our discussion of Theorem 1 we expect $\hat{\alpha}$ to fall with $\bar{v}_0$. The intuition is that, as the manager's information advantage widens, the additional costs in terms of the forgone excess fund returns that higher fulcrum fees entail outweigh their additional risk-alignment benefits. As a result, we observe in Panel A that investors reduce the rewards and penalties for relative out-

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and underperformance as $\sqrt{\bar{v}_0}$ rises by lowering the weight $\hat{\alpha}/\hat{k}$ of fulcrum fees in the optimal contract. Perhaps more surprisingly, we also observe that, as the fulcrum fee falls, the distance between the benchmark and the passive fund portfolios $|\hat{\phi}^Y - \phi^P|$ widens. The result points to a certain degree of substitutability within the optimal contract between the fulcrum fee and the benchmark composition, according to which more "specialized" benchmarks partially subsume the role of the fulcrum fee in tilting the fund portfolio in the desired direction.

In practice, the benchmarks stipulated in performance fee schedules typically represent an unlevered position either in a market index such as the S&P 500, or in a money market instrument such as the 3-month T-bill rate. To account for this empirical regularity, we solve for the optimal linear contract under the constraint that the benchmark includes no short positions ("long-only") in either the riskfree or the risky assets ($0 \leq \phi^Y \leq 1$). Since this constraint is not binding for relatively risk-averse investors ($\gamma_h > \gamma$, Panel (b)), Fig. 2 plots the resulting optimal constrained fulcrum fee and benchmark allocation to the stock for relatively risk-tolerant investors ($\gamma_h < \gamma$, Panel (a)) only.[22] Importantly, fulcrum fees remain optimal ($\hat{\alpha} > 0$) in this constrained case. The optimal benchmark, which allocates 100% weight in the stock, resembles the type of benchmarks commonly observed, for example, in the mutual fund industry.

Lastly, Table 1 highlights that the results are robust to alternative calibrations of the model. In line with our previous observations, the fulcrum fee $\hat{\alpha}$ is positive and the benchmark allocation $\hat{\phi}^Y$ in the stock substantially differs from the unconditionally efficient allocation ϕ^P across investment horizons T and degrees of information advantage of the active manager $\bar{v}_0$, for both the unconstrained- and constrained-benchmark cases.

5.3.2 Value of Benchmark-Adjusted Fees

According to Lemma 1, a linear contract implies a loss in the ex-ante value that households can extract from the manager's information. In this section, we quantify both this loss and the relative gains that, according to Theorem 1, households can attain under fulcrum fees over purely proportional fees.

Figure 3 plots the CER of households, in excess of their CER under optimal passive management, under the optimal linear contracts (solid and dash-and-dot lines) and the pure-proportional fee contracts (dashed line) of Fig. 1. We present the excess CER as a percentage of the excess CER

[22] The optimal proportional fee $\hat{k}$ is, as argued in Section 5.3.1, independent of the choice of benchmark. Thus, it does change from the optimal unconstrained to the optimal constrained contracts.

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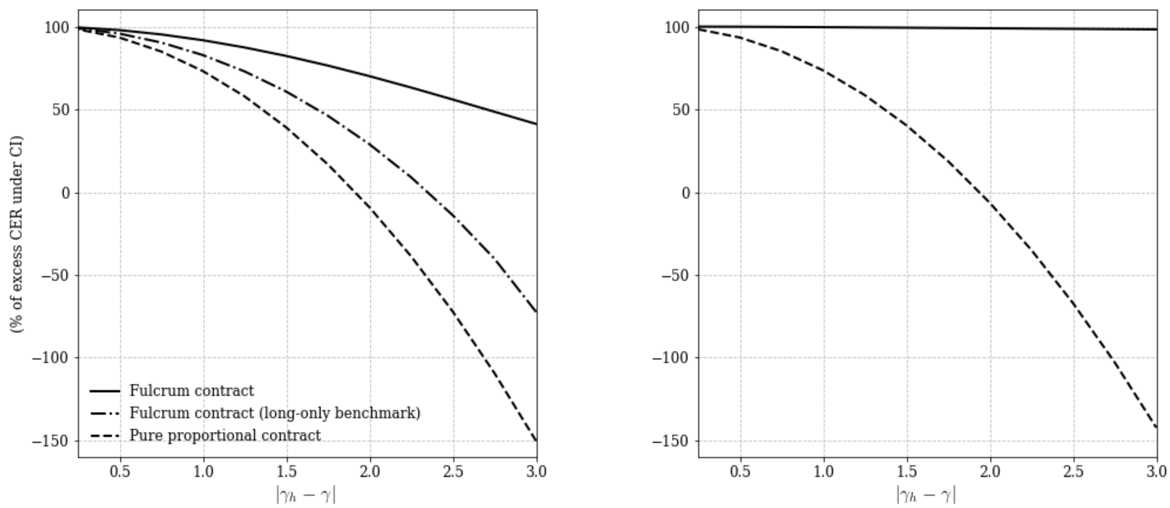
under our reference CI case or, equivalently, as a fraction of the *maximum ex-ante value that households can extract from the manager's information*. For example, when $\gamma_h = 3$ ($|\gamma_h - \gamma| = 2$), households earn an ex-ante CER of 6.91% per year on the optimally designed passive fund. This return would increase to 7.56% if they could purchase the private signal of the manager (CI case), leading to an excess CER of 65 bps. The figure illustrates the relative merits of the different contract types for the active manager as a percentage of this excess CER in the CI case. A negative number means that households lose money by delegating their wealth to an active manager that is subject to the corresponding fee contract, instead of investing it in their preferred passively managed fund.



(a) Risk-Tolerant Investors ($\gamma_h = 3$) (b) Risk-Averse Investors ($\gamma_h = 7$)

Figure 3: **Value of Active Management.**

The figure plots the excess certainty equivalent returns (CER) from delegation under the optimal linear benchmark-adjusted contracts both without (solid line) and with (dash-and-dot line) benchmark constraints, and under the pure-proportional fee contract (dashed line), for relatively risk-tolerant ($\gamma_h < 5$, Panel (a)) and risk-averse ($\gamma_h > 5$, Panel (b)) fund investors. Excess CER are computed with respect to households' CER from delegation to a passively managed fund and reported as a percentage of the excess CER under the reference CI case. The rest of the model parameters are as in Fig. 1.

A first result from Fig. 3 is that linear benchmarking can be highly valuable to delegating investors. Both relatively risk-tolerant ($\gamma_h < 5$) and risk-averse ($\gamma_h > 5$) households realize a higher fraction of the ex-ante value of the manager's private information under optimal linear benchmarking than under the optimal pure-proportional contract. As anticipated from our discussion in Section 5.2, the gains of adding fulcrum fees to a pure-proportional fee contract increase with the risk aversion misalignment $|\gamma_h - \gamma|$. For example, these gains equal 18.6 and 26.1 percentage points,

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respectively, for households with relative risk aversion coefficients of 4 (relatively risk tolerant) and 6 (relatively risk averse). For either lower or higher coefficients of relative risk aversion (RRA) for households, equal to 3 or 7, the gains of fulcrum fees relative to purely proportional fees rise to 79.2 and 105.4 percentage points, respectively.

A second result is that, by including a fulcrum fee with respect to an appropriately designed benchmark in the fee contract, households can extract almost the entire ex-ante value of the active manager's information. This is especially the case for relatively risk-averse investors. Following the result in Lemma 1, the fraction of this value that investors of the active fund realize under *any* linear contract falls with the misalignment in risk aversion with the fund manager. This fall is steep under purely proportional fees but much less so when the optimal benchmark-adjusted fee is added to the contract. This implies that relatively risk-tolerant investors with RRA coefficient of between 2 and 5 realize more than 41% of the ex-ante value of the manager's information under our baseline parameterization. Relatively risk-averse investors with RRA coefficient higher than 5, on the other hand, extract at least 98.3% of this value by using the optimal linear contract.[23]

We take a closer look at how the value of the manager's information is extracted under the optimal versus the pure-proportional fee contracts, for each realization η of the signal, in Figure 4. The value of the signal, represented by the CER that households realize under the CI case in excess of the passive alternative, increases with the distance $|\eta - \bar{m}_0|$ from its mean. Purely proportional fees allow households to extract a high fraction of the value of very high signals, and almost the entire value of very low signals, both of which have low probability of occurrence. However, they do a poor job at extracting the value of more likely high and low signals around the mean $\bar{m}_0$. In fact, if active funds only charge proportional fees, households are better off delegating to passive funds when these signals are realized. In contrast, if active funds add the appropriate fulcrum fees, households can extract nearly all the value of the mid-range signals and always improve on the CER delivered by passive funds. For relatively risk-averse investors, the possibility of extracting almost the entire value of the manager's information extends to very high and low signal realizations, dominating purely proportional fees for each realization of $\tilde{\eta}$.

Thirdly, we observe that the optimal pure-proportional fee contract can entail substantial ex-ante losses to active fund investors. Both sufficiently risk tolerant ($\gamma_h \leq 3$) and sufficiently risk averse

[23] Under symmetric information, moral hazard problems and portfolio constraints, Dybvig et al. (2010) also argue that linear contracts entail little losses relative to the optimal contract between an investor and a manager with CRRA preferences. Our results in this section extend theirs to a delegation context of asymmetric information about asset returns between the fund manager and her investors.

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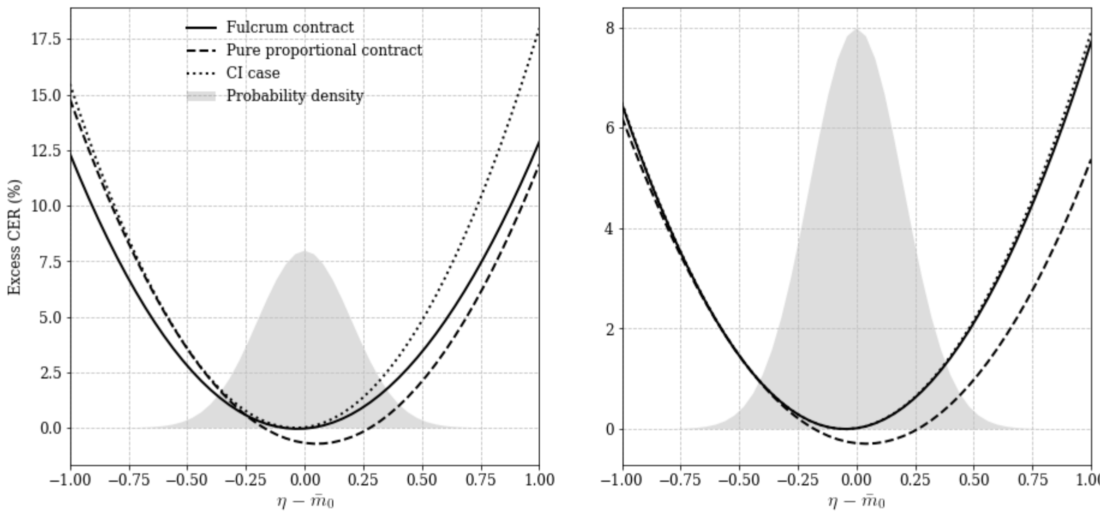

(a) Risk-Tolerant Investors ($\gamma_h = 3$) (b) Risk-Averse Investors ($\gamma_h = 7$)

Figure 4: **Conditional Value of Active Management.**

The figure plots households' annualized excess certainty equivalent returns (CER) under the contractible-information (CI) and active management cases for each realization of the manager's signal $\tilde{\eta}$. Excess CER are computed with respect to households' CER from delegation to a passively managed fund. The optimal fulcrum contracts for the risk-tolerant and risk-averse investors are, respectively, $(\hat{k}/T, \hat{\theta}, \hat{\alpha}/\hat{k}, \hat{\phi}^Y) = (1.433 \times 10^{-3}, 1, 0.244, 2.44)$ and $(1.474 \times 10^{-3}, 1, 0.429, 0.04)$. The corresponding pure-proportional contracts are $(1.433 \times 10^{-3}, 1, 0, 0)$ and $(1.474 \times 10^{-3}, 1, 0, 0)$. The rest of the model parameters are as in Fig. 1.

($\gamma_h \geq 7$) households earn negative CERs in excess of passive investing when delegating to active managers with purely proportional fees. Intuitively, from these investors' viewpoints the manager can take either insufficient or excessive risks in response to absolute-performance compensation. The ensuing losses on their utility can more than offset the average value they extract from the manager's signal even if, as seen from Fig. 4, this value is close to 100% for low signals. Ex ante, the optimal passive fund is a better alternative.

In fact, following Proposition 4, for sufficiently risk-tolerant or risk-averse investors *all* pure-proportional fee active funds, regardless of their fees, can be dominated by passive funds. Figure 5 shows, in particular, that low levels of information asymmetry about the stock returns $\bar{v}_0$, as well as good expected market conditions $\bar{m}_0$, favor the dominance of pure-proportional fee active funds by passive funds for a wide ranges of risk aversion misalignments.

Table 2, which reports the value of active management for the model parameterizations of Table 1, indicates that the ex-ante value that households extract from the manager's information under the optimal linear contract increases with the information advantage of the manager, and is substantial (greater than 70% for risk-tolerant investors, nearly 100% for risk-averse investors)

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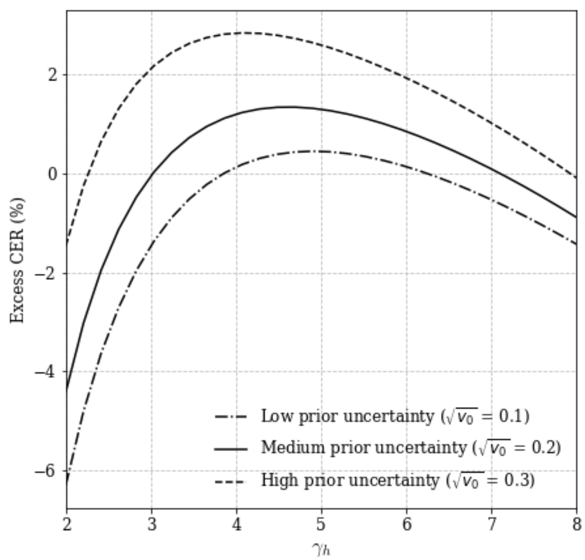
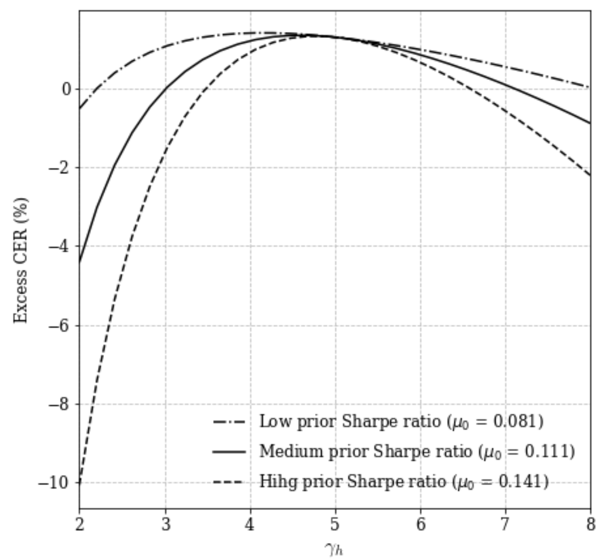


(a) Effect of Information Asymmetry



(b) Effect of Expected Market Conditions

Figure 5: **Value of Purely Proportional Fees in Active Management.**
The figure plots the effects of the manager's information advantage (Panel (a)) and the expected market conditions (Panel (b)) on the annualized *gross-of-fees* certainty equivalent return (CER) that an active fund with no fulcrum fee ($\alpha = 0$) delivers, in excess of the *net-of-fees* certainty equivalent return delivered by the optimal passive fund. Model parameters are as in Fig. 1.

when the advantage is sufficiently large (e.g., $\sqrt{\bar{v}_0} > 0.1$). The table also shows that, for both relatively risk-tolerant and risk-averse investors, the benefits of fulcrum over pure-proportional fee arrangements, and of active over passive management under the optimal fulcrum contract, generalize to the different investment horizons T and degrees of managerial information advantage $\bar{v}_0$ in our alternative scenarios, and are economically large in all cases.

Panel (a) of Figure 3 and Panel B of Table 2 show that the economic benefits of benchmarking are sizable also when the benchmark is constrained to take unlevered positions in the underlying assets. Table 2 shows that, as we approach the symmetric-information case ($\bar{v}_0 \to 0$) the benefits of linear benchmarking vanish along with the value of active management, in which case fund investors are better off investing their wealth in the passive alternative. The result highlights the importance for the optimality of linear benchmarking as a risk-sharing device, when passive funds are available, of considering the skill level of active managers.

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				Excess CER (% of Excess CI CER)				
					Panel A: Unconstrained Benchmark		Panel B: Constrained Benchmark	
γ_h	T	$\sqrt{\bar{v}_0}$	Excess CER in CI case (bps)	pure-proportional Fee Contract (1)	Optimal Fulcrum Contract (2)	(2) - (1)	Optimal Fulcrum Contract (3)	(3) - (1)
3	1	0.1	16.8	< -250	63.4	> 100.0	-11.3	> 100.0
		0.2	66.0	-16.9	72.0	88.9	19.8	36.7
		0.3	146.0	41.3	76.2	34.9	45.3	4.0
	3	0.1	17.0	< -250	58.3	> 100.0	-4.5	> 100.0
		0.2	64.7	-9.2	70.1	79.2	28.9	38.1
		0.3	138.6	48.0	76.5	28.4	53.7	5.6
	5	0.1	17.3	< -250	56.7	> 100.0	1.8	> 100.0
		0.2	63.5	-2.3	70.1	72.4	36.6	38.8
		0.3	132.2	53.5	77.7	24.3	60.2	6.7
7	1	0.1	7.2	< -250	98.8	> 100.0	98.8	> 100.0
		0.2	28.2	-15.9	99.5	> 100.0	99.5	> 100.0
		0.3	62.1	42.4	99.7	57.3	99.7	57.3
	3	0.1	7.2	< -250	97.2	> 100.0	97.2	> 100.0
		0.2	27.4	-6.4	99.0	> 100.0	99.0	> 100.0
		0.3	58.2	50.6	99.5	48.9	99.5	48.9
	5	0.1	7.2	< -250	96.1	> 100.0	96.1	> 100.0
		0.2	26.7	1.9	98.7	96.8	98.7	96.8
		0.3	54.9	56.9	99.5	42.6	99.5	42.6

Table 2: **Value of Active Management under Baseline and Alternative Scenarios.** This table reports annualized excess certainty equivalent returns (CER) from delegation under the contractible-information (CI) case, the optimal pure-proportional fee contracts, and the optimal fulcrum contracts of Table 1. Excess CER are computed with respect to households' CER from delegation to a passively managed fund. Values are reported for different combinations of household's risk aversion, time horizons T, and degrees of managerial information advantage $\bar{v}_0$. In Panel A, we impose no additional constraint on the contract space beyond those introduced in Section 4. In Panel B, we constrain the benchmark to adopt a long-only position in both the riskfree and risky assets ($0 \leq \phi^Y \leq 1$). The rest of the model parameters are as in Fig. 1.

6 Extensions and Practical Implications

6.1 Asymmetric Performance Fees and Imperfect Signals

In an accompanying Internet Appendix we generalize our setup of Section 3 to (i) allow the manager to have *partial* (instead of *complete*) information about the return fundamental $\tilde{\eta}$, and (ii) enlarge the managerial contract space to include an *asymmetric* benchmark-adjusted performance fee.

Our findings under this setup further strengthen the argument for fulcrum fees in asset management. First, the optimal contract is always linear, as it includes a positive fulcrum fee for both

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relatively risk-tolerant and risk-averse investors but no asymmetric performance fees. Since linear benchmarking serves the purpose of risk-alignment better than nonlinear benchmarking and at lower cost, the margin of outperformance increases with the risk aversion misalignment $|\gamma_h - \gamma|$.[24] Although dominated by fulcrum fees, we find that the optimal asymmetric performance fee schedule is preferable to proportional fee-only contracts, especially for risk-averse investors.

Second, although the benefits of fulcrum fees increase with the precision of the manager's information, the optimality of these fees does not depend on the assumption of complete information for the manager. The optimal fulcrum fee contract allows risk-averse fund investors to realize almost the entire ex-ante value of the manager's information even when this information is imperfect. It also delivers positive excess CERs from (sufficiently informed) active management to risk-tolerant investors, even though in their case the long-only constraint on the benchmark is binding.

6.2 Larger Investment Opportunity Set for Households

In our setup, households make a binary decision to allocate their entire wealth to either the active or the passive funds. Thus, their investment opportunity set could be expanded to allow for simultaneous allocations to active *and* passive funds, as well as to the riskfree asset. We note that such a change to our setting does not unravel the optimality of benchmark-adjusted fees in the linear managerial contract. The reason is twofold. First, in choosing the optimal ETF composition ϕ^P, households directly decide the fraction of their wealth to allocate in the riskfree asset. Second, and more importantly, allowing households to mix up their active and passive fund holdings will not perfectly substitute the role of the optimal managerial compensation in solving the portfolio alignment and risk-sharing problems that we study. This follows from the asymmetry in information about asset returns between households and the manager, which may (and will likely) result in a too low allocation of households to an active fund with no benchmark-adjusted fees for certain realizations of this signal, and a too high allocation for other realizations.

This can be clearly seen in Fig. 3 and Table 2, where for reasonable parameterizations of the model there is no possible combination of a pure-proportional fee active fund and a passive fund under which households achieve higher utility (CERs) than under the optimal linear contract. For example, when delegating to an active fund manager with a short horizon and moderate information

[24] When investors do not internalize the effect of their decisions on fund fees and no information asymmetries exist, Cuoco and Kaniel (2011) find instead that asymmetric performance fees can dominate both proportional and fulcrum fees.

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advantage ($T = 1, \sqrt{\bar{v}_0} = 0.2$), the risk-tolerant households ($\gamma_h = 3$) of Table 2 earn, under purely proportional fees, a negative CER in excess of the optimal ETF alternative (column (1)). Thus, the problem of allocating these households' wealth between an active fund subject to no benchmarking and the ETF has a corner solution with a 100% allocation to the ETF. However, households could do better by allocating their entire wealth to the benchmarked active fund (column (2)) instead, where they would earn positive excess CERs relative to their 100%-ETF allocation. Thus, the optimality of benchmarking in our setup does not depend on the particular investment opportunity set for households that we assume.

6.3 Comparison with a Portfolio-Aligning Nonlinear Contract

In Section 5.2 (footnote 19) we argued that the nonlinear managerial contract $s(W_T) = (\kappa W_T)^{(1-\gamma_h)/(1-\gamma)}$, for some constant $\kappa > 0$, induces the manager to select the CI portfolio (16) and achieves perfect portfolio alignment. Because under nonlinear contracts the exposure of the manager's pay to unwanted (either excessive or insufficient) risk cannot be fully hedged, securing the manager's participation could require a higher average compensation (as summarized by κ) than under a linear contract. By how much $s(W_T)$ improves risk sharing over the optimal linear contract then depends on how its portfolio-aligning benefits compare to this extra compensation.

In unreported analysis, we solve for the CER of households under the contract $s(W_T)$ that secures the manager her reservation utility ($c = 1$). Across model parameterizations, the losses of the optimal linear contract relative to the portfolio-aligning nonlinear contract are approximately the same as computed in Section 5.3.2 relative to the CI case. As we argued in that section, these losses are generally small for relatively risk-tolerant, and nearly zero for relatively risk-averse fund investors. In practice, linear contract are presumably easier to implement than nonlinear variants. A predominance of the benefits of this simplicity over the low or insignificant portfolio misalignment costs of linear fee structures might explain their prevalence, as well as the nonexistence of nonlinear fee contracts such as $s(W_T)$, in practice.[25]

[25] Massa and Patgiri (2009) report that one third of mutual funds in the U.S. have a concave fee structure in which the percentage advisory fee decreases as the total assets increase above prespecified thresholds. At first glance, this type of contract resembles $s(W_T)$ for $\gamma_h < \gamma$. However, $s(\cdot)$ is a smooth function of total assets whereas the fee structure of U.S. funds is piecewise linear, with kinks at the total asset thresholds. Larsen (2005) characterizes the manager's optimal portfolio policy under this type of kinked incentives. This policy differs from the CI policy (16) of households.

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6.4 Practical Implications

Our results have a number of practical implications:

Recent trends in asset management. Our findings imply that active fund managers should optimally include in their fee schedules a fulcrum component, whose size depends on both their information advantage (investment skills) and the risk profile of the investors to which the fund is catered. In particular, (i) as the information advantage shrinks the weight of the fulcrum component in the optimal fee schedule should increase; and (ii) for a sufficiently small information advantage, passive funds eventually dominate active funds for any misalignment in risk aversion between manager and investors.

In a context of highly efficient financial markets and widespread information dissemination (i.e., shrinking information advantage by any party), these results help explain two recent trends in the asset management industry. The first is the adoption of fulcrum fees by prominent asset management companies in the last few years, which motivates this paper (see Section 1).[26] The second is the increasing preference of investors for passive over active funds in the U.S., where purely proportional fees are the norm (see, e.g., the Investment Company Institute 2020 Factbook).[27]

To further illustrate how our model can help understand these trends, we calibrate the bargaining power constant c to approximately match the weighted average expense ratio of 52 basis points per year ($\hat{k}/T = 0.0052$) for U.S. equity mutual funds as of 2019 (Investment Company Institute 2020 Factbook). Under our baseline parameterization, we find that a relatively risk-averse (respectively, risk-tolerant) investor gains a CER of 39 (44) basis points per year by switching from an active fund with purely proportional fees to the passive alternative. By contrast, the same investor gains 10 (loses 7) basis points if the active fund charges the optimal fulcrum fee instead.[28]

Benchmark design. Accounting for leverage restrictions, our results prescribe relatively simple rules for the design of the benchmark specified in the fulcrum fee, namely an all-equity index (e.g., the S&P 500 Index) for investors with aggressive investment profile and a low-risk index (e.g., a LIBOR benchmark) for more conservative investors. We expect these rules to generalize, potentially even for the case of unlevered benchmarks, to a multiasset setup in which the riskiness of the

[26] In practice, the fulcrum component of fees can have an upper and a lower limit on size so that the total fees can never be negative (Elton et al., 2003).

[27] Available at the Investment Company Institute's website: https://www.ici.org/

[28] The coefficients of relative risk aversion we use for the risk-tolerant and risk-averse investors in this exercise are those of Tables 1 and 2.

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benchmark can be altered also by changing its portfolio allocation across the different risky assets. Besides aligning with the type of benchmarks commonly found in practice, these prescriptions are robust to the specific level of relative risk aversion of the investors, as variations in their risk profiles do not substantially change the optimal composition of the benchmark portfolio in Section 5.3.1.

Implicit managerial incentives. The implications of our findings go beyond the analysis of explicit fulcrum fees in the compensation of asset management companies. There is broad consensus among finance academics that the annual flow of funds into mutual funds displays an increasing profile around fund performance relative to a benchmark.[29] Most mutual funds charge a fixed annual percentage of the funds under management, so the total revenue of the fund resembles the profile of the flow of money. This profile fits the type of benchmark-adjusted performance fees that we address in this paper, and strengthens the importance of analyzing the relative merits of benchmark-adjusted contracts.

Symmetric versus asymmetric performance fees. Our results are also relevant to the ongoing debate on acceptable incentives for fund managers. The use of performance fees by mutual funds has been allowed only recently in many European countries (e.g., in 2004 in the U.K.) and has attracted scrutiny ever since.[30] In the U.S., prior to 1971 mutual funds could use either fulcrum or "bonus" (asymmetric) performance fees. In 1971, the SEC ruled that if investment companies use performance-based compensation contracts, these could not be of the asymmetric type. Our findings suggest that the prohibition does not entail welfare losses necessarily, and is most likely welfare-improving, for delegating investors.

Our results do not support the use of option-like fees for the purposes of either aligning the optimal portfolios of fund managers and investors or improve risk-sharing.[31] However, we cannot rule out that they still help resolve effort inducement problems between the two parties (as in Li and Tiwari (2009) and Dybvig et al. (2010)), from which our model abstracts away.

[29] See footnote 2 for references.

[30] See, e.g., the renewed call among European authorities for a performance fee ban: http://www.ft.com/cms/s/0/630fefc4-2fd9-11e2-ae7d-00144feabdc0.html#axzz4IhH7AVDY.

[31] As discussed in Section 6.1, see Cuoco and Kaniel (2011) for a setup in which option-like fees can still help with this purposes.

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7 Conclusions

The adoption of symmetric benchmark-adjusted fees by active fund managers has found limited support in the portfolio delegation literature. In particular, prior research has demonstrated, under a particular type of preferences, the irrelevance of linear benchmarking to align the optimal portfolios of fund managers and their investors or to improve risk sharing between the two parties. In this paper we show that these results are overturned in situations in which an active fund manager has better information than her fund investors, a misalignment in relative risk aversion between the two parties exists, and passive fund alternatives are available to the investors.

Under these realistic premises, linear benchmarking makes possible for investors to align risk tolerances at the cost of distorting the manager's informed investment policy towards an inefficient portfolio. We show that this tradeoff resolves in favor of the inclusion of a benchmark-adjusted (fulcrum) performance component in the optimal linear fee schedule of fund companies. From investors' perspective, a simple fee proportional to the value of total final wealth leads managers to take either excessive or insufficient risk in their portfolio. Through the inclusion of fulcrum fees, investors can affect the manager's portfolio in a predictable way for any realization of the private information at no extra cost of compensating the manager for exposure to relative-performance risk. We illustrate how, by choosing the optimal benchmark-adjusted performance fees and benchmark composition, investors can derive from active management a utility nearly as high as if they could trade on the same private information as the manager. We show that, under certain conditions, linear benchmarking is also necessary for active management to be a viable alternative to passive investing. We further provide conditions under which asymmetric benchmark-adjusted performance fees cannot improve over their symmetric counterparts and characterize how the optimal contract depends on the risk-aversion misalignment and the precision of the manager's information.

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Appendix

A Proofs

We start by introducing an auxiliary result:

Lemma A1. *Let $z \sim \mathbf{N}(0, \sigma_z^2)$, and let $\rho, c, \bar{z} \in \mathbb{R}$. We have:*

(i) $E\left[e^{\rho z}\mathbb{1}_{\{z\leq\bar{z}\}}\right] = e^{\frac{\rho^2\sigma_z^2}{2}}\mathcal{N}\left(\frac{\bar{z}-\rho\sigma_z^2}{\sigma_z}\right)$,

(ii) $E\left[e^{-\rho(z-c)^2}\mathbb{1}_{\{z\leq\bar{z}\}}\right] = \frac{e^{-\frac{\rho c^2}{1+2\rho\sigma_z^2}}}{\sqrt{1+2\rho\sigma_z^2}}\mathcal{N}\left(\frac{\bar{z}-\frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}c}{\sigma_z/\sqrt{1+2\rho\sigma_z^2}}\right)$,

where $\mathcal{N}(.)$ is the standard normal cumulative distribution function.

Proof. Follows from direct integration against the normal density, using the change of variables $\tilde{z} = \frac{z-\rho\sigma_z^2}{\sigma_z}$ for part (i) and $\check{z} = \frac{z-\frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}}{\sigma_z/\sqrt{1+2\rho\sigma_z^2}}$, $\check{z}_l = \frac{\bar{z}-\frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}}{\sigma_z/\sqrt{1+2\rho\sigma_z^2}}$ for part (ii). □

Proof of Proposition 1. The dynamic self-financing condition (8) can be re-expressed as a static budget constraint (see e.g. Karatzas and Shreve, 2001), so for a given realization η of the manager's signal and contract $\mathscr{C} \equiv (k, \theta, \alpha, \phi^Y)$ problem (7)-(8) becomes:

$$\max_{W_T} E\left[u\left((1-\theta)kw\beta_T + \theta kW_T + \alpha(W_T - Y_T)\right)\middle|\,\tilde{\eta} = \eta\right], \tag{A1}$$

$$s.t. \qquad E_0\left[\pi_T W_T\right] = w, \tag{A2}$$

where for notational simplicity we omit the superscripts $(\eta; \mathscr{C})$ that associate the processes with the realized value of $\tilde{\eta}$ and with the contract parameters $\mathscr{C}$. For the remainder of this proof, conditional expectations as of time $t \in [0, T]$ (e.g., $E_0[\cdot]$) are conditional on the realization η of $\tilde{\eta}$.

The optimal terminal wealth $\hat{W}_T$ satisfies the first-order condition: $\partial u\left(\hat{W}_T\right)/\partial W_T = \lambda\pi_T$ for a Lagrange multiplier λ of the budget constraint (A2) that depends on η. The first-order condition for the manager's portfolio problem leads to:

$$\hat{W}_T = (\theta k + \alpha)^{\frac{1}{\gamma}-1}(\lambda\pi_T)^{-\frac{1}{\gamma}} + \frac{\alpha Y_T - (1-\theta)kw\beta_T}{\theta k + \alpha}. \tag{A3}$$

Let Q be the risk-neutral probability and $B_t^Q = B_t + \eta t$, $t \in [0, T]$, denote a standard Brownian motion under Q. The manager's budget constrain can be rewritten as

$$e^{-rT}E_0^Q[\hat{W}_T] = w, \tag{A4}$$

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where $E^Q[\cdot]$ denotes the expectation under Q. Given that both π_T and Y_T are log-normally distributed with constant mean and variance, the expectation in (A4) can be computed explicitly to solve for λ as:

$$\lambda = ((1-\theta)kw + \theta kw + \alpha(w-y))^{-\gamma}(\theta k + \alpha)e^{-(\gamma-1)\left(r+\frac{\eta^2}{2\gamma}\right)T}. \tag{A5}$$

Plugging λ back into (A3) and remembering that $w = y$ we get:

$$\hat{W}_T = \frac{kw}{\theta k + \alpha}e^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T}(\pi_T^\eta)^{-\frac{1}{\gamma}} + \frac{\alpha Y_T - (1-\theta)kw\beta_T}{\theta k + \alpha}. \tag{A6}$$

Replacing the optimal terminal wealth (A6) into the manager's compensation (5) and rearranging gives (11), for $g(\cdot)$ as defined in the Proposition.

Since $e^{-rt}W_t$ is a martingale process under Q, we obtain the optimal interim wealth $\hat{W}_t$ as:

$$\begin{aligned}
\hat{W}_t =& e^{-r(T-t)}E_t^Q[\hat{W}_T] \\
=& e^{-r(T-t)}\frac{kw}{\theta k + \alpha}e^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T}(\pi_t^\eta)^{-\frac{1}{\gamma}}E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\right] + \\
&+ \frac{\alpha}{\theta k + \alpha}e^{-r(T-t)}E_t^Q[Y_T] - \frac{(1-\theta)kw\beta_t}{\theta k + \alpha}.
\end{aligned} \tag{A7}$$

Since the discounted benchmark process is a martingale under Q, $e^{-r(T-t)}E_t^Q[Y_T] = Y_t$. Conditional on time-$t$ information, the process π_T/π_t is log-normally distributed with constant mean and variance, so the expectation in the first term of (A7) can be computed explicitly to yield (10).

To derive the investment policy (9) replicating the optimal portfolio value (10), note that the state-price deflator π_t is a function of t and B_t^Q only. Therefore, the manager's optimal portfolio value can be rewritten as $\hat{W}_t = f(t, B_t^Q)$, for $f \in C^{1,2}$. Applying Itô's Lemma the diffusion term of $d\hat{W}_t$ is:

$$\frac{\eta}{\gamma}\left(\hat{W}_t - \frac{\alpha Y_t - (1-\theta)kw\beta_t}{\theta k + \alpha}\right) + \frac{\alpha}{\theta k + \alpha}\phi^Y\sigma Y_t. \tag{A8}$$

Equating this expression to the diffusion term of dW_t ($\hat{W}_t\hat{\phi}_t\sigma$) and rearranging leads to the optimal portfolio (9). □

Proof of Proposition 2. If $\phi_t = \phi$ for all $t \in [0, T]$ in (8), households' wealth is a geometric Brownian motion process with end-of-period value:

$$W_T^\eta = w\exp\{(r + \phi\sigma\eta - 0.5(\phi\sigma)^2)T + \phi\sigma B_T\}. \tag{A9}$$

Households' ex-ante utility from investing the fixed portfolio ϕ in the stock over the investment

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period is:

$$E^{\tilde{\eta},B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right]=E^{\tilde{\eta}}\left[E_0^{B_T}\left[\left.\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right|\tilde{\eta}=\eta\right]\right], \tag{A10}$$

where for clarity we add a superscript in each expectation operator that indicates the variable whose distribution is used to take expectations (e.g., $E^{\tilde{\eta},B_T}$ is the expectation with respect to the joint distribution of $(\tilde{\eta}, B_T)$).

For a given realization η of $\tilde{\eta}$, W_T^{η} is log-normally distributed with fixed mean and variance. This allows the inner expectation in (A10) to be computed as:

$$E_0^{B_T}\left[\left.\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right|\tilde{\eta}=\eta\right]=\frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\exp\{-(\gamma_h-1)(r+\phi\sigma(\eta-0.5\gamma_h\phi\sigma))T\}. \tag{A11}$$

The RHS of (A11) is log-normally distributed with constant mean and variance, so the expectation in (A11) can be computed as:

$$\begin{aligned}E^{\tilde{\eta},B_T}\left[\frac{\left((1-k_m)W_T^{\tilde{\eta}}\right)^{1-\gamma_h}}{1-\gamma_h}\right]=&\frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\times\exp\{-(\gamma_h-1)(r+\phi\sigma\bar{m}\\&-0.5(\phi\sigma)^2(\gamma_h+(\gamma_h-1)\bar{v}_0T))T\}.\end{aligned} \tag{A12}$$

Households' problem of choosing an ETF can be seen as the problem of finding the portfolio ϕ^P that maximizes their ex-ante utility (A12), i.e.:

$$\max_{\phi}\frac{((1-k_m)w)^{1-\gamma_h}}{1-\gamma_h}\exp\{-(\gamma_h-1)(r+\phi\sigma\bar{m}-0.5(\phi\sigma)^2(\gamma_h+(\gamma_h-1)\bar{v}_0T))T\}. \tag{A13}$$

The objective function is globally concave. Computing the first-order condition for (A13) and solving for ϕ yields the optimal ETF (12). Replacing (12) back into households' ex-ante utility (A11) gives households' reservation utility (13).

Similarly, the manager's ex-ante utility from the fixed portfolio ϕ is given by:

$$E^{\tilde{\eta},B_T}\left[\frac{\left(k_mW_T^{\tilde{\eta}}\right)^{1-\gamma}}{1-\gamma}\right]=\frac{(k_mw)^{1-\gamma}}{1-\gamma}\exp\{-(\gamma-1)(r+\phi\sigma\bar{m}-0.5(\phi\sigma)^2(\gamma+(\gamma-1)\bar{v}_0T))T\}. \tag{A14}$$

Plugging households' chosen passive portfolio (12) into (A14) gives, after some algebra, the manager's reservation utility (14).

□

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Proof of Proposition 3. From Eq. (17), the after-fee end-of-period wealth of households is:

$$W_{CI,T}^{\tilde{\eta}} = (1 - k_{CI}(\tilde{\eta}))we^{-rT+\left(1-\frac{1}{2\gamma_h}\right)\frac{\tilde{\eta}^2}{2\gamma_h}T+\frac{\tilde{\eta}}{\gamma_h}B_T}.$$

Given η, $W_{CI,T}^{\eta}$ is log-normally distributed with constant mean and variance:

$$E_0\left[\frac{(W_{CI,T}^{\tilde{\eta}})^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right] = \frac{((1-k_{CI}(\eta))w)^{1-\gamma_h}}{1-\gamma_h}\exp\left\{-(\gamma_h-1)\Big(r+\frac{\eta^2}{2\gamma_h}\Big)T\right\}$$

Households' expected utility is then:

$$\begin{aligned} U_{h,CI}(w) \equiv E\left[\frac{(W_{CI,T}^{\tilde{\eta}})^{1-\gamma_h}}{1-\gamma_h}\right] =& E\left[E_0\left[\frac{(W_{CI,T}^{\tilde{\eta}})^{1-\gamma_h}}{1-\gamma_h}\middle|\tilde{\eta}=\eta\right]\right] \\ =& \frac{w^{1-\gamma_h}}{1-\gamma_h}\int_{-\infty}^{+\infty}(1-k_{CI}(x))^{1-\gamma_h}e^{-(\gamma_h-1)\left(r+\frac{x^2}{2\gamma_h}\right)T}\mathbf{n}_\eta(x)dx, \end{aligned}$$

where $\mathbf{n}_\eta(\cdot)$ is the normal probability density function of $\tilde{\eta}$.

Following the same reasoning, the manager's end-of-period wealth is:

$$k_{CI}(\eta)\hat{W}^{\eta;0,\phi^Y} = k_{CI}(\eta)we^{-rT+\left(1-\frac{1}{2\gamma}\right)\frac{\eta^2}{2\gamma}T+\frac{\eta}{\gamma}B_T},$$

so

$$E_0\left[\frac{(k_{CI}(\tilde{\eta})\hat{W}^{\tilde{\eta};0,\phi^Y})^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right] = \frac{(k_{CI}(\eta)w)^{1-\gamma}}{1-\gamma}\exp\left\{-(\gamma-1)\Big(r+\frac{\eta^2}{2\gamma}\Big)T\right\}. \tag{A15}$$

From Eqs. (12) and (A11), the manager's utility under passive management in state η is:

$$\frac{(k_m w)^{1-\gamma}}{1-\gamma}\exp\left\{-(\gamma-1)\left(r+\frac{\bar{m}}{\gamma_h+(\gamma_h-1)\bar{v}_0T}\left(\eta-\frac{\gamma\bar{m}}{2(\gamma_h+(\gamma_h-1)\bar{v}_0T)}\right)\right)T\right\} \tag{A16}$$

Equating (A15) to c times (A16) and solving for $k_{CI}(\eta)$ gives (18). To obtain (20), note that the equality between (A15) and c times (A16) holds state by state. Therefore, the manager's ex-ante utility in the CI case has to equal c times his reservation utility (14). □

Proof of Theorem 1. First, we show that for a positive load-fee contract to be optimal it must also include positive fulcrum fees with respect to a riskless benchmark, hence be equivalent to a contract with proportional and fulcrum fees only. Thus, load fees are either suboptimal or redundant within the class of contracts (5), and $\hat{\theta}$ can be set to equal 1 at an optimum.

To show this, we note that for an arbitrary contract $\mathscr{C}$ the optimal after-fee wealth of households

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at the end of the period is:

$$\begin{aligned}\hat{W}_T^{\tilde{\eta};\mathscr{C}} - X_T^{\tilde{\eta};\mathscr{C}} =& \frac{1-(\theta k+\alpha)}{\theta k+\alpha} X^{\tilde{\eta};\mathscr{C}} + \frac{\alpha Y_T^{\tilde{\eta};\phi^Y} - (1-\theta)kw\beta_T}{\theta k+\alpha} \\ =& \frac{1-(\theta k+\alpha)}{\theta k+\alpha} kw \exp\left\{ r + (1 - \frac{1}{2\gamma}\frac{\tilde{\eta}^2}{\gamma})T + \frac{\tilde{\eta}}{\gamma} B_T \right\} \\ &+ \frac{\alpha y \exp\left\{ \left(r - \frac{1}{2}(\phi^Y\sigma)^2 + \phi^Y\sigma\tilde{\eta}\right)T + \phi^Y \sigma B_T \right\} - (1-\theta)kw\beta_T}{\theta k+\alpha} \end{aligned} \tag{A17}$$

where in the second and third lines we used Eq. (11) and wrote $\pi_T^{\tilde{\eta}}$ and $Y_T^{\tilde{\eta};\phi^Y}$ explicitly in terms B_T. Households derive negative infinite utility in states in which (A17) is nonpositive. Thus, a necessary condition for a fee contract to be optimal is that it avoids states of nonpositive after-fee wealth for households, i.e. (given $w = y$):

$$\begin{aligned}&\big(1-(\theta k+\alpha)\big)k \exp\left\{ r + \big(1 - \frac{1}{2\gamma}\frac{\tilde{\eta}^2}{\gamma}\big)T + \frac{\tilde{\eta}}{\gamma} B_T \right\} + \\ &\qquad \alpha \exp\left\{ \big(r - \frac{1}{2}(\phi^Y\sigma)^2 + \phi^Y\sigma\tilde{\eta}\big)T + \phi^Y\sigma B_T \right\} > (1-\theta)k\beta_T. \end{aligned}$$

for all realizations of $(\tilde{\eta}, B_T)$. Since $\tilde{\eta}$ and B_T are independently distributed normal variables, for any $\theta < 1$ and risky benchmark $\phi^Y \neq 0$ there exist nonzero-probability events under which this condition is not satisfied (e.g., when ϕ^Y and η are positive the left hand side of the condition tends to zero as $B_T \to -\infty$). Thus, for a positive load fee $1-\theta^* > 0$ to be optimal it must be that the benchmark is riskless, $\phi^Y = 0$, in which case $Y_T^{\tilde{\eta};0} = y\beta_T$ is constant for all realizations of $(\tilde{\eta}, B_T)$. Moreover, the associated fulcrum fee α^* must be positive and greater than $(1-\theta^*)k^*$, for the corresponding proportional fee k^*. Letting $\mathscr{C}^* = (k^*, \theta^*, \alpha^*, 0)$ be the corresponding positive-load fee contract, the manager's compensation is:

$$\begin{aligned} X_T^* =& (1-\theta^*)k^* w\beta_T + \theta^* k^* W_T + \alpha^*(W_T - w\beta_T) \\ =& (\theta^* k^* + \alpha^*)W_T - (\alpha^* - (1-\theta^*)k^*)w\beta_T \\ =& (k^{**} + \alpha^{**})W_T - \alpha^{**} w\beta_T \\ =& k^{**} W_T + \alpha^{**}(W_T - w\beta_T), \end{aligned}$$

where $k^{**} = k^*$ and $\alpha^{**} \equiv \alpha^* - (1-\theta^*)k^*$. Thus, any feasible positive-load fee contract $\mathscr{C}^*$ is equivalent to a proportional-plus-fulcrum fees contract $\mathscr{C}^{**} = (k^{**}, 1, \alpha^{**}, 0)$ with no load fees, and without loss of generality we set $\hat{\theta} = 1$ at the optimum.

Next, we find the optimal proportional fee $\hat{k}$. From (11), the only contract parameter that affects the manager's optimal compensation $\hat{X}_T^{\eta;k}$ is the base management fee k. Let U^k be the manager's

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expected utility for a given fee k, i.e:

$$U^k \equiv E\left[\frac{(\hat{X}_T^{\tilde{\eta};k})^{1-\gamma}}{1-\gamma}\right].$$

Given η, $\hat{X}_T^{\eta;k} = kw\exp\{(r+(1-1/(2\gamma))\eta^2/\gamma)T+\eta/\gamma B_T\}$ is log-normally distributed with constant mean and variance, so:

$$E_0\left[\frac{(\hat{X}_T^{\tilde{\eta};k})^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right] = \frac{(kw)^{1-\gamma}}{1-\gamma}\exp\left\{-(\gamma-1)\Big(r+\frac{\eta^2}{2\gamma}\Big)T\right\}.$$

The manager's expected utility is then:

$$\begin{aligned} E\left[E_0\left[\frac{(\hat{X}_T^{\tilde{\eta};k})^{1-\gamma}}{1-\gamma}\middle|\tilde{\eta}=\eta\right]\right] =& \frac{(kw)^{1-\gamma}}{1-\gamma}\exp\{-(\gamma-1)rT\} \\ &\times E\left[\exp\left\{-\frac{\gamma-1}{2\gamma}T\Big(\eta-\bar{m}+\bar{m}\Big)^2\right\}\right]. \end{aligned}$$

Letting $z=\eta-\bar{m}\sim\mathbf{N}(0,\bar{v}_0)$, $\rho=-\frac{\gamma-1}{2\gamma}T$, $c=-\bar{m}$ and $\bar{z}=\infty$, applying Lemma A1 and rearranging, we get

$$U^k = \frac{(kw)^{1-\gamma}}{1-\gamma}\frac{\exp\left\{-(\gamma-1)\left(r+\frac{1}{\gamma+(\gamma-1)\bar{v}_0T}\frac{\bar{m}^2}{2}\right)T\right\}}{\sqrt{1+\Big(1-\frac{1}{\gamma}\Big)\bar{v}_0T}}.$$

The optimal management fee (21) then results from solving for the value of k that equates U^k to $c\bar{U}$.

To complete the proof we note that, setting k and θ to their optimal values $\hat{k}$ and $\hat{\theta}$, the optimization program (6) reduces to finding an interior solution $\{\hat{\alpha},\hat{\phi}^Y\}$ to the following (unconstrained) maximization problem:

$$\max_{\{\alpha,\phi^Y\}} E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};(\hat{k},1,\alpha,\phi^Y)}-\hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right], \tag{A18}$$

and verifying that the solution satisfies the participation constraint of households (HH's PC). In (A18), $\hat{W}_T^{\tilde{\eta};(\hat{k},1,\alpha,\phi^Y)}$ and $\hat{X}_T^{\tilde{\eta};\hat{k}}$ are as given by Eqs. (10) and (11), and expectations are with respect to the joint distribution of $(\tilde{\eta}, B_T)$.

Rewriting (A17) in terms of π and Y:

$$\hat{W}_T^{\eta;(\hat{k},1,\alpha,\phi^Y)} - \hat{X}_T^{\eta;\hat{k}} = \frac{1-(\hat{k}+\alpha)}{\hat{k}+\alpha}\hat{k}we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\eta^2}{2\gamma}\right)T}(\pi_T^{\eta})^{-\frac{1}{\gamma}} + \frac{\alpha}{\hat{k}+\alpha}Y_T^{\eta;\phi^Y}. \tag{A19}$$

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For a given (α, ϕ^Y), define households' indirect utility function V as

$$V(\alpha, \phi^Y) \equiv E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};(\hat{k},1,\alpha,\phi^Y)} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right]. \tag{A20}$$

The first-order conditions for an interior solution $(\hat{\alpha}, \hat{\phi}^Y)$ to the maximization problem (A18) are:

$$\frac{\partial V(\hat{\alpha}, \hat{\phi}^Y)}{\partial \alpha} = \frac{1}{(\hat{k}+\hat{\alpha})^2} E\left[\left(\hat{W}_T^{\tilde{\eta};(\hat{k},1,\alpha,\phi^Y)} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\left(\hat{k}Y_T^{\tilde{\eta};\hat{\phi}^Y} - \hat{k}we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] = 0 \tag{A21}$$

$$\frac{\partial V(\hat{\alpha}, \hat{\phi}^Y)}{\partial \phi^Y} = \frac{1}{\hat{k}+\hat{\alpha}} E\left[\left(\hat{W}_T^{\tilde{\eta};(\hat{k},1,\alpha,\phi^Y)} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\hat{\alpha}Y_T^{\tilde{\eta};\hat{\phi}^Y}\left(\sigma(\tilde{\eta}-\hat{\phi}^Y\sigma)T + \sigma B_T\right)\right] = 0 \tag{A22}$$

Clearly, $\alpha = 0$ satisfies (A22) for any ϕ^Y. To also satisfy (A21), it must be that:

$$\begin{aligned}
\frac{\partial V(0, \hat{\phi}^Y)}{\partial \alpha} =& E\left[\left(\hat{W}_T^{\tilde{\eta};0,\hat{\phi}^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{-\gamma_h}\left(Y_T^{\tilde{\eta};\hat{\phi}^Y} - we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] \\
=& E\left[\left((1-\hat{k})we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h}\right. \\
&\left.\times\left(Y_T^{\tilde{\eta};\hat{\phi}^Y} - we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)\right] \\
=& 0.
\end{aligned} \tag{A23}$$

According to (A23), $\alpha = 0$ satisfies the first-order conditions if and only if

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h} Y_T^{\tilde{\eta};\hat{\phi}^Y}\right] = E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right] \tag{A24}$$

Using iterated expectations, both sides of (A24) can be computed explicitly by first taking expectations relative to the distribution of B_T for a given η, and then taking expectations with respect to the distribution of $\tilde{\eta}$ (i.e., following the approach in (A10) using the result in Lemma A1). This

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procedure yields:

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{-\gamma_h} Y_T^{\tilde{\eta};\hat{\phi}^Y}\right] = \gamma w^{1-\gamma_h} \times \frac{\exp\left\{-\frac{(\gamma-\gamma_h)^2\bar{v}_0 T\frac{(\phi^Y\sigma)^2}{2}+(\gamma-\gamma_h)\gamma\phi^Y\sigma\bar{m}-(2\gamma-\gamma_h-1)\gamma_h\frac{\bar{m}^2}{2}}{\gamma^2+(2\gamma-\gamma_h-1)\gamma_h\bar{v}_0 T}T\right\}}{\sqrt{\gamma^2+(2\gamma-\gamma_h-1)\gamma_h\bar{v}_0 T}}, \tag{A25}$$

and

$$E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right] = \gamma w^{1-\gamma_h}\frac{\exp\left\{-\frac{(\gamma_h-1)(2\gamma-\gamma_h)}{\gamma^2+(\gamma_h-\gamma)(2\gamma-\gamma_h)\bar{v}_0 T}\frac{\bar{m}^2}{2}T\right\}}{\sqrt{\gamma^2+(\gamma_h-\gamma)(2\gamma-\gamma_h)\bar{v}_0 T}}. \tag{A26}$$

Note that, for all ϕ^Y:

$$V(0,\phi^Y) = E\left[\frac{\left(\hat{W}_T^{\tilde{\eta};0,\phi^Y} - \hat{X}_T^{\tilde{\eta};\hat{k}}\right)^{1-\gamma_h}}{1-\gamma_h}\right] = \frac{((1-\hat{k})w)^{1-\gamma_h}}{1-\gamma_h} E\left[\left(we^{\left(1-\frac{1}{\gamma}\right)\left(r+\frac{\tilde{\eta}^2}{2\gamma}\right)T}(\pi_T^{\tilde{\eta}})^{-\frac{1}{\gamma}}\right)^{1-\gamma_h}\right]. \tag{A27}$$

Equation (A27) shows that $V(0,\phi^Y)$ does not depend on ϕ^Y, so changing ϕ^Y (given that any value of ϕ^Y satisfies (A22) when $\alpha = 0$) cannot increase or decrease the expected utility of households. In turn, this implies that if $\alpha = 0$ solves problem (A18) then (A24) has to be satisfied for all values of ϕ^Y.[32] Comparing (A25) and (A26), this is the case if and only if $\gamma = \gamma_h$. Thus, $\alpha = 0$ is optimal if and only if $\gamma = \gamma_h$. For $\gamma \neq \gamma_h$, (A24) is not satisfied for some values of ϕ^Y (e.g., for $\phi^Y = 0$). By continuity of $V(\cdot,\cdot)$ in α and ϕ^Y at $(0,\phi^Y)$, there exists some $\bar{\alpha} \neq 0$ and $\bar{\phi}^Y$ such that $V(\bar{\alpha},\bar{\phi}^Y) > V(0,\phi^Y)$ for all ϕ^Y. Moreover, If a solution $(\hat{\alpha},\hat{\phi}^Y)$ exists to problem (A18), then it has $\hat{\alpha} \neq 0$. □

Proof of Lemma 1. That purely proportional fees implement households' CI portfolio (16) when $\gamma_h = \gamma$ follows immediately from setting $\alpha = 0$ in (9) and $\gamma_h = \gamma$ in (16), and comparing the two.

[32] To see this, suppose that (A21) is satisfied for $\hat{\phi}^Y$ but not for some $\bar{\phi}^Y \neq \hat{\phi}^Y$. Without loss of generality, assume $\partial V(0,\bar{\phi}^Y)/\partial\alpha > 0$. Since $V(0,\bar{\phi}^Y) = V(0,\hat{\phi}^Y)$, $\partial V(0,\bar{\phi}^Y)/\partial\alpha > 0$ implies that there exists some $\epsilon > 0$ such that households can improve on their indirect utility relative to $(\alpha,\phi^Y) = (0,\hat{\phi}^Y)$ by choosing $(\alpha,\phi^Y) = (\epsilon,\bar{\phi}^Y)$. Thus, $(\alpha,\phi^Y) = (0,\hat{\phi}^Y)$ cannot maximize (A18). By a similar argument it cannot minimize (A18) either, which means that $(\alpha,\phi^Y) = (0,\hat{\phi}^Y)$ corresponds to a saddle point of $V(\cdot)$.

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For $\gamma_h \neq \gamma$, setting $\alpha = 0$ leads the manager to choose a portfolio

$$\hat{\phi}_t^{\eta;(\hat{k},1,0,\phi^Y)} = \frac{\eta}{\gamma\sigma} \neq \frac{\eta}{\gamma_h\sigma} = \phi_{CI}^{\eta}. \tag{A28}$$

For $\alpha \neq 0$, perfect alignment of the manager's and households CI portfolio would require choosing a fulcrum fee $\breve{\alpha}$ and a benchmark composition $\breve{\phi}^Y$ such that

$$\frac{\breve{\alpha}}{\hat{k} + \breve{\alpha}} \frac{R_t^Y}{R_t^{\hat{W}}} \left(\breve{\phi}^Y - \frac{\eta}{\gamma\sigma} \right) = (\gamma - \gamma_h) \frac{\eta}{\gamma_h \gamma \sigma} \tag{A29}$$

for all $(\eta, R_t^Y / R_t^{\hat{W}})$-states. Condition (A29) represents a continuum of equations with only two unknowns (α, ϕ^Y). Because no solution to the associated system exists, no linear contract $\mathscr{C}$ can perfectly align the manager's portfolio with the CI portfolio of households. □

Proof of Proposition 4. From (10), the end-of-period active fund value under the pure-proportional (APP) fee contract is:

$$W_T^{APP} = w \exp\left\{ rT + \left(1 - \frac{1}{2\gamma}\right) \frac{\eta^2}{\gamma} T + \frac{\eta}{\gamma} B_T \right\}$$

Given a proportional fee k, households' ex-ante utility under this contract is:

$$U_{h,APP} \equiv E^{\tilde{\eta}, B_T} \left[\frac{\left((1-k) W_T^{APP}\right)^{1-\gamma_h}}{1-\gamma_h} \right] = E^{\tilde{\eta}} \left[E_0^{B_T} \left[\frac{\left((1-k) W_T^{APP}\right)^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\eta} = \eta \right] \right].$$

For a given realization η of $\tilde{\eta}$, W_T^{APP} is log-normally distributed with fixed mean and variance. This allows the inner expectation to be computed as:

$$E_0^{B_T} \left[\frac{\left((1-k) W_T^{APP}\right)^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\eta} = \eta \right] = \frac{((1-k)w)^{1-\gamma_h}}{1-\gamma_h} \exp\left\{ -(\gamma_h - 1)\left(r + \left(1 - \frac{\gamma_h}{2\gamma}\right) \frac{\eta^2}{\gamma} \right) T \right\}.$$

The households' expected utility is then:

$$\begin{aligned} E\left[E_0 \left[\frac{\left((1-k) W_T^{APP}\right)^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\eta} = \eta \right] \right] =& \frac{((1-k)w)^{1-\gamma_h}}{1-\gamma_h} \exp\{-(\gamma_h - 1) rT\} \\ & \times E\left[\exp\left\{ -(\gamma_h - 1) \frac{2\gamma - \gamma_h}{2\gamma^2} T \left(\eta - \bar{m} + \bar{m} \right)^2 \right\} \right]. \end{aligned}$$

Letting $z = \eta - \bar{m} \sim \mathbf{N}(0, \bar{v}_0), \rho = -(\gamma_h - 1)\frac{2\gamma - \gamma_h}{2\gamma^2} T, c = -\bar{m}$ and $\bar{z} = \infty$, applying Lemma A1 to

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the expectation on the RHS and rearranging, we get:

$$U_{h,APP}(w) = \gamma \frac{((1-k)w)^{1-\gamma_h}}{1-\gamma_h} \frac{\exp\left\{-(\gamma_h - 1)\left(r + \frac{2\gamma-\gamma_h}{\gamma^2+(\gamma_h-1)(2\gamma-\gamma_h)\bar{v}_0 T}\frac{\bar{m}^2}{2}\right)T\right\}}{\sqrt{\gamma^2 + (\gamma_h - 1)(2\gamma - \gamma_h)\bar{v}_0 T}}.$$

The certainty equivalent return of this ex-ante utility, $CER^{APP} \equiv 1/w[(1-\gamma_h)U_{h,APP}(w)]^{\frac{1}{1-\gamma_h}}$, is:

$$CER^{APP} = \gamma^{\frac{1}{1-\gamma_h}} \frac{(1-k)\exp\left\{\left(r + \frac{2\gamma-\gamma_h}{\gamma^2+(\gamma_h-1)(2\gamma-\gamma_h)\bar{v}_0 T}\frac{\bar{m}^2}{2}\right)T\right\}}{\sqrt{(\gamma^2 + (\gamma_h - 1)(2\gamma - \gamma_h)\bar{v}_0 T)^{\frac{1}{1-\gamma_h}}}}.$$

In turn, the certain equivalent return of the reservation utility of households (13), $CER^P \equiv 1/w[(1-\gamma_h)U_{h,P}(w)]^{\frac{1}{1-\gamma_h}}$, is

$$CER^P = (1-k_m)\exp\left\{\left(r + \frac{1}{\gamma_h + (\gamma_h - 1)\bar{v}_0 T}\frac{\bar{m}^2}{2}\right)T\right\}.$$

The passive fund dominates the pure-proportional fee active fund with proportional fee k iff $\ln(CER^{APP}/CER^P) < 0$, i.e., iff:

$$\begin{aligned}\ln(1-k) <& \ln(1-k_m) + \frac{1}{\gamma_h - 1}\ln\gamma - \frac{1}{2(\gamma_h-1)}\ln\left(\gamma^2 + (\gamma_h - 1)(2\gamma-\gamma_h)\bar{v}_0 T\right) \\ &+ \left(\frac{1}{\gamma_h + (\gamma_h-1)\bar{v}_0 T} - \frac{2\gamma - \gamma_h}{\gamma^2 + (\gamma_h-1)(2\gamma-\gamma_h)\bar{v}_0 T}\right)\frac{\bar{m}^2}{2}T. \qquad \text{(A30)}\end{aligned}$$

Since $\ln(1-k) \leq 0$ for any $k \geq 0$, a sufficient condition for the dominance of any active funds with proportional-only fees by the passive fund is that the RHS of (A30) is positive, leading to condition (23). □

B Irrelevance of fulcrum fees under CARA preferences

We state and prove the following:

Proposition B1. *When households and the manager have constant absolute risk aversion (CARA) preferences with coefficients γ_h and γ respectively, a pure-proportional fee contract $\hat{\mathscr{C}} = (\hat{k}, \hat{\theta}, \hat{\alpha}, \hat{\phi}^Y) = (\gamma_h/(\theta\gamma), \theta, 0, \phi^Y)$, $\theta \in (0,1]$, $\phi^Y \in \mathbb{R}$, achieves, for each realization η of $\tilde{\eta}$, perfect alignment of the actively managed portfolio with the contractible-information portfolio of households. In contrast, nonzero fulcrum fees imply a misalignment between the two portfolios.*

Proof. The utility function of households is:

$$u_h(w) = -\frac{1}{\gamma_h}e^{-\gamma_h w}, \qquad \text{(B1)}$$

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whereas the utility function of the manager is:

$$u(w) = -\frac{1}{\gamma} e^{-\gamma w}. \tag{B2}$$

Following the approach in the proof of Proposition 1, for each realization η of $\tilde{\eta}$ the manager solves:

$$\max_{W_T} E\left[e^{-\gamma((1-\theta)kw\beta_T + \theta k W_T + \alpha(W_T - Y_T))} \middle| \tilde{\eta} = \eta \right], \tag{B3}$$

$$s.t. \qquad e^{-rT} E_0^Q \left[W_T \right] = w, \tag{B4}$$

where for notational simplicity we omit the superscripts $(\eta; \mathscr{C})$ that associate the processes with the realized value of $\tilde{\eta}$ and with the contract parameters $\mathscr{C}$. For the remainder of this proof, conditional expectations as of time $t \in [0, T]$ (e.g., $E_0[\cdot]$) are conditional on the realization η of $\tilde{\eta}$.

The manager's optimal terminal wealth $\hat{W}_T$ satisfies the first-order condition: $\partial u\left(\hat{W}_T\right)/\partial W_T = \lambda \pi_T$ for a Lagrange multiplier λ of the budget constraint (B4). The first-order condition for the manager's portfolio problem (B3) leads to:

$$\hat{W}_T = \frac{1}{\gamma(\theta k + \alpha)} (\ln(\theta k + \alpha) - \ln(\lambda \pi_T)) + \frac{\alpha}{\theta k + \alpha} Y_T - \frac{(1-\theta)kw\beta_T}{\theta k + \alpha}. \tag{B5}$$

Since $e^{-rt} W_t$ is a martingale process under Q, we obtain the optimal interim wealth $\hat{W}_t$ as:

$$\begin{aligned} \hat{W}_t =& e^{-r(T-t)} E_t^Q [\hat{W}_T] \\ =& \frac{\left(r - \frac{\eta^2}{2}\right)(T-t) + \ln \frac{\theta k + \alpha}{\lambda} - \gamma(1-\theta)kw\beta_T}{\gamma(\theta k + \alpha)} e^{-r(T-t)} + \frac{\alpha}{\theta k + \alpha} Y_t - \frac{e^{-r(T-t)}}{\gamma(\theta k + \alpha)} \ln \pi_t. \end{aligned} \tag{B6}$$

To derive the manager's optimal investment policy $\hat{\varphi}_t^\eta$, note that the manager's optimal portfolio value can be rewritten as $\hat{W}_t = f(t, B_t^Q)$, for $f \in C^{1,2}$. Applying Itô's Lemma the diffusion term of $d\hat{W}_t$ is:

$$\frac{e^{-r(T-t)}}{\theta k + \alpha} \frac{\eta}{\gamma} + \phi^Y \sigma \frac{\alpha}{\theta k + \alpha} Y_t. \tag{B7}$$

Equating this expression to the diffusion term of dW_t ($\hat{\varphi}_t^\eta \sigma$) we obtain the manager's optimal amount $\hat{\varphi}^\eta$ in the risky asset portfolio:

$$\hat{\varphi}_t^\eta = \frac{1}{\theta k + \alpha} \left(e^{-r(T-t)} \frac{\eta}{\gamma \sigma} + \alpha \phi^Y Y_t \right). \tag{B8}$$

In the CI case of Section 5.1, households solve:

$$\max_{W_T} E\left[e^{-\gamma_h W_T} \middle| \tilde{\eta} = \eta \right], \tag{B9}$$

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$$s.t. \qquad e^{-rT} E_0^Q [W_T] = w. \tag{B10}$$

Note that problem (B9)-(B10) is the same as the manager's problem (B3)-(B4) for $\theta k = \gamma_h/\gamma$ and $\alpha = 0$. Households' CI portfolio can then be computed from (B8) as:

$$\varphi_{CI,t}^{\eta} = e^{-r(T-t)} \frac{\eta}{\gamma_h \sigma}. \tag{B11}$$

This implies that households can achieve perfect portfolio alignment by choosing $\hat{k} = \gamma_h/(\theta\gamma)$ and $\hat{\alpha} = 0$, for $\theta \in (0, 1]$. If $\alpha \neq 0$, then no value of the fulcrum fee α and the benchmark allocation in the stock ϕ^Y can equate Eqs. (B8) and (B11) across all η-states, implying a misalignment between the managed portfolio and the CI portfolio of households under non-zero fulcrum fees. □

C Model Parameterizations

We use the following baseline and alternative parameterizations in the numerical analysis of our model. We identify the riskless asset β with the 3-month U.S. Treasury bill and the stock S with a broad-based market portfolio. We set the real riskless interest rate r equal to 3%. For the prior value $\bar{m}$ of the market price of risk and for the market volatility σ, we use historical estimates during the sample period January 1980-December 2006. This corresponds to a relatively recent and long period over which the hypothesis of normality of annual returns cannot be rejected.[33] Following Brennan and Xia (2001), we set the prior value for the market excess return $\mu - r$ equal to the sample mean return of the Fama and French (1996) market portfolio during the period, 8.1%. The corresponding standard deviation of the market portfolio, σ, equals 15.8%. We set the baseline prior variance $\bar{v}_0$ equal to the square of the standard error of the sample mean market price of risk. This standard error equals 0.2 for the period 1980-2006 and corresponds to a standard error for the mean return of 3.2%, in line with baseline values used in the literature. Our alternative parameterizations of $\bar{v}_0$ consider both low ($\sqrt{\bar{v}_0} = 0.1$) and high ($\sqrt{\bar{v}_0} = 0.3$) levels of uncertainty corresponding, respectively, to small and large information advantages of the manager relative to households.

We consider investment horizons T of 1 to 5 years. We take the middle of the range, $T = 3$, as our baseline value, as it agrees with the average performance evaluation period for fulcrum fees in the US mutual fund industry (Cuoco and Kaniel, 2011). Based on data from the Investment Company Institute, we set the management fee k_m for passive funds (ETFs) to 15 basis points (bps). In line with standard practice in the industry (see, e.g., Elton et al., 2003), we constrain the benchmark-linked performance fee to be non-negative ($\alpha \geq 0$) across all parameterizations. To facilitate the comparison of the value of the different contract arrangements, we set the relative

[33] The Jarque-Bera test of normality results in a p-value of 16.6% during this period.

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bargaining power constant c to 1 for most of our analysis.

We assume a coefficient of relative risk aversion $\gamma = 5$ for the manager in order to approximately match the mean estimate of Koijen (2014) in a setup similar to ours. Following the dispersion in the relative risk aversion coefficients of households that Kimball et al. (2008) extract from the Health and Retirement Survey, we consider both relatively risk tolerant ($\gamma_h < 5$) and risk averse ($\gamma_h > 5$) fund investors.

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Internet Appendix to Accompany Carrot *and* Stick: A Risk-Sharing Rationale for Fulcrum Fees in Active Fund Management

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In this Appendix, we generalize our analysis in "Carrot *and* Stick: A Risk-Sharing Rationale for Fulcrum Fees in Active Fund Management" (henceforth, referred to as to RSRFF) to (i) allow the manager to have *partial* (instead of *complete*) information about the return fundamental $\tilde{\eta}$, and (ii) enlarge the managerial contract space to include an *asymmetric* benchmark-adjusted performance fee.

A More General Information and Contract Structures

A.1 Imperfect Signal

We assume that the manager's information about the stock fundamentals can be *imperfect.* More precisely, at $t = 0$ she has access to a noisy signal $\tilde{\theta} = \tilde{\eta} + \tilde{\epsilon}$ about the market price of risk $\tilde{\eta}$, where $\tilde{\epsilon}$ is an independent and normally distributed noise term: $\tilde{\epsilon} \sim \mathbf{N}(0, \sigma_\epsilon^2)$. The realized value θ of the signal affects the manager's perception about the dynamics of the stock price S, the fund's assets under management (AUM) W and the value Y of the benchmark. For notational simplicity, however, we omit the superscript indicating the dependence of these processes on θ and on the contract parameters that we introduce next.

A.2 Asymmetric Performance Fees

The fund charges a fee rate f_T that ties the manager's compensation to performance according to $f_T = f(R_T^W, R_T^Y; \kappa_1, \kappa_2, \bar{\kappa}_3, \bar{\delta})$, with $R_T^W \equiv W_T/W_0$, $R_T^Y \equiv Y_T/Y_0$, $\kappa_1, \kappa_2, \kappa_3, \bar{\delta} > 0$,

$$f(x, y; k_1, k_2, k_3, \delta) = k_1 x + k_2(x - y) + k_3(x - \delta y)^+, \tag{IA.1}$$

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and $x^+ \equiv \max(x, 0)$.

Fee contract (IA.1) is similar to the specification of Cuoco and Kaniel (2011) and consists of three components: a *proportional* fee $\kappa_1 R_T^W$, a fulcrum performance fee $\kappa_2(R_T^W - R_T^Y)$, and an *asymmetric* benchmark-linked performance fee $\bar{\kappa}_3(R_T^W - \bar{\delta} R_T^Y)^+$. The proportional and fulcrum fee rates κ_1 and κ_2 are identical to the fee rates k and $k\alpha$, respectively, of Section 3.2 in RSRFF. The asymmetric performance fee rate $\bar{\kappa}_3$ rewards the fund's excess performance over the benchmark, scaled by a threshold $\bar{\delta}$. Relative to the class of fee contracts considered in RSRFF, (IA.1) encompasses also the type of incentives fees vastly used in the hedge fund industry, as well as implicit asymmetric incentives (e.g., investors' flows as a function of past relative performance) between a fund manager and the fund owners (see the literature review in Section 2 of RSRFF).

Letting $\delta \equiv \bar{\delta} W_0 / Y_0$, total managerial compensation can be expressed as:

$$\begin{aligned} X_T \equiv f_T W_0 &= \kappa_1 W_T + \kappa_2 (W_T - \delta Y_T) + \bar{\kappa}_3 (W_T - \delta Y_T)^+ \\ &= f\left(W_T, Y_T; \kappa_1, \kappa_2, \bar{\kappa}_3, \delta\right). \end{aligned} \tag{IA.2}$$

A management fee contract $\mathscr{C}$ specifies the different fee rates κ_1, κ_2 and $\bar{\kappa}_3$, the benchmark's stock allocation ϕ^Y and the threshold δ. Given a contract $\mathscr{C}$ and a realization θ of the signal at $t = 0$, the fund manager chooses a dynamic investment policy ϕ_t representing the fraction of the fund's wealth W_t allocated in the risky asset over the investment period $t \in [0, T]$. The policy seeks to maximize the manager's conditional expected utility over end-of-period compensation X_T:

$$E\left[\left.\frac{X_T^{1-\gamma}}{1-\gamma}\right| \tilde{\theta} = \theta\right], \tag{IA.3}$$

subject to the self-financing constraint for assets under management:

$$dW_t = W_t \left(r + \phi_t \sigma \eta\right) dt + W_t \phi_t \sigma dB_t, \tag{IA.4}$$

and initial wealth $W_0 = w$.

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A.3 Nonlinear Contracting Problem

Under asymmetric performance fees, the contracting problem of Section 4 in RSRFF can be rewritten as:

$$\max_{\{\kappa_1,\kappa_2,\bar{\kappa}_3,\phi^Y,\delta\}} E\left[\frac{\left(\hat{W}_T - \hat{X}_T\right)^{1-\gamma_h}}{1-\gamma_h}\right] \tag{IA.5}$$

$$s.t. = \begin{cases} \forall\theta: \quad \hat{W}_T = \arg\max_{W_T} \tilde{E}\left[\frac{f\left(W_T, Y_T; \kappa_1, \kappa_2, \bar{\kappa}_3, \phi^Y, \delta\right)^{1-\gamma}}{1-\gamma}\middle| \tilde{\theta} = \theta\right], & \text{(M's ICC)} \\ E\left[\frac{(\hat{X}_T)^{1-\gamma}}{1-\gamma}\right] \geq \bar{U}, & \text{(M's PC)} \\ E\left[\frac{\left(\hat{W}_T - \hat{X}_T\right)^{1-\gamma_h}}{1-\gamma_h}\right] \geq U_{h,P}, & \text{(HH's PC)} \end{cases}$$

where the expectations in (IA.5), (M's PC) and (HH's PC) are with respect to the joint distribution of $(\tilde{\eta}, \tilde{\epsilon}, B_T)$. As in Section 4.2 of RSRFF, the reservation utilities $\bar{U}$ and $U_{h,P}$ of, respectively, the manager and households, are given by the utility they derive under passive management of the households' wealth—i.e., the ETF the manager offers following the portfolio composition that households optimally choose. Because neither $\bar{U}$ nor $U_{h,P}$ depend on θ, the explicit expressions for these reservation utilities remain the same as those in Section 4.2 of RSRFF. We next analyze the active manager's incentive compatibility constraint (M's ICC) by solving for the optimal response $\hat{W}_T$ to a given contract $\mathscr{C}$.

B Optimal Active Fund Portfolio under Asymmetric Fees

B.1 Filtering Problem

As long as $\sigma_\epsilon < \infty$ the realization of the private signal θ provides the manager with a more accurate initial assessment, relative to the common prior $\mathbf{N}(\bar{m}, \bar{v}_0)$ she shares with households, of the realized market price of risk η. Following the projection theorem for normal variables, after observing θ the manager updates her prior of η to a normally distributed variable with conditional mean:

$$m \equiv E[\tilde{\eta}|\tilde{\theta} = \theta] = \frac{\sigma_\epsilon^2 \bar{m} + \bar{v}_0 \theta}{\bar{v}_0 + \sigma_\epsilon^2}, \tag{IA.6}$$

and conditional variance $v_0 \equiv \text{var}[\tilde{\eta}|\tilde{\theta} = \theta] = \bar{v}_0 \sigma_\epsilon^2 / (\bar{v}_0 + \sigma_\epsilon^2)$. Note that the more accurate $\tilde{\theta}$ (the smaller σ_ϵ), the more the manager relies on her signal to estimate η, and the greater the value of her private information relative to the information of investors. Indeed, as the signal's noise falls

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$(\sigma_\epsilon \to 0)$ the correlation between $\tilde{\theta}$ and $\tilde{\eta}$ increases,[1] approaching the perfect (complete) information case in which $m = \eta$ and $v_0 = 0$ in the limit.[2]

From then on, at any $t \in [0, T]$ the manager updates her estimate of η based on the information flow $\mathcal{F}_t^S \equiv \sigma\{S_u, 0 \le u \le t\}$ according to Bayes' rule. Given the updated prior $\mathbf{N}(m, v_0)$, the distribution of $\tilde{\eta}$ conditional on $\mathcal{F}_t^S$ and $\tilde{\theta}$ is Gaussian, with conditional mean $\tilde{\eta}_t \equiv E\left[\tilde{\eta}|\tilde{\theta} = \theta, \mathcal{F}_t^S\right]$ and variance $v_t \equiv E\left[(\tilde{\eta} - \tilde{\eta}_t)^2|\tilde{\theta} = \theta, \mathcal{F}_t^S\right]$ satisfying (see, e.g., Liptser and Shirayayev, 2001):

$$\begin{cases} d\tilde{\eta}_t = v_t d\tilde{B}_t, \\ dv_t = -v_t^2 dt, \end{cases} \tag{IA.7}$$

with $\tilde{\eta}_0 = m$ and v_0 as initial values for $\tilde{\eta}_t$ and v_t, respectively. $\tilde{B}_t$ is a standard Brownian motion with respect to $\mathcal{F}_t^S$, with dynamics given by $d\tilde{B}_t = 1/\sigma\left[dS_t/S_t - (r + \sigma\tilde{\eta}_t)dt\right] = dB_t + (\eta - \tilde{\eta}_t)dt$.

B.2 The Manager's Optimal Portfolio

Under parameter uncertainty, markets are still complete with respect to the observable states of the economy (a single risky asset S driven by a single Brownian motion $\tilde{B}$). Absent arbitrage opportunities, the manager sees financial markets as driven by a unique state-price deflator π with dynamics $d\pi_t = -r\pi_t dt - \pi_t\tilde{\eta}_t d\tilde{B}_t$. Let $B_t^Q = \tilde{B}_t + \int_0^t \tilde{\eta}_s ds = B_t + \eta t$ denote the risk-neutral Brownian motion, and $\tau \equiv T - t$ the time remaining until the end of the period. Let $\kappa_3 \equiv \kappa_2 + \bar{\kappa}_3$ and $\kappa_4 \equiv \kappa_2 + \delta\bar{\kappa}_3$ be the sum of the benchmark-linked performance fees, scaled by the threshold δ in the case of κ_4. For a given compensation contract, the manager's optimal fund value $\hat{W}$ and investment strategy $\hat{\phi}$ are characterized in closed-form in the following:

Proposition IA.1. *Let $\tilde{\gamma}(t) \equiv \gamma + (\gamma - 1)v_t(T - t)$ and $V(t) \equiv \gamma/\tilde{\gamma}(t)(1 + v_t(T - t))$. Define the constants $\psi_1 \equiv (\kappa_1 + \kappa_2)^{\frac{1}{\gamma} - 1}$ and $\psi_2 \equiv (\kappa_1 + \kappa_3)^{\frac{1}{\gamma} - 1}$, and let $\psi_3 \equiv \kappa_2/(\kappa_1 + \kappa_2)$ and $\psi_4 \equiv \kappa_4/(\kappa_1 + \kappa_3)$ be the relative weight of the fulcrum fee in a linear contract, and the relative weight of the benchmark-linked incentive fees in a general fee contract, respectively. For an arbitrary contract*

[1]Given the orthogonality assumption between $\tilde{\eta}$ and $\tilde{\epsilon}$, $\text{cov}(\tilde{\eta}, \tilde{\theta}) = \text{cov}(\tilde{\eta}, \tilde{\eta} + \tilde{\epsilon}) = \bar{v}_0$. Thus, $\text{corr}(\tilde{\eta}, \tilde{\theta}) \equiv \rho(\tilde{\eta}, \tilde{\theta}) = \frac{\bar{v}_0}{\sqrt{\bar{v}_0}\sqrt{\bar{v}_0 + \sigma_\epsilon^2}} = \sqrt{\frac{\bar{v}_0}{\bar{v}_0 + \sigma_\epsilon^2}}$, and $\rho(\tilde{\eta}, \tilde{\theta}) \to 1$ as $\sigma_\epsilon \to 0$.

[2]Naturally, keeping σ_ϵ constant the value of the manager's private information to investors also rises with $\bar{v}_0$, when there is a greater overall uncertainty that the manager's signal can help reduce.

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$\mathscr{C} \equiv (\kappa_1, \kappa_2, \bar{\kappa}_3, \phi^Y, \delta)$, the manager's optimal fund value at time $t \in [0, T]$ is given by:

$$\hat{W}_t = (\lambda \pi_t)^{-\frac{1}{\gamma}} e^{-\left(1-\frac{1}{\gamma}\right) r\tau} g\left(\frac{1}{\gamma}, t, \tilde{\eta}_t; T\right) \left[\psi_2 - (\psi_2 - \psi_1)\left(\mathcal{N}(d_{1,t}) - \mathcal{N}(d_{2,t})\right)\right] + Y_t \left[\psi_4 - (\psi_4 - \psi_3)\left(\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})\right)\right] \quad \text{(IA.8)}$$

and her optimal risk exposure is given by:

$$\hat{\phi}_t = \frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma} + \omega_t \left(\phi^Y - \frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma}\right) + \Phi_t, \quad \text{(IA.9)}$$

where the conditional mean $\tilde{\eta}$ and variance v of the market price of risk are given by:

$$\begin{cases} \tilde{\eta}_t = v_t \left(B_t^Q + \frac{m}{v_0}\right), \\ v_t = \frac{v_0}{1 + v_0 t}, \end{cases} \quad \text{(IA.10)}$$

the Lagrange multiplier λ solves $\hat{W}_0 = w$, $\mathcal{N}(.)$ is the standard normal cumulative distribution function,

$$g(\zeta, t, x; T) \equiv \sqrt{\frac{(1 + v_t(T-t))^{1-\zeta}}{1 + (1-\zeta) v_t(T-t)}} \exp\left\{-\frac{\zeta}{2} \frac{(1-\zeta)(T-t)}{1 + (1-\zeta) v_t(T-t)} x^2\right\},$$

$$\omega_t \equiv \frac{Y_t \left[\psi_4 - (\psi_4 - \psi_3)\left(\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})\right)\right]}{\hat{W}_t} \geq 0, \quad \text{(IA.11)}$$

$$\Phi_t \equiv \frac{1-\omega_t}{\sigma} \sqrt{\frac{V(t)}{\tau}} \frac{(\psi_2 - \psi_1)\left(\mathcal{N}'(d_{1,t}) - \mathcal{N}'(d_{2,t})\right)}{\psi_2 - (\psi_2 - \psi_1)\left(\mathcal{N}(d_{1,t}) - \mathcal{N}(d_{2,t})\right)} + \frac{\omega_t}{\sigma\sqrt{\tau}} \frac{(\psi_4 - \psi_3)\left(\mathcal{N}'(d_{3,t}) - \mathcal{N}'(d_{4,t})\right)}{\psi_4 - (\psi_4 - \psi_3)\left(\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})\right)}, \quad \text{(IA.12)}$$

and

$$d_{1,t} \equiv \frac{\frac{\tilde{\gamma}(t)\sigma}{v_t} V(t)\left(\phi^Y - \frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma}\right) + \varphi(\lambda)}{\sqrt{V(t)\tau}}, \quad d_{2,t} \equiv d_{1,t} - 2\frac{\varphi(\lambda)}{\sqrt{V(t)\tau}},$$
$$d_{3,t} \equiv \frac{\frac{\tilde{\gamma}(t)\sigma}{v_t}\left(\phi^Y - \frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma}\right) + \varphi(\lambda)}{\sqrt{\tau}}, \quad d_{4,t} \equiv d_{3,t} - 2\frac{\varphi(\lambda)}{\sqrt{\tau}},$$

for the function $\varphi(\cdot)$ as given in the proof.

Proof. The manager's inference problem is the same as that of the uninformed managers in Sotes-Paladino (2017), so equation (IA.10) follows directly from his Proposition 1. For a realization θ of her signal at $t = 0$ and a given contract $\mathscr{C} \equiv (\kappa_1, \kappa_2, \bar{\kappa}_3, \phi^Y, \delta)$, the manager's problem (IA.3) can

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be equivalently stated as:

$$\max_{W_T} \tilde{E}\left[u\left(\kappa_1 W_T + \kappa_2(W_T - Y_T) + \bar{\kappa}_3(W_T - \delta Y_T)^+; \gamma\right)\right], \tag{IA.13}$$

$$s.t. \qquad \tilde{E}\left[\pi_T W_T\right] = w. \tag{IA.14}$$

$\tilde{E}(\cdot)$ denotes the expectation with respect to the equivalent probability $\tilde{P}$ under which $\tilde{B}$ is a standard Brownian motion. Although omitted for notational simplicity, this expectation—as well as all expectations in the remainder of this proof—are conditional on the realization θ of $\tilde{\theta}$. For $\bar{\kappa}_3 > 0$, $u(\cdot;\gamma)$ is globally non-concave and non-differentiable at $W_T = \delta Y_T$. This is due to the presence of a kink in the compensation function $f(\cdot, y; \kappa_1, \kappa_2, \bar{\kappa}_3, \delta)$ at $x = \delta y$. The kink implies that the manager is risk-loving in the interval over which $u(\cdot;\gamma)$ is non-concave and prefers to take on gambles that result in values of W_T in either extreme of this interval. In order to use standard optimization techniques, we first need to construct the concavification $\tilde{u}(\cdot)$ of $u(\cdot;\gamma)$ (i.e. the smallest concave function v satisfying $v \geq u$). Following the approach in Basak and Makarov (2014), the concavified objective function can be obtained as:[3]

$$\tilde{u}(W_T) = \begin{cases} \frac{1}{1-\gamma}\left[(\kappa_1+\kappa_2)W_T - \kappa_2 Y_T\right]^{1-\gamma}, & W_T < \underline{W} \\ a + b(Y_T)W_T, & \underline{W} \leq W_T \leq \overline{W} \\ \frac{1}{1-\gamma}\left[(\kappa_1+\kappa_2+\bar{\kappa}_3)W_T - (\kappa_2+\bar{\kappa}_3\delta)Y_T\right]^{1-\gamma}, & \overline{W} \leq W_T \end{cases} \tag{IA.15}$$

where:

$$\underline{W} \equiv (\kappa_1+\kappa_2)^{\frac{1}{\gamma}-1} b(Y_T)^{-\frac{1}{\gamma}} + \frac{\kappa_2}{\kappa_1+\kappa_2} Y_T,$$

$$\overline{W} \equiv (\kappa_1+\kappa_2+\bar{\kappa}_3)^{\frac{1}{\gamma}-1} b(Y_T)^{-\frac{1}{\gamma}} + \frac{\kappa_2+\bar{\kappa}_3\delta}{\kappa_1+\kappa_2+\bar{\kappa}_3} Y_T,$$

and

$$b(x) = (\kappa_1+\kappa_2+\bar{\kappa}_3)^{1-\gamma}\left\{\frac{\gamma-1}{\gamma}\frac{1}{\kappa-1}\left[\frac{\kappa_2+\bar{\kappa}_3\delta}{\kappa_1+\kappa_2+\bar{\kappa}_3} - \frac{\kappa_2}{\kappa_1+\kappa_2}\right]x\right\}^{-\gamma},$$

for

$$\kappa \equiv \left(\frac{\kappa_1+\kappa_2+\bar{\kappa}_3}{\kappa_1+\kappa_2}\right)^{1-\frac{1}{\gamma}} > 1,$$

and

$$a = \frac{1}{1-\gamma}\left[(\kappa_1+\kappa_2)\underline{W} - \kappa_2 Y_T\right]^{1-\gamma} - b(Y_T)\underline{W}.$$

[3]See also Cuoco and Kaniel (2011) and the references therein.

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Replacing u for $\tilde{u}$ in problem (IA.13), the optimal terminal wealth $\hat{W}_T$ needs to satisfy the first order condition: $\partial\tilde{u}(\hat{W}_T)/\partial W_T = \lambda\pi_T$ for a Lagrange multiplier λ of the budget constraint (IA.14). Using the constants defined in Section A.2, a solution can be characterized as:

$$\hat{W}_T = \begin{cases} (\kappa_1+\kappa_2)^{\frac{1}{\gamma}-1}(\lambda\pi_T)^{-\frac{1}{\gamma}} + \frac{\kappa_2}{\kappa_1+\kappa_2}Y_T, & \lambda\pi_T > b(Y_T) \qquad (\mathcal{R}_I) \\ (\kappa_1+\kappa_3)^{\frac{1}{\gamma}-1}(\lambda\pi_T)^{-\frac{1}{\gamma}} + \frac{\kappa_4}{\kappa_1+\kappa_3}Y_T, & \lambda\pi_T \leq b(Y_T) \qquad (\mathcal{R}_{II}) \end{cases} \tag{IA.16}$$

Using the definition of $b(\cdot)$ above and the explicit expression for the state-price deflator π_T in Appendix A of Sotes-Paladino (2017)

$$\pi_T = e^{-rT+\frac{1}{2}\int_0^T \tilde{\eta}_s^2 ds - \int_0^T \tilde{\eta}_s dB_s^Q} = \sqrt{1+v_0T}\, e^{-rT+\frac{m^2}{2v_0}-\frac{v_T}{2}\left(B_T^Q+\frac{m}{v_0}\right)^2}, \tag{IA.17}$$

region $\mathcal{R}_I$ is given by all values $\tilde{\eta}_T$ such that:

$$\frac{\gamma\phi^Y\sigma}{v_T} - \varphi(\lambda) < \frac{\tilde{\eta}_T}{v_T} < \frac{\gamma\phi^Y\sigma}{v_T} + \varphi(\lambda), \tag{IA.18}$$

where

$$\varphi(x) \equiv \frac{1}{\sqrt{v_T}}\sqrt{\frac{(m-\gamma\sigma\phi^Y)^2}{v_0} + 2(\gamma-1)\left[r+\frac{\gamma}{2}\left(\phi^Y\sigma\right)^2\right]T + 2\ln\left(xb(y)\sqrt{1+v_0T}\right)}.$$

Region $\mathcal{R}_{II}$ is the relative complement of $\mathcal{R}_I$ in $\mathbb{R}$.

Under the risk-neutral measure Q the deflated wealth $e^{-rt}\hat{W}_t$ is a martingale (as is the deflated benchmark value $e^{-rt}Y_t$), so using (IA.16) the optimal wealth $\hat{W}_t$, for all $t \in [0,T]$, is given by:

$$\begin{aligned} \hat{W}_t =& e^{-r(T-t)}E_t^Q\left[\hat{W}_T\right] \\ =& e^{-r(T-t)}\left\{(\kappa_1+\kappa_2)^{\frac{1}{\gamma}-1}(\lambda\pi_t)^{-\frac{1}{\gamma}}E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\mathbb{1}_{\mathcal{R}_I}\right] + \frac{\kappa_2 Y_t}{\kappa_1+\kappa_2}E_t^Q\left[\left(\frac{Y_T}{Y_t}\right)\mathbb{1}_{\mathcal{R}_I}\right]\right. \\ &\left. + (\kappa_1+\kappa_3)^{\frac{1}{\gamma}-1}(\lambda\pi_t)^{-\frac{1}{\gamma}}E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\mathbb{1}_{\mathcal{R}_{II}}\right] + \frac{\kappa_4 Y_t}{\kappa_1+\kappa_3}E_t^Q\left[\left(\frac{Y_T}{Y_t}\right)\mathbb{1}_{\mathcal{R}_{II}}\right]\right\}. \end{aligned} \tag{IA.19}$$

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The first and second expectations on the RHS are:

$$E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\mathbb{1}_{\mathcal{R}_I}\right]=\sqrt{\frac{(1+v_t(T-t))^{1-\frac{1}{\gamma}}}{1+\left(1-\frac{1}{\gamma}\right)v_t(T-t)}}$$
$$\times e^{\frac{r}{\gamma}(T-t)-\frac{1}{2\gamma}\frac{(1-\frac{1}{\gamma})(T-t)}{1+(1-\frac{1}{\gamma})v_t(T-t)}\tilde{\eta}_t^2}\left[\mathcal{N}(d_{1,t})-\mathcal{N}(d_{2,t})\right],$$

and

$$E_t^Q\left[\left(\frac{Y_T}{Y_t}\right)\mathbb{1}_{\mathcal{R}_I}\right]=e^{r(T-t)}\left[\mathcal{N}(d_{3,t})-\mathcal{N}(d_{4,t})\right], \tag{IA.20}$$

with

$$\begin{aligned} d_{1,t} &\equiv \frac{\frac{\tilde{\gamma}(t)\sigma}{v_t}V(t)\left(\phi^Y-\frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma}\right)+\varphi(\lambda)}{\sqrt{V(t)\tau}}, \\ d_{2,t} &\equiv d_{1,t}-2\frac{\varphi(\lambda)}{\sqrt{V(t)\tau}}, \\ d_{3,t} &\equiv \frac{\frac{\tilde{\gamma}(t)\sigma}{v_t}\left(\phi^Y-\frac{\tilde{\eta}_t}{\tilde{\gamma}(t)\sigma}\right)+\varphi(\lambda)}{\sqrt{\tau}}, \text{ and} \\ d_{4,t} &\equiv d_{3,t}-2\frac{\varphi(\lambda)}{\sqrt{\tau}}, \end{aligned}$$

for $\tilde{\gamma}(t)$ and $V(t)$ as defined in the Proposition. These results follow from applying Lemma A1 in Appendix A of Sotes-Paladino (2017) to:

$$\begin{aligned}\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}} &= e^{\frac{r}{\gamma}(T-t)-\frac{1}{2\gamma}\int_t^T\tilde{\eta}_s^2ds+\frac{1}{\gamma}\int_t^T\tilde{\eta}_s dB_s^Q} \\ &= \sqrt{(1+v_t(T-t))^{-\frac{1}{\gamma}}}\,e^{\frac{r}{\gamma}(T-t)-\frac{1}{2\gamma}\frac{\tilde{\eta}_t^2}{v_t}+\frac{1}{\gamma}\frac{v_T}{2}\left(B_T^Q-B_t^Q+\frac{\tilde{\eta}_t}{v_t}\right)^2}\end{aligned}$$

and to $Y_T/Y_t=e^{\left[r-\frac{1}{2}\left(\phi^Y\sigma\right)^2\right](T-t)+\phi^Y\sigma\left(B_T^Q-B_t^Q\right)}$. Similar computations give the third and fourth expectations on the RHS as:

$$E_t^Q\left[\left(\frac{\pi_T}{\pi_t}\right)^{-\frac{1}{\gamma}}\mathbb{1}_{\mathcal{R}_{II}}\right]=\sqrt{\frac{(1+v_t(T-t))^{1-\frac{1}{\gamma}}}{1+\left(1-\frac{1}{\gamma}\right)v_t(T-t)}}$$
$$\times e^{\frac{r}{\gamma}(T-t)-\frac{1}{2\gamma}\frac{(1-\frac{1}{\gamma})(T-t)}{1+(1-\frac{1}{\gamma})v_t(T-t)}\tilde{\eta}_t^2}\left[1+\mathcal{N}(d_{2,t})-\mathcal{N}(d_{1,t})\right],$$

and

$$E_t^Q\left[\left(\frac{Y_T}{Y_t}\right)\mathbb{1}_{\mathcal{R}_{II}}\right]=e^{r(T-t)}\left[1+\mathcal{N}(d_{4,t})-\mathcal{N}(d_{3,t})\right]. \tag{IA.21}$$

Plugging the four expectations in (IA.19), using constants ψ_1-ψ_4 as defined in the Proposition

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and rearranging, we get equation (IA.8). The Lagrange multiplier λ results from solving $\hat{W}_0 = w$, for $\hat{W}_t$ in (IA.8) evaluated at $t = 0$.

In order to derive the investment policy (IA.9) replicating the optimal portfolio value (IA.8), note that the latter can be rewritten as $\hat{W}_t = h(t, \tilde{\eta}_t; T)$, with $d\tilde{\eta}_t = -v_t\tilde{\eta}_t + v_t dB_t^Q$, for some function $h \in C^{1,2}$. Applying Itô's Lemma the diffusion term of $d\hat{W}_t$ is

$$v_t \frac{\partial d\hat{W}_t}{\partial \tilde{\eta}_t}.$$

Under the risk-neutral measure, $d\hat{W}_t$ satisfies the self-financing constraint:

$$d\hat{W}_t = \hat{W}_t r dt + \hat{W}_t \hat{\phi}_t \sigma dB_t^Q.$$

Equating diffusion terms:

$$\hat{\phi}_t = \frac{v_t}{\hat{W}_t \sigma} \frac{\partial d\hat{W}_t}{\partial \tilde{\eta}_t}. \tag{IA.22}$$

Substituting the derivative of (IA.8) with respect to $\tilde{\eta}_t$ in (IA.22) gives the optimal fund stock allocation (IA.9). □

C Optimal Fee Contract

C.1 Value of the Manager's Noisy Signal to Households

As in Section 5.1 of RSRFF, we first quantify the value of the manager's private information to the households in the noisy signal case. This value is given by the ex ante utility (certainty equivalent) that households derive under the hypothetical first-best scenario in which the realization of $\tilde{\theta}$ can be contracted upon. Under this *symmetric information* (SI) case, we assume that at (state-dependent) cost $k_{SI}(\tilde{\theta})w$ the households can purchase the (reliable) revelation of the private information θ from the manager, right after it is realized at $t = 0$.

Upon completing the transaction, both households and the manager have symmetric (incomplete) information about asset fundamentals over the investment period $[0, T]$. Households then solve, for each realized value θ of $\tilde{\theta}$, the following problem:

$$\max_{\{\phi_t\}_{t\in[0,T]}} E_0 \left[\frac{(W_T)^{1-\gamma_h}}{1-\gamma_h} \middle| \tilde{\theta} = \theta \right], \tag{IA.23}$$

subject to the self-financing constraint (IA.4) and initial wealth $W_0^\eta = (1 - k_{CI}(\theta))w$. The solution

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is given in the following:

Lemma 1. *When households observe the realization θ of the signal $\tilde{\theta}$ at $t = 0$, their optimal portfolio strategy $\phi_{SI,t}$ and wealth $W_{SI,t}$ for $t \in [0, T]$ are given by*

$$\phi_{SI,t} = \frac{1}{\gamma_h + (\gamma_h - 1)v_t\tau}\frac{\tilde{\eta}_t}{\sigma} = \frac{\tilde{\eta}_t}{\gamma_h\sigma} - \frac{(\gamma_h - 1)v_t\tau}{\gamma_h + (\gamma_h - 1)v_t\tau}\frac{\tilde{\eta}_t}{\gamma_h\sigma}, \tag{IA.24}$$

$$W_{SI,t} = (\lambda_{SI}\pi_t)^{-\frac{1}{\gamma_h}} e^{-\left(1-\frac{1}{\gamma_h}\right)r\tau} g\left(\frac{1}{\gamma_h}, t, \tilde{\eta}_t; T\right), \tag{IA.25}$$

where $\lambda_{SI} = \left[g\left(\frac{1}{\gamma_h}, 0, m; T\right)/((1 - k_{CI})w)\right]^{\gamma_h}$*, and $\tilde{\eta}_t$, v_t and $g(\cdot)$ are given in Proposition IA.1.*

Proof. Problem (IA.23) is identical to problem (IA.13) for $\gamma = \gamma_h, \kappa_1 = 1$, and $\kappa_2 = 0 = \bar{\kappa_3}$ (for any values of ϕ^Y and δ). For a given λ_{SI}, equations (IA.24) and (IA.25) then follow from plugging these parameter values in equations (IA.9) and (IA.8) of Proposition IA.1. The Lagrange multiplier follows from solving for λ_{SI} in (IA.8) for $t = 0$, with $W_{SI,0} = w$. □

A comparison of equation (IA.9) with equation (IA.24) shows that, for an arbitrary contract $\mathscr{C}$, the optimal portfolio $\hat{\phi}$ chosen by the manager will differ from the portfolio ϕ_{SI} desired by households whenever $\gamma \neq \gamma_h$. As in RSRFF, it can be shown that no contract, within the class considered here, can induce the manager to choose portfolio ϕ_{SI}.

We assume that households can invest in their desired portfolio (IA.24) by allocating their wealth in an ETF with stock weight $\phi_t^P = \phi_t^{SI}$, and do so at no cost in this idealized case. Their end-of-period wealth is $W_{SI,T}$, for W_{SI} as characterized by (IA.25). Their ex-ante utility is:

$$U_{h,SI}(w) \equiv E\left[\frac{(W_{SI,T})^{1-\gamma_h}}{1-\gamma_h}\right],$$

where the expectation is taken with respect to the joint distribution of $(\tilde{\eta}, \tilde{\epsilon}, B_T)$.

In turn, the manager collects the amount $k_{SI}(\theta))w$ at $t = 0$ from the households and invests this amount over the investment period $[0, T]$ according to her own preferences. The manager's final wealth is $k_{SI}\hat{W}_T$, for $\hat{W}$ as characterized by (IA.8) for $\kappa_2 = \bar{\kappa}_3 = 0$. Her ex-ante utility is:

$$U_{SI}(w) \equiv E\left[\frac{(k_{SI}W_{SI,T})^{1-\gamma}}{1-\gamma}\right],$$

where the expectation is taken with respect to the joint distribution of $(\tilde{\eta}, \tilde{\epsilon}, B_T)$.

Given the initial wealth w, for each realization of $\tilde{\theta}$ the base fee $k_{SI}(\tilde{\theta})$ splits the aggregate

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value created by the active management of the households' and the manager's portfolios between the two parties. Setting $k_{SI}(\tilde{\theta})$ to the value that equalizes the manager's utility in the SI and passive management cases state-by-state we obtain the *maximum value that households can extract from the manager's noisy signal*. We identify this quantity with the certainty equivalent of $U_{h,SI}$ of households in excess of the certainty equivalent of their reservation utility $U_{h,P}$.

C.2 The Optimal Nonlinear Contract

In solving for the optimal fee schedule of the possibly nonlinear type (IA.2), we seek to answer three main questions related to our results in RSRFF:

(i) Does the fulcrum fee still improve over base fee-only contracts when the manager has partial information?

(ii) Do asymmetric performance fees similarly improve upon fulcrum fees for the purpose of risk-sharing?

(iii) Can investors still extract most of the ex-ante value of the manager's information?

Problem (IA.5) has no analytical solution. However, expression (IA.8), evaluated at $t = T$, identifies the optimal fund value for a given contract $\mathscr{C}$ and realization of the triple $(\tilde{\eta}, \tilde{\epsilon}, B_T)$ in closed form. Using this expression and the expression for Y_T (as given in the proof of Proposition IA.1), it is straightforward to compute the expectations in (IA.5) (via numerical integration over a multivariate normal distribution), and in turn to use standard optimization algorithms to find a global optimum for the contracting problem. We do this for appropriate versions of our baseline and alternative model parameterizations in RSRFF, under the constraint in sections 5.3.1 and 5.3.2 that the benchmark is unlevered.[4] For ease of computation and comparability with other studies (e.g., Li and Tiwari, 2009), we fix the performance threshold δ to equal 1. We present our results in Figure IA.1 and Table IA.1 below.

Figure IA.1 shows that, even when the manager's signal is imperfect, linear benchmarking (fulcrum performance fees) are valuable to both relatively risk-tolerant (Panel (a)) and risk-averse (Panel (b)) households. The figure depicts the utility derived by households under the optimal

[4] The baseline parameterization is as described in Section 5.3 of RSRFF, to which we add the assumption that $\sigma_\epsilon = 0.2$, impying a relatively conservative correlation of 0.09 between the manager's signal θ and the realized market price of risk η. The alternative paramterizations in Table 1 are as in Section 5.3 of RSRFF, except that we now fix $\bar{v}_0$ and vary σ_ϵ between 0.00 (most informed manager) and 0.30 (least informed manager).

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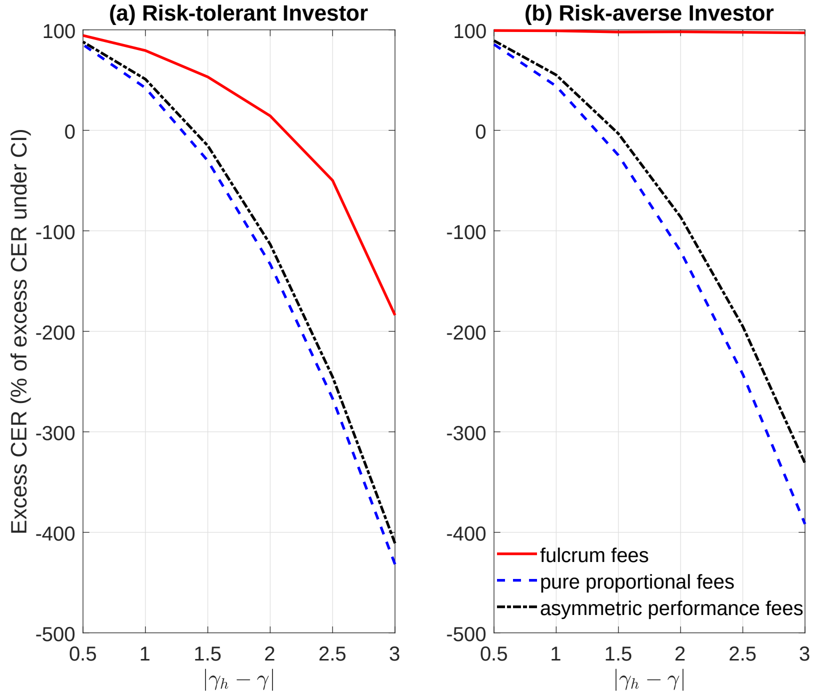

Figure IA.1: **Benefits of Active Management under Optimal Benchmarking**.
The figure plots the excess certainty equivalent returns (CER) from delegation under the optimal linear benchmark-adjusted contract (red solid line, $\kappa_1, \kappa_2 > 0, \bar{\kappa}_3 = 0$), under the pure proportional fee contract (blue dashed line, $\kappa_1 > 0, \kappa_2 = 0 = \bar{\kappa}_3$), and under the optimal asymmetric benchmark-adjusted contract (black dash-and-dot line, $\kappa_1, \bar{\kappa}_3 > 0, \kappa_2 = 0$). Values are reported for relatively risk-tolerant ($\gamma_h < 5$, Panel (a)) and risk-averse ($\gamma_h > 5$, Panel (b)) fund investors. Excess CER are computed with respect to households' CER from delegation to a passively managed fund, and reported as a percentage of the excess CE returns under the reference symmetric information (SI) case. The model parameters are: $T = 3, \sigma_\epsilon = 0.2, r = 3\%, \sigma = .0158, \bar{m} = 0.513, \bar{v}_0 = 0.037, \gamma_h = 5, \delta = 1$.

contract of different basic types: proportional-only fees ($\kappa_1 > 0, \kappa_2 = 0 = \bar{\kappa}_3$), linear benchmarking ($\kappa_1, \kappa_2 > 0, \bar{\kappa}_3 = 0$), and asymmetric benchmarking ($\kappa_1, \bar{\kappa}_3 > 0, \kappa_2 = 0$). This utility is expressed as certainty equivalent (CE) rate of returns in excess of the return to a passive strategy under each contract type, and presented as a percentage of the excess CE returns under the reference SI case. As before, we refer to this as the *ex-ante value that households extract form the manager's information*. Clearly, fulcrum performance fees improve not only over base fee-only contracts but *also over asymmetric performance fees* for both type of investors. Since linear benchmarking serves the purpose of risk-alignment better than nonlinear benchmarking and at lower cost, the margin of outperformance increases with the risk aversion misalignment $|\gamma_h - \gamma|$. While dominated by fulcrum fees, Fig. IA.1 also indicates that the optimal asymmetric performance fee schedule can dominate proportional fee-only contracts, especially among risk-averse investors.

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			Optimal Contract				Excess CER
γ_h	T	σ_ϵ	κ_1	κ_2	$\bar{\kappa_3}$	ϕ^Y	(%)
3	1	0.0	14.8	1.54	0.00	1.00	17.2
		0.1	14.8	2.27	0.00	1.00	11.5
		0.2	14.8	5.19	0.00	1.00	2.2
		0.3	14.8	9.95	0.00	1.00	-4.8
	5	0.0	13.9	1.25	0.00	1.00	32.7
		0.1	14.0	1.68	0.00	1.00	29.2
		0.2	14.1	2.59	0.00	1.00	24.4
		0.3	14.1	3.50	0.00	1.00	21.9
7	1	0.0	14.9	0.44	0.00	0.05	99.1
		0.1	14.9	0.51	0.00	0.11	98.6
		0.2	14.9	0.50	0.00	0.10	98.7
		0.3	15.0	0.46	0.00	0.06	98.7
	5	0.0	14.6	0.47	0.00	0.07	97.9
		0.1	14.7	0.48	0.00	0.08	97.8
		0.2	14.7	0.52	0.00	0.09	97.5
		0.3	14.8	0.58	0.00	0.12	97.2

Table IA.1: **Optimal General Managerial Contract and Households' Derived Utility.** The table reports the optimal contract $(\kappa_1, \kappa_2, \bar{\kappa}_3, \phi^Y)$ and associated excess certainty equivalent returns (CER) from delegation for different combinations of households' relative risk-aversion γ_h, time horizons T, and quality of manager's private information σ_ϵ. Benchmark-linked fees κ_2 and $\bar{\kappa}_3$ are expressed as a multiple of the proportional fee κ_1. Excess CER are computed with respect to households' CER from delegation to a passively managed fund, and presented as percentage of their excess CER under the SI case. The rest of the model parameters are: $r = 3\%, \sigma = .0158, \bar{m} = 0.513, \sqrt{\bar{v}_0} = 0.192, \gamma = 5, \delta = 1$.

These observations are further verified in Table IA.1, which shows that the optimal contract is *always linear*, as the optimal asymmetric performance fee $\bar{\kappa}_3$ equals 0 across all parameterizations. As in Section 5.3, the optimal contract restricted to long-only benchmarks has an all-equity allocation for risk-tolerant fund investors, while it is predominantly risk-free for their risk-averse counterparts. The optimal fulcrum fee contract allows the latter type of households to realize almost the full value of the manager's information even when this information is imperfect. It also delivers positive excess CE returns from (sufficiently informed) active management to risk-tolerant investors, even though in their case the long-only constraint on the benchmark is binding.

Summing up, our analysis in this Appendix shows that even when we enlarge the contract space to allow for nonlinear fees, and relax the assumption of complete information for the manager, the optimal fee schedule is of the fulcrum performance type. In particular, in relation to our motivating questions, fulcrum fees: (i)-(ii) improve over both base fee-only contracts and asymmetric performance fees; and (iii) allow fund investors to extract either a substantial fraction (risk-tolerant

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type) or almost all (risk-averse type) of the ex-ante value of the manager's information.

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