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Robust Optimal Macroprudential Policy

Federico Bennett (Duke University)

Giselle Montamat (Uber)

Francisco Roch (IMF)

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Robust optimal macroprudential policy*

Federico Bennett — Duke University
Giselle Montamat — Uber
Francisco Roch — IMF

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Abstract

We consider how fear of model misspecification on the part of the planner and/or the households affects welfare gains from optimal macroprudential taxes in an economy with occasionally binding collateral constraints as in Bianchi (2011). In this setup, the decentralized equilibrium may differ from the social planner's equilibrium both because of the pecuniary externalities associated with the collateral constraint and because of the paternalistic imposition of the planner's beliefs when designing policy. When robust agents have doubts about the model, they create endogenous worst-case beliefs by assigning a high probability to low-utility events. The ratio of worst-case beliefs of the planner over the household's captures the degree of paternalism. We show that this novel channel could render the directions of welfare gains from a policy intervention ambiguous. However, our quantitative results suggest that doubts about the model need to be large in order to make a "laissez-faire regime" better than an intervention regime.

JEL Classification D62, E32, E44, E62, F32, F41, G01, H21

Keywords Robust Control, Model Uncertainty, Optimal Taxation, Sudden Stops, Financial Crises.

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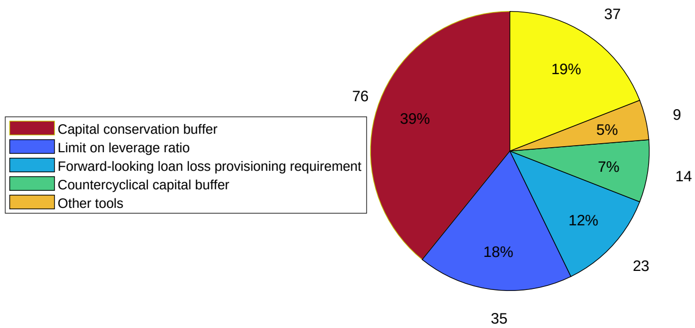
1 Introduction

Since the global financial crisis there has been a new push in favor of macroprudential regulation in order to safeguard financial stability both in advanced and emerging countries. Figure 1 provides information on macroprudential policy measures taken by IMF member countries from a survey conducted in 2018. It shows that most of these tools are aimed at managing liquidity and currency mismatches in the balance sheets of the banking sector, as well as restricting credit (e.g., capital buffers and loan to value caps).[1] The survey also reports that there are, on average, nine macroprudential instruments in place per country (as of 2018). Following the seminal papers by Bianchi and Mendoza (2010) and Bianchi (2011), several studies have analyzed how different financial frictions can lead to inefficient overborrowing which justifies the implementation of these regulatory measures. The average tax on debt reported in the literature varies from 3.6 to 5 percent, which almost eliminates the long-run probability of financial crises. However, the welfare gains are usually not too large and sometimes even ambiguous. The benefits of macroprudential policies are even less clear in the midst of uncertainty about the nature of the shocks the economy might be subject to. Thus, much debate remains on whether prudential taxes should at all be imposed given the possibility that the model regulators have in mind may not be an accurate characterization of the real world. Both in academic and policy circles, it is relevant to analyze how robust these optimal prudential taxes are to potential misspecifications of the model that governs the shocks to the economy. Our paper informs this debate by concluding that, even when regulators design policy based on an "incorrect" model, for *reasonable levels of* misspecification macroprudential policies are generally welfare improving.

This paper illustrates how macroprudential policies that are robust to model misspecification (Hansen and Sargent, 2011) generally lead to welfare gains, which can indeed be significant when the policy maker engages in prudent behavior while the household does not. Specifically, robust policies refer to the case where the decision maker is not sure about the probabilities that are governing

[1] BIS-FSB-IMF (2016) defines macroprudential policy as the use of primarily prudential tools to limit systemic risk which pursues three interlocking objectives: (i) contain build-up of cyclical vulnerabilities, (ii) limit structural vulnerabilities, and (iii) build resilience.

Figure 1: Number of Broad Based Tools



Note: This figure shows the number of broad based tools reported by 141 countries in the Macroprudential Policy Survey of 2018. Numbers denote frequency of measures reported; percentages denote the share among total measures reported.

the evolution of shocks in the economy. Instead of considering just one probability distribution (a reference model), the planner will account for the fact that there is a set of distributions for shocks that could fit the data as well as the reference model, and should implement a policy that is optimal under the worst case scenario (i.e., the policy that does not create vulnerabilities if one of the non-reference models turns out to be the true one).[2] In the words of Hansen and Sargent (VOX - 2019): "Prudent decision making should acknowledge what we do not know... Policy makers should strive to quantify the dimensions of their ignorance and adjust their decisions accordingly."

We evaluate the robustness of macroprudential instruments by introducing fear of model misspecification into the canonical model of optimal macroprudential policies developed in Bianchi (2011). This paper rationalizes and justifies the need for macroprudential policy on the basis of a financial friction that generates sudden stops within the business cycle. This friction takes the form of an occasionally binding collateral constraint that generates a pecuniary externality. Specifically, the value of the collateral that constrains households' borrowing depends on aggregate indebtedness. However, given that individuals are price takers, they fail to internalize how their borrowing decisions impact the price of the collateral. This externality leads to inefficient overborrowing. Instead,

[2]The optimal policy under the worst-case scenario guarantees that one is not worse-off compared to the worst-case outcome if the actual probability distribution turns out to be another one. You are, however, worse-off than the outcome derived from the optimal policy under that true distribution.

the planner internalizes the pecuniary externality and increases precautionary savings that can reduce both the magnitude and likelihood of a crisis when the collateral constraint becomes binding. Bianchi (2011) shows that the efficient allocation can be achieved through the implementation of macroprudential debt taxes.

Our contribution is to adapt this workhorse model to an environment in which there is a fear of model uncertainty from both the planner and/or households following the framework in Hansen and Sargent (2011). We take the model in Bianchi (2011) as our benchmark model, in which both the planner and household make their optimal decisions under the true probability distribution. Under model misspecification, agents fear that the transition probabilities they are considering may not be the correct ones. Thus, they will now make their optimal decisions by considering a set of alternative probabilities that are not too far from the reference model.[3] Under robustness, agents make their optimal decisions as if an "evil" agent chooses the worst-case density among all those possible distributions surrounding the reference distribution. In this setup, the decentralized equilibrium may differ from the social planner's equilibrium both because of the pecuniary externalities associated with the collateral constraint and because of the paternalistic imposition of the planner's beliefs when designing policy. When robust agents have doubts about the model, they create endogenous worst-case beliefs by assigning a high probability to low-utility events. The ratio of worst-case beliefs of the planner over the household's captures the degree of paternalism.

When the views of the world differ between planner and household, or these are different than the true underlying model, intervention via macroprudential taxes can result in smaller welfare gains or even losses. These losses can occur when the social planner imposes, by means of its paternalistic motive, an incorrect view of the world. While this is a theoretical possibility, in our quantitative analysis we find that, for a reasonable degree of robustness, even when "mistaken", a social planner's intervention generates welfare gains. The inneficiency introduced by imposing an incorrect view of the world is small relative to the efficiency gains from internalizing the pecuniary externality.

[3] To discipline how far they can be from the reference distribution, Hansen and Sargent suggest using the notion of relative entropy.

Moreover, if it were the case that the robust social planner's beliefs are correct, welfare gains can be sizeable and indeed larger than those implied without considering robustness.

Our results are four-fold. First, we find that introducing fear of model misspecification leads the planner to increase savings, for precautionary motives, in every possible state of the world when the collateral constraint is not binding. The reason behind this result is that she is now assigning higher probabilities to worse shocks tomorrow relative to the case without robustness. We note, however, that these precautionary motives are weaker than those of a non-robust planner that instead features higher risk aversion. Decisions under robustness and under higher risk aversion are very similar close to the binding region because a robust planner is assigning a high probability to future negative shocks and thus behaves similarly to a high risk-averse agent. But the further away the economy is from this binding region, the worst-case scenario is a more favorable one and so a robust planner chooses levels of debt higher than would a more risk-averse agent who remains cautious.

Second, we find that the optimal taxes over the region at which the collateral constraint is close to binding depend on the planner's degree of robustness relative to the household's. For instance, if we model a planner who fears misspecification while households make their decisions under the reference model, then the difference between the planner and households' debt levels widens and leads to higher macroprudential taxes than in Bianchi (2011). For the purposes of correcting pecuniary externalities, taxes are needed when bond holdings are close to the region where the constraint is binding. In addition, for the purpose of saving as a precaution against the worst-case scenario in the future, this widening of the optimal bond holdings justifies the introduction of higher taxes to reduce households' borrowing over all the non-binding region. In the long-run, a robust planner imposes taxes that are about 5 percentage points higher than a non-robust planner.

Third, our main contribution is to provide an answer to whether a planner should ex-ante intervene and implement a macroprudential tax when there is model uncertainty. Although taxes are higher than in the non-robust case, these will not result in larger welfare gains if shocks are truly governed by the reference model. If households also make choices that are robust to model misspecification, the gains are significantly reduced. This is because the household already saves more as a precaution against future bad shocks, indirectly addressing the overborrowing problem due to the pecuniary externality. A highly-robust planner "underborrows" and induces welfare losses. However, if the worst-case scenario that the robust agents fear is the one that actually governs the evolution of shocks, then welfare gains are among the largest we find. Overall, we find that an ex-ante intervention is generally welfare improving even in cases in which the social planner is "wrong"' (i.e., her beliefs do not coincide with the actual model generating the shocks). Doubts about the model need to be implausibly high in order to maker a "laissez-faire regime" better than an intervention regime.

Fourth, we find that the long run probability of a crisis is further reduced by a robust planner's allocation relative to what a non-robust planner achieves because introducing model uncertainty leads to higher savings overall. This result holds even if the social planner is wrong about the underlying probability model generating the shocks.

This paper relates mainly to two strands of the literature. First, following the seminal contribution of Bianchi (2011), a vast literature has emerged that rationalizes macroprudential policies as measures to minimize the effects of distortions arising from financial frictions. Relative to Bianchi (2011), Benigno et al (2013) analyze a production economy model in which the pecuniary externality arises because agents fail to internalize the effect of their decisions on the relative price of non-tradable goods. Boz et al (2012) highlight the importance of considering the information set of policymakers in the design of macroprudential policies. Schmitt-Grohe and Uribe (2017) characterize the optimal capital control policy in an economy with a collateral constraint in which both tradable and non-tradable goods have collateral value, and find that, contrary to the general wisdom, the optimal prudential policy is procyclical. Acosta et al (2019) extend the model of Bianchi (2011) by incorporating an Ss-type cost of implementing macroprudential taxes to generate "sticky" optimal capital controls. Basu et al (2020) develop a micro-founded model that characterizes optimal monetary policy, capital controls, and foreign exchange intervention. While all these papers justify the use of macroprudential policies, they do not consider the robustness of the optimal policy (in the sense of Hansen and Sargent, 2011) which is the focus of our paper.

Second, our study relates to the literature on robust control methods pioneered by Hansen and Sargent (2011). A growing theoretical macro literature extend canonical models to the case in which the social planner and/or private agents fear model misspecification and search for robust policies under worst-case scenarios. Pouzo and Presno (2016) and Roch and Roldan (2022) show that the introduction of robust international lenders have imporant quantitative implications in sovereign default models. Adam and Woodford (2012) introduce the robustness framework in a New Keynesian model to analyze optimal robust monetary policy. Bidder and Smith (2012) develop an algorithm to apply robust control methods within nonlinear DSGE models, and show that the interaction between time varying risk and robustness provides an amplification mechanism for volatility shocks (which they interpret as animal spirits). Karantounias (2013, 2020) show that the degree of paternalism and pessimistic expectation management are the key forces shaping optimal policy in setups with model uncertainty. Finally, Young (2012) is the paper closest to ours as it also studies robust macroprudential policy. They consider a production economy with occasionally binding constraints as well. But we differ from them in two aspects: (i) they take into consideration policies that are not prudential in spirit, but rather they focus more on interventions that occur at the time of a crisis (i.e., when the collateral constraint binds); and, (ii) we examine more carefully how to interpret results when we have planner and/or agents that actually disagree on the view of the world that they have.

The rest of this paper proceeds as follows. First, Section 2 lays out the canonical model of pecuniary externalities developed by Bianchi (2011) and adds model misspecification following the approach of Hansen and Sargent (2011). Section 3 presents the quantitative results. Finally, Section 4 concludes.

2 A model of ocassionally binding constraints with robustness

In this section, we extend the canonical model of occasionally binding constraints à la Bianchi (2011) to allow for model uncertainty by applying the robustness framework of Hansen and Sargent (2011).

We discuss how to interpret fear of misspecification and what it entails in terms of welfare gains when the household and the planner share different views of the world.

Time is discrete. We study an infinite-horizon small open economy that receives a stochastic endowment stream of tradable goods, y_t^T, and non-tradable goods, y_t^N. Let $y_t \equiv (y_t^T, y_t^N)$ be a discrete-state Markov chain with transition probabilities given by $\pi(y_{t+1}|y_t)$ for all $t > 0$, and y_0 given. There is a representative household who derives utility from the consumption of tradable goods, c_t^T, and non-tradable goods, c_t^N. In particular, the agent's period payoff function takes the form

$$u(c_t^T, c_t^N) = \frac{1}{1-\sigma}\left(\left[\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}\right]^{-1/\eta}\right)^{1-\sigma}$$

where σ is the coefficient of relative risk aversion, $\frac{1}{1+\eta}$ is the elasticity of substitution between consumption of tradable and non-tradable goods, and $\omega \in (0,1)$ is a share parameter.

Households can only issue a single non-contingent, one-period, risk-free bond available to the agent denominated in terms of the tradable good that pays a fixed interest rate r. Their sequential budget constraint is then given by

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N \tag{1}$$

where b_{t+1} denotes the bond holdings chosen by the household in period t (negative values denote debt), and p_t^N denotes the relative price of non-tradable goods in terms of tradable goods.

An occasionally-binding borrowing constraint limits households' debt to a fraction κ^N of non-tradable income and a fraction κ^T of tradable income. Then, the collateral constraint is given by

$$b_{t+1} \geq -(\kappa^N p_t^N y_t^N + \kappa^T y_t^T) \tag{2}$$

2.1 Equilibrium and fears of model misspecification

Robust households distrust the probability model which dictates the evolution of y_t. They entertain $\pi(y_{t+1}|y_t)$ as their reference model but fear that it is misspecified. Thus, a robust household doubts the joint density $\pi(y^t)$ and considers a set of alternative distorted distributions $\tilde{\pi}(y^t)$ for all (t, y^t), where $y^t = (y_0, ..., y_t)$ denotes the history of realizations of the endowment up to t. These alternative probability models are assumed to be absolutely continuous with respect to the reference model and, by means of the Radon-Nikodym theorem, can be represented in terms of a non-negative random variable $M_t(y^t)$ that is a measurable function of y^t given by

$$M_t(y^t) = \begin{cases} \frac{\tilde{\pi}(y^t)}{\pi(y^t)} & \text{if } \pi(y^t) > 0 \\ 1 & \text{if } \pi(y^t) = 0 \end{cases} \quad (3)$$

and $M_0 = 1$. Let E_t be the expecation operator with respect to the reference model conditional on the information set at period t. It follows that M_t is a martingale with respect to the reference model, i.e., $E_t(M_{t+1}) = M_t$, and we set $M_0 = 1$. We can decompose M_t by defining the conditional likelihood ratio $m_{t+1} \equiv \frac{M_{t+1}(y^{t+1})}{M_t(y^t)}$ as

$$m_{t+1}(y_{t+1}|y^t) = \begin{cases} \frac{\tilde{\pi}(y_{t+1}|y^t)}{\pi(y_{t+1}|y_t)} & \text{if } \pi(y^{t+1}) > 0 \\ 1 & \text{if } \pi(y^{t+1}) = 0 \end{cases} \quad (4)$$

with $m_{t+1} \geq 0$ and $E_t(m_{t+1}) = 1$, to guarantee that the distorted probability is a well-defined probability measure. The martingale representation twists the measures implicit in the reference model so as to obtain absolutely continuous measures that represent distorted models contemplated by the household. Under these twisted measures we are able to take expectations with respect to the alternative distribution $\tilde{\pi}(y^t)$: $E_{\tilde{\pi}}(X_{t+1}|y^t) = E(m_{t+1}X_{t+1}|y_t)$, for some random variable X_t.

Following Hansen and Sargent (2011), we assume that a robust household features multiplier preferences to capture concerns about potential model misspecification:

$$\min_{m_{t+1}, M_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t M_t \{ u(c_t^T, c_t^N) + \beta \theta_A E_t[m_{t+1} log(m_{t+1})] \} \right] \quad (5)$$

subject to $M_{t+1} = m_{t+1}M_t$, $E_t(m_{t+1}) = 1$, $M_0 = 1$, and $\theta_A \geq \underline{\theta}$.

Multiplier preferences lead our households to distort their reference model and seek rules that perform well under a variety of possible models that are statistically close to the reference model.[4] A common metaphor is that households choose their actions to maximize utility while a fictitious 'evil agent' chooses a probability distribution (or equivalently a sequence of conditional likelihood ratios, m_{t+1}) to minimize that same utility. As a result, the action chosen performs well across a range of possible specification errors of the reference model. The second term in (5) represents the relative entropy associated with a given distortion. Thus, the overall utility includes a gain from the entropy of the distribution used with respect to the reference model (resulting in an entropy cost incurred by the evil agent), which limits the size of the distortions that are considered. The key parameter is the marginal cost of relative entropy, θ_A, which regulates the degree to which the minimizing agent is penalized for contemplating alternative models that are too far from the reference model.

Let V be the value function of the household, B_t the aggregate level of debt, $\Gamma(B_t, y_t)$ the household's perceived law of motion for aggregate bond holdings, and $\tilde{p}^N(B_t, y_t)$ the household's forecast price function for non-tradable goods. Invoking results in Hansen and Sargent (2011), the recursive representation of the robust household's max-min problem is given by

$$V(B_t, b_t, y_t) = \underset{c_t^N, c_t^T, b_{t+1}}{max} \ \underset{m_{t+1}}{min} \ u(c_t^T, c_t^N) + \beta E\left[\theta_A m_{t+1} log\left(m_{t+1}\right) + m_{t+1} V(B_{t+1}, b_{t+1}, y_{t+1})\right] \tag{6}$$

subject to

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N$$

$$b_{t+1} \geq -(\kappa^N p_t^N y_t^N + \kappa^T y_t^T)$$

$$B_{t+1} = \Gamma(B_t, y_t) \ , \ p_t^N = \tilde{p}^N(B_t, y_t) \ , \ E[m_{t+1}] = 1 \ , \ m_{t+1} \geq 0 \ , \ \theta_A \in (\underline{\theta}, +\infty)$$

The solution to the inner minimization problem in (6) yields the worst-case beliefs of the robust household, denoted with asterisks in the expression below

$$m_{t+1}^* = \frac{e^{-V(B_{t+1}, b_{t+1}, y_{t+1})/\theta_A}}{E[e^{-V(B_{t+1}, b_{t+1}, y_{t+1})/\theta_A}]}$$

[4] Stzralecki(2011) shows that a large class of uncertainty-averse preferences can be related to the axiomatization of multiplier preferences.

Note that the household assigns higher probability distortions to low utility states, which are associated with bad shocks in the endowment of tradable and non-tradable goods. Also, as $\theta_A \to \infty$ the probability distortions to the reference model vanish, and (5) collapses to standard expected utility. Substituting this solution into the original problem, we can express the Bellman equation in (6) with the familiar risk-sensitive form

$$V(B_t, b_t, y_t) = \max_{c_t^N, c_t^T, b_{t+1}} u(c_t^T, c_t^N) - \beta\theta_A log\left(E[e^{-V(B_{t+1}, b_{t+1}, y_{t+1})/\theta_A}|B_t, b_t, y_t]\right) \tag{7}$$

where the expectation is with respect to the transition density given by the reference model.

Definition 1. *A recursive competitive equilibrium is defined by a price function $\tilde{p}^N(B_t, y_t)$, a law of motion for aggregate bond holdings ($B_{t+1} = \Gamma(B_t, y_t)$), belief distortions m_{t+1}^*, and household decision rules ($b_{t+1}(B_t, y_t, b_t)$, $c^T(B, y, b)$, $c^N(B_t, y_t, b_t)$) with the associated value function ($V(B_t, y_t, b_t)$), such that:*

- *Given $\tilde{p}^N(B_t, y_t)$, $\Gamma(B_t, y_t)$, and m_{t+1}^*, decision rules and value function solve the household's maximization problem in (6).*
- *Given $\tilde{p}^N(B_t, y_t)$, $\Gamma(B_t, y_t)$, household's decision rules and value function, m_{t+1}^* solve the alter ego's minimization problem in (6).*
- *Markets clear:*

$$y_t^N = c^N(B_t, y_t, b_t) \tag{8}$$

$$\Gamma(B_t, y_t) + c^T(B_t, y_t, b_t) = y_t^T + B_t(1+r) \tag{9}$$

- *The perceived law of motion of aggregate bond holdings coincides with the actual law of motion induced by the household's choice, and the price of non-tradables taken as given equals the one obtained from the optimal decisions:*

$$B_{t+1} = \Gamma(B_t, y_t) = b_{t+1}(B_t, y_t, B_t) \tag{10}$$

$$\tilde{p}^N(B_t, y_t) = \frac{1-\omega}{\omega}\left(\frac{y_t^T + B_t(1+r) - \Gamma(B_t, y_t)}{y_t^N}\right)^{\eta+1} \tag{11}$$

2.2 Social Planner's Problem

A planner is constrained by the same credit limit of the households and chooses the level of debt but lets the goods markets clear competitively (and this determines quantities and prices). The households receive the proceeds from the credit choices of the planner in a lump sum fashion. Formally, this means that the planner solves a utility maximization problem subject to the collateral constraint (2), resource constraints (i.e. market clearing (8)-(9)) and an implementability constraint (i.e. competitive prices (11)).

Similarly to households, we also attribute fears of model misspecification to the planner. A robust planner also distrusts the probability model which dictates the evolution of y_t. Drawing parallels with the previous section, we endow the robust planner with multiplier preferences, where θ_R is the planner's penalty parameter that captures the degree of robustness and $n_{t+1}(y_{t+1}|y^t)$ is the conditional likelihood ratio. Again, as $\theta_R \to \infty$ the probability distortions to the reference model vanish, which reduces the multiplier preferences to the expected utility preferences of the Bianchi (2011) planner.

Definition 2. *A recursive constrained-efficient equilibrium is defined by a price of non-tradables $p^N(B_t, y_t)$, the planner's law of motion for bond holdings $B(B_t, y_t)$ and its value function $W(B_t, y_t)$, and the household's decision rules $c^T(B_t, y_t)$, $c^N(B_t, y_t)$ -given the planner's decision for debt-, such that:*

- *Bond holdings, decision rules and value function solve the planner's recursive optimization problem. A robust planner considers:*

$$W(B_t, y_t) = \max_{B_{t+1}, c_t^T} u(c_t^T, y_t^N) - \beta\theta_R log\left(E[e^{-W(B_{t+1}, y_t)/\theta_R}|B_t, y_t]\right) \tag{12}$$

subject to

$$B_{t+1} + c_t^T = B_t(1+r) + y_t^T$$

$$B_{t+1} \geq -(\kappa^N p^N(B_t, y_t) y_t^N + \kappa^T y_t^T)$$

$$c_t^T \geq 0$$

$$p^N(B_t, y_t) = \frac{1-\omega}{\omega} \left(\frac{y_t^T + B_t(1+r) - B_{t+1}}{y_t^N} \right)^{\eta+1}$$

Consumption of non-tradables is given by $c^N(B_t, y_t) = y_t^N$.

The presence of the relative price of non-tradables in the borrowing constraint introduces a pecuniary externality in the model which makes the decentralized equilibrium suboptimal. In Bianchi (2011), without model uncertainty, the key difference between the planner and the household is that the planner internalizes the effect that the choice of debt has on the price of non-tradables via the level of tradable consumption it permits. This mechanism becomes clear from inspecting the household and planner's optimality conditions in periods when the collateral constraint is not binding. Recall that our setup converges to Bianchi (2011) as $\theta_A \to \infty$ and $\theta_R \to \infty$. In this case, the Euler equation for consumption in the decentralized equilibrium is given by

$$u_t^T = \beta(1+r)E\left[u_{t+1}^T\right] \tag{13}$$

where u_t^T denotes the marginal utility with respect to tradables in period t. The Euler equation for the planner is:

$$u_t^T = \beta(1+r)E[u_{t+1}^T + \mu_{t+1}^{SP}\psi_{t+1}] \tag{14}$$

where $\psi_t \equiv \kappa^N \frac{\partial p_t^N}{\partial c_t^T} y_t^N$ reflects how much the value of collateral changes when tradable consumption increases; and μ_t^{SP} is the shadow price of relaxing the credit constraint of the planner's problem, which is positive when the constraint binds. Comparing these Euler equations we can see that the marginal benefit of decreasing borrowing for the household (i.e., higher expected consumption tomorrow due to an increase in savings) differs from the marginal benefit of a planner (i.e., in addition to higher consumption tomorrow due to higher savings, the planner also accounts for higher consumption tomorrow due to a more relaxed collateral constraint). The externality from extra borrowing that the agent is not internalizing is given by the discounted expected marginal utility cost of a collateral constraint that is tighter tomorrow due to a lower price of non-tradables. Therefore, in competitive equilibrium, there is inefficient overborrowing in normal times by private agents that do not internalize the effect of their borrowing decisions on the value of collateral.

Thus, there is a role for macroprudential policy to induce the household to internalize this externality and implement the constrained efficient allocation. For periods when the collateral constraint is not currently binding, the tax on debt, τ_t, that implements the planner's allocation is such that both the household's and the planner's Euler equations are satisfied when evaluated at the planner's allocation. The Euler for consumption in the regulated decentralized equilibrium is then given by

$$u_t^T = \beta(1+r)(1+\tau_t)E[u_{t+1}^T]$$

where

$$\tau_t = \frac{E[\mu_{t+1}^{SP}\psi_{t+1}]}{E[u_{t+1}^T]} \tag{15}$$

Introducing the possibility of either household and/or planner fearing model misspecification adds an additional tension for macroprudential policy, which we analyze in the following section.

2.3 Optimal policy and model uncertainty

As discussed in the Introduction, there is a growing theoretical literature that extends canonical macroeconomic models to the case in which the social planner and/or private agents fear model misspecification and analyze the implications for optimal policy. For instance, several recent papers introduce the multiplier preferences of Hansen and Sargent (2011) into a framework à la Lucas and Stokey (1983) and characterize optimal fiscal policy under various alternative assumptions about the doubts of the social planner and household. Karantounias (2013) considers a government that fully trusts the reference model and uses it to design policy, while facing households that form pessimistic expectations. Instead, Ferriere and Karantounias (2019) study the case of a planner that adopts the perspective of the household in evaluating welfare. Finally, Karantounias (2020) examine the case of a government that can doubt the reference model more, the same, or less than the household.

A similar analysis can be found in Benigno and Paciello (2014) in the context of optimal monetary policy.

This literature has dissected two prominent forces through which the different assumptions about the concerns of model misspecification of the social planner/household shape the optimal policy: (i) the ratio of worst-case beliefs of the planner over the private sector's, which captures paternalism; and (ii), the extent to which the endogenous worst-case beliefs of the private sector can be managed by the social planner. The paternalistic force is present whenever the household and the planner differ in their beliefs (i.e., when $\theta_A \neq \theta_R$), and refers to the fact that the planner imposes its own expected utility criterion when choosing the efficient allocation. The larger the disagreement in beliefs, the larger the paternalistic motive of the planner becomes. The pessimistic expectation management is absent in our setup because the collateral constraint is static and, thus, the private sector endogenous beliefs do not enter the constraints that the planner is facing. However, the paternalism channel is still relevant in our model and adds an additional motive (on top of the pecuniary externality) for which the decentralized equilibrium may differ from the constrained efficient allocation.

We can gain intuition about the two fundamental mechanisms in our model by reconsidering the household and planner's Euler equations when allowing for model uncertainty (i.e., $\theta_A, \theta_R < \infty$). When the collateral constraint is not currently binding, the Euler equation of a household who doubts the model becomes

$$u_t^T = \beta(1+r)E\left[m_{t+1}^* u_{t+1}^T\right] \tag{16}$$

where

$$m_{t+1}^* = \frac{e^{-V(B_{t+1}, b_{t+1}, y_{t+1})/\theta_A}}{E[e^{-V(B_{t+1}, b_{t+1}, y_{t+1})/\theta_A}]}$$

Similarly, if the credit constraint does not currently bind, the Euler equation of a planner who doubts the model becomes

$$u_t^T = \beta(1+r)E[n_{t+1}^*(u_{t+1}^T + \mu_{t+1}^{SP}\psi_{t+1})] \tag{17}$$

where

$$n_{t+1}^* = \frac{e^{-W(B_{t+1},y_{t+1})/\theta_R}}{E[e^{-W(B_{t+1},y_{t+1})/\theta_R}]}$$

In contrast to the case without model uncertainty, from (16) and (17) we note that now there are two forces that could make the competitive equilibrium different from the social planner equilibrium. We still observe the pecuniary externality motive that the planner internalizes in (17), captured by the term $\mu_{t+1}^{SP}\psi_{t+1}$. Moreover, when introducing the possibility of either household and/or planner fearing model misspecification, the difference in beliefs (m_t^* versus n_t^*) creates an additional potential source of discrepancy between the decentralized and constrained efficient allocations. This is our main theoretical contribution relative to Bianchi (2011).

These two channels also manifest themselves in the expression for the optimal macroprudential tax that implements the planner's allocation. If the collateral constraint is currently not binding, the opimal tax is given by

$$1 + \tau = \frac{E[n_{t+1}^*(u_{t+1}^T + \mu_{t+1}^{SP}\psi_{t+1})]}{E[m_{t+1}^* u_{t+1}^T]} \tag{18}$$

Note that the optimal tax is not necessarily non-negative anymore due to the paternalism of the planner. Moreover, even if the collateral constraint has zero probability of being binding in the next period (i.e., $\psi_{t+1} = 0$), the tax is not zero anymore either due to the paternalism channel. In this case, the expression for the optimal tax becomes

$$1 + \tau = \frac{E[n_{t+1}^* u_{t+1}^T]}{E[m_{t+1}^* u_{t+1}^T]} \tag{19}$$

The optimal tax will be negative (positive) when θ_A is greater (lower) than θ_R. $\theta_A > \theta_R$ implies that the household is more robust than the planner, and thus chooses higher levels of precautionary savings. A paternalistic planner then imposes its view of the world and introduces a subsidy ($\tau < 0$) to implement her allocation. The tax rate will become zero if the beliefs are the same ($\theta_A = \theta_R$).

3 Quantitative Results

We follow closely the calibration from Bianchi (2011) that uses data from Argentina, summarized in Table 1. We re-calibrate only the discount factor and the share of collateralized tradable to achieve a probability of a sudden stop of 13% and an average debt-to-GDP ratio of 30% under the decentralized equilibrium without government intervention. While Bianchi targets a probability of 5%, we focus on a higher value following evidence from Jeanne and Ranciere (2011) and Furceri et al (2011) that sudden stops are twice as frequent, with probabilities of around 10% and as high as 20%. This probability justifies the implementation of higher macroprudential taxes, consistent with the empirical evidence. In Appendix 5.2 we also present the results using Bianchi's exact calibration for comparability.

In terms of risk aversion and the degree of misspecification, we consider a low and a high value both to assess the sensitivity of the results to the different parameterizations and to discuss the differences between introducing robustness in the decision-making process versus simply increasing an agent's risk aversion. Our baseline calibration considers a standard risk-aversion of 2, and we later double this to compare results relative to those of a robust agent. In order to discipline the choice of θ, we compute detection error probabilities under the allocation of the decentralized equilibrium. These capture the probability with which an agent is able to distinguish between the worst-case and the reference densities. We search for a level of robustness (i.e., a value of θ) such that an agent would not "easily" distinguish between the two, so that it remains plausible for her to want to make a robust choice against the possibility that the model is not the reference one. Based on this analysis, we take $\theta = 0.35$ as a baseline value in our calibration, and later compare with results from increasing the level of robustness to $\theta = 0.15$.

Table 1: Calibration

Parameter	Value	Source
Utility function		
Risk aversion σ	2	Stardard, Bianchi (2011)
Intratemp elast of substitution $1/(1+\eta)$	0.83	Bianchi (2011), conservative
Discount factor β	0.8	To match data moments - Bianchi (2011): 0.91
Share of tradables in CES ω	0.31	Bianchi (2011), to match data moments
Interest rate		
World risk-free interest rate r	0.04	Stardard, Bianchi (2011)
Collateral constraint		
Relative quality of collateral κ^N/κ^T	1	Bianchi (2011)
Share of collateralized tradable κ^T	0.31	To match data moments - Bianchi (2011): 0.32
Robustness		
Degree of misspecification θ	0.35	To target a detection error probability of $\approx 20\%$

Note: Our main calibration follows closely that of Bianchi (2011), with two differences: we consider a lower discount factor and a slightly lower share of collateralized tradable to match a probability of sudden stop of about 13% and a debt-to-GDP ratio of about 30%. To analyze the sensitivity of results to a higher risk aversion and degree of robustness, we also present results computed with a risk aversion of 4 and a $\theta = 0.15$.

Following Bianchi (2011), we assume that the endowment process follows a first-order bivariate autoregressive process

$$log(y_t) = \rho log(y_{t-1}) + \epsilon_t \tag{20}$$

with $|\rho| < 1$ and $\epsilon_t \equiv (\epsilon_t^T, \epsilon_t^N) \sim N(0, V)$. This will be the reference model shared by the household and social planner. The parameter values that govern the endowment process are chosen to mimic the cyclical components of the HP detrended tradable and non-tradable GDP in Argentina from 1965 to 2007.[5] The estimates of ρ and V are

$$\rho = \begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix} \quad V = \begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}.$$

[5] The GDP data comes from the World Development Indicators database. Tradable GDP consists of manufacturing and primary products while the rest of the components of GDP fall under non-tradables.

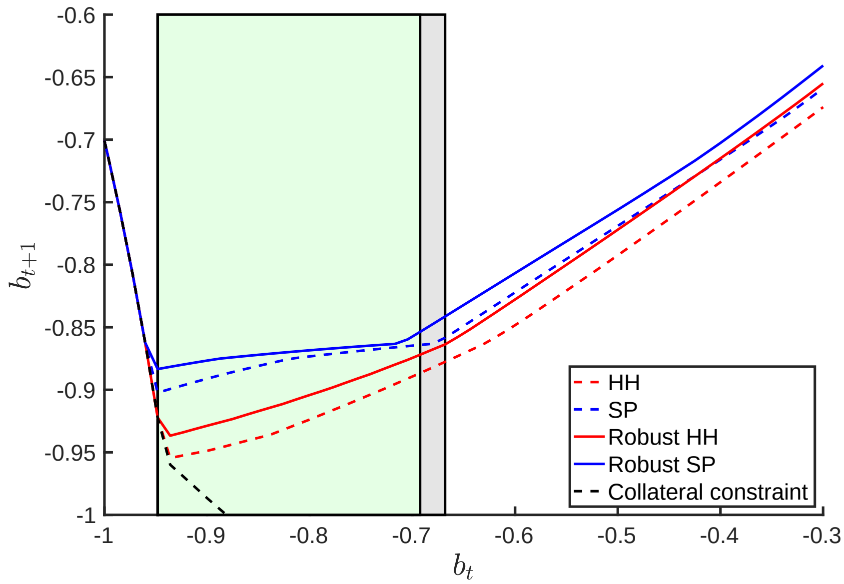
3.1 Debt policy: household vs planner

Figure 2 shows the optimal borrowing decisions of a planner and a household when both tradable and non-tradable endowments are one standard deviation below mean. The dashed lines show the debt policies for a household and a planner that do not fear misspecification and replicate the findings in Bianchi (2011) with the noted change in the calibration. The solid lines depict the debt policies for these agents under robustness. The green area that overlaps a larger grey area indicates the region over which macroprudential taxes are required to decentralize the allocation of a robust planner in a context in which households also make robust decisions. The grey area is the corresponding tax region for a planner and a household that are not robust. In Section 3.4 we discuss how these regions compare, we present the optimal taxes and also extend this analysis for the mixed cases (when one agent makes robust choices but the other does not).

The decisions of both household and planner share in common a U-shaped pattern. Without a borrowing constraint, consumption smoothing would imply a monotonically increasing policy for bond holdings: the higher the current debt level (lower current bond holdings), the higher the debt that is optimally chosen. When there is a collateral constraint and it binds, however, the choice of bond holdings increases with a higher current debt. This is because a higher debt today implies a lower tradable consumption given a choice of next-period bond holdings. This lowers the price of non-tradables, tightening the collateral constraint and leading to a lower level of borrowing. This binding region is smaller for robust agents.

In the non-binding area, a planner accumulates larger bond holdings compared to a household, to serve as precautionary savings against the possibility of the constraint binding in the future. Robust agents will also save more compared to their non-robust versions over the entire non-binding region in a precautionary attitude against a higher probability of negative shocks in the future. Notably, a robust household at some point overtakes the non-robust social planner and choses higher levels of savings given current levels of debt that are far from the binding region. Closer to the binding area, a non-robust planner still chooses higher savings to account for the externalities on the value

Figure 2: Debt policies of (robust) household and (robust) planner



Note: This plot depicts the optimal debt policy of a planner and a household as a function of current debt holdings, for a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables. For the robust agents, the degree of misspecification considered is characterized by $\theta = 0.35$. The green area is the region of current debt holdings for which a tax is needed to implement a robust planner's allocation in a decentralized economy that features a robust household. It overlaps a wider grey area that corresponds to the tax region for non-robust agents. Parameters are calibrated to generate a probability of crisis of 13% under the debt policy of a non-robust household and the reference distribution of shocks.

of collateral.

The tax areas are regions in which the constraint for the private agents will bind tomorrow with positive probability (i.e. given the household's choice of debt, there will be some states y' tomorrow for which the constraint will bind). We note that a robust planner will tax a robust household over a slightly narrower region of debt (the green shaded area) than a non-robust planner with respect to a non-robust household (the grey area). While it is not entirely obvious whether taxes are larger when both agents are robust relative to when they are not (note that the gap between debt choices is very similar), the difference in debt choices of a robust planner compared to a non-robust household is significantly larger. A robust planner borrows considerably less in the unconstrained region close to the binding point compared to a non-robust household, and this would suggest that much larger taxes are needed to implement the socially optimal allocation, and over a much larger tax region. We explore this further in Section 3.4.

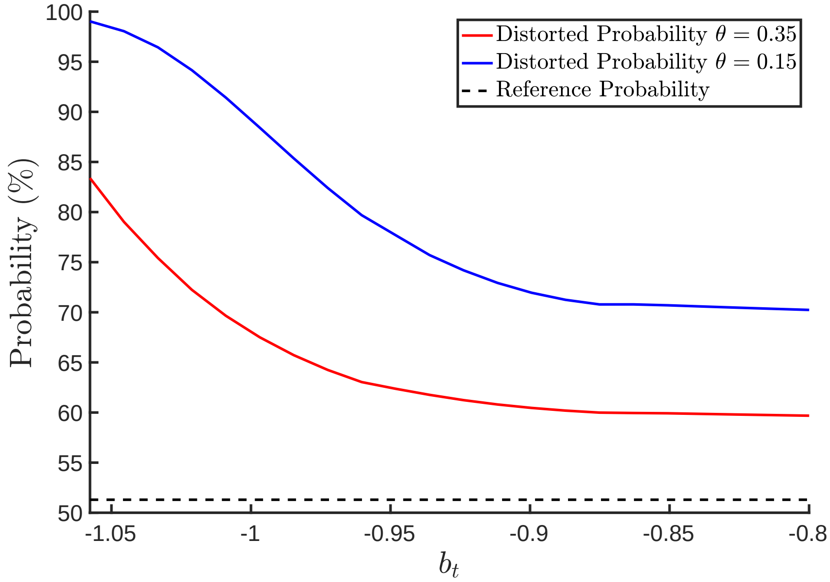
3.2 Distorted Probabilities

Consider a state in which both tradable and non-tradable endowments are one standard deviation below mean (as in Figure 2). Figure 3 shows the reference and distorted probabilities that the economy transitions to a worse state in the next period. These probabilities are plotted as a function of the current bond holding (b_t). The black dashed line indicates the probability under the reference model, which does not depend on b_t. The red and blue lines correspond to the distorted probabilities when $\theta = 0.35$ and $\theta = 0.15$ respectively (i.e., the probabilty under which an agent that has a preference for robustness forms expectations). Note that the distortion is in the direction of the worst-case: the perceived probability that income will be low is higher than the reference probability for all values of b_t. Moreover, in contrast with the reference probability, the distortion is not invariant to the endogenous state b_t. Specifically, for states where the households are more indebted, the distortion is higher (the blue and red curves are decreasing in b_t). When the stakes are higher, namely when a future low realization of income can make the household financially constrained and hence unable to smooth consumption, the agent is willing to entertain larger deviations from the reference distribution. As expected, a higher preference for robustness (lower θ) leads to larger distortions as evidenced by the blue line ($\theta = 0.15$) departing from the reference probability more than the red line ($\theta = 0.35$).

3.3 Simulations

Based on the allocations that result from the optimal debt policies, Bianchi (2011) simulates the centralized and decentralized economies to analyze how the difference in bond accumulation of households and planner affect the long-run distribution of debt, and the probability and magnitude of sudden stops. We follow this approach to compare the results from policies that respond to non-robust and robust choices. In line with Bianchi (2011), we obtain $80,000$ period allocations given some history of realized shocks for the endowment process. These shocks are obtained from the

Figure 3: Reference and Distorted Probabilities



Note: This plot depicts the reference and distorted probabilities that next period's state is worse than the current one (both treadeable and non-tradable income are one standard deviation below mean). These probabilities are shown as a function of the endogenous level of bondholdings. The dashed black line shows the reference probability, while the red and blue lines show the distorted probabilities induced by a preference for robustness with $\theta = 0.35$ and $\theta = 0.15$ respectively.

reference probability model but, when analyzing welfare gains, we also report the results from simulations that use worst-case probabilities viewed from the perspective of the household.[6] Based on a history of shocks, we obtain the simulated series for all relevant macroeconomic variables when the debt policy follows the optimal choices of: a non-robust household, a non-robust planner, a robust household or a robust planner. With respect to robust policies, we consider both a value for θ of 0.35 (benchmark calibration) and of 0.15 (which implies a higher degree of misspecification than the benchmark). For all the other parameters, we use the calibration that renders a probability of crisis of 13% under the household's non-robust allocation and the reference distribution of shocks (but we also report the results that replicate Bianchi (2011) for comparability in Appendix 5.2).

A sudden stop happens when the collateral constraint binds and this leads to an increase in net

[6] There is a different matrix of "worst-case" transition probabilities for each possible current state. This is because what constitutes a worst-case scenario for a household, based on her optimal debt policy, depends on the current level of bond holdings and how close the economy is to the binding region for the collateral constraint.

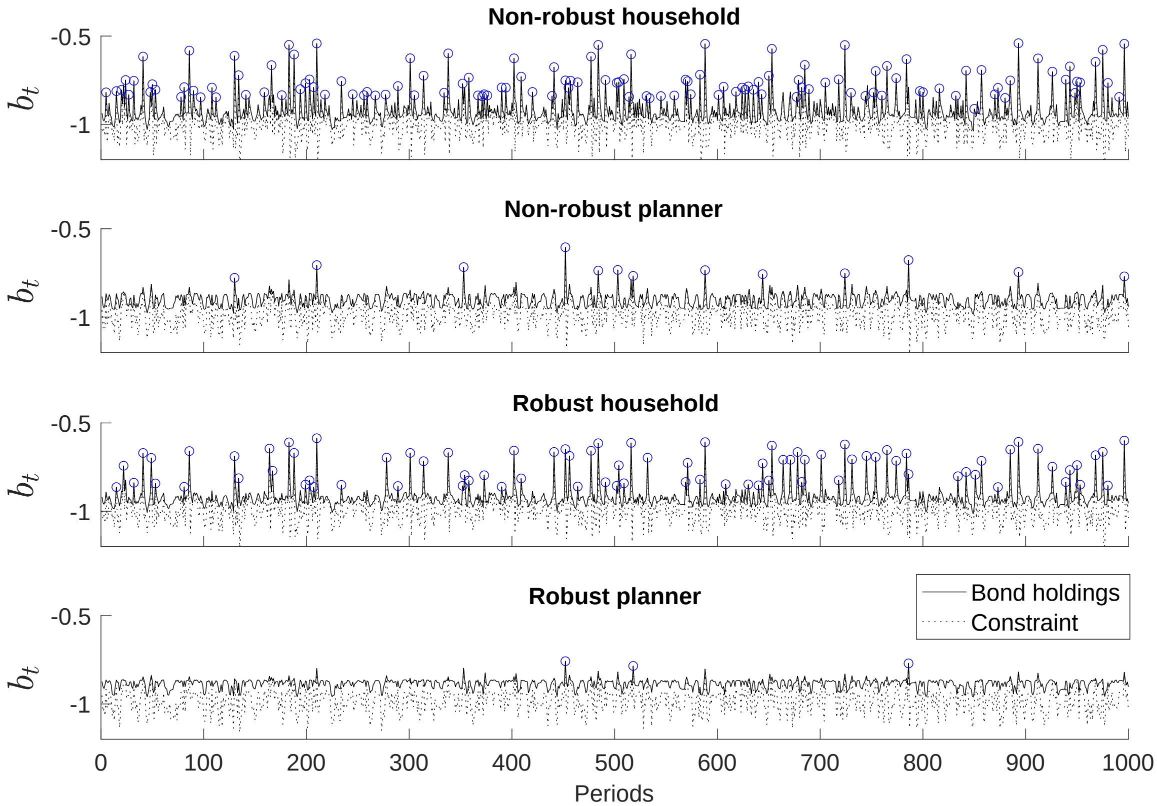
capital outflows (i.e. an increase in bond holdings) that exceeds *one standard deviation* of net capital outflows according to the long-run distribution of bond holdings of the decentralized economy (i.e., under the non-robust household's allocation). Figure 4 shows the evolution of the simulated time series of bond holdings for an interval of 1000 periods, based on the (robust) household's and the (robust) planner's chosen allocations. The shocks are realizations from the reference model and the robust allocations are based on a degree of misspecification of $\theta = 0.35$. Figure 8 in the Appendix shows the corresponding results when shocks are derived from a worst-case distribution. As expected, a planner is able to reduce the number of periods in which a sudden stop occurs, and robust agents can also achieve this relative to their non-robust counterparts.

Table 2 compares the simulation results for the two economies.[7] Using the generated data, we take the time series for debt and we count the number of periods with a sudden stop. To measure the severity of a crisis, we follow Bianchi (2011) and consider the following experiment: we find the median financial crisis in the decentralized equilibrium (i.e., under the non-robust household's allocation), defined as that which features the median current account reversal. We then backtrack the level of debt two periods before and simulate the response of a robust household and of a planner (both a robust one and a non-robust one) that are faced with that same initial level of debt and the same set of shocks over the next three periods. Finally, we report a set of standard deviations and correlations with GDP.

In line with the results from Bianchi (2011), a social planner that internalizes the effect of borrowing on the value of collateral is able to significantly reduce the occurrence of sudden stops, from 13% to 1.5% in our baseline calibration (from 5% to 0.3% for Bianchi's calibration in Appendix 5.2). The severity of the crisis is also greatly reduced, as evidenced by a much lower reduction in consumption (4% versus 9%, where percentages are relative to the long-run mean), a lower depreciation of the real exchange rate (8% versus 20%), and a smaller current account reversal (2% versus 8%). A robust household's allocation is able to improve on the results of a non-robust household for the simple

[7] Table 7 in Appendix 5.2 replicates the results in Bianchi (2011).

Figure 4: Evolution of bond holdings based on reference distribution of shocks



Note: These plots show the evolution of bond holdings from simulations that consider a specific history of shocks to tradable and non-tradable endowments based on the reference distribution. The same series of shocks is fed into the four cases depicted here. The first plot considers the optimal debt choice of a non-robust household, the second that of a non-robust planner, the third shows the choices from a robust household and the fourth from a robust planner. The blue circles indicate a sudden stop (i.e. the constraint binds *and* there is a one standard deviation reversal of the current account).

reason that due to fear of misspecification the robust agent will always save more for precautionary motives. A robust household, even if they still do not account for the pecuniary externalities, can decrease the probability of a sudden stop to 9% and halve the reduction in consumption and in the depreciation of the real exchange rate (though there are no significant improvements in terms of median current account reversal). A robust planner is able to improve on the results of a non-robust planner, bringing the probability of a crisis to 0.7% and reducing the severity of these episodes. Whether these improvements imply gains in welfare is still something that needs to be assessed and we address this in Section 3.5.

Table 2: Probability and magnitude of a sudden stop

	HH	SP	HH Robust $\theta = 0.35$	SP Robust $\theta = 0.35$
Long-run prob of crisis* (%):	12.70	1.50	8.58	0.65
Long-run prob of constraint binding (%):	19.35	2.86	11.90	1.73
Average debt as % of GDP:	29.18	28.56	29.04	28.08
Decrease in consumption in median crisis:	-8.81	-3.72	-4.91	-2.76
Depreciation RER in median crisis:	-20.06	-7.93	-10.71	-5.28
CA reversal in median crisis	7.94	2.25	7.94	1.19
Std dev of consumption:	0.07	0.06	0.07	0.05
Std dev of RER:	0.37	0.20	0.30	0.15
Std dev of CA (as % of GDP):	0.05	0.02	0.04	0.01
Corr consumption with GDP:	0.87	0.83	0.76	0.58
Corr RER with GDP:	0.91	0.75	0.79	0.63
Corr CA (as % of GDP) with GDP:	-0.89	-0.70	-0.77	-0.61

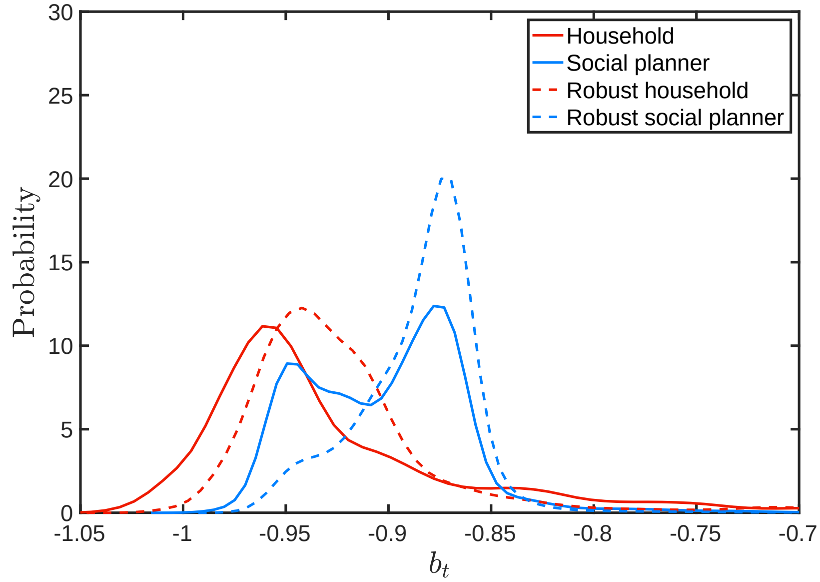
Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the reference distribution. A sudden stop happens when the constraint binds *and* there is a one standard deviation reversal of the current account. Columns 1 and 2 report the results from economies that feature non-robust agents, while columns 3 and 4 are based on the optimal choices of a robust household and planner.

The precautionary savings of the planner imply that the long-run (ergodic) distribution of bond holdings for the planner assigns a higher probability to lower levels of debt (high bond holdings) compared to the private agents (Figure 5 and Figure 9 in the Appendix). Notably, on average, both planner and private agents -whether robust or not- tend to hold the same level of debt (30%). However, the planner reduces exposure to very high levels of debt (low bond holdings) that would make the collateral constraint bind and trigger a sudden stop when there is a negative shock to output. This same observation holds when comparing a robust agent against her non-robust counterpart.

3.4 Macroprudential taxes

The fact that the difference in optimal bond holding decisions between a non-robust household and a robust social planner widens over the non-binding region, suggests that in this case the macroprudential taxes that would be required for decentralizing the planner's allocation are higher compared to a situation in which the planner is non-robust. This is explained by the paternalism channel

Figure 5: Distribution of debt holdings



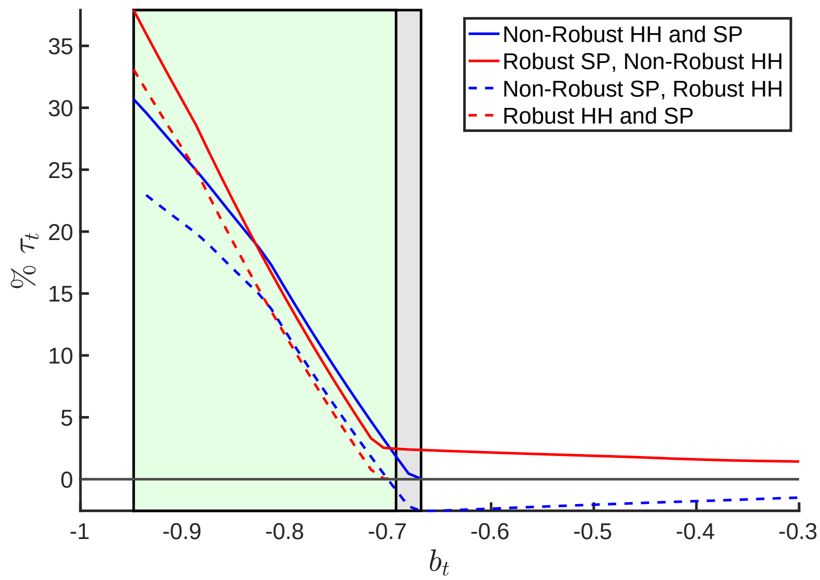
Note: Distribution of debt holdings based on robust and non-robust allocations by household and planner. The parametrization used for this plot implies a probability of crisis of 13%.

discussed in Section 2.3. We find that in all states of nature and across the entire bond grid, the macroprudential taxes that are required to decentralize the allocation of a robust planner are larger, and are set over the entire set of possible current bond holdings where the constraint is not yet binding. Figure 6 illustrates this point by comparing the optimal taxes and the tax region across the endogenous state-space (debt holding) imposed by a non-robust/robust planner when households are non-robust/robust conditional on a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables.

In order to compare the average tax over a long-run horizon required to decentralize a planner's allocation across different cases, we compute the mean tax over the simulations from the previous section. Table 3 summarizes the cyclical properties of the macroprudential tax that implements the planner's allocation when the economy is subject to shocks from the reference distribution, and also indicates the probability that a tax is implemented.[8] Our baseline calibration that renders a crisis

[8]Table 8 in the Appendix presents these results under the calibration in Bianchi (2011).

Figure 6: Tax regions



Note: This figure shows the macroprudential taxes that are required to implement a social planner's allocation in a decentralized economy conditional on a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables. Taxes are functions of the current level of debt (the x-axis of the plots). The red lines are taxes that are implemented by a robust planner's allocation (with $\theta = 0.35$ and the blue lines refer to a non-robust planner. Solid lines are taxes imposed on non-robust households while dashed lines refer to robust households. The light gray area is the tax region for taxes from a non-robust planner when implemented on a non-robust household, and it overlaps with the wider narrower green area that indicates the tax region for a robust planner implmented on robust household. The calibration used for computing the optimal debt policies underlying these taxes is our baseline specification that delivers a 13% probability of crisis and an average debt over GDP of 30%.

probability of 13% already justifies macroprudential taxes that are more than twice as large as those in Bianchi (2011) without model uncertainty, at around 16%. A robust planner will further induce an increase of almost 5 percentage points, to 20%; and even more so the higher the degree of robustness (i.e., lower θ). Moreover, the probability that a tax is needed to implement the social planner's allocation is much higher (88% compared to 78%). This was to be expected, as we noted before that the region of current debt over which a tax is imposed widens. Taxes remain fairly stable though if both agents fear misspecification, compared to the case when neither does, although the probability that one is implemented increases. When the household is robust but the planner is not, then even a subsidy (a negative tax) could be required to implement the planner's allocation because a household that fears misspecification and is optimizing against worst-case probabilities will tend to

Table 3: Taxes (simulations)

	$\theta = 0.35$	$\theta = 0.15$
Average tax (%)		
1) non-robust HH v non-robust SP	15.97	15.97
2) non-robust HH v robust SP	20.11	22.26
3) robust HH v robust SP	17.00	15.72
4) robust HH v non-robust SP	9.29	0.16
Prob of tax region (%)		
1) non-robust HH v non-robust SP	77.71	77.71
2) non-robust HH v robust SP	87.91	93.01
3) robust HH v robust SP	84.81	90.69
4) robust HH v non-robust SP	74.59	74.59
Tax correlation to GDP		
1) non-robust HH v non-robust SP	-0.65	-0.65
2) non-robust HH v robust SP	-0.65	-0.59
3) robust HH v robust SP	-0.77	-0.70
4) robust HH v non-robust SP	-0.59	-0.61

Note: Baseline parameterization that implies a probability of crisis of 13%. The numbers represent the average macroprudential tax that implements the planner's allocation over the 80,000 periods of the simulation. The realization of shocks to tradable and non-tradable goods is based on the reference model.

underborrow and hold more savings than are required to internalize the pecuniary externality. Finally, we find that the fear of model misspecification not only changes the tax levels but also affects their cyclicality. Note that when the household and/or the planner have preferences for robustness, the correlation of taxes and income become less countercyclical than in the case when neither the household nor the planner feature model uncertainty. This happens due to the fact that the difference in optimal bond holding decisions between a non-robust household and a robust social planner widens over the non-binding region, as discussed earlier.

3.5 Welfare gains and losses

Should a policymaker intervene ex-ante and impose a macroprudential tax policy when there is model uncertainty? How does this depend on who (planner and/or household) fears misspecification? What are the implications of implementing the policy when doubts about the reference model were unjustified? In order to answer these questions, in this section we assess the welfare gains (or losses) from the planner's intervention for various cases, presented in Table 4.[9] We use a compensating variation approach as in Lucas (1987). We compute the constant proportional increase in consumption (the percent subsidy to consumption) that would make agents indifferent between remaining in a decentralized equilibrium without government intervention and adopting the planner's socially optimal allocation. This consumption subsidy, which we call $\gamma(B, y)$, is state dependent (that is, while the proportional increase in consumption is constant over time, the γ that can achieve the welfare from the planner's allocation will depend on what initial state we compute it at).

Whenever the planner and household differ in their beliefs, we need to take a stance about what model (i.e., distribution of shocks) we will consider to measure welfare gains (whose computation involves expectations). This matters especially if we are trying to decide whether a planner's intervention is optimal or not: while the planner will address the externality that leads to overborrowing, she might solve for an optimal allocation under a probability model that is incorrect and/or that the households do not share. Then, it might be the case that under an alternative model, welfare does not improve or, at the very least, welfare gains are very small.

There are four models on the table: (i) the reference model (the probability distribution of the exogenous shocks that agents doubt); (ii) the worst-case model in the eyes of the planner; (iii) the worst-case model in the eyes of the household; and, (iv) the true data-generating process (DGP). The welfare gains presented in Table 4 are calculated under two models. First, under the "Reference probabilities" for which we equate the DGP with the reference model. Second, under the "Worst-case probabilities" for which we equate the DGP with the worst-case model in the eyes of a robust household with $\theta \in \{0.15, 0.35\}$.[10]

Let $c(DE; \theta_A)$ denote the allocation from the DE (decentralized economy) when the household

[9] Table 9 in the Appendix shows the results under the calibration of Bianchi (2011).

[10] While in principle the worst-case distribution induced by the allocation of the decentralized equilibrium can be a very different object than the one induced by the social planner's allocation, we find that quantitively the differences are not substantial. Thus, in the paper we only consider the worst-case model in the eyes of the household as results do not vary significantly under planner's worst-case model.

Table 4: Average welfare gains (simulations)

	$\theta = 0.35$	$\theta = 0.15$
Reference probabilities		
1) non-robust HH v non-robust SP	0.27	0.27
2) non-robust HH v robust SP	0.24	0.20
3) robust HH v robust SP	0.09	-0.02
4) robust HH v non-robust SP	0.11	0.04
Worst-case probabilities		
5) non-robust HH v non-robust SP	0.55	1.04
6) non-robust HH v robust SP	0.56	1.17
7) robust HH v robust SP	0.31	0.34
8) robust HH v non-robust SP	0.26	0.12

Note: The numbers represent the mean welfare gain over the periods in the simulation when a tax is needed to decentralize the planner's allocation. For the first set of results ("Reference probabilities"), the realization of shocks to tradable and non-tradable goods in the simulations is based on the reference model. The second block of results ("Worst-case probabilities") considers the household's worst-case distribution. The parametrization implies a probability of crisis of 13%. Results are presented for two values of θ, representing different preferences for robustness (lower θ implies more robustness).

has doubts about the model that correspond to θ_A (only the household's beliefs matter for this allocation). Let $c(SP;\theta_R)$ denote the social planner's allocation when the planner has fears about the model that correspond to θ_R. Let $V(c|\theta)$ denote the utility of allocation c, calculated from the risk-sensitive recursion (7) with parameter θ.

First, we consider the case where both the planner and the household share the same beliefs, i.e. $\theta_R = \theta_A = \bar{\theta} \leq \infty$, and these are correct. Recall that $\bar{\theta} = \infty$ corresponds to a model without robustness. In this case, for any $\bar{\theta}$ we have that $V(c(DE,\bar{\theta})|\bar{\theta}) < V(c(SP,\bar{\theta})|\bar{\theta})$ because the planner's intervention corrects the pecuniary externality while the paternalism channel is irrelevant (because planner and household agree). The welfare gains associated with these cases correspond to rows 1 ($\bar{\theta} = \infty$), and 7 ($\bar{\theta} \in \{0.15, 0.35\}$) in Table 4. When $\bar{\theta} = \infty$ and indeed the shocks are generated following the reference distribution, for the calibration that renders a probability of sudden stops of 13% in the decentralized economy with no government intervention, welfare gains from implementing the planner's allocation are twice as large as those obtained by Bianchi (2011)—0.27% of permanent consumption. These numbers are still rather small, but consistent with the literature's finding, as noted in Bianchi (2011), that the welfare cost of business cycles is small.

Second, imagine that the planner and the household doubt the model equally ($\theta_R = \theta_A = \bar{\theta} < \infty$), but the actual uncertainty is generated by the reference model. This corresponds to one situation where the policymaker and household are actually "wrong". In this case, we cannot establish theoretically the ordering between the allocations, i.e., $V(c(DE, \bar{\theta})|\theta = \infty) \lesseqgtr V(c(SP, \bar{\theta})|\theta = \infty)$. The welfare gains associated with this case correspond to row 3 in Table 4. If both agents fear misspecification but shocks follow the reference distribution, then the gains from implementing a planner's allocation are significantly reduced. This is because the household already saves more as a precaution against future bad shocks, and so this indirectly addresses the problem of overborrowing due to pecuniary externalities. Any further gains from a planner that internalizes this externality does not increase welfare substantially (0.09%). In fact, if the degree of robustness is sufficiently high (for example, $\theta = 0.15$), then implementing the planner's allocation decreases welfare. A robust planner "overshoots" in savings.[11]

Third, imagine that neither the planner nor the household doubt the model ($\theta_R = \theta_A = \infty$), but the actual uncertainty is generated by the worst-case model. This corresponds to another situation where the policymaker and household are actually "wrong". Formally, this would correspond to $V(c(DE, \theta_A = \infty)|\theta = \delta) \lesseqgtr V(c(SP, \theta_R = \infty)|\theta = \delta)$. Quantitatively, however, we do find that intervention leads to sizeable welfare gains, as seen in row 5 in Table 4. We find that the welfare gains from implementing the social planner's allocation if the worst-case scenario takes place is 0.55, the largest among the cases we have considered. In our notation, this implies that $V(c(DE, \theta_A = \infty)|\theta < \infty) < V(c(SP, \theta_R = \delta)|\theta < \infty)$. As shown in Figure 3, under the worst-case model the distortions are such that higher probabilities are assigned to those states in which income is low and the collateral constraint binds. It is precisely in those states that is very valuable to arrive with relatively lower debt levels and, thus, when the intervention by the social planner would generate largest welfare gains. Despite paternalism being absent in this case, the social planner's macroprudential motive inadvertently acts in the same direction required to "correct" the house-

[11]Indeed values below $\theta^R = 0.18$ obtain welfare losses in this case. The DEP implied by $\theta^R = 0.18$ is 8.5%.

hold's beliefs.

Fourth, when planner and household disagree on their beliefs ($\theta_R \neq \theta_A$) and uncertainty is generated under the household's model, then there are two forces to consider: on the one hand, welfare might decrease because the planner is solving for an allocation under the "wrong" model but, on the other hand, welfare might be improved because she is correcting the pecuniary externality. In our notation, these cases correspond to $V(c(DE, \theta_A = \infty)|\theta = \infty) \lesseqgtr V(c(SP, \theta_R = \delta)|\theta = \infty)$ and $V(c(DE, \theta_A = \delta)|\theta = \delta) \lesseqgtr V(c(SP, \theta_R = \infty)|\theta = \delta)$. The welfare gains associated with these cases correspond to rows 2 and 8 in Table 4, respectively. Importantly, we find that a robust planner's allocation will still increase welfare even if this is measured assuming shocks are realized according to the reference model that the households have in mind. This was not evident a priori: a robust planner will choose an allocation that generates a lower bound of lifetime discounted utility, in the sense that implementing that allocation will result in larger welfare if shocks are actually drawn from the worst-case distribution rather than the reference model. However, if indeed shocks follow the reference model, then it would have been optimal to pick an allocation based on this belief. We know that a robust planner will correct the externality, and so this should increase welfare compared to a competitive equilibrium, but we do not know how much welfare is lost from not choosing the optimal allocation based on the correct data generating process. Our quantitative results indicate that the gains are larger than the costs. Consider row 2 in Table 4: a robust planner that fears misspecification with $\theta_R = 0.35$ may still achieve a welfare gain almost as large as a non-robust planner when shocks follow the reference distribution (0.24% of permanent consumption compared to the 0.27% for a non-robust planner). The higher the degree of robustness, however, the lower the welfare gain (0.20% for a $\theta_R = 0.15$).[12]

Finally, consider rows 4 and 6, in these cases the planner and household disagree while shocks follow the social planner's model. Here, the social planner's allocation improves welfare by means of both channels. Optimal policy internalizes the pecuniary externality at the same time it "corrects"

[12] There is a value of θ_R below which welfare losses are obtained and thus a status-quo regime would be preferred over intervention. This value is $\theta^R \approx 0.01$ which implies an EDP below 1%.

the household's beliefs. In our notation, these cases correspond to $V(c(DE, \theta_A = \delta)|\theta = \infty) < V(c(SP, \theta_R = \infty)|\theta = \infty)$ and $V(c(DE, \theta_A = \infty)|\theta = \delta) < V(c(SP, \theta_R = \delta)|\theta = \delta)$ repectively. Indeed these cases are those where welfare gains are maximum within each block of Table 4.

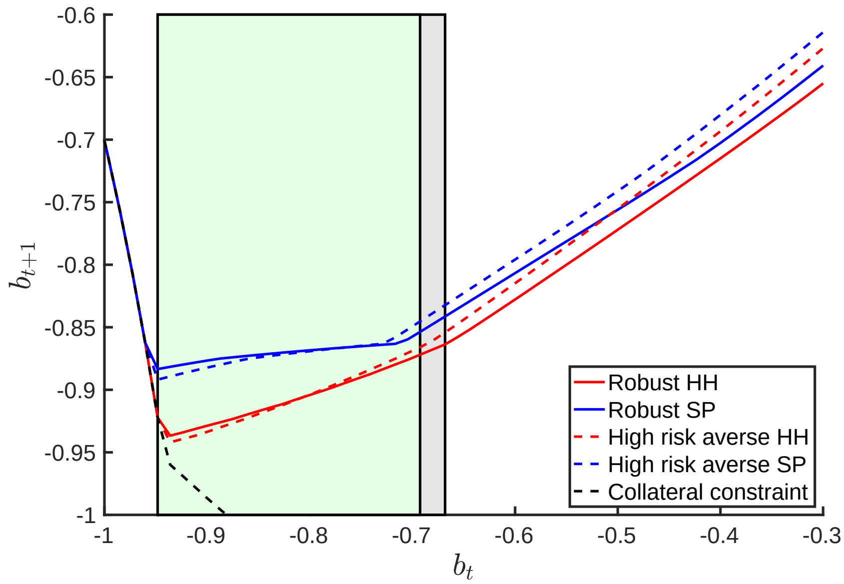
3.6 Comparing robustness with higher risk aversion

One interesting question is how the decisions of a robust agent compare to those of an agent with CRRA preferences that is simply more risk averse than in the benchmark calibration. To this end, we increase the CRRA risk aversion coefficient, σ, from 2 to 4. Figure 7 shows that they tend to make very similar decisions when debt is high and the collateral constraint is close to binding. However, for levels of debt further away from the binding region, a highly risk averse agent continues to be conservative in terms of savings, while the robust agent no longer fears very negative shocks in the following period and thus accumulates larger amounts of debt. Recall that a robust agent's worst-case scenario of probabilities for the following period depends on the current state; if the economy is far from the binding region, then there is less concern about negative future shocks. Table 5 compares the results from the simulations. Notably, both a robust and a highly risk averse household and planner arrive at similar probabilities of a crisis (around 8.5% based on a household's debt policy, and this is reduced to 0.7% for a planner). This is not surprising given that their debt choices are extremely similar close to the binding region. In terms of the severity of a median crisis, both types of planners are able to halve the reduction in consumption, although the median crisis in the context of highly risk averse agents renders a median financial crisis that is more severe compared to that which arises when agents are robust.

3.7 Doubts about κ

So far, we have considered the case where households and/or the social planner face ambiguity aversion with respect to the endowment process. As in Ferriere and Karantounias (2013) we take seriously the idea that economic agents face uncertainty about the cycle that cannot be specified with a unique probability measure, and are averse towards it. We believe that this is a reasonable assumption for

Figure 7: Debt policies of household and planner - comparing robustness with high risk aversion



Note: This plot depicts the optimal debt policy of a planner and an agent as a function of current debt holdings, for a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables. For the robust agents, the degree of misspecification considered is characterized by $\theta = 0.35$. Non-robust agents in this plot have a high risk aversion of $\sigma = 4$.

emerging economies whose business cycle is tightly linked to commodity prices that follow uknown processes, which are precisely the type of countries prone to the sudden stops analyzed in this model. Yet, one may think of considering other sources of ambiguity.

In this subsection we explore a variation of the model that allows the planner and/or households to have worries about aspects of the financial system. In particular, we consider economic agents that have a preference for robustness with respect to future values of κ, the pleadgeable fraction of their endowment. To this end, following the modeling approach in Bianchi, Boz and Mendoza (2012), we assume that κ_t is time varying and switches between two values κ^H and $\kappa^L < \kappa^H$ following an exogenous markov switching process that is independent from the endowment's process. In this variation, both households and the planner fully trust the probability model for the endowment process but have doubts about the process that governs κ_t. Details regarding the model setup, solution, and calibration are relegated to Appendix 5.3.

Table 5: Probability and magnitude of a sudden stop

	HH $\theta = 0.35$ $\sigma = 2$	SP $\theta = 0.35$ $\sigma = 2$	HH $\theta = 0.15$ $\sigma = 2$	SP $\theta = 0.15$ $\sigma = 2$	HH $\theta = \infty$ $\sigma = 4$	SP $\theta = \infty$ $\sigma = 4$
Long-run prob of crisis* (%):	8.58	0.65	5.90	0.17	8.74	0.78
Long-run prob of constraint binding (%):	11.90	1.73	6.28	0.59	11.64	2.09
Average debt as % of GDP:	29.04	28.08	28.56	27.66	28.88	28.06
Decrease in consumption in median crisis:	−4.91	−2.76	−2.57	−2.11	−12.60	−7.37
Depreciation RER in median crisis:	−10.71	−5.28	−4.62	−3.40	−13.65	0.76
CA reversal in median crisis	7.94	1.19	7.94	0.46	7.06	1.02
Std dev of consumption:	0.07	0.05	0.06	0.05	0.06	0.05
Std dev of RER:	0.30	0.15	0.23	0.11	0.30	0.16
Std dev of CA (as % of GDP):	0.04	0.01	0.02	0.01	0.04	0.01
Corr consumption with GDP:	0.76	0.58	0.65	0.52	0.84	0.83
Corr RER with GDP:	0.79	0.63	0.67	0.56	0.86	0.64
Corr CA (as % of GDP) with GDP:	−0.77	−0.61	−0.63	−0.51	−0.85	−0.52

Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the reference distribution. A sudden stop happens when the constraint binds *and* there is a one sandard deviation reversal of the current account. The first two columns report the results from economies that feature a robust household and planner with $\theta = 0.35$, columns 3 and 4 refer to agents with a higher degree of robustness ($\theta = 0.15$) and the final two columns concern non-robust agents with a higher risk aversion ($\sigma = 4$, double the value of our baseline calibration).

The central focus of this paper is to understand whether interventions by a social planner are still welfare improving under model uncertainty. As discussed in our baseline model, the answer depends on the discrepancy in the pessimistic beliefs between the planner and private agents, and whether their beliefs coincide or not with the actual model generating the shocks. Thus, in this subsection we only present the welfare evaluations for the various scenarios analyzed in Section 3.5 in Table 6.[13] Similarly to our baseline model, we find that an ex-ante intervention is welfare improving for plausible degrees of robustness, even if the planner is "wrong" (i.e., their beliefs are not justified by the probability model generating the shocks). Moreover, the ordinal relation across cases 1-4 and 5-8 in table 6 is maintained across the two forms of considering model uncertainty.

[13] In Appendix 5.3 we provide the remaining simulation results.

Table 6: Average welfare gains (simulations) - Baseline Model vs Stochastic κ

	Baseline Model	Stochastic κ
Reference probabilities		
1) non-robust HH v non-robust SP	0.27	0.39
2) non-robust HH v robust SP	0.24	0.37
3) robust HH v robust SP	0.09	0.21
4) robust HH v non-robust SP	0.11	0.22
Worst-case probabilities		
5) non-robust HH v non-robust SP	0.55	0.50
6) non-robust HH v robust SP	0.56	0.51
7) robust HH v robust SP	0.31	0.34
8) robust HH v non-robust SP	0.26	0.32

Note: The numbers represent the mean welfare gain over the periods in the simulation when a tax is needed to decentralize the planner's allocation. For the first set of results ("Reference probabilities"), the realization of shocks to tradable and non-tradable goods in the simulations is based on the reference model. The second block of results ("Worst-case probabilities") considers the household's worst-case distribution. The first column repeats the results from our baseline model from Table 4. The second column shows the welfare gains under the variation with stochastic κ.

4 Conclusion

Following the 2008 global financial crisis, macroprudential regulation has become a well-established policy to contain credit booms and reduce financial fragility in both advanced and emerging countries. Against this background, starting with the seminal contributions of Bianchi and Mendoza (2010) and Bianchi (2011), an increasing number of studies have focused on understanding how these policies affect the transmission mechanism driving financial crises. While this literature shows the optimality of adopting macroprudential policies, the prescribed taxes usually generate small welfare gains. From a policy perspective, this raises the question of whether such policies would still remain optimal once the decision maker acknowledges that there may exist a vast set of alternative statistical models with unkown forms that could fit the data nearly as well as the reference model. This paper attempts to answer this question and, thus, characterizes robust optimal macroprudential policy in the context of Bianchi (2011)'s small open economy with occasionally binding collateral constraints.

Without model uncertainty, there exist welfare gains from internalizing how borrowing decisions in good times affect the value of collateral during a crisis. When there is fear of model misspecification by the planner, however, the decentralized equilibrium may differ from the social planner's equilibrium both because of the pecuniary externalities associated with the collateral constraint and because of the paternalistic imposition of the planner's beliefs when designing policy. When the actual shocks are generated by the worst-case distribution and both the household and the planner share the same doubts about the model, the intervention by the planner will be welfare improving due to the correction of the pecuniary externality. However, in situations in which the actual shocks are generated by the reference (worst-case) model and the regulator and household doubt (do not doubt) the model, it is theoretically ambiguos whether an intervention by the planner would be welfare improving. In our quantitative analysis we find that, for reasonable degrees of robustness, the interventions are welfare improving even if the planner/household is actually "wrong". However, in some of these cases the welfare gains can become negative for higher degrees of model misspecification.

Our findings illustrate the importance of acknowledging ambiguity aversion (Knightian uncertainty) in the design and evaluation of macroprudential policies. In this paper we take a particular approach by endowing both the planner and the household with multiplier preferences (Hansen and Sargent, 2011): our agents act as if they evaluate plans using a worst case belief that minimizes the sum of expected utility and a smooth function that penalizes deviations from a reference belief. However, it is not the only framework for analyzing model uncertainty. For instance, Ilut and Schneider (2014) describe preferences by the multiple priors utility axiomatized by Gilboa and Schmeidler (1989), and use a recursive version of the multiple priors utility model. In their model, instead, agents act as if they evaluate plans using a worst case probability drawn from a set of multiple beliefs. A prominent alternative to robust control is to formulate the decision problem from a Bayesian perspective by specifying a set of priors over fully articulated parameter and model spaces as in Batini et al. (2006). Considering the implications of different ambiguity aversion models for the optimal policy design constitutes an interesting avenue for future research.

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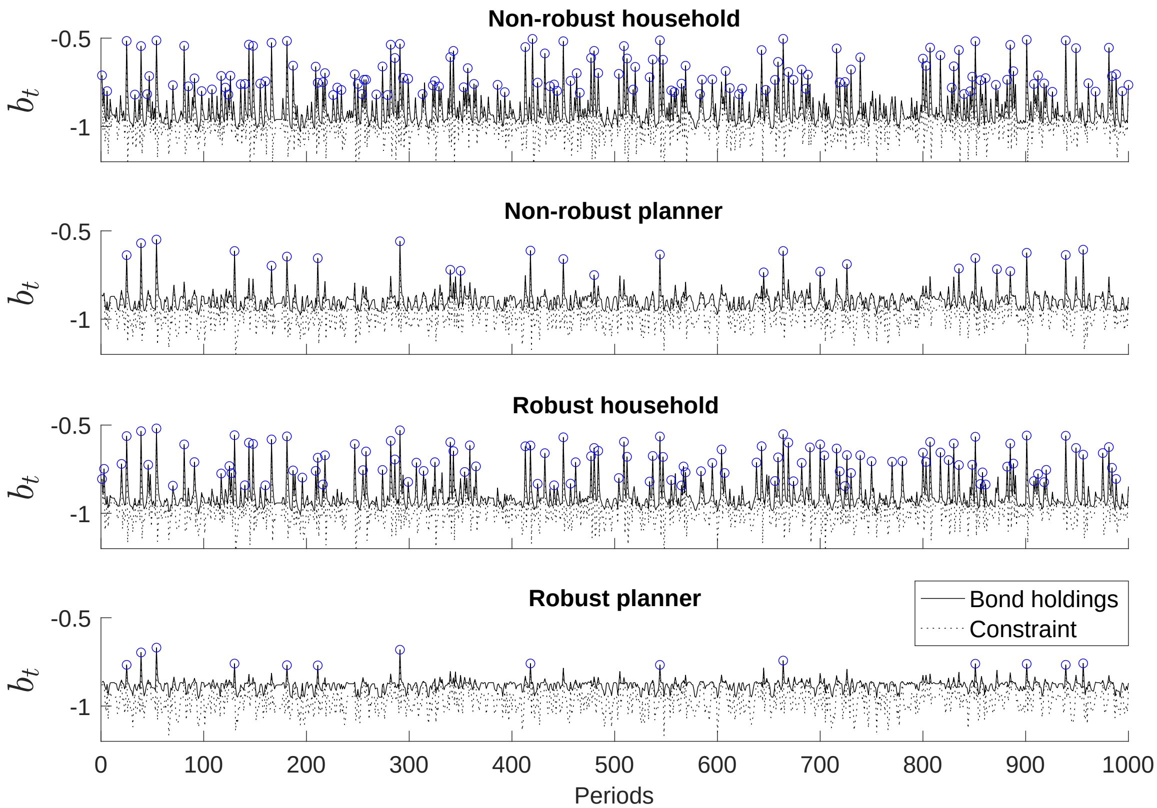
5 Appendix

5.1 Additional Figures

Figure 8: Evolution of bond holdings based on worst-case distribution of shocks



Note: These plots show the evolution of bond holdings from simulations that consider a specific history of shocks to tradable and non-tradable endowments based on the worst-case distribution of shocks that a robust household considers. The same series of shocks is fed into the four cases depicted here. The first plot considers the optimal debt choice of a non-robust household, the second that of a non-robust planner, the third shows the choices from a robust household and the fourth from a robust planner. The blue circles indicate a sudden stop (i.e. the constraint binds *and* there is a one standard deviation reversal of the current account). The parametrization used for the simulation is that of Bianchi (2011), which implies a 5% probability of crisis under the descentralized equilibrium with no intervention.

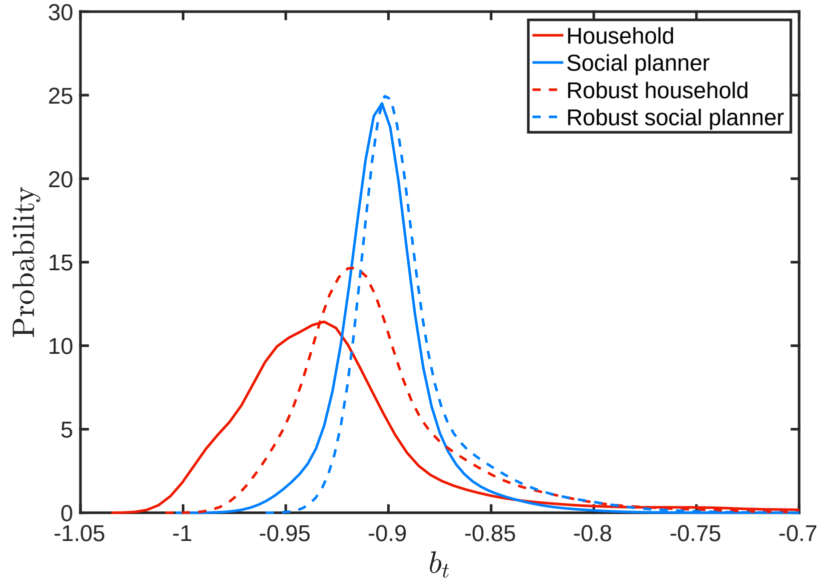
5.2 Results under the calibration in Bianchi (2011)

Table 7: Probability and magnitude of a sudden stop

	HH	SP	HH Robust $\theta = 0.35$	SP Robust $\theta = 0.35$
Long-run prob of crisis* (%):	5.16	0.34	3.73	0.03
Long-run prob of constraint binding (%):	5.93	0.53	4.45	0.11
Average debt as % of GDP:	29.35	28.63	28.74	28.25
Decrease in consumption in median crisis:	-16.69	-10.34	-12.52	-10.25
Depreciation RER in median crisis:	-19.17	-2.03	-8.12	-1.62
CA reversal in median crisis	7.83	0.23	7.83	0.04
Std dev of consumption:	0.06	0.05	0.05	0.05
Std dev of RER:	0.23	0.11	0.16	0.1
Std dev of CA (as % of GDP):	0.03	0.01	0.01	0.01
Corr consumption with GDP:	0.83	0.85	0.73	0.62
Corr RER with GDP:	0.79	0.44	0.65	0.41
Corr CA (as % of GDP) with GDP:	-0.75	-0.08	-0.52	0.13

Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the reference distribution using the calibration of Bianchi (2011). A sudden stop happens when the constraint binds *and* there is a one standard deviation reversal of the current account. Columns 1 and 2 report the results from economies that feature non-robust agents, while columns 3 and 4 are based on the optimal choices of a robust household and planner.

Figure 9: Distribution of debt holdings



Note: Distribution of debt holdings based on robust and non-robust allocations by household and planner. The parametrization used for this plot is that of Bianchi (2011), which implies a 5% probability of crisis under the descentralized equilibrium with no intervention.

Table 8: Taxes (simulations)

	$\theta = 0.35$	$\theta = 0.15$
Average tax (%)		
1) non-robust HH v non-robust SP	7.27	7.27
2) non-robust HH v robust SP	6.33	5.94
3) robust HH v robust SP	5.66	3.76
4) robust HH v non-robust SP	3.68	−0.42
Prob of tax region (%)		
1) non-robust HH v non-robust SP	80.11	80.11
2) non-robust HH v robust SP	94.31	95.74
3) robust HH v robust SP	70.06	44.89
4) robust HH v non-robust SP	91.84	92.35
Tax correlation to GDP		
1) non-robust HH v non-robust SP	−0.74	−0.74
2) non-robust HH v robust SP	−0.60	−0.61
3) robust HH v robust SP	−0.59	−0.45
4) robust HH v non-robust SP	−0.65	−0.67

Note: The parameterization is that of Bianchi (2011) and implies a probability of crisis of 5%. The numbers represent the average macroprudential tax that implements the planner's allocation over the 80,000 periods of the simulation. The realization of shocks to tradable and non-tradable goods is based on the baseline distribution.

Table 9: Average welfare gains (simulations)

	$\theta = 0.35$	$\theta = 0.15$
Reference probabilities		
1) non-robust HH v non-robust SP	0.14	0.14
2) non-robust HH v robust SP	0.13	0.11
3) robust HH v robust SP	0.03	-0.01
4) robust HH v non-robust SP	0.03	0.02
Worst-case probabilities		
5) non-robust HH v non-robust SP	0.34	0.77
6) non-robust HH v robust SP	0.34	0.79
7) robust HH v robust SP	0.11	0.07
8) robust HH v non-robust SP	0.10	0.04

Note: The numbers represent the mean welfare gain over the periods in the simulation when a tax is needed to decentralize the planner's allocation. For the first set of results ("Reference probabilities"), the realization of shocks to tradable and non-tradable goods in the simulations is based on the reference model. The second block of results ("Worst-case probabilities") considers the household's worst-case distribution. The parametrization is that of Bianchi (2011) which leads to a probability of crisis of 5% in the decentralized equilibrium with no intervention. Results are presented for two values of θ, representing different preferences for robustness (lower θ implies more robustness).

5.3 A model with regime-switching collateral constraints

This appendix presents a model where households face model misspecification concerns with respect to the financial constraints they may face tomorrow. To do this, we first modify the baseline model such that the ability to borrow (κ) switches between periods where households can borrow a larger fraction of tradable and non-tradable income and periods where the constraint is tighter in that this fraction is lower. This is modelled by allowing $\kappa_t^T = \kappa_t^N = \kappa_t \in \{\kappa^H, \kappa^L\}$ with $\kappa^H > \kappa^L$. Over time, κ_t follows an exogenous markov process independent from the endowment process, denote $Pr(\kappa_{t+1}^i | \kappa_t^j) = \pi_{ij}$ with $i, j \in H, L$. Note that this setup can be generalized to an arbitrary number of values for κ or a AR process could be used instead. In order to ease exposition, this appendix will only deal with the two-state markov-switching case.

As in the main text, we then introduce concerns for model mispecification by means of a preference for robustness. Unlike in the main text, the probability distortions induced by this preference will not affect expectations with respect to the endowment process, instead, robustness will be placed solely on the markov switching process. This is as if agents were absolutely confident in how their endowment process is distributed but face ambiguity with respect to the likelihood of facing tigher (or looser) constraints in the future.

These modifications imply that the robust household's decentralized problem in its risk-sensitive representation is:

$$V(b_t, B_t, y_t, \kappa_t) = \max_{c^N, c^T, b_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} - \beta\theta_A \log(E_\kappa[e^{-\frac{1}{\theta_A} E_y[V(b_{t+1}, B_{t+1}, y_{t+1}, \kappa_{t+1}) | b_t, B_t, y_t, \kappa_{t+1}]} | b_t, B_t, y_t, \kappa_t])$$

$$\text{subject to: } c_t \equiv \left[\omega\left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-1/\eta}$$

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N$$

$$b_{t+1} \geq -\kappa_t(p_t^N y_t^N + y_t^T)$$

which is the equivalent to (7).

The robust social planner's problem in its risk-sensitive representation is:

$$W(B_t, y_t, \kappa_t) = \max_{c_t^T, b_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} - \beta\theta_R \log(E_\kappa[e^{-\frac{1}{\theta_R} E_y[W(B_{t+1}, y_{t+1}, \kappa_{t+1}) | B_t, y_t, \kappa_{t+1}]} | B_t, y_t, \kappa_t])\Big]$$

$$\text{subject to: } c_t \equiv \left[\omega\left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-1/\eta}$$

$$B_{t+1} + c_t^T = B_t(1+r) + y_t^T$$

$$B_{t+1} \geq -\kappa_t(p_t^N y_t^N + y_t^T)$$

$$p_t^N = \frac{(1-\omega)}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}$$

$$c_t^N = y_t^N$$

which is the equivalent to (12).

Note first that, in both cases, κ_t is time varying as denoted by its time subscript and κ_t is included as a state variable given its relevance in forecasting κ_{t+1}. Also note that expectation with respect to κ_{t+1} and to y_{t+1} are nested, represented by $E_\kappa[\cdot_{t+1}]$ and $E_y[\cdot_{t+1}|\kappa_{t+1}]$ respectively. Finally, note that risk sensitivity is placed only on the expectation E_κ, indicating that the preference for robustness is only affecting this expectation. If we momentarily ignore the expectation of κ and endow the decision maker with certainty with respect to κ_{t+1}, we have that

$$-\beta\theta e^{-\frac{1}{\theta}\log(E_y[G(\cdot)])} = \beta E_y[G(\cdot)], \quad G \in \{V, W\}$$

and the problem reduces to a standard Bellman equation without a preference for robustness.

Aside from these distinctions, the differences between the decentralized and social planner's problem are the same as discussed in the main text. The Euler equations that correspond to each problem are

$$\beta(1+r)E_\kappa[m^*_{t+1}E_y[u^T_{t+1}|b_t, B_t, y_t, \kappa_{t+1}]|\kappa_t] = u^T_t + \mu_t$$
$$\beta(1+r)E_\kappa[n^*_{t+1}E_y[u^T_{t+1} + \mu^{SP}_{t+1}\psi_{t+1}|b_t, B_t, y_t, \kappa_{t+1}]|\kappa_t] = u^T_t + \mu^{SP}_t$$

with

$$m^*_{t+1} = \frac{e^{-\frac{1}{\theta_A}E_y[V(b_{t+1},B_{t+1},y_{t+1},\kappa_{t+1})|b_t,B_t,y_t,\kappa_{t+1}]}}{E_\kappa[e^{-\frac{1}{\theta_A}E_y[V(b_{t+1},B_{t+1},y_{t+1},\kappa_{t+1})|b_t,B_t,y_t,\kappa_{t+1}]}|\kappa_t]}$$
$$n^*_{t+1} = \frac{e^{-\frac{1}{\theta_R}E_y[W(B_{t+1},y_{t+1},\kappa_{t+1})|B_t,y_t,\kappa_{t+1}]}}{E_\kappa[e^{-\frac{1}{\theta_R}E_y[W(B_{t+1},y_{t+1},\kappa_{t+1})|B_t,y_t,\kappa_{t+1}]}|\kappa_t]}$$

these are the equivalent expressions for (16) and (17).

Optimal policy, follows directly exactly as in the main text and the pecuniary externality and paternalistic motives are elucidated as before:

$$1+\tau = \frac{E_\kappa[n^*_{t+1}E_y[u^T_{t+1} + \mu^{SP}_{t+1}\psi_{t+1}|b_t, B_t, y_t, \kappa_{t+1}]|\kappa_t]}{E_\kappa[m^*_{t+1}E_y[u^T_{t+1}|b_t, B_t, y_t, \kappa_{t+1}]|\kappa_t]}$$

5.3.1 Calibration

To show that the message drawn from the welfare implications under robustness are qualitatively the same as those discussed in the main text, we calibrate the modified model using the same criteria as in the main text with a couple of differences. First, we let κ^H and κ^L deviate from the value for κ chosen in the main calibration symmetrically, i.e. $\kappa = 0.31$ remains the midpoint between $\kappa^H = \kappa + \Delta/2$ and $\kappa^L = \kappa - \Delta/2$. Where the additional parameter Δ represents the difference between periods of high vs low collateral plegeability. Note that this model nests the baseline in the sense that setting $\Delta = 0$ recovers the baseline model without time-varying kappa. The value of Δ is set to 0.031, 10% of the value of the midpoint. Another difference is that a choice must be made for the transition probabilities of the markov switching process. Here we follow Mendoza and Terrones (2008) who find that the mean duration of credit booms in industrialized economies is of about 7 years, this implies that the probability of a κ^H persisting each period is $\pi^{HH} \approx 0.85$. As in Bianchi, Boz, and Mendoza (2012) we treat the markov switching process symmetrically and set $\pi^{LL} = 0.85$. The parameter governing the preference for robustness $\theta = 0.025$ is chosen following the same criteria as in the main text, we target a detection error probability above 20%. Table 10 summarizes the calibration of parameters that differ from those shown in table 1.

Table 10: Calibration

Parameter	Value	Source
Utility function		
Discount factor β	0.75	To match data moments
Collateral constraint		
Share of collateralized tradable κ^H	0.326	To match data moments
Share of collateralized tradable κ^L	0.294	To match data moments
Robustness		
Degree of misspecification θ	0.025	To target a detection error probability of $> 20\%$

5.3.2 Simulation Results

Below, Table 11 presents the simulation results for the model with regime-switching collateral constraints (stochastic kappa). Comparing these to the results from the baseline model (table 2), one can immediately see that the calibration of both sets of simulations yields comparable simulated statistics.

Table 11: Probability and magnitude of a sudden stop with time-varying κ

	(1) HH	(2) SP	(3) Robust HH	(4) Robust SP
Long-run prob of crisis* (%):	13.31	2.35	12.35	2.33
Long-run prob of constraint binding (%):	21.15	5.79	18.56	4.79
Average debt as % of GDP:	29.00	28.45	28.93	28.32
Decrease in consumption in median crisis:	−13.17	−3.38	−9.77	−3.38
Depreciation RER in median crisis:	−23.30	1.43	−15.19	1.67
CA reversal in median crisis	10.94	−0.28	10.94	−0.37
Std dev of consumption:	0.08	0.06	0.07	0.06
Std dev of RER:	0.45	0.24	0.40	0.22
Std dev of CA (as % of GDP):	0.06	0.03	0.05	0.02
Corr consumption with GDP:	0.88	0.62	0.82	0.61
Corr RER with GDP:	0.93	0.7	0.88	0.69
Corr CA (as % of GDP) with GDP:	−0.91	−0.65	−0.85	−0.65

Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the reference distribution. The simulated model features time-varying κ_t which follows a markov switching process. A sudden stop happens when the constraint binds *and* there is a one standard deviation reversal of the current account. Columns 1 and 2 report the results from economies that feature non-robust agents, while columns 3 and 4 are based on the optimal choices of a robust household and planner that face doubts regarding the process governing κ_t.