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Authority and Specialization under Informational Interdependence*

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Abstract

I study the relationship between a firm's organization and its ability to aggregate information under interdependence. The firm must adapt the design of two products to innovations over two attributes, such that information about each attribute affects the design of both products. Agents have access to imperfect information but must incur costs to obtain it. The principal can delegate decision-making authority to any agent, and such authority includes private communication channels with other players. I characterize the optimal organizational structure under informational spillovers. The possibility of specialization leads to three novel intuitions. First, the principal benefits from agents' restricted access to information because it reduces profitable deviations from truthful communication. Second, an agent's specialization depends on his preferences, the costs of acquiring information, and its expected influence on decisions. Third, delegation leads to suboptimal information aggregation because some agents' acquisition decisions fail to internalize the interdependence.

Keywords: Multidimensional Cheap Talk, Industrial Organization, Delegation, Organizational Design.
JEL: D21, D83.

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1 Introduction

When a principal in charge of many decisions needs to aggregate soft information from agents, incentives for communication depend on their preferences over decisions (Battaglini, 2002) and on how information affects these decisions (Levy and Razin, 2007; Ambrus and Takahashi, 2008). In cases where information about a single issue influences many decisions, the aggregation of preferences can lead to informational spillovers that will affect communication (Farrell and Gibbons, 1989; Goltsman and Pavlov, 2011). The allocation of authority—who decides what—will define the degree of preference misalignment between each agent and decision-maker, and determine the amount of information effectively transmitted (Alonso et al., 2008; Rantakari, 2008; Deimen and Szalay, 2019). Besides communication, authority also affects incentives to acquire information (Aghion and Tirole, 1997; Rantakari, 2012). This paper studies specialization in this environment, the conditions under which it emerges, and its consequences for the quality of information aggregated.

Consider information aggregation in Multinational Corporations (MNC). The ability to generate and exploit innovations in such organizations is dispersed among subunits (Andersson et al., 2002; Monteiro and Birkinshaw, 2017) but requires that they forge close relationships with local partners which, in turn, creates conflicts with organizational goals (Andersson et al., 2005; Ecker et al., 2013). Preference misalignment will affect the transmission of information that is valuable across the organization, creating an 'innovation-integration dilemma' (Mudambi, 2011).

If decisions are independent, the headquarters delegates control over a decision to a subsidiary if the information it aggregates from business partners compensates for the preference misalignment (Dewan et al., 2015).[1] If there is informational interdependence, on the other hand, delegating control over one decision may affect incentives to acquire and transmit information relevant for other decisions. These incentives depend on how the interdependence aggregates conflicts of interest between senders and receivers—i.e. whether such interdependence benefits or harms information transmission. Some evidence from the International Business literature supports this claim: a subsidiary's autonomy is positively correlated with how much of the knowledge it creates spills over to sister units (Andersson et al., 2007; Kähäri et al., 2017; Kunisch et al., 2019).

In this paper I construct a theoretical model of authority under informational interdependence. A MNC must decide on the design of two products and needs information about two different attributes. There are n subsidiaries, each represented by its manager. Each manager has access to local information about each attribute, which can refer to innovations that would be profitable locally, or to local consumers' tastes over attributes, among others. Managers' preferences over the design of both products are common knowledge. Information can be transmitted trough costless, non-verifiable (cheap talk) messages. The MNC's headquarters can allocate *product mandates* to any of its business

[1] See also Dessein (2002); Alonso et al. (2015).

units—i.e. it can fully delegate the design of any product to a subsidiary *and* allow for private communication channels with other subsidiaries. The headquarters can also retain authority over any decision. The degree of preference misalignment (conflict of interest) between each manager and decision-maker determines the amount of information the former transmits to the latter.

I first analyze a baseline scenario, in which each manager privately observes local information about both attributes. Delegating authority over a decision can lead to two types of informational gains. Direct gains refer to the additional information a manager receives in equilibrium as compared to what the headquarters would receive if she controlled that decision (Aghion and Tirole, 1997 and Dessein, 2002, among others). Indirect gains, by contrast, refer to the additional information *the headquarters* receives when she delegates one decision and retains authority on the other. These gains only occur under informational interdependence: there must exist managers whose preferences are misaligned with the headquarters in one dimension but aligned in the other. Hence, when indirect informational gains are sufficiently large and concentrate on one decision, the optimal organizational structure involves delegation of the high-conflict decision. But interdependence can lead to a different kind of spillovers. Preference misalignment between managers and headquarters may be large in both dimensions and, still, the *aggregate conflict of interest* be small. In such cases, information affects decisions in a way such that the conflicts of interest counteract each other,[2] leading to centralization as the optimal organizational structure. The mechanism behind these results resembles that of 'subversion' and 'mutual discipline' in public communication with multiple audiences (Farrell and Gibbons, 1989; Goltsman and Pavlov, 2011), which have found empirical support (Battaglini and Makarov, 2014).[3]

My main analysis involves endogenous information acquisition. I thus extend the baseline framework allowing managers to decide on what information to observe once decision rights have been allocated. Managers have access to imperfect information about each attribute and must incur in a cost to observe it. The investment is worth making only if the expected utility gains from a given piece of information compensate its costs. But individual gains are decreasing in the amount of information the decision-maker receives on the equilibrium path. As a consequence, there exists an upper bound on the amount of information any decision-maker receives in equilibrium.

Specialization in this context arises when an agent acquires information about only one of the attributes. In contrast with recent results, Proposition 3 shows that specialization enhances communication incentives under any organizational structure. Observing information about only one attribute reduces a manager's set of deviations from truthful communication and information transmission is incentive compatible for a larger set of preferences. Therefore, the headquarters benefits from restricting a manager's access to information he is not expected to reveal in equilibrium.

[2]More precisely, in an equilibrium where a manager with such preferences truthfully reveal his information, any deviation leads to utility gains in one dimension and losses in the other, both of similar magnitude.

[3]It also relates to 'persuasive cheap talk' in multidimensional communication (Chakraborty and Harbaugh, 2010), and the idea of issue linkage as a strategy to reach agreements in international negotiations (Davis, 2004; Trager, 2011).

Besides, a manager's decision to specialize responds to different incentives. Proposition 4 characterizes the three different motives behind specialization, which depend the allocation of authority, communication incentives, and information costs. Then, an agent specializes when he is only willing to reveal information about the state which is more relevant for the low-conflict decision (if there is any); or when he is willing to reveal information about both states but doing so would be too costly; or when, under centralization, he is willing to reveal information about *any single state* but not about both. In the latter two cases, he acquires the piece of information with the largest expected payoff.

I also investigate the case of uncertainty in agents' (decision-specific) biases, to assess the potential for information aggregation of each organizational structure. The analysis features a single subsidiary with access to n local partners whose preferences are ex-ante unknown to the headquarters. The local partners expected to engage in informational cooperation are those for whom the expected gains from knowledge creation and transmission compensate the associated costs. Proposition 5 shows that when the subsidiary controls only one decision, the amount of information it expects to aggregate is lower and more concentrated on the attributes salient for that decision. In other words, some partners' investments in information fail to internalize the effects on decisions outside the subsidiary's control.

The previous results have important implications for the design of knowledge-based organizations with multiple subunits. For instance, the International Business literature has identified firms that resolved the innovation-integration dilemma, where knowledge transfers across subsidiaries occur; yet, the mechanisms have not been fully understood (Monteiro and Birkinshaw, 2017). The effects of informational spillovers on the optimal allocation of authority account for a plausible theoretical mechanism. Indeed, there is some evidence pointing that way: several papers document a positive correlation between product or managerial mandates granted to subsidiaries and knowledge transfers from these subsidiaries to sister units (Andersson et al., 2005; Ecker et al., 2013; Kunisch et al., 2019).

Most importantly, the results on specialization offer normative implications related to subunits' role as knowledge producers. Firstly, the positive effects on information transmission imply that knowledge creation at the subsidiary level may benefit from access to highly-specialized local partners. When deciding on mandates to enhance knowledge production, however, headquarters need to ponder on the nature of local partners' expertise and the connections between them.[4] Secondly, if knowledge creation depends on local partners' decisions to engage in informational cooperation with subsidiaries, then restrictive mandates will more likely attract partners who fail to internalize the informational synergies across the firm. Such narrow mandates will then lead to inefficient specialization of subunits.

Related Literature. In multidimensional cheap talk, the receiver can extract all the information from perfectly informed senders by restricting individual influence to the dimension of common interest (Battaglini, 2002). When there are as many perfectly-informed senders as decisions, the receiver can

[4]Specialization benefits communication only if agents know about a limited set of decision-relevant issues.

commit to ignore part of the information provided by each sender because it is provided (in equilibrium) by the others. However, this equilibrium commitment power relies on the assumption of perfect information. Levy and Razin (2007) show that, when senders observe noisy signals about the state, the interdependence between decisions can lead to communication breakdown due to the aggregation of decision-specific preferences (see also Ambrus and Takahashi 2008; Chakraborty and Harbaugh 2010). However, this aggregation of preferences across dimensions can also benefit communication (Farrell and Gibbons, 1989; Chakraborty and Harbaugh, 2007; Goltsman and Pavlov, 2011).[5] My paper shows delegation substitutes for the receiver's ability to ignore information in Battaglini (2002) and, more generally, analyzes how the allocation of authority helps to manage informational interdependence.[6]

Strategic communication with informational interdependence has received some recent attention in the work of Deimen and Szalay (2019). Their paper focuses on a unidimensional decision problem with two payoff-relevant states of the world, such that principal and agent disagree about the state upon which the decision has to be calibrated. Specialization then harms communication. In my paper, the effect of interdependence on communication is not monotonic because conflicts of interest are state-independent.[7] The delegation of authority may lead to inefficient information acquisition, but the underlying mechanisms are starkly different from those in Deimen and Szalay (2019).

The literature on organizational design shows that the benefits from delegation are increasing in the sender's informational advantage and decreasing in his bias (Dessein, 2002).[8] This intuition has important implications for the organization of legislative debate (Gilligan and Krehbiel, 1987; Krishna and Morgan, 2001a; Dewan et al., 2015), fiscal authority in federations (Kessler, 2014), and bureaucracies (Epstein and O'Halloran, 1994; Gailmard and Patty, 2012). In practice, however, these environments typically feature complex decision-making problems characterized by multiple, interrelated decisions and information dispersed among many interested agents. My paper shows how such complexity affects organizational design and information acquisition.

Part of the literature has addressed questions similar to mine. Several papers focus on multi-divisional firms, which trade off the need for adaptation to local shocks with the need for coordination between divisions. Communication frictions may lead to inefficiencies in terms of giving up benefits from the specialization of production (Dessein and Santos, 2006) or the need for coordination through scheduled tasks instead of using communication (Dessein et al., 2016). When divisional managers' information is not verifiable, the allocation of decision rights—along with non-separability of preferences and divisional conflict of interest—shapes incentives for communication (Alonso et al., 2008,

[5] Battaglini and Makarov (2014) find evidence about the mechanisms in Goltsman and Pavlov (2011).

[6] Koessler and Martimort (2012) show that in the optimal delegation scheme à la Holmström, a principal can induce an agent to fully-reveal his type trading-off across decision dimensions.

[7] A recent contribution by Lipnowski and Ravid (2020) characterizes the limits of cheap talk communication when the sender's motives are state independent.

[8] Agastya et al. (2014) show that residual uncertainty, in addition to sender-type uncertainty, can improve information transmission and, thus, lead to centralization being preferred to delegation when the former is influential.

2015; Rantakari, 2008).[9] I focus on interdependence arising exclusively from information and not tasks, a relevant distinction when analyzing the determinants of optimal delegation (Dobrajska et al., 2015).[10] Besides matching many real-world environments, my approach also helps study the acquisition of imperfect information in the context of strategic communication (for recent contributions with unidimensional decision problems see Di Pei, 2015; Migrow and Severinov, 2021.)

Regarding how the allocation of decision rights affects incentives to acquire information, Aghion and Tirole (1997) show delegation motivates agents but results in a loss of control for the principal (see also Argenziano et al. 2016). In some cases, however, delegation may discourage information acquisition, either because the agent prefers to put more effort into information that benefits him personally (Rantakari, 2012; Deimen and Szalay, 2019), or because he no longer has to convince a principal with divergent opinions about the best course of action (Che and Kartik, 2009).[11] My paper shows that having the choice about which information to observe enhances communication incentives by means of specialization. It also shows that delegation leads to suboptimal information acquisition because agents fail to internalize the informational interdependencies across the organization.

The next section presents the basic set-up, while the baseline model with no information acquisition can be found in section 3. In section 4, I integrate the allocation of decision-rights with endogenous information acquisition. Section 5 describes the connections with and implication for the literature on the organization of knowledge-based multinational corporations. Section 6 concludes.

2 Basic Set-up

Players and preferences. An organization consists of a principal, P, and n agents. The principal needs to decide on two issues, denoted by $\mathbf{y} \in \mathbb{R}$, for which she needs information about two state variables θ_1 and θ_2 (to be defined later). Informational interdependence arises because taking an optimal decision on each issue depends on having information about both states. I represent the decision-specific uncertainty as two composite states $\delta_1(\theta_1, \theta_2)$ and $\delta_2(\theta_1, \theta_2)$. Preferences for player $i = \{P, 1, \ldots, n\}$ are given by:

$$U^i(\boldsymbol{\theta}, \mathbf{y}, \mathbf{b}^i) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^i\right)^2 - \left(y_2 - \delta_2(\theta_1, \theta_2) - b_2^i\right)^2$$

Where $\mathbf{b}^i \in \mathbb{R}^2$ represents i's bias vector, which is normalized to $\mathbf{b}^P = (0, 0)$ for the principal.

[9] Several recent papers have documented the positive relationship between delegation of authority and the need for adaptation to local volatility in a context of asymmetric information (Acemoglu et al., 2007; Guadalupe and Wulf, 2010; Dessein et al., 2019; Aghion et al., 2021), which also depends on the need for coordination between decisions (McElheran, 2014; Lo et al., 2016). My results are consistent with the first of these mechanisms.

[10] They find evidence that informational interdependence relates to less delegation due to lower monitoring costs.

[11] Bester and Krähmer (2008) show that the incentive effects of delegation depends on whether the agent's costs are incurred before or after the decision is taken.

Restricting the analysis to two decisions and two states is without loss of generality; however, this is not true for the assumption of additively-separable preferences. In a companion paper (Habermacher, 2022), I analyse a more general model and discuss the case of non additive-separable preferences.

Information structure. Pay-off relevant states, θ_1 and θ_2, are uniformly distributed with support in the interval $[0, 1]$, and $\theta_1 \perp \theta_2$. *Informational interdependence* means that information about any single state can be useful for both decisions. In the example of multinational corporations, the states can be interpreted as different technological attributes relevant to several of the firm's products. Innovation on a specific technology, then, would affect the design of different products; but would do so to a different extent for each product depending on how salient that attribute is. The composite states δ_1 and δ_2 capture informational interdependence:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_1\, \theta_1 + (1 - w_2)\, \theta_2 \\ (1 - w_1)\, \theta_1 + w_2\, \theta_2 \end{bmatrix}$$

The uniform distribution of states represents the canonical example in Crawford and Sobel (1982) and has been extensively used in the cheap talk literature.[12] Without loss, I take $w_1, w_2 \in [0.5, 1]$, meaning that the state θ_1 $[\theta_2]$ is (weakly) more salient for y_1 $[y_2]$. This formulation also assumes the normalization of the aggregate influence of each state across decisions, which allows me to restrict the analysis to the effect of interdependence and preferences on incentives for communication (and later information acquisition). Assuming independent states allows me to isolate the effect of informational interdependence through $\boldsymbol{\delta}$, which is linear and captured by:

$$\text{Corr}(\delta_1, \delta_2) = \frac{[w_1(1 - w_2) + (1 - w_1)w_2]}{(w_1^2 + (1 - w_1)^2)(w_2^2 + (1 - w_2)^2)}$$

Agents' signals and communication. Agents have access to noisy, non-verifiable information about both states. Each agent observes one signal associated with each state (two in total). Let $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S} \equiv \{0, 1\}^2$ be i's signals, and $\tilde{\mathbf{S}} \in \mathcal{S}$ be the vector of realizations. Signals are independent across players, conditionally on $\boldsymbol{\theta}$. The prior probability distribution for each signal is characterized by $\Pr(\tilde{S}_1^i = 1) = \theta_1$ and $\Pr(\tilde{S}_2^i = 1) = \theta_2$.[13]

Each agent sends private, non-verifiable (cheap talk) messages to decision-maker $j = \{P, 1, .., n\}$. Let $\mathbf{m}_j^i\left(\mathbf{S}^i\right) \in \{0, 1\}^2$ denote agent i's message to decision-maker j, in charge of $y_d = \{y_1, y_2\}$. Note that i's (pure) message strategies associated with each signal can take one of two forms (up to relabelling messages): the truthful one, $m_j^i(S_r^i) = \tilde{S}_r^i$ for all S_r^i, and the babbling one, $m_j^i(\tilde{S}_r^i = 0) = m_j^i(\tilde{S}_r^i = 1)$.

[12] For a more general formulation with a Beta prior, see Migrow (2021).

[13] A similar information structure, for unidimensional problems with one state variable, has been used in Morgan and Stocken (2008); Galeotti et al. (2013) among others.

Besides, the complete set of message strategies is not just based on babbling or revealing information on separate dimensions independently. An agent could, for instance, truthfully reveal both signals for some realizations and send the babbling message for others. Such message strategies arise because states are orthogonal and information about one state does not reveal information about the other. I call these strategies 'dimensional non-separable' (DNS).

Let $\mathbf{m}_j = \{..., \mathbf{m}_j^i, ...\}$ denote the matrix containing all the messages decision-maker j receives from agents (including himself if applicable). The updated expectation and variance for each state depend on the number of agents revealing the corresponding signal truthfully, $k_r(j) \leq n$, and the the number of those agents who report a 1, $\ell_r(j)$, for $\theta_r = \{\theta_1, \theta_2\}$, as follows.

$$E\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)}{(k_r(j)+2)} \qquad \mathrm{Var}\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)(k_r(j)-\ell_r(j)+1)}{(k_r(j)+2)^2(k_r(j)+3)}$$

Allocation of decision rights. Decision-rights are allocated before each agent learns his information. Formally, the principal decides on a set of assignments that grants decision making authority over the set of decisions. The assignment grants complete jurisdiction over the delegated decision, such that authority over each decision is granted to a unique individual. Decision-makers are also granted the possibility of private communication with each agent. I assume decision-makers cannot communicate the information transmitted to them by other agents. Different allocations of decision-rights lead to different organizational structures. I group these structures into three categories: under *centralization* the principal decides on both issues; under *full delegation* the principal grants authority to two different agents, each of them assigned to a different decision; under *partial delegation* the principal retains authority over one issue and delegates the other to an agent.

At this point two clarifications are necessary. First, I am not considering the case of delegation of both decisions to a single agent. Incentives for communication in such a case use the same measure of conflict of interest as under centralization. Hence, interdependence plays no role beyond the basic trade-off between informational gains and loss of control. On the contrary, the two forms of delegation considered in the paper involve different measures of conflict of interest. The second clarification relates to the distinction between delegation of decision authority and decentralization on the access to information. In my framework, authority can be centralized or decentralized, the latter is called 'delegation' throughout the paper. Information, on the contrary, is always decentralized because it is dispersed among agents.

Equilibrium concept. The equilibrium concept is pure-strategies Perfect Bayesian Equilibria (equilibrium, henceforth). A full characterization including mixed strategies is cumbersome and does not provide much insight beyond the pure-strategies case. Because communication is cheap talk, there typically exist multiple equilibria. I select among equilibrium message strategies on the basis of the decision-maker's ex-ante expected utility.[14]

3 Authority and Communication Incentives

In this section I characterize the role of informational interdependence in communication incentives and organizational design. To do so, I abstract from information acquisition decisions by endowing each agent with one signal per state. I first describe incentives for communication under each organizational structure (the formal analysis is left to the appendix) and, then, characterize the optimal organizational structure. The figure below outlines the timing of the game.

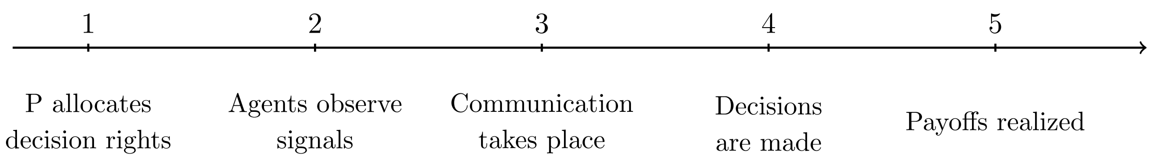


Figure 1: Timing of the Organizational Structure Game

To keep track of who decides what, let $j_1, j_2 \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively. An equilibrium of the game defined by the allocation of authority is characterized by the triple $\left(\mathbf{y}^*, \mathbf{m}^*_{j_1}, \mathbf{m}^*_{j_2}\right)$, representing the vector of decisions and the vectors of message strategies to j_1 and j_2, respectively. Optimal actions satisfy:

$$y_1^* = w_1\, E(\theta_1|\mathbf{m}^*_{j_1}) + (1-w_2)\, E(\theta_2|\mathbf{m}^*_{j_1}) + b_1^{j_1} \qquad y_2^* = (1-w_1)\, E(\theta_1|\mathbf{m}^*_{j_2}) + w_2\, E(\theta_2|\mathbf{m}^*_{j_2}) + b_2^{j_2}$$

Note that centralization means $j_1 = j_2 = P$, and the biases are equal to zero. From the principal's perspective, delegation of decision-rights has two payoff-relevant consequences. On the one hand, it implies a biased agent decides on her behalf, which results in a biased decision. On the other hand, individual incentives for communication depend on the conflict of interest between the agent and each decision-maker. Different organizational structures (and decision-makers) then result in different communication incentives. Agent i's optimal message strategy to decision-maker j solves:

$$\mathbf{m}_j^{i*}(\mathbf{S}^i, \mathbf{b}^i, b_1^{j_1}, b_2^{j_2}) = \arg\max_{\mathbf{m}_j^i} \left\{ E\Big[-\left(y_1\left(m^i_{j_1}, \mathbf{m}^{-i}_{j_1}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(m^i_{j_2}, \mathbf{m}^{-i}_{j_2}\right) - \delta_2 - b_2^i\right)^2 \Big| \mathbf{S}^i \Big] \right\}$$

To further simplify notation, let $k_r^{\mathrm{C}} \equiv \{k_r(j)|\, j_1 = j_2 = P\}$ denote the number of truthful messages about $\theta_r = \{\theta_1, \theta_2\}$ the principal receives under centralization; let $k_r^{\mathrm{P1}} \equiv \{k_r(j_1)|\, j_1 = P\}$ be the number of messages received when she decides on y_1 only, and $k_r^{\mathrm{P2}} \equiv \{k_r(j_2)|\, j_2 = P\}$ when she decides on y_2 only. For when P does not decide at all, let $k_r^{\mathrm{D1}} \equiv \{k_r(j_1)|\, j_1 \neq P\}$ and $k_r^{\mathrm{D2}} \equiv \{k_r(j_2)|\, j_2 \neq P\}$

[14] See Crawford and Sobel (1982); Chen et al. (2008).

refer to the number of truthful messages for decision-makers of y_1 and y_2, respectively. I keep $k_r^j \equiv k_r^*(j)$ for a generic decision-maker $j = \{P, 1, ..., n\}$. Note that the principal's expected utility from different allocations of decision rights depends on the amount of information the different decision-makers are expected to receive on the equilibrium path.

Incentives for communication under delegation. I first describe agent i's incentives to reveal information to decision-maker j in charge of $y_d = \{y_1, y_2\}$ only. Communication between i and j is private; then, incentives depend on the conflict of interest between them, $|b_d^i - b_d^j|$. But since i is imperfectly informed, incentives for communication decrease on the number of other agents who are expected to be truthful to j on the equilibrium path (Morgan and Stocken, 2008; Galeotti et al., 2013). In this framework, however, agents observe signals about two independent states, which introduces a third mechanism.

Because of informational interdependence, incentives can be affected by specific signals realizations. When i observes $\mathbf{S}^i = \{(0, 1)\}$, for instance, the way in which he will affect j's beliefs depends on the information he is expected to reveal in equilibrium. Given both the positive correlation between decisions and the unidimensional conflict of interest between i and j, one of such signals always moves the decision in the direction of b_d^i. Therefore, i has stronger incentives to follow this 'favourable information' for all possible message strategies.[15] Such incentives to deviate from truth-telling lead to a *credibility loss* in the form of a tighter IC constraint as compared to the case in which i only observes the information he is willing to reveal on-path (see Habermacher, 2022). The extent of the credibility loss depends on i's conjecture about the influence of his favourable signal; hence, incentives for communication now depend on how much information *about both states* other agents are expected to reveal on-path.

Lemma A1 and Proposition A2 in the appendix A characterize equilibrium communication between i and j, I only describe the intuitions here. When i is expected to reveal one signal on-path there are three determinants of his communication incentives: the conflict of interest with j, how many agents reveal the same information on-path, and how many of them reveal the other signal (this is due to the credibility loss). When i is expected to reveal both signals on-path, the influence he will have on the decision depends on his information. For $\mathbf{S}^i = \{(0, 0), (1, 1)\}$, the influence of each signal reinforces that of the other and, thus, i's expected marginal utility from this message strategy will be larger than the alternative of revealing any of them individually. Agent i in principle has stronger incentives to reveal both signals than either of them individually. For $\mathbf{S}^i = \{(0, 1), (1, 0)\}$, however, the influence of each signal on the decision counteracts that of the other, leading to the *credibility loss*. As a consequence, the incentive-compatibility (IC) constraints associated to revealing both signals and those associated

[15] For example, suppose $b_1^i > 0$ and i is expected to reveal information about θ_1 only. For $\mathbf{S}^i = \{(0, 1)\}$, the fact that i 'knows' announcing $S_1^i = \{0\}$ will overshoot the update on y_d makes his IC constraint tighter.

to revealing only one of them hold for the same set of bias vectors. The equilibrium under delegation then features only two message strategies: full revelation, $\mathbf{m}_j^{i*} = \{\{(0,0)\};\{(1,1)\};\{(1,0)\};\{(0,1)\}\}$, and babbling, $\mathbf{m}_j^{i*} = \{\{(0,0);(1,1);(1,0);(0,1)\}\}$.[16]

Incentives for communication under centralization. Interdependence means that information transmitted to the principal under centralization affects both decisions. But an agent's influence on decisions depends on the type and amount of information he transmits. To see this, note that information about θ_1 has a larger influence on y_1 such that the bias on the first dimension weighs more heavily in determining i's incentives to reveal S_1^i only. If he reveals both signals, on the contrary, the overall influence is more balanced and so are the weights of b_1^i and b_2^i on the IC constraints. Different message strategies are then associated to different measures of conflict of interest between i and P.

The possibility of different measures of conflict of interest results in two main differences with respect to delegation. First, there exists a non-empty set of bias vectors for which the equilibrium message strategy is to reveal information about one state only. In such cases, the relevant measure of conflict of interest for revealing S_1^i is $\beta_1 = w_1\, b_1^i + (1-w_1)\, b_2^i$, while that for revealing S_2^i is $\beta_2 = (1-w_2)\, b_1^i + w_2\, b_2^i$ (Lemma A2). The second difference with delegation relates to the effects of 'ambiguous information'—i.e. $\mathbf{S}^i = \{(0,1),(1,0)\}$. Note that truthful revelation of $\mathbf{S}^i = \{(0,1)\}$ or $\mathbf{S}^i = \{(1,0)\}$ would 'move' optimal decisions in opposite directions with respect to the prior. For instance, if i credibly announces $\mathbf{m}_P^i = \{(0,1)\}$, y_1^* would update towards 0 and y_2^* towards 1. When $\mathrm{sign}(b_1^i) = \mathrm{sign}(b_2^i)$, such a message leads to utility gains in one dimension and losses in the other. In addition, there exists a set of bias vectors for which the equilibrium message strategy consists of full revelation when signals do not coincide, and announcing the (equilibrium) non-influential message otherwise. In summary, the most informative equilibrium under centralization features the following message strategies: full revelation, revelation of information about one state, full revelation of some signal realization and non-influential messages for the rest of signal realizations, and babbling.[17] I now focus on the analysis of the optimal organizational structure.

Optimal Organizational Structure. Based on the well-know result of existence of at least one (babbling) equilibrium in cheap talk games, each possible profile of preferences $\mathbf{B} = \{\mathbf{b}^1, ..., \mathbf{b}^n\}$ has an associated set of equilibria, each characterized by the number of truthful messages each decision-maker receives about both states. In allocating decision rights, the principal effectively chooses among the different equilibria induced by $\mathbf{B}$. Let $\mathrm{Var}(\theta_r|\mathbf{m}_j)$ denote the ex-ante residual variance associated to

[16] One could think that i may want to fully reveal signals when they coincide and announce the corresponding babbling message when they do not—i.e. $\mathbf{m}_j^i = \{\{(0,0)\};\{(1,1)\};\{(1,0);(0,1)\}\}$. In a companion paper I show the set of bias vectors for which this is incentive compatible is a strict subset of that for which full revelation is.

[17] For a more thorough discussion of communication incentives under centralization see Habermacher (2022).

state $\theta_r = \{\theta_1, \theta_2\}$ in the equilibrium in which decision-maker $j = \{P, 1, ..., n\}$ receives $\mathbf{m}_j^*$ messages.[18] The principal's ex-ante expected utility for decision-makers j_1 and j_2 is then given by:

$$\begin{aligned} E\left[U^{\mathrm{P}}(\boldsymbol{\theta}, \mathbf{B}); \mathbf{m}^*\right] = &- \left[(b_1^{j_1})^2 + (w_1)^2 \operatorname{Var}(\theta_1 | \mathbf{m}_{j_1}^*) + (1-w_2)^2 \operatorname{Var}(\theta_2 | \mathbf{m}_{j_1}^*)\right] \\ &- \left[(b_2^{j_2})^2 + (1-w_1)^2 \operatorname{Var}(\theta_1 | \mathbf{m}_{j_2}^*) + (w_2)^2 \operatorname{Var}(\theta_2 | \mathbf{m}_{j_2}^*)\right] \end{aligned} \tag{1}$$

Equation (1) represents the residual variance in cheap talk games with quadratic preferences (see Galeotti et al., 2013; Habermacher, 2022).The first term in square brackets represents the principal's ex-ante expected utility associated with y_1 when the decision-maker is j_1. His decision will be biased under delegation and, thus, the principal's utility will be affected. But her payoff also depends on her expectations about the precision of the resulting decision, which is a function of how much information j_1 receives from other players in equilibrium. Hence, as is well-know in the literature, optimal delegation arises when informational gains must compensate for the loss of control.

But informational interdependence can lead to a different source of informational gains. Note that delegation 'breaks' the interdependence because, in such cases, communication incentives only depend on decision-specific conflict of interest. As a consequence, there may be agents willing to reveal information *to the principal* under delegation (of one decision) but not under centralization. This happens, for instance, when agents' biases are very large in one dimension and small in the other. If the principal delegates the high-conflict decision and retains authority over the low-conflict decision, such agents will reveal information to her despite not willing to do so in case she retains authority on both issues. Such informational gains from delegation are *indirect*, since they do not arise because the decision-maker aggregates more information than the principal but because some agents are affected by *negative informational spillovers* (Levy and Razin, 2007). The proposition below defines both types of informational gains arising in this game, and characterizes the necessary conditions for each to emerge.

Lemma 1. *Given the profile of preferences* $\mathbf{B}$*, the equilibrium allocation of authority involves delegating decision* $y_d = \{y_1, y_2\}$ *if there exists an agent* $j_d = \{1, ..., n\}$ *such that delegation leads to at least one of the following informational gains:*

- ***Direct Informational Gains****, that is:*

$$DIG_{j_d}(y_d) \equiv (w_d)^2 \left[Var(\theta_d | \mathbf{m}_{\mathrm{C}}^*) - Var(\theta_d | \mathbf{m}_{j_d}^*) \right] + (1-w_{\text{-}d})^2 \left[Var(\theta_{\text{-}d} | \mathbf{m}_{\mathrm{C}}^*) - Var(\theta_{\text{-}d} | \mathbf{m}_{j_d}^*) \right] \geq (b_d^{j_d})^2$$

- ***Indirect Informational Gains****, that is:*

$$IIG(y_{\text{-}d}) \equiv (1-w_d)^2 \left[Var(\theta_d | \mathbf{m}_{\mathrm{C}}^*) - Var(\theta_d | \mathbf{m}_{\mathrm{P}2}^*) \right] + (w_{\text{-}d})^2 \left[Var(\theta_{\text{-}d} | \mathbf{m}_{\mathrm{C}}^*) - Var(\theta_{\text{-}d} | \mathbf{m}_{\mathrm{P}\text{-}d}^*) \right] \geq 0$$

[18] Formally, $\operatorname{Var}(\theta_r | \mathbf{m}_j^*) \equiv E\left[\left(E(\theta_r | \mathbf{m}_j^*) - \theta_r\right)^2; \mathbf{m}_j\right] = \frac{1}{6(k_r^j + 2)}$.

For $y_{\text{-}d} \neq y_d = \{y_1, y_2\}$, $\theta_{\text{-}d} \neq \theta_d = \{\theta_1, \theta_2\}$, and $w_{\text{-}d} \neq w_d = \{w_1, w_2\}$, and $\text{P}_{\text{-}d}$ indexes the principal deciding on $y_{\text{-}d}$.

Proof. All proofs can be found in Appendix A □

Delegation can benefit the principal in two (non-exclusive) ways.[19] First, if there is an agent whose preferences are more central than the principal's in a specific dimension, he will be able to aggregate more information than her when having authority on that decision. Then, direct informational gains arise when such agent aggregates strictly more information than the principal under centralization. Note that these gains relate to the decision that has been delegated, and reflect the mechanism widely studied by the literature (see Aghion and Tirole, 1997; Dessein, 2002 among others).

The second mechanism through which the principal can benefit from delegation involves *indirect gains*. Suppose there is an agent whose preferences feature a very large conflict of interest with the principal in the first dimension (i.e. b_1^i is large), but with no conflict of interest in the second dimension (i.e. $b_2^i = 0$). Suppose, also, that the former bias is so large that there cannot be credible communication between this agent and the principal when she decides on both issues. In such a case, if the principal *delegates* y_1 but *retains authority over* y_2, agent i's communication with the principal will only affect the decision in which preferences are aligned. In other words, delegating the conflictive decision leads to more information transmitted to the principal, which will be relevant for other decisions. These are *indirect informational gains*, as they are associated to decisions other than the delegated one.

I now analyze how informational spillovers affect the optimal allocation of authority. To do so, I first introduce the intuition of *positive informational spillovers*. Suppose an agent whose preferences feature large biases in both dimensions, with $b_1^i > 0$ and $b_2^i < 0$. Due to the positive correlation between decisions, if i has information $\mathbf{S}^i = \{(0,0);(1,1)\}$ and were to reveal any of these signals truthfully, the principal's beliefs about optimal decisions would move in the same direction with respect to the prior. This means that, by misleading the principal, i would obtain utility gains in one dimension but losses in the other. Such a compromise could lead to improved credibility for i under centralization despite conflict of interest being large on each dimension separately. The proposition below shows how informational spillovers affect the allocation of authority.

Proposition 1. *Let $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$ denote a profile of biases for n informed agents, with associated profile $\mathbf{B}^{n+p} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+p})$, in which the preferences for the first n agents coincide with $\mathbf{B}^n$. There exists a sufficiently large $\mathfrak{b} \in \mathbb{R}_+$ for which it is true that:*

1. *For $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+p} = (\mathfrak{b}, 0)$; if there is an agent j_1 with $b_1^{j_1} \leq \tilde{b}$, then there exists a sufficiently large p such that the optimal organizational structure in the game with $n+p$ agents is partial*

[19] The optimal allocation of decision rights is fully characterized in Proposition A3. Full delegation is optimal when there are two different agents with central preferences and no informational spillovers associated with retaining authority. The allocation of decision rights depends then on the different communication equilibria induced by the profile of preferences.

delegation of y_1 only.

2. *For $\mathbf{b}^{n+1} = ... = \mathbf{b}^{\frac{2n+p+1}{2}} = (-\mathfrak{b}, \mathfrak{b})$ and $\mathbf{b}^{\frac{2n+p+1}{2}} = ... = \mathbf{b}^{n+p} = (\mathfrak{b}, \mathfrak{b})$; then, there exists a sufficiently large p such that the optimal organizational structure in the game with $n+p$ agents is centralization.*

Where $\tilde{b} \equiv \frac{[w_1^2+(1-w_1)^2]+[w_2^2+(1-w_2)^2]}{6(n+2)} - \frac{w_1^2+(1-w_2)^2}{18}$.

If negative spillovers associated to one dimension are sufficiently large, the principal prefers to delegate the high-conflict decision and retain authority over the low-conflict decision in equilibrium. This requires that there exists an agent whose preferences on the delegated decision are not so large, but this restriction applies only if there are no direct informational gains from delegation of y_1. If positive spillovers are sufficiently large, the principal retains authority on both decisions in equilibrium. Informational spillovers in Proposition 1 are captured by the preferences of the 'additional agents,' such that p reflects the intensity of these spillovers.

Negative spillovers lead to partial delegation because the principal finds optimal to 'get rid of' the controversial decision in order to induce the additional agents to reveal the information they have. Positive spillovers, on the other hand, lead to centralization because the additional agents are willing to play dimensional non-separable strategies under centralization. Agents whose decision-specific biases have different signs fully reveal their signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$ and announce the babbling message for the other possible realizations; while agents whose biases have the same sign fully reveal their signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$ and announce the babbling message otherwise. In both cases, the additional information the principal expects to receive from the p agents brings her a higher expected utility than the optimal allocation under the original profile of preferences.

Having characterized incentives for communication and the role of informational interdependence on the allocation of authority, I now proceed to the paper's main analysis involving endogenous information acquisition.

4 Endogenous Information Acquisition

This section introduces endogenous information acquisition to the basic set-up of section 2. First, I present the extended model and derive the two incentive compatibility constraints involved in the decision to acquire information. With these, I show that information costs can limit the informational gains from delegation. I then characterize the equilibrium strategies for a generic agent and show the cases for specialization. Finally, I analyze the case of unknown biases in order to study how much information each organizational structure aggregates in expectation.

4.1 Basic set-up with endogenous information acquisition

Each agent has access to one binary trial per state and decides which realizations to observe, if any.[20] Formally, let $\mathfrak{s}^i \in \left\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\right\}$ be agent i's information acquisition decision. Note that i's type is given by the realizations of both signals but he decides how much to know about it.

Definition 1. *The information structure for agent i in the game with endogenous information acquisition consists of the following elements: $\mathbf{S}^i = (S_1^i, S_2^i)$ are the signals available to him, $\tilde{\mathbf{S}}^i = (\tilde{S}_1^i, \tilde{S}_2^i)$ the realization of the corresponding signals (his type), and $\mathfrak{s}^i \in \left\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\right\}$ the information he actually decides to observe.*

The function $C(\mathfrak{s})$ captures the information costs, and satisfies $C\left(\{\tilde{S}_1, \tilde{S}_2\}\right) > C\left(\{\tilde{S}_1\}\right) = C\left(\{\tilde{S}_2\}\right) > C\left(\emptyset\right) = 0$. The principal has no direct access to information. The preferences of agent $i = \{1, \ldots, n\}$ are given by:

$$U^i(\boldsymbol{\theta}, \mathbf{b}^i, \mathfrak{s}^i) = -\sum_{y_d=\{y_1,y_2\}} \left(y_d - \delta_d(\theta_1, \theta_2) - b_d^i\right)^2 - C(\mathfrak{s}^i)$$

Whereas the principal has similar preferences with no cost of acquisition and biases on each dimension normalized to zero. Figure 2 shows the timing of the game. The allocation of decision rights is observed by all agents. Knowing who decides what, each agent chooses the information he will observe. I assume *overt information acquisition*; that is, individual decisions (but not information) are common knowledge. In Section, 6 I discuss the implications of relaxing this assumption.

The communication stage is similar to the previous section. Let $k_r^j \equiv k_r^*(\mathbf{m}_j^*(\mathfrak{s}^*))$ be the number of truthful messages decision-maker $j = \{j_1, j_2\}$ receives in equilibrium.

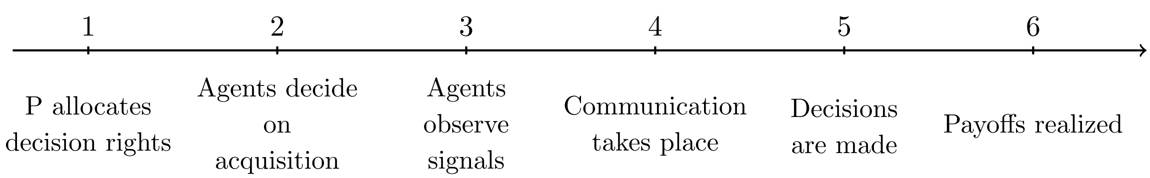


Figure 2: Timing of the Org. Structure / Info Acquisition game.

An equilibrium in this game is then characterized by the decision vector, $\mathbf{y}_d^*$, and collections of messages and acquisition strategies for each agent and decision-maker j, $\mathbf{m}_j^* = \{\ldots, \mathbf{m}_j^{i*}, \ldots\}$ and $\mathfrak{s}^* = \{\ldots, \mathfrak{s}^{i*}, \ldots\}$. The expressions for optimal actions and messages are similar to those of the previous section, noting that $k_r^*(\mathbf{m}^*(\mathfrak{s}^*))$, $y_d^*(\mathbf{m}_j^*(\mathfrak{s}^*))$, and $\mathbf{m}_j^{*i}\left(\mathfrak{s}^i, \mathbf{m}^{-i}(\mathfrak{s}^{-i})\right)$. Agent i's information

[20] In principle, agents could have the choice on how much information about each state to observe, involving information acquisition at the intensive and the extensive margins. Here, however, I focus on the extensive margin, meaning that each agent decides whether to observe at most one binary signal per state. In section 6, I discuss some implications of allowing agents to acquire information on the intensive margin.

acquisition strategy is given by:

$$\mathfrak{s}^{i*} = \arg\max_{\mathfrak{s}^i} \left\{ E\Big[-\left(y_1\left(\mathbf{m}^i_{j_1}(\mathfrak{s}^i), \mathbf{m}^{-i}_{j_1}\right) - \delta_1 - b^i_1\right)^2 - \left(y_2\left(\mathbf{m}^i_{j_2}(\mathfrak{s}^i), \mathbf{m}^{-i}_{j_2}\right) - \delta_2 - b^i_2\right)^2 \Big] - C(\mathfrak{s}^i) \right\}$$

The expectation is based on equilibrium beliefs. Agent i's equilibrium message strategy depends on the information he acquired in an earlier stage of the game and his conjecture about other agents' message strategies.[21] The signals i acquired thus affect his communication strategy. When he acquires information about both states, the IC constraints for communication are the same as in the previous section. When he acquires information about one state, however, the IC constraints change significantly because his incentives to reveal information are not affected by beliefs about the other state. This kills the credibility loss described in the previous section and, *ceteris paribus*, truthful communication is incentive compatible for a larger set of bias vectors. I now proceed to the details of these arguments.

Incentives to acquire information. Costly information acquisition means that each agent will invest in a signal only if he expects to benefit from it. In equilibrium i only acquires signals he is willing to reveal; for if he fails to reveal any piece of information (off-path), no other agent will change his equilibrium message strategy but he still bears the costs.[22] The lemma below formalizes this: incentive compatibility at the acquisition stage requires incentive compatibility at the communication stage.

Lemma 2. *Let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ be equilibrium strategy profiles for the principal and all agents. The equilibrium is characterized by the number of truthful messages decision-makers receive, $k_1^{j_d}\left(\mathbf{m}^*_{j_d}(\mathfrak{s}^*)\right)$ and $k_2^{j_d}\left(\mathbf{m}^*_{j_d}(\mathfrak{s}^{i*})\right)$, for $j_d = \{j_1, j_2\}$. Then, i's equilibrium information acquisition strategy, $\mathfrak{s}^{i*}$, satisfies:*

- *$S_r \in \mathfrak{s}^{i*}$ only if truthful revelation to j_d is incentive compatible, given $k_r^{j_d}\left(\mathbf{m}^*_{j_d}(\mathfrak{s}^*)\right)$;*
- *$\{S_1, S_2\} \in \mathfrak{s}^{i*}$ only if full revelation to j_d is incentive compatible, given $k_1^{j}\left(\mathbf{m}^*_{j_d}(\mathfrak{s}^*)\right)$ and $k_2^{j_d}\left(\mathbf{m}^*_{j_d}(\mathfrak{s}^*)\right)$.*

The main implication of Lemma 2 is that the choice of organizational structure will affect agents' incentives for information acquisition because it determines the relevant communication IC constraints. Incentives to acquire information depend on being influential at the communication stage, but credibility hinges on both the conflict of interest and the number of other agents expected to reveal similar information on-path. Hence, agents acquire information they expect to reveal on the equilibrium path, given the profile of biases and the allocation of decision rights.

[21] Note that information acquisition is observable at the communication stage, which simplifies the beliefs space.

[22] Formally, i's incentives for communication depend on having acquired the signal, $\mathbf{b}^i$, and on his conjecture about k_1^j and k_2^j. Then, for i acquiring S_r^i *off-path* to change another agent h's conjecture about k_r^j, b^i should be such that he is willing to reveal that signal. In such a case, h (off-path) conjecture for k_r^j should be larger than the equilibrium value, but then i would be willing to reveal S_r^i in equilibrium and would have acquired it.

A second element affecting incentive compatibility at the acquisition stage relates to information costs. Utility gains from revealing a given piece of information are decreasing in the number of other agents revealing the same information (k_r^j). Given costs are strictly positive, there is a maximum number of agents for whom the utility gains of revealing that piece of information compensate its costs (see Proposition 2).

Lemma 3. *Let k_r^{C} denote i's conjecture about the information the principal receives about $\theta_r = \{\theta_1, \theta_2\}$ from other agents under centralization; while $k_r^{j_d}$ refers to the case of delegation when decision-maker is $j_d = \{j_1, j_2\}$. Then,*

- *Acquisition is cost-effective for i under **centralization** if:*

$$C(S_r^i) \leq \frac{(w_r)^2 + (1 - w_r)^2}{6(k_r^{\mathrm{C}} + 2)(k_r^{\mathrm{C}} + 3)} \tag{2}$$

- *Under **delegation**, when i is willing to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to decision-maker $j_d = \{y_1, y_2\}$ only, acquisition is cost-effective if:*

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^{j_d} + 2)(k_r^{j_d} + 3)} \tag{3}$$

When i is willing to reveal information about θ_r to both decision-makers, cost-effectiveness is:

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^{j_d} + 2)(k_r^{j_d} + 3)} + \frac{(1 - w_{dr})^2}{6(k_r^{j\text{-}d} + 2)(k_r^{j\text{-}d} + 3)} \tag{4}$$

Where $w_{dr} = \{w_{11}, w_{21}, w_{12}, w_{22}\}$; with $w_{11} = w_1$, $w_{21} = (1-w_1)$, $w_{22} = w_2$, and $w_{12} = (1-w_2)$.

The right-hand sides of the expressions above represent the ex-ante expected utility gains from revealing one signal under centralization and delegation, respectively. An agent acquires a signal if anticipates an (equilibrium) large influence on the decision(s) under consideration. Effectively having such influence depends on incentive compatibility at the communication stage and on the number of other agents who reveal the same information to the same decision-maker on-path. Under centralization, any piece of information about a state influences both decisions, as shown in the numerator of (2). Under delegation, however, the influence of the same piece of information depends on whom i reveals information to. If i does so to one decision-maker only, his influence depends on the salience of that state for the decision under consideration, as shown in the numerator of (3). If, on the other hand, i reveals the information to both decision-makers on-path, cost-effectiveness is given by (4). Finally, dimensional non-separable (DNS) message strategies face more restrictive cost-effectiveness conditions because i expects to reveal information for half of the possible signal realizations. The costs of acquiring both signals must be sufficiently low for such a strategy to be cost-effective.

I now present how cost-effectiveness restricts the aggregate investment in information.

Proposition 2. *Let $\theta_r = \{\theta_1, \theta_2\}$ denote the state which is salient for decision $y_r = \{y_1, y_2\}$, and $j_r \neq j_{\text{-}r} = \{j_1, j_2\}$ the decision-maker for y_r under delegation. Then, under* ***centralization****, the maximum number of agents acquiring information about θ_r in any equilibrium is given by:*

$$K_r^{\mathrm{C}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_r)^2 + (1-w_r)^2]}{6\,C(S_r^i)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{5}$$

Under ***delegation****, the maximum amount of information on θ_r available for decision-making is:*

$$K_r^{j_r} = \left\lfloor \left[\frac{1}{4} + \frac{(w_r)^2}{6\,C(S_r^i)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \qquad \text{and} \qquad K_r^{j_{\text{-}r}} = \left\lfloor \left[\frac{1}{4} + \frac{1-(w_r)^2}{6\,C(S_r^i)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{6}$$

And the maximum amount of information about θ_r that can be aggregated under delegation is:

$$K_r^{\mathrm{D}} = \max\left\{ K_r^{j_r} + K_r^{j_{\text{-}r}}\, ;\, K_r^{\mathrm{C}} \right\} \tag{7}$$

Corollary 1. $K_r^{\mathrm{C}} > K_r^{j_r} > K_r^{j_{\text{-}r}}$*, for all $w_r \in [0.5, 1)$ and $\theta_r = \{\theta_1, \theta_2\}$.*

Expressions (5) and (7) represent the maximum number of agents for whom investing in a signal is cost-effective under centralization and delegation, respectively. Any influential agent affects both decisions under centralization, but under delegation this is true only for agents whose preferences are such that they reveal information to both decision-makers. If the latter were true for sufficiently many agents under delegation, the cost-effectiveness condition will be the same as for centralization. Additionally, under delegation there may exist a subset of agents willing to acquire information about θ_r to reveal it to j_r, while a different subset of agents acquire the same information to reveal it to $j_{\text{-}r}$. In this case, $K_r^{\mathrm{D}} = K_r^{j_r} + K_r^{j_{\text{-}r}}$ such that $K_r^{j_{\text{-}r}} > 0$ for sufficiently small costs. Numerical simulations show that $K_r^{j_r} + K_r^{j_{\text{-}r}} > K_r^{\mathrm{C}}$ for sufficiently small costs and relatively low interdependence. This, however, does not mean that more information is available for decision-making under delegation.

Corollary 1 shows that, despite delegation may aggregate more information overall, cost-effectiveness imposes a weaker constraint regarding the information available for each decision. In other words, the decision-maker under centralization can potentially decide with more information about both states than any decision-maker under delegation. This claim abstracts from specific profiles of preferences, but points towards some potential inefficiencies due to the reduced expected marginal benefit from investment in information about a state under delegation. I explore such inefficiencies in section 4.3.

Covert information acquisition. Proposition OA.1 in the online appendix is the equivalent to Lemma 3 for the case of covert information acquisition. In such a case, acquiring a signal is incentive compatible if robust to two deviations. On the one hand, inducing a more-informed decision(s) must compensate the potential saving on information costs; hence, such costs must not be so large. On the other hand, when an agent acquires information about one state on-path, this strategy must be immune to acquiring information on the other state also. Doing so would allow the agent to adapt his message to both signals which, given the interdependence, could be profitable via implementing an off-path DNS message strategy. As a consequence, information costs must not be 'too low' either.

4.2 Equilibrium information acquisition and specialization

The fact that information is costly can lead some agents to acquire less of it than what they find incentive compatible to communicate, but such 'underinvestment' can benefit communication in some cases. When i acquires information about one state, for instance, his incentives are not affected by information about the other state, which kills the *credibility loss* described in section 3. This enlarges the set of biases for which revealing that piece of information is incentive compatible, as compared to the case in which i observes information about both states. The proposition below states the result.

Proposition 3. *Let $\mathbf{k}^j = \{k_1^j, k_2^j\}$, where $k_r^j = k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ be i's conjecture about other agents revealing information about $\theta_r = \{\theta_1, \theta_2\}$ to decision-maker $j = \{P, 1, ..., n\}$. There exists a set of bias vectors, $\mathfrak{B}_r^j = \mathfrak{B}_r^j(\mathbf{b}^j, \mathbf{k}^j)$, such that if $\mathbf{b}^i \in \mathfrak{B}_r^j$, then revealing information about θ_r is incentive compatible when $\mathfrak{s}^i = \{\tilde{S}_r^i\}$, but is not incentive compatible when $\mathfrak{s}^i = \{\tilde{S}_1^i, \tilde{S}_2^i\}$. Moreover, the set $\mathfrak{B}_r^j$ depends on the organizational structure.*

Acquiring information about one state eliminates the possibility of ambiguous information—i.e. when i's willingness to reveal information about one state is negatively affected by what he knows about the other state. *Specialization* then works as a commitment device for i: he will not know when revealing information about one state has an 'excessive' influence against his preferences.[23] Proposition 3 also implies that, for a given set of biases, the principal prefers that i specializes even when he has free access to information.

Corollary 2. *Let $C(S_1^i) = C(S_2^i) = 0$. If agent i's preferences satisfy:*

$$|\beta_1^i| \in \left(\frac{w_1^2 + (1-w_1)^2}{2}\left[\frac{1}{(k_1^{\mathrm{C}}+3)} - \frac{\rho_1}{(k_2^{\mathrm{C}}+3)}\right] \; ; \; \frac{w_1^2 + (1-w_1)^2}{2\,(k_1^{\mathrm{C}}+3)}\right]$$

Then, in the most informative equilibrium under centralization i acquires and truthfully reveals information about θ_1 only.

The principal prefers a less informed agent because it guarantees he will not be tempted to lie when observing favourable information that cannot be credibly conveyed in equilibrium. Note that the

[23] I.e. he does not observe information about the other state which would move decisions toward his biases.

two results above hinge on the assumption that information acquisition decisions are observable. In Section 6, I discuss the implications of relaxing it and show that specialization still increases credibility when the cost of information is not too low. These results have implications for how firms organize subunits' access to information, since increasing the cost of some types of information may improve the quality of communication.

Having discussed the informational benefits of specialization, I now analyze the different conditions that induce an agent to specialize using an example with two agents.

The different drivers of specialization. To simplify the exposition let assume $n = 2$ and marginal costs are linear, $C(\mathfrak{s}^i) = c \times (\# \mathfrak{s}^i)$. I focus on the centralization equilibrium in which agent $i = \{1, 2\}$ acquire information about state θ_i; that is, $\mathfrak{s}^1 = \{\tilde{S}_1^1\}$ and $\mathfrak{s}^2 = \{\tilde{S}_2^2\}$. Note that in such equilibrium the principal is (ex-post) more informed than each of the agents since $k_1^{\mathrm{C}} = 1$ and $k_2^{\mathrm{C}} = 1$. A similar situation, in the form of a generalist-specialist information structure, has been analyzed by Alonso et al. (2015) without endogenous acquisition of information. The proposition below formalizes my result and panel (a) in Figure 3 illustrates the set of biases for which $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ under centralization.

Proposition 4 (Specialization under centralization)**.** *Suppose that there are only two agents and the marginal cost of each signal is linear and equal to c. For each agent $i = \{1, 2\}$ and the associated state $\theta_i = \{\theta_1, \theta_2\}$, there is a cost threshold $\bar{c}_i = \frac{w_i^2 + (1-w_i)^2}{36}$, such that for $c \leq \bar{c}_i$ there exists an equilibrium under centralization in which agent i acquires and reveals information about state θ_i only.*[24] *Moreover, such equilibrium arises in the following cases:*

1. ***Driven by preferences:*** *if $c \leq \bar{c}_i$ and revealing information about θ_i is incentive compatible for agent i in equilibrium, but revealing information about θ_{-i} is not.*

2. ***Driven by influence:*** *if $c \leq \bar{c}_i$, and revealing information about any state individually is incentive compatible for agent i but revealing information about both is not, and*

$$\frac{w_{-i}^2 + (1 - w_{-i})^2}{72} \leq \frac{w_i^2 + (1 - w_i)^2}{36} \tag{8}$$

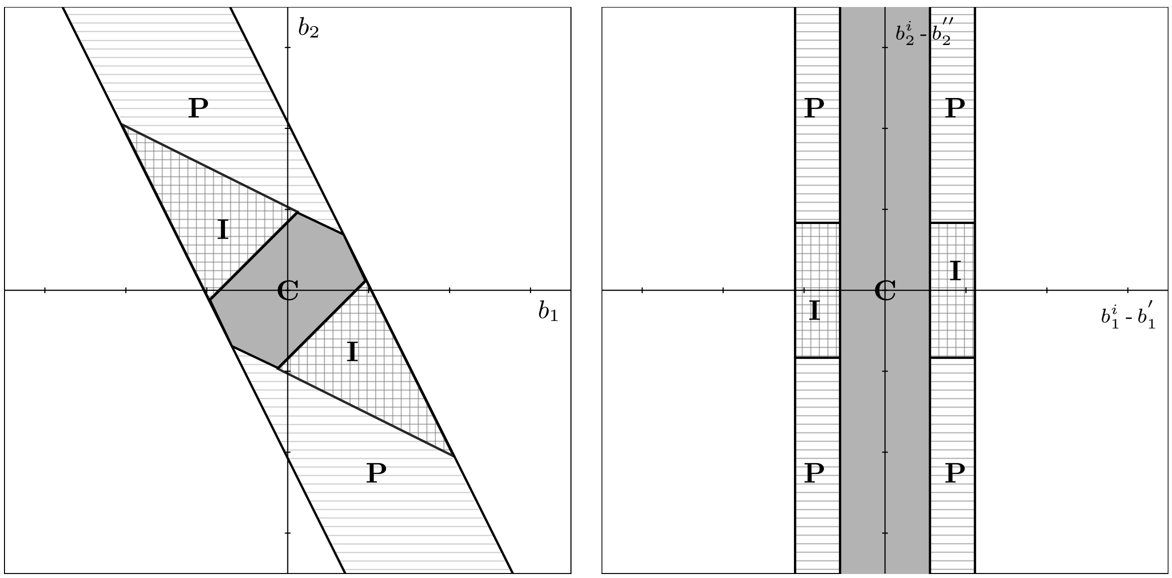
3. ***Driven by costs:*** *for $\underline{c}_i = \frac{w_i^2 + (1-w_i)^2}{72}$, if $c \in (\underline{c}_i, \bar{c}_i]$, revealing information about both states is incentive compatible for agent i, and (8) holds.*

If an agent's preferences are such that he finds incentive compatible to reveal information about one state only, specialization arises naturally if doing so is cost-effective. This case is shown by the the striped area in panel (a) of Figure 3. If, however, i is also willing to reveal information about

[24]Formally, the equilibrium consists in $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $m^{1*} = \big\{\{(0,0),(0,1)\},\{(1,0),(1,1)\}\big\}$ for agent 1, and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$ and $m^{2*} = \big\{\{(0,0),(1,0)\},\{(0,1),(1,1)\}\big\}$ for agent 2.

the other state, specialization has somewhat stricter requirements. For the case in which he is willing to reveal information about any, but only one of the states, he chooses according to the expected influence of each strategy. Condition (8) shows the case in which agent i finds more profitable to acquire information about state θ_i, given the other agent will acquire and reveal information about the other state in equilibrium. The cross-hatched region in panel (a) of Figure 3 illustrates this case. Specialization is *driven by the expected larger influence* on the principal's beliefs, given the equilibrium strategy of the other agent.[25]

Figure 3: Specialization with 2 agents — Driven by preferences (**P**), influence (**I**), and costs (**C**).



(a) $i = 1$ specializes in θ_1 – Centralization (b) $i = 1$ specializes in θ_1 – Delegation

Notes: $w_1 = w_2 = \frac{2}{3}$ and $k_1^* = k_2^* = 1$

Finally, when the agent is willing to reveal information about both states in equilibrium, specialization can only arise if acquiring both signals is too costly. An in the previous case, the information the agent invests in depends on the expected influence of each alternative. Whether i decides to observe a signal about θ_1 or θ_2 depends on what the other agent is expected to do. The solid grey region in panel (a) illustrates the case of specialization *driven by costs*.

Specialization under delegation follows the same intuitions. I describe them using Panel (b) in Figure 3. Recall that information about θ_1 affects y_1 more than y_2. Therefore, agent 1 specializes on θ_1 when his preferences on the first dimension are sufficiently close to the decision-maker of y_1. Indeed, whenever his preferences are close to the decision-maker of y_2, agent 1 prefers to acquire information

[25] An alternative equilibrium exists when both agents' bias vectors lie on cross-hatched regions. The strategies $\mathfrak{s}^{1*} = \{\tilde{S}_2^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_1^2\}$ can also be sustained; agents thus face a coordination problem for which there is no clear selection criterion—the principal is ex-ante (and ex-post) indifferent between any of these. Both equilibria involve specialization mainly because no agent is willing to reveal both signals.

about θ_2. The intuitions for the different drivers of specialization (preferences, influence, and costs) are the same as for centralization, and are illustrated in the equivalent regions of panel (b).

4.3 Organizational Design on Incentives to Acquire Information

So far, the optimal allocation of authority and the amount of information it aggregates depended on the profile of agents' biases, $\mathbf{B}$. This section analyzes how much information each organizational structure is expected to aggregate abstracting from any specific profile of preferences. To do so, I assume that the principal does not know the 'identity' of the agents at the authority allocation stage. The amount of information she expects a given organizational structure to aggregate thus depend on the intensity of incentives it provides—i.e. on the probability of finding an agent whose preferences are sufficiently aligned with the designed decision-maker. As in Rantakari (2012) and Deimen and Szalay (2019), I find delegation leads to inefficient information acquisition. The inefficiency here arises not because of differences in preferences between sender and receiver, but because of the expected net return of information about each state, given its influence on decisions the latter controls.

Suppose that the headquarters (principal) opens a new subsidiary in a geographic area known for its R&D potential. There are n potential local partners with access to information, but the headquarters is uncertain about their preferences over the firm's products; that is, each partner has $\mathbf{b}^i$ uniformly distributed in $[-\mathfrak{b}, \mathfrak{b}]^2$ for a given $\mathfrak{b} \in \mathbb{R}_+$. I also take that the principal does not observe the realization of these biases when deciding on authority. Despite the uncertainty concerning the subsidiary's potential for knowledge creation, the headquarters still have to define several elements related to its organization. Among them, the headquarters needs to appoint a local manager whose "identity" (decision-specific preferences) will affect his ability to form ties with local business partners with access to valuable information. I assume the headquarters faces a sufficiently large pool of candidates for the position, such that her choices will consider each candidates' ability to aggregate information (in expectation) and the loss of control associated to his preferences over decisions. Formally, for each position the principal can choose a j such that $\mathbf{b}^j \in [-\mathfrak{b}, \mathfrak{b}]^2$. In addition to choosing decision-makers, the principal decides on their authority over the two decisions.

The assumptions just presented are meant to simplify the analysis and, thus, focus on qualitative results about general informational consequences of the different organizational structures. They also reflect an scenario in which the principal's uncertainty about the prospects of information aggregation are maximal within a given interval of decision-specific preferences. More general assumptions on the (joint) distribution of biases would lead to an analysis similar to that of the previous sections, in the sense that optimal organizational structures will depend on different moments of the distribution.

After the local manager is appointed, the biases for the n local business partners are revealed to the organization. Product-relevant information must be produced (or acquired) at a cost for the agents.

As shown in the previous section, costly information acquisition sets a limit on the informational gains the principal can obtain from delegation (Proposition 2). At this point, it is worth recalling that centralization describes the case in which authority over both decisions lies on one player, while delegation features two decision-makers who cannot communicate between them.

From an agent's perspective, the possibility of acquiring information affects his communication incentives. On the one hand, the expected marginal benefit of any signal under centralization is weakly larger than under delegation: the agent expects to influence both decisions in the former case, whilst this is not necessarily true in the latter. The proposition below shows that such differences lead to lower investment in information under delegation. On the other hand, each piece of information affects decisions differently. Recall that, by assumption, state θ_1 [θ_2] is more salient for decision y_1 [y_2]. For any given decision under delegation, the agent typically expects a larger marginal utility when acquiring information about the salient state. This has consequences on the expected investment in information across states.

Proposition 5. *Let $j_d = \{j_1, j_2\}$ denote the decision-maker of $y_d = \{y_1, y_2\}$ under delegation. Suppose that $\mathbf{b}^i \sim U[-\mathfrak{b}, \mathfrak{b}]^2$ and $C(S_1^i) = C(S_2^i) = C(S)$, for all $i = \{1, ..., n\}$. Then, there exists two cost threshold, $\bar{c} \geq \underline{c} > 0$ such that:*

1. *For $C(S) \leq \bar{c}$; then, $E\big(k_d^{\mathrm{C}}\big) > E\big(k_d^{j_d}\big)$ if and only if $w_1, w_2 \in [0.5, 1)$; and*
2. *For $C(S) \leq \underline{c}$; then, $\Big|E\big(k_d^{j_d}\big) - E\big(k_{\text{-}d}^{j_d}\big)\Big| > \big|E\big(k_d^{\mathrm{C}}\big) - E\big(k_{\text{-}d}^{\mathrm{C}}\big)\big|$ if $w_d = w_{\text{-}d} \in (0.5, 1]$.*

For $d = \{1, 2\}$; where $\bar{c} \equiv \frac{(w_1)^2 + (1-w_1)^2}{72}$ and $\underline{c} \equiv \frac{(w_1)^2}{72}$.

Corollary 3. *Suppose that $\mathbf{b}^i \sim U[-\mathfrak{b}, \mathfrak{b}]^2$ and $C(S_1^i) = C(S_2^i) \leq \bar{c}$, for all $i = \{1, ..., n\}$. Then, delegation aggregates the same amount of information than centralization in expectation if and only if there is no informational interdependence—i.e. $w_1 = w_2 = 1$.*

Delegation aggregates less information than centralization in expectation and, moreover, that information will concentrate on the attributes that are more salient for each decision. In other words, the argument that delegation enhances incentives for information acquisition ex-ante (Aghion and Tirole, 1997) does not hold under informational interdependence when agents have access to noisy signals about multiple states. In environments like these, decision-makers under delegation not only can expect to receive information from fewer agents, but also they must expect to lose perspective because the information received will be more focused on the issues salient for the set of decision they control. Both results relate to the influence each agent expects to have over decisions, and the associated expected 'returns' of information under each organizational structure.

How much information a decision-maker expects to aggregate depends on the probability of finding agents whose preferences make acquiring and revealing such information incentive compatible. Results in Proposition 5 stem from the fact that incentive compatibility of acquisition and communication is more restrictive under delegation than centralization. To see this, consider cost-effectiveness. Under centralization, revealing information about one state affects both decisions; while under delegation, the same is true only for agents who expect to be truthful to both decision-makers. The probability of finding such agents is a subset of the total probability of agents revealing to j_d only. As the amount of information about both states a decision-maker expects to receive increases, the mass of agents truthful to both decision-makers decreases relative to those expected to reveal information to one decision-maker only. As a result, the maximum amount of information each decision-maker can aggregate under delegation is strictly smaller than under centralization—i.e. $K_r^{j_d} < K_r^{\mathrm{C}}$ for all $j_d = \{j_1, j_2\}$ and $\theta_r = \{\theta_1, \theta_2\}$.

Secondly, the probability of finding an agent willing to reveal information is proportional to the ex-ante expected returns of truthful communication. This feature is not exclusive to the uniform distribution of biases, since truthful communication is a necessary condition for information acquisition (Lemma 2). However, with the uniform distribution, the probability of finding an agent willing to reveal a given piece of information *equals* the mass of biases that satisfy the associated IC constraint, relative to the total area of the square with sides of length $2\,\mathfrak{b}$. Given that the expected utility gains from investing in a signal are larger under centralization, the associated IC constraints hold for a larger set of biases than under delegation.

In summary, more agents are expected to acquire and reveal information about a given state under centralization because of its larger expected influence. Agents' investment is also expected to be more balanced across states: under delegation, agents expect a larger influence on a given decision from acquiring information about the salient state. Despite Proposition 5 states the last result for $w_1 = w_2$, numerical simulations in the appendix show it holds for a larger set of parameters. In particular, for sufficiently small information costs it holds for all $w_d \geq w_{\text{-}d}$.

Finally, Corollary 3 states that for delegation to aggregate the same amount of information than centralization in expectation, decisions must be independent. Note that in such a case it is true that $\left|E\left(k_d^{j_d}\right) - E\left(k_{\text{-}d}^{j_d}\right)\right| > \left|E\left(k_d^{\mathrm{C}}\right) - E\left(k_{\text{-}d}^{\mathrm{C}}\right)\right|$ but, because decisions are independent, it is somewhat efficient that decision-makers fully specialize on information relevant for the decision each takes.

5 Implications for knowledge-based multidivisional organizations

The environment in this paper can be interpreted as the problem the headquarters of a multinational corporation faces in relation subsidiaries with access to product-relevant information. The International Business literature has found that a subsidiary's ability to identify and assimilate new knowledge depends on forging close relationships with local business partners, which can create conflicts with organizational goals (Andersson et al., 2005, 2007; Mudambi, 2011). The literature knows this trade-off as the innovation-integration dilemma: "*[t]he more that headquarters exercises legal rights as principal to monitor and control the subsidiary, the lower the level of subsidiary innovation.*" (Mudambi, 2011). While the literature was able to identify firms that have resolved the dilemma with some evidence of knowledge transfers, the mechanisms have not been yet fully understood (Monteiro and Birkinshaw, 2017).

Lemma 1 and Proposition 1 offer a plausible theoretical mechanism for a solution of the innovation-integration dilemma. When a subsidiary has access to information that is valuable for the whole organization, granting authority over decisions that will allow closer relationships with local partners can create the local embeddedness necessary to produce such information. At the same time, retaining authority on other decisions for which that information is useful enables the integration with the rest of the organization. This mechanism is consistent with some findings the literature has not previously linked with the dilemma—i.e. the positive correlation between product and management mandates granted to subsidiaries, and knowledge spillovers from such subsidiaries to sister units (Andersson et al., 2007; Ecker et al., 2013; Kunisch et al., 2019).

In Section 4, I extended the analysis introducing two dimensions of the problem that, to the extent of my knowledge, have not been addressed elsewhere. Firstly, Proposition 3 states that the possibility of specialization enlarges the set of biases for which it is incentive compatible to reveal information about one state only. Local knowledge creation in multinational corporations would then benefit from highly-specialized local business partners. Put it differently, headquarters need to ponder on the nature of local partners' expertise when facing the innovation-integration dilemma for a given subsidiary. Proposition 4 suggests three alternatives in that direction: specialization emerges when local partners seek innovations specific to products or processes they care the most, or when there is plenty of information about innovations specific to a subset of decisions they care, or when access to some of those innovations is too costly. Note that the benefits of restricting agents' direct access to information arise if they have no indirect means of obtaining it—i.e. communication between local partners and managers better be private, and local partners better not talk to each other.

The second dimension relates to the long-run informational effects of delegation. Despite strategic decisions regarding local mandates and social controls typically consider the contemporaneous potential for knowledge creation, such institutions can unleash dynamics that will affect their own informational efficacy. The information a subsidiary expects to aggregate will come from agents whose gains from the relationship at least compensate its costs (in this case, acquiring information). In such environments, Proposition 5 shows that subsidiaries with restrictive mandates will fail to internalize informational synergies, typically leading to inefficient specialization and narrow assessments of innovations on the boundaries of their competencies. By these means, the dynamics of informational synergies should also be a key consideration in the allocation of authority among subunits.

6 Conclusion

Most types of organizations feature some form of informational interdependence: divisions in multi-divisional firms possess information that is relevant for decisions in other divisions; business units in multinational corporations develop innovations that are useful across products and markets; information necessary to design different provisions of a policy are typically dispersed across policy-related institutions like legislative committees and governmental agencies. If agents with access to costly information have interests on the many decisions affected by it, acquisition and communication will be strategic. Therefore, the influence each agent may have on the decisions—and, thus, the ability to grant such influence—will affect incentives to acquire and reveal information. This paper studied the allocation of authority under informational interdependence and endogenous information acquisition.

Because of interdependence, incentives to reveal a given piece of information to a decision-maker depend on how it affects the decisions she controls and how it shapes the conflict of interest with the sender. I showed that the principal finds optimal to delegate control over controversial decisions if that improves the transmission of information on other, less controversial ones. When senders have extreme preferences in all dimensions, however, interdependence can 'discipline' conflict of interests such that more information is transmitted under centralization.

In many real-world environments, agents may need to obtain the information previous to the communication stage. I found that specializing in specific topics, issues, or fields of knowledge, improves incentives for communication of that information because it reduces the instances in which deviations from truth-telling are profitable. I then extended the model to analyze each organizational structure's potential for information aggregation independent of the specific profile of agents' preferences. Under delegation, among agents who are expected to transmit information to a decision-maker, few will internalize its effects in decisions outside her control. Therefore, decision-makers are expected to aggregate less information under delegation, and it will be concentrated on salient issues.

I used my model to derive implications for the allocation of authority among subunits in knowledge-based organizations. The International Business literature has documented the role of knowledge spillovers in the allocation of regional mandates (Andersson et al., 2002, 2007; Kähäri et al., 2017; Kunisch et al., 2019) much in line with the results Proposition 1. At a normative level, Proposition 3 shows that restricting managers' access to information can improve communication with the rest of the organization, while Proposition 5 shows that narrow mandates can hamper a subsidiary's 'absorptive capacity'—i.e. its ability to recognize the value of new, external information, assimilate it, and apply it to commercial ends. These implications are subject to future empirical work.

My analysis is within the confines of a specific model. Naturally, there are features of richer environments that cannot be captured by the particulars of my model. Consider, as an example, the case of agents having access to different amounts of information. Based on recent results (Förster, 2021; Habermacher, 2022), it seems that most of my main conclusions will remain intact. In the margin, the principal will prefer delegation to a better informed agent for the sake of making a better use of his information. Another form of asymmetry relates to decisions having different importance for different players. Under its conventional interpretation (see Rantakari, 2008, for example), higher (lower) salience will amplify (dampen) the effects that decision-specific biases have on communication—i.e. deeming important a controversial decision will hinder incentives to inform those in charge of taking it. In addition, such differences could open new possibilities for informational gains under interdependence when, for instance, the principal grants control of a decision that she finds relatively unimportant but has high salience for the agent in question. I hope future research will make progress on dropping these as well as other restrictions I had to make for tractability.

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Appendix A Complementary results and proofs of main results

Generic communication IC constraints

Proposition A1 (Proposition 1 in Habermacher, 2022)**.** *Consider sender i's message strategy which consists of $m_t^{i*}(S_t^i) = \{S_t^i\}$ for $t \in \tau$ and $m_r^{i*}(S_r^i = 0) = m_r^{i*}(S_r^i = 1)$ for all $r \in T^i \setminus \{\tau\}$. Then, such $\mathbf{m}^{i*}$ is incentive compatible if and only if, for all possible deviations $\mathbf{m}^{i'}$ and given $\mathbf{m}^{\text{-}i*}$, it is true that:*

$$\sum_{d=1}^{D} \sum_{t \in \tau} (b_d^i \, w_{d,t}) \, \Delta_t \leq \sum_{d=1}^{D} \frac{1}{2} \Big(\sum_{t \in \tau} w_{d,t} \, \Delta_t \Big) \left[\Big(\sum_{t \in \tau} w_{d,t} \, \Delta_t \Big) - 2 \Big(\sum_{r \in T^i \setminus \{\tau\}} w_{d,r} \, \pi_r \Big) \right] \tag{9}$$

Where $\tau = \{0, 1, 2\}$ represents the set of states for which i truthfully reveals his information in equilibrium. In addition, $\Delta_t = E(\theta_t | \mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}) - E(\theta_t | \mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*})$ and $\pi_r = E(\theta_t | S_r^i, \mathbf{m}^{\text{-}i*}) - E(\theta_t | \mathbf{m}^{\text{-}i*})$.

Note that in the case of delegation the sum over decisions involves only the one in charge of decision-maker j_d, such that the left-hand side of (9) becomes $b_d^i \, (w_d \, \Delta_d + (1 - w_d) \, \Delta_{\text{-}d})$.

Proof. See Habermacher (2022). The online appendix includes a transcript of the proof. □

I now derive the expressions for Δ_r and π_r. Suppose that the decision-maker holds k_r^* signals about state θ_r, and let ℓ_r^* denote the number of such signals that equal 1; then the conditional pdf is:

$$f(\ell_r^* | \theta_r, k_r^*) = \frac{k_r^*!}{\ell_r^*!(k_r^* - \ell_r^*)!} \theta_r^{\ell_r^*} (1 - \theta_r)^{k_r^* - \ell_r^*}$$

And her posterior is:

$$h(\theta_r | \ell_r^*, k_r^*) = \frac{(k_r^* + 1)!}{\ell_r^*!(k_r^* - \ell_r^*)!} \theta_r^{\ell_r^*} (1 - \theta_r)^{k_r^* - \ell_r^*}$$

Consequently:

$$E(\theta_r | \ell_r^*, k_r^*) = \frac{(\ell_r^* + 1)}{(k_r^* + 2)}$$
$$\mathrm{Var}(\theta_r | \ell_r^*, k_r^*) = \frac{(\ell_r^* + 1)(k_r^* - \ell_r^* + 1)}{(k_r^* + 2)^2 (k_r^* + 3)}$$

For $\theta_r = \{\theta_1, \theta_2\}$. After some algebra I get $E\left(\theta_r | S_r^i = 0, \mathbf{m}^{\text{-}i}\right) = \frac{(k_r+2)}{2(k_r+3)}$ and $E\left[E\left(\ell_r, \theta_r | S_r^i = 1, \mathbf{m}^{\text{-}i}\right)\right] = \frac{(k_r+4)}{2(k_r+3)}$. As for Δ_r and π_r, note that if i does not reveal information about θ_r, then $\Delta_r = 0$ and $\pi_r \neq 0$; in particular,

$$\Delta_r(\tilde{S}_r^i = 0, \mathbf{m}^{\text{-}i*}) = \frac{1}{(k_r + 3)} \qquad \pi_r(\tilde{S}_r^i = 0, \mathbf{m}^{\text{-}i*}) = -\frac{1}{2(k_r + 3)}$$
$$\Delta_r(\tilde{S}_r^i = 1, \mathbf{m}^{\text{-}i*}) = -\frac{1}{(k_r + 3)} \qquad \pi_r(\tilde{S}_r^i = 1, \mathbf{m}^{\text{-}i*}) = \frac{1}{2(k_r + 3)}$$

Equilibrium communication under delegation

Lemma A1 (Incentive Compatibility of Communication.)**.** *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which the principal delegates decision $y_d = \{y_1, y_2\}$ to agent $j_d = \{j_1, j_2\}$. Let $\theta_r = \{\theta_1, \theta_2\}$ [θ_{-r}] denote the state that is more [less] salient for decision y_d, and w_r the associated weight such that $r = d = \{1, 2\}$.*

Then, truthful communication (of one or both signals) is incentive compatible if and only if:

$$|b_r^i - b_r^{j_d}| \leq \frac{1}{2}\left|\frac{w_r}{\left(k_r^{j_d}+3\right)} - \frac{(1-w_{\text{-}r})}{\left(k_{\text{-}r}^{j_d}+3\right)}\right| \tag{10}$$

Also, fully revealing information when $\tilde{\mathbf{S}}^i = \{(0,0);(1,1)\}$ and announcing a non-influential message for the other realizations is incentive compatible if and only if:

$$|b_r^i - b_r^{j_d}| \leq \frac{1}{4}\left[\frac{w_r}{\left(k_r^{j_d}+3\right)} + \frac{(1-w_{\text{-}r})}{\left(k_{\text{-}r}^{j_d}+3\right)}\right] \tag{11}$$

Proof. The IC constraints for revealing information about one states and full revelation follow directly from replacing the corresponding Δ_r and π_r on equation (9), noting that there is just one decision under j_d's control. For the strategy associated to (11), note that i fully reveals his information when signals coincide and plays the available non-influential message otherwise. Hence, deviations consists of type $(0,0)$ and $(1,1)$ announcing the other influential message, or either of them announcing the non-influential one (and vice-versa). The first set of deviations would move both decisions in the same direction, given the interdependence. The second will move decisions towards or from the prior, but they will also move in the same direction. Because the expected influence on the latter deviations is smaller than the former, the associated IC constraint is tighter and, hence, prevails. □

Proposition A2 (Equilibrium Communication under delegation)**.** *In the receiver-optimal equilibrium $(\mathbf{y}^*, \mathbf{m}_{j_1}^*, \mathbf{m}_{j_2}^*)$, i's message strategy to decision-maker $j = \{j_1, j_2\}$ is:*

- *Full revelation, $m_{j_d}^i = \{\{(0,0)\},\{(1,0)\},\{(0,1)\},\{(1,1)\}\}$, if and only if (10) holds.*
- *The DNS strategy, $m_{j_d}^i = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$, if and only if (11) holds, and*

$$\left[\frac{w_r}{\left(k_r^{j_d}+3\right)} + \frac{(1-w_{-r})}{\left(k_{-r}^{j_d}+3\right)}\right] > \frac{1}{2}\left|\frac{w_r}{\left(k_r^{j_d}+3\right)} - \frac{(1-w_{-r})}{\left(k_{-r}^{j_d}+3\right)}\right|.$$

- *The babbling strategy, $m_j^i = \{(0,0),(1,0),(0,1),(1,1)\}$, otherwise.*

Equilibrium communication in the case of one decision is characterized by (10). Because of the focus on the receiver-optimal equilibrium, full revelation dominates message strategies in which i reveals one signal since their IC constraints are the same. Similarly, the dimensional non-separable message strategy characterized by (11) holds for most parameter values for which (10) also holds, so there is little loss of generality in overlooking the former.

Communication IC constraints under centralization

Lemma A2 (Incentive Compatibility of Communication under Centralization)**.** *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$, truthful communication is be incentive compatible for agent i in the following cases:*

- *Revealing information about state $\theta_r = \{\theta_1, \theta_2\}$ only, if:*

$$|\beta_r^i| \leq \frac{(w_r)^2 + (1-w_r)^2}{2}\left[\frac{1}{(k_r^{\mathrm{C}}+3)} - \frac{\rho_r}{(k_{-r}^{\mathrm{C}}+3)}\right] \tag{12}$$

- *Revealing information about both states for all possible realization of his signals (Full Revelation):*

◇ *For realizations* $\tilde{\mathbf{S}}^i = \{(0,0);(1,1)\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} + \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| \leq \frac{1}{2}\left[\frac{(w_1)^2+(1-w_1)^2}{(k_1^{\mathrm{C}}+3)^2} + \frac{(w_2)^2+(1-w_2)^2}{(k_2^{\mathrm{C}}+3)^2} + \frac{2[w_1(1-w_2)+w_2(1-w_1)]}{(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)}\right] \tag{13}$$

◇ *For realizations* $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} - \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| \leq \frac{1}{2}\left[\frac{(w_1)^2+(1-w_1)^2}{(k_1^{\mathrm{C}}+3)^2} + \frac{(w_2)^2+(1-w_2)^2}{(k_2^{\mathrm{C}}+3)^2} - \frac{2[w_1(1-w_2)+w_2(1-w_1)]}{(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)}\right] \tag{14}$$

- *Revealing information about both states for some realization of his signals and announcing an uninformative message otherwise (dimensional non-separable communication strategy), if*

◇ *For realizations* $\tilde{\mathbf{S}}^i = \{(0,0);(1,1)\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} + \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| \leq \frac{1}{4}\left[\frac{(w_1)^2+(1-w_1)^2}{(k_1^{\mathrm{C}}+3)^2} + \frac{(w_2)^2+(1-w_2)^2}{(k_2^{\mathrm{C}}+3)^2} + \frac{2[w_1(1-w_2)+w_2(1-w_1)]}{(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)}\right] \tag{15}$$

◇ *For realizations* $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} - \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| \leq \frac{1}{4}\left[\frac{(w_1)^2+(1-w_1)^2}{(k_1^{\mathrm{C}}+3)^2} + \frac{(w_2)^2+(1-w_2)^2}{(k_2^{\mathrm{C}}+3)^2} - \frac{2[w_1(1-w_2)+w_2(1-w_1)]}{(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)}\right] \tag{16}$$

Where $\beta_r = b_r^i w_r + b_{-r}^i(1-w_r)$, *and* $\rho_r = \frac{w_1(1-w_2)+(1-w_1)w_2}{w_r^2+(1-w_r)^2} \in [0,1]$.

Proof. See Habermacher (2022). □

Optimal Organizational Structure

Proposition A3 (Optimal Organizational Structure)**.** *Let define the informational gains as follows:*

$$DIG_{j_d}(y_d) \equiv (w_d)^2\big[Var(\theta_d|\mathbf{m}_{\mathrm{C}}^*) - Var(\theta_d|\mathbf{m}_{j_d}^*)\big] + (1-w_{\text{-}d})^2\big[Var(\theta_{\text{-}d}|\mathbf{m}_{\mathrm{C}}^*) - Var(\theta_{\text{-}d}|\mathbf{m}_{j_d}^*)\big]$$
$$IIG(y_d) \equiv (w_d)^2\big[Var(\theta_d|\mathbf{m}_{\mathrm{C}}^*) - Var(\theta_d|\mathbf{m}_{\mathrm{P}_d}^*)\big] + (1-w_{\text{-}d})^2\big[Var(\theta_{\text{-}d}|\mathbf{m}_{\mathrm{C}}^*) - Var(\theta_{\text{-}d}|\mathbf{m}_{\mathrm{P}_d}^*)\big]$$

Given the vector of preferences, $\mathbf{B} = \{\mathbf{b}^1, \dots, \mathbf{b}^n\}$, *agent* i, *and decision-makers* j_1 *and* j_2*; the organizational structure that maximizes the principal's ex-ante expected utility is:*

- ***Full delegation****; that is, agents* j_1 *and* j_2 *decide on* y_1 *and* y_2*, respectively, if and only if:*

$$DIG_{j_d}(y_d) - (b_d^{j_d})^2 > \max\Big\{DIG_i(y_d) - (b_d^i)^2\,,\; IIG(y_d)\,,\; -\{DIG_{j_{\text{-}d}}(y_{\text{-}d}) - (b_2^{j_{\text{-}d}})^2\}\Big\}$$

Such that $j_d = \{j_1, j_2\}$ *and* $i \neq j_d$.

- ***Partial delegation****; hat is, agent* j_d *decides on* y_d *and the principal retains decision authority over* $y_{\text{-}d}$*; if and only if there exist both Direct and Indirect informational gains such that:*

1. $DIG_{j_d}(y_d) - (b_d^{j_d})^2 > \max\Big\{DIG_i(y_d) - (b_d^i)^2\,,\; IIG(y_d)\,,\; -IIG(y_{\text{-}d})\Big\}$ *for any* $i \neq j_d$*; and*
2. $IIG(y_{\text{-}d}) > \max\Big\{DIG_i(y_{\text{-}d}) - (b_{\text{-}d}^i)^2\,,\; -\{DIG_{j_d}(y_d) - (b_d^{j_d})^2\}\Big\}$ *for any* $i \neq j_d$.

- ***Centralization****; that is, the principal decides on both issues, if and only if there are no agent i and j such that:*

  1. $DIG_j(y_d) - (b_d^j)^2 + IIG(y_{\text{-}d}) > 0$*; nor*
  2. $DIG_{j_1}(y_1) - (b_1^{j_1})^2 + DIG_{j_2}(y_2) - (b_2^{j_2})^2 > 0$

Proof. Given expression (1), choosing $j_1, j_2 = \{P, 1, ..., n\}$ as decision-makers of y_1 and y_2 is optimal if and only if the expected utility of doing to is larger than retaining authority over both decisions:

$$\begin{aligned} &-\Big[(b_1^{j_1})^2 + (w_1)^2 \operatorname{Var}(\theta_1|\mathbf{m}_{j_1}) + (1-w_2)^2 \operatorname{Var}(\theta_2|\mathbf{m}_{j_1})\Big] - \Big[(b_2^{j_2})^2 + (1-w_1)^2 \operatorname{Var}(\theta_1|\mathbf{m}_{j_2}) + (w_2)^2 \operatorname{Var}(\theta_2|\mathbf{m}_{j_2})\Big] \geq \\ &\geq -\left[(w_1)^2 + (1-w_1)^2\right] \operatorname{Var}(\theta_1|\mathbf{m}_{\text{C}}) - \left[(w_2)^2 + (1-w_2)^2\right] \operatorname{Var}(\theta_2|\mathbf{m}_{\text{C}}) \end{aligned} \tag{17}$$

The rest of the proof consists of algebra that reflects the following arguments. Full delegation is then optimal if there are two agents j_1 and j_2 such that the associated reduction in residual variances more than compensate $b_1^{j_1}$ and b^{j_2}, these gains are maximal among all alternative allocations of authority to other agents, and strictly larger than if the principal retained any single decision (IIG).

Partial delegation is optimal in either of two (non-exclusive) cases. First, when DIGs exists only for one decision the principal prefers to retain authority on the other. Indeed, for sufficiently large DIGs she may be willing to tolerate some informational losses on the retained decision (as compared to centralization). Secondly, partial delegation is optimal when IIGs are large. Such informational gains must again compensate for the bias on the delegated decision, which may even result in an informational loss with respect to centralization. In the latter case, the reduction of the residual variance on the retained decision is large, which requires the presence of negative informational spillovers (Proposition 1).

Finally, centralization is optimal when any potential informational gain is small, such that it does not compensate the loss of control on the delegated decision(s). □

Proof of Lemma 1

Proof. The organizational structure that maximizes the principal's ex-ante expected utility depends on the information each player would aggregate if a decision (or both) were under his control. Informational gains thus arise then when more agents reveal information to a decision-maker as compared to those revealing information to the principal under centralization. Consider direct informational gains first. From expression (17), a necessary condition for optimal delegation of y_1 is:

$$\begin{aligned} &(w_1)^2\big[\operatorname{Var}(\theta_1|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_1|\mathbf{m}^*_{j_1})\big] + (1-w_2)^2\big[\operatorname{Var}(\theta_2|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_2|\mathbf{m}^*_{j_1})\big] - (b_1^{j_1})^2 \geq \\ &\quad \geq -(w_2)^2\big[\operatorname{Var}(\theta_2|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_2|\mathbf{m}^*_{j_2})\big] - (1-w_1)^2\big[\operatorname{Var}(\theta_1|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_1|\mathbf{m}^*_{j_2})\big] + (b_2^{j_2})^2 \end{aligned}$$

In other words, the informational gains in the first dimension must compensate the decision-maker's bias and the potential informational losses in the second dimension. Note that such informational losses can arise even if the principal retains authority on y_2—i.e. when there are some positive informational spillovers under centralization. Assume there are no informational gains associated to y_2: either $\big[\operatorname{Var}(\theta_2|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_2|\mathbf{m}^*_{j_2})\big] < 0$, or $\big[\operatorname{Var}(\theta_1|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_1|\mathbf{m}^*_{j_2})\big] < 0$, or both. Then, a necessary condition for delegating y_1 to agent j_1 is:

$$(w_1)^2\big[\operatorname{Var}(\theta_1|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_1|\mathbf{m}^*_{j_1})\big] + (1-w_2)^2\big[\operatorname{Var}(\theta_2|\mathbf{m}^*_{\text{C}}) - \operatorname{Var}(\theta_2|\mathbf{m}^*_{j_1})\big] - (b_1^{j_1})^2 \geq 0.$$

Let now consider the case of indirect informational gains. Suppose there are no direct informational gains from delegating y_1, but agent j_1 minimizes the principal's losses associated to information and bias in that dimension. In such case, delegation of y_1 is optimal if and only if she benefits from retaining control over y_2, that is:

$$\begin{aligned}(w_2)^2\big[\mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{P}2})\big] + (1-w_1)^2\big[\mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{P}2})\big] \geq \\ -(w_1)^2\big[\mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}^*_{j_1})\big] - (1-w_2)^2\big[\mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}^*_{j_1})\big] + (b_1^{j_1})^2\end{aligned}$$

Where $\mathbf{m}^*_{\mathrm{P}2}$ represents equilibrium communication with the principal when she decides on y_2 only. Assuming there are no informational gains associated to y_1: either $\big[\mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}^*_{j_1})\big] < 0$ or $\big[\mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}^*_{j_1})\big] < 0$ or both. A necessary condition for delegating y_1 to agent j_1 is:

$$(w_2)^2\big[\mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}^*_{\mathrm{P}2})\big] + (1-w_1)^2\big[\mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}^*_{\mathrm{P}2})\big] \geq 0$$

□

Proof of Proposition 1

Proof. Let $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$ denote a given profile of biases for n informed agents. Associated to $\mathbf{B}^n$ there is a duple $(\mathbf{k}^{j_1}, \mathbf{k}^{j_2})$ such that $\mathbf{k}^{j_d} \in \mathbb{N}^2$ represent the equilibrium truthful messages decision-makers receive. Also, let $\mathbf{B}^{n+p} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+p})$ denote the profile in which the preferences of the first n agents are the same as in $\mathbf{B}^n$. Suppose without loss that agents $n+1$ to $n+p$ do not transmit additional information to the decision-makers or any other agent under $\mathbf{B}^n$. For a sufficiently large $\mathfrak{b} \in \mathbb{R}_+$, consider the preferences of agents $n+1$ to $n+p$ in the following cases:

1. $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, 0)$.

   (a) First, note that agents $n+1$ to $n+p$ have maximal incentives to reveal information to the principal, such that all of them will reveal both signals. Suppose the optimal organizational structure under $\mathbf{B}^n$ is full delegation; also, that $DIG(y_1) \geq (b_1^{j_1})^2$. Let $k_r^{\mathrm{P}2}$ denote the number of agents willing to reveal information about state θ_r to the principal when she decides only on y_2 under $\mathbf{B}^n$ (note that it not necessarily equal to k_r^{C}). Hence, under $\mathbf{B}^{n+p}$, the principal prefers to retain authority on y_2 rather than delegating it to j_2 (the decision-maker of y_2 under $\mathbf{B}^n$) if and only if the amount of information he receives, $k_r^{\mathrm{P}2}+p$, satisfies:

$$\begin{aligned}-\frac{(1-w_1)^2}{6}\frac{1}{(k_1^{\mathrm{P}2}+p+2)} - \frac{w_2^2}{6}\frac{1}{(k_2^{\mathrm{P}2}+p+2)} \geq -\frac{(1-w_1)^2}{6}\frac{1}{(k_1^{j_2}+2)} - \frac{w_2^2}{6}\frac{1}{(k_2^{j_2}+2)} - (b_2^{j_2})^2 \\ \iff \frac{(1-w_1)^2}{6}\left[\frac{1}{(k_1^{j_2}+2)} - \frac{1}{(k_1^{\mathrm{P}2}+p+2)}\right] + \frac{w_2^2}{6}\left[\frac{1}{(k_2^{j_2}+2)} - \frac{1}{(k_2^{\mathrm{P}2}+p+2)}\right] \geq -(b_2^{j_2})^2\end{aligned}$$

   Then, for any $k_1^{j_2}, k_2^{j_2}, k_1^{\mathrm{P}2}, k_2^{\mathrm{P}2} \leq n$, there exists a p for which the above holds.

   (b) Now suppose the optimal organizational structure under $\mathbf{B}^n$ is centralization. Also, suppose that $DIG(y_1) < (b_1^{j_1})^2$; there are no informational gains associated to delegation of y_1 to j_1. For the principal to delegate y_1 under $\mathbf{B}^{n+p}$, the loss of control this dimension must be compensated by indirect informational gains associated to y_2. Then:

$$\frac{(1-w_1)^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)}-\frac{1}{(k_1^{\mathrm{P}2}+p+2)}\right]+\frac{w_2^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)}-\frac{1}{(k_2^{\mathrm{P}2}+p+2)}\right]\geq$$
$$\geq\frac{w_1^2}{6}\left[\frac{1}{(k_1^{j_1}+2)}-\frac{1}{(k_1^{\mathrm{C}}+2)}\right]+\frac{(1-w_2)^2}{6}\left[\frac{1}{(k_2^{j_1}+2)}-\frac{1}{(k_2^{\mathrm{C}}+2)}\right]+(b_1^{j_1})^2$$

Note that $DIG(y_1)<(b_1^{j_1})^2$ means the RHS is negative. Delegation is optimal if there exists an agent j_1 whose preferences satisfy:

$$(b_1^{j_1})^2\leq\frac{[w_1^2+(1-w_1)^2]+[w_2^2+(1-w_2)^2]}{6(n+2)}-\frac{w_1^2+(1-w_2)^2}{18}$$

The expression above reflects the minimum informational gains in the worst-case scenario for delegation to be optimal; that is, n agents fully reveal information under centralization ($k_r^{\mathrm{C}}=n$), j_1 does not receive any signals from other agents under $\mathbf{B}^n$ ($k_r^{j_1}=1$), and the indirect informational gains are maximal ($p=\mathfrak{b}$). Then, there exists a finite p such that delegation of y_1 and retaining authority over y_2 is preferred by the principal over centralization.

2. $\mathbf{b}^{n+1}=...=\mathbf{b}^{\frac{2n+p+1}{2}}=(-\mathfrak{b},\mathfrak{b})$ and $\mathbf{b}^{\frac{2n+p+1}{2}+1}=...=\mathbf{b}^{n+p}=(\mathfrak{b},\mathfrak{b})$.

   Equations (15) and (16) imply that senders with the preferences above have maximal incentives to play (different) equilibrium DNS strategies. In particular, those with preferences in the first group satisfy:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)}+\frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right|=\left|\frac{w_1b_1^i+(1-w_1)b_2^i+(1-w_2)b_1^i+w_2b_2^i}{(k^{\mathrm{C}}+3)}\right|=\left|\frac{b_1^i+b_2^i}{(k^{\mathrm{C}}+3)}\right|=0$$

   And those in the second group:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)}-\frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right|=\left|\frac{w_1b_1^i+(1-w_1)b_2^i-(1-w_2)b_1^i-w_2b_2^i}{(k^{\mathrm{C}}+3)}\right|=\left|\frac{b_1^i-b_2^i}{(k^{\mathrm{C}}+3)}\right|=0$$

   Note that this holds independently of the number of agents revealing information to the principal under $\mathbf{B}^n$; hence, a sufficiently large p guarantees that, under $\mathbf{B}^{n+p}$, it is true that $k_1^{\mathrm{C}}=k_2^{\mathrm{C}}$. It follows that there exists a sufficiently large p such that the principal prefers centralization.

□

Proof of Lemma 2

Proof. The proof proceeds by contradiction. I focus on the centralization case, since delegation follows the same logic. Let $(\mathbf{y}^*,\mathbf{m}^*,\mathfrak{s}^*)$ be the equilibrium strategy profiles for the receiver and all agents, respectively. The equilibrium is characterized by $\mathbf{k}^{j_1}$ and $\mathbf{k}^{j_2}$.

Acquisition of S_1^i. Suppose that i's equilibrium acquisition strategy has $S_1^i\in\mathfrak{s}^{i*}$ but condition (12) does not hold for S_1^i. In such a case, revealing information about θ_1 is not incentive compatible for i despite he acquired information about it. Other agents base their message strategies on conjectures about $k_1^{j_1}$ and $k_1^{j_2}$, but i does not count as revealing any information. At the information acquisition stage, i's expected payoff of $\mathfrak{s}^{i*}$ is thus given by:

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)),\delta,b^i\right)\right]=-E\left[\left(\mathbf{y}_1\left(m^{i*}(\mathfrak{s}^{i*}),\mathbf{m}^{-i*}\right)-\delta_1-b_1^i\right)^2+\left(\mathbf{y}_2\left(m^{i*}(\mathfrak{s}^{i*}),\mathbf{m}^{-i*}\right)-\delta_2-b_2^i\right)^2\right]-C(\mathfrak{s}^{i*})$$

Now, consider the following deviation: $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*}\backslash\{S_1\}$. Note that this deviation does not affect $\mathbf{k}^{j_1}$ nor $\mathbf{k}^{j_2}$, and i's overall influence on decisions does not change—i.e. $y_d\left(m^i(\hat{\mathfrak{s}}^i),\mathbf{m}_j^{-i}\right) = y_d\left(m^i(\mathfrak{s}^{i*}),\mathbf{m}_j^{-i}\right)$. Note also that $C(\mathfrak{s}^{i*})>C(\hat{\mathfrak{s}}^i)$, given $\#\mathfrak{s}^{i*}>\#\hat{\mathfrak{s}}^i$. Consequently,

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)),\delta,b^i\right)\right]-E\left[U^i\left(\mathbf{y}\left(m^i(\hat{\mathfrak{s}}^i),\mathbf{m}^{-i*}\right),\delta,b^i\right)\right]=-C(\mathfrak{s}^{i*})+C(\hat{\mathfrak{s}}^i)<0$$

So, $\hat{\mathfrak{s}}^i$ is a profitable deviation from $\mathfrak{s}^{i*}$. $\Rightarrow\Leftarrow$

Acquisition of both signals. The proof is similar to the previous one, with i's equilibrium strategy $\mathfrak{s}^{i*}=\{S_1^i,S_2^i\}$. If conditions for full revelation, (12), (13), and (14) fail to hold, then not acquiring the information i is not willing to reveal on path is a profitable deviation from $\mathfrak{s}^{i*}$. □

Equilibrium Information Acquisition and Cost-effectiveness condition

Before the proof of Lemma 3, I derive the IC constraints associated to i's information acquisition.

Observation. *Let $k_r^{j_d}\equiv k_r^{j_d}\left(\mathbf{m}_{j_d}^i(\mathfrak{s}^{i*},\mathfrak{s}^{-i}),\mathbf{m}_{j_d}^{-i}(\mathfrak{s}^{i*},\mathfrak{s}^{-i})\right)$ and $\hat{k}_r^{j_d}\equiv k_r^{j_d}\left(\mathbf{m}_{j_d}^i(\hat{\mathfrak{s}}^i,\mathfrak{s}^{-i}),\mathbf{m}_{j_d}^{-i}(\mathfrak{s}^i,\mathfrak{s}^{-i})\right)$ for $\theta_r=\{\theta_1,\theta_2\}$. Recall that $w_r\geq\frac{1}{2}$ for $r=d=\{1,2\}$, that j_d decides over y_d, and $-d\neq d=\{1,2\}$. Let $\mathfrak{s}^i$ denote i's information acquisition strategy in an equilibrium characterized by $(\mathbf{y},\mathbf{m},\mathfrak{s})$. Then, i's ex-ante expected utility from $\mathfrak{s}^i$ is given by:*

$$E\left[U^i\left(\mathbf{m},\mathfrak{s}^i,\mathfrak{s}^{-i},\boldsymbol{\delta},\mathbf{b}^i\right)\right]=-\left[(b_1^i)^2+(b_2^i)^2\right]-\sum_{d=\{1,2\}}\left[\frac{(w_d)^2}{6\left(k_d^{j_d}+2\right)}+\frac{(1-w_{-d})^2}{6\left(k_{-d}^{j_d}+2\right)}\right]$$

Now, let $(\mathbf{y},\mathbf{m},\mathfrak{s})$ be equilibrium strategy profiles. Then, $\mathfrak{s}^i$ is incentive compatible for agent i if and only if, for every alternative $\hat{\mathfrak{s}}^i$:

$$\sum_{y_d=\{y_1,y_2\}}\sum_{\theta_r=\{\theta_1,\theta_2\}}\frac{(w_{dr})^2}{6}\left[\frac{1}{\left(\hat{k}_r^{j_d}+2\right)}-\frac{1}{\left(k_r^{j_d}+2\right)}\right]\geq\left[C(\mathfrak{s}^i)-C(\hat{\mathfrak{s}}^i)\right] \tag{18}$$

Where $w_{dr}=\{w_{11},w_{21},w_{12},w_{22}\}$; with $w_{11}=w_1$, $w_{21}=(1-w_1)$, $w_{22}=w_2$, and $w_{12}=(1-w_2)$.

Proof of Lemma 3 and Proposition 2

Proof. I first derive the cost-effectiveness condition (2) and then the maximum number of agents for which acquiring a given piece of information is cost-effective. In order to derive cost-effectiveness (CE, henceforth), I consider each possible acquisition strategy in equilibrium.

The number of agents revealing truthfully their signals in equilibrium, k_r, includes i's message strategy when he acquires it.[26] Two clarification are in order. The first relates to equilibrium coordination. Suppose that there are more than one agent who find incentive compatible to reveal information about θ_1 when k_1 other agents are expected to do so, but not when k_1+1 agents are. In such a case

[26] In equilibria in which i does not acquire S_1^i, k_1 does not count him; but in any deviation in which he does acquire it, then $\hat{k}_1=k_1^*+1$.

there may be acquisition of information that is not revealed in equilibrium, but I assume agents can adjust their message strategies such that the equilibrium number of agents revealing information is k_1. As a consequence, any of such deviations will result in $\hat{k}_1 = k_1 + 1$.

The second clarification relates to what happens when i acquires a signal and does not reveal it. Since other agents' message strategies will depend on conjectures about k_r, i not revealing the signal acquired off-path does not affect their equilibrium behaviour at the communication stage. In other words, Lemma 2 holds: i gains nothing from acquiring a signal he will not reveal.

Centralization. Let first consider the acquisition of both signals in equilibrium; that is $\mathfrak{s}^i = \{S_1^i, S_2^i\}$. By Lemma 2, such acquisition strategy can only lead to full revelation or a DNS message strategy at the communication stage. I then evaluate expression (18) for each possible deviation:

1) Deviation to not acquiring information: $\tilde{\mathfrak{s}}^i = \{\emptyset\}$.

$$\frac{(w_1)^2 + (1 - w_1)^2}{6(k_1 + 2)(k_1 + 3)} + \frac{(1 - w_2)^2 + (w_2)^2}{6(k_2 + 2)(k_2 + 3)} \geq C(S_1^i, S_2^i)$$

When agent i fully reveals his information at the communication stage. Now, if i plays DNS message strategies on-path, he expects to reveal information for half of the possible realizations, such that the CE conditions becomes:

$$\frac{(w_1)^2 + (1 - w_1)^2}{6(k_1 + 2)(k_1 + 3)} + \frac{(1 - w_2)^2 + (w_2)^2}{6(k_2 + 2)(k_2 + 3)} \geq 2\, C(S_1^i, S_2^i) \tag{19}$$

2) Deviation to acquiring information about θ_1 only: $\tilde{\mathfrak{s}}^i = \{S_1^i\}$.

$$\frac{(w_2)^2 + (1 - w_2)^2}{6(k_2 + 2)(k_2 + 3)} \geq C(S_2^i)$$

From which the deviation to acquiring information about θ_2 only can be inferred.

Now consider the acquisition strategy $\mathfrak{s}^{i*} = \{\tilde{S}_1^i\}$. The IC constraints become:

3) Deviation to not acquiring information: $\tilde{\mathfrak{s}}^i = \{\emptyset\}$.

$$\frac{(w_1)^2 + (1 - w_1)^2}{6(k_1 + 2)(k_1 + 3)} \geq C(S_1^i)$$

4) Deviation to acquiring information about θ_2 only: $\tilde{\mathfrak{s}}^i = \{S_2^i\}$. Recall that $C(S_1) = C(S_2)$.

$$\frac{(w_1)^2 + (1 - w_1)^2}{6(k_1 + 2)(k_1 + 3)} \geq \frac{(w_2)^2 + (1 - w_2)^2}{6(k_2 + 2)(k_2 + 3)}$$

5) Deviation to acquiring information about both states: $\tilde{\mathfrak{s}}^i = \{S_1^i, S_2^i\}$.

$$\frac{(w_2)^2 + (1 - w_2)^2}{6(k_2 + 2)(k_2 + 3)} < C(S_2^i)$$

Note that case 3) illustrates the necessary condition to acquire any individual signal $S_r^i = \{S_1, S_2\}$, since it implies case 1) (in which it holds for both signals) and case 4) (in which the agent acquires the signal that would have the highest influence). This case corresponds to equation (2).

Before proceeding to the delegation case, I derive the expression for the maximum number of agents willing to acquire information under centralization. The maximum number of agents who will acquire information about θ_r is the largest k_r for which the CE condition hold—i.e. equation (2). Re-arranging it, I get the following polynomial:

$$-(k_r)^2 - 5k_r - \left[6 - \frac{(w_r)^2 + (1-w_r)^2}{6\,C(S_r^i)}\right] \geq 0$$

Then, solving for the highest positive root I get K_r^{C} in (5).

Delegation. Because there are two decision-makers under delegation, the communication IC constraint that is a necessary condition for acquisition (Lemma 2) may refer to any of them (or both). Hence, CE requires that i is willing to reveal information to at least one decision-maker $j_d = \{j_1, j_2\}$.

$$\frac{(w_{dr})^2}{6(k_r^{j_d}+2)(k_r^{j_d}+3)} \geq C(S_r^i) \tag{3}$$

Where $\theta_r = \{\theta_1, \theta_2\}$ depending on incentive compatibility of communication. Note that $w_{11} = w_1$, $w_{21} = (1-w_1)$, $w_{22} = w_2$, and $w_{12} = (1-w_2)$. Now, it is IC to reveal information about one state both decision-makers the CE condition looks as follows:

$$\frac{(w_d)^2}{6(k_d^{j_d}+2)(k_d^{j_d}+3)} + \frac{(1-w_d)^2}{6(k_d^{j_{\text{-}d}}+2)(k_d^{j_{\text{-}d}}+3)} \geq C(S_d^i) \tag{4}$$

Now consider CE for acquisition of both signals. There are two cases in which i would acquire information about both states under delegation. First, when i is willing to reveal at least one signal to a different decision-maker, condition (3) must hold for each decision-maker. Second, when i is willing to reveal both signals to a single j_d, the IC constraint becomes

$$\frac{(w_d)^2}{6(k_d^{j_d}+2)(k_d^{j_d}+3)} + \frac{(1-w_d)^2}{6(k_d^{j_{\text{-}d}}+2)(k_d^{j_{\text{-}d}}+3)} \geq C(S_1^i, S_2^i)$$

The above holds only if equation (3) holds for each signal with the corresponding decision-maker. I now proceed to derive the maximum number of agents for which acquiring a given signal is CE under delegation. Agent i's maximal incentives to acquire information about θ_r takes place when he is willing to reveal that information to both decision-makers at the communication stage. In the extreme case in which all agents have preferences perfectly aligned with both decision-makers, the CE condition for each of them coincides with that of centralization; hence, $K_r^{\mathrm{D}} = K_r^{\mathrm{C}}$. However, this reflects the total amount of information about θ_r aggregated under delegation.

One can think of the case in which, say, $n/2$ agents are willing to reveal information about θ_r to decision-maker j_r while the rest of the agents are willing to reveal the same information to $j_{\text{-}r}$. Note that agents in the latter group will also find IC to reveal information about $\theta_{\text{-}r}$ to the same decision-maker (see equation (22)); hence, for small costs and large n then

$$K_r^{\mathrm{D}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_r)^2]}{6\,C(S_r^i)}\right]^{1/2} - \frac{5}{2} \right\rfloor + \left\lfloor \left[\frac{1}{4} + \frac{[(1-w_r)^2]}{6\,C(S_r^i)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 2$$

Numerical simulations show that there exists range of cost and interdependence parameters for which $K_r^{\mathrm{D}} > K_r^{\mathrm{C}}$ in this case. However, the possibility that delegation aggregates more information about a single state does not imply that more precise decisions. Note that the **maximum amount of information** a decision-maker can receive about a state is governed by condition (3) when $d = r$. Therefore, the maximum amount of information about θ_r that can be used for decision-making is:

$$K_r^{j_r} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_r)^2]}{6\,C(S_r^i)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{20}$$

And it is easy to check that $K_r^{j_r} < K_r^{\mathrm{C}}$ for all $w_r = [0.5, 1)$. $\square$

Proof of Corollary 1

Proof. Let $(\mathfrak{s}, \mathbf{m}, \mathbf{y})$ denote a generic equilibrium in which $\kappa \in (0, n)$ is the maximum number of agents willing to reveal S_r to both decision-makers. For any of such agents, CE under delegation is given by:

$$C(S_r^i) \leq \frac{(w_r)^2 + (1 - w_r)^2}{6(\kappa + 2)(\kappa + 3)}$$

But for any other agent, the CE condition is at most:

$$C(S_r^i) \leq \frac{(w_r)^2}{6(k_r^{j_d} + 2)(k_r^{j_d} + 3)}$$

For $r = d$, where $k^{j_d} = \kappa + \tilde{k}^{j_d}$ and $\tilde{k}^{j_d}$ represents the number of agents willing to reveal information about θ_r to j_d only. Note that if $C(S_r^i) > \frac{(w_r)^2}{6(\kappa+2)(\kappa+3)}$, agents willing to reveal S_r to at most one decision-maker will not find CE to acquire information about θ_r.

Equation (5) determines the maximum number of agents for which acquiring S_r is CE under centralization. Hence, $C(S_r^i) \leq \frac{(w_r)^2+(1-w_r)^2}{6(\kappa+3)(\kappa+4)}$ implies that $K_r^{\mathrm{C}} \geq \kappa + 1$. Then, $K_r^{\mathrm{D}} = \kappa < K_r^{\mathrm{C}}$. $\square$

Proof of Proposition 3

Proof. Let $*$ denote equilibrium strategies. Let $S_r \in \mathfrak{s}^{i*}$ and $S_{\text{-}r} \notin \mathfrak{s}^{i*}$ for $\theta_r \neq \theta_{\text{-}r}$, and $k_r^j(\mathbf{m}_j(\mathfrak{s}))$ be i's conjecture about other agents revealing their information about θ_r to decision-maker j. Then, agent i's IC constraint for revealing S_r^i is:

- When $j = P$ decides on both issues (centralization),

$$|\beta_r^i| \leq \frac{(w_r)^2 + (1 - w_r)^2}{2\,(k_r^{\mathrm{C}} + 3)} \tag{21}$$

- When $j = j_d$ decides on y_d only,

$$|b_d^i - b_d^{j_d}| \leq \frac{w_{dr}}{2\big(k_r^{j_d} + 3\big)} \tag{22}$$

  Note that $w_{11} = w_1$, $w_{21} = (1 - w_1)$, $w_{22} = w_2$, and $w_{12} = (1 - w_2)$.

I will derive the IC constraint for case of delegation, as centralization follows the same argument but requires more algebra (see Habermacher, 2022 for a reference). Let $w_{11} = w_1$, $w_{21} = (1-w_1)$, $w_{22} = w_2$, and $w_{12} = (1 - w_2)$. Also, let $\nu_{dr}^{i*} = E(\theta_r|\mathbf{m}_j^*)$ and $\hat{\nu}_{dr}^i = E(\theta_r|\hat{\mathbf{m}}_j)$ denote agent i's expectations of j's posterior beliefs about θ_r in equilibrium when he plays strategies m_j^{i*} and $\hat{m}_j$, respectively; and let

$\nu^i_{dr} = E(\theta_r|S^i_r, \mathbf{m}^{\text{-}i}_j)$ denote what the posterior should be if i revealed his information about θ_r. When i acquires S^i_1 only, $\mathbf{m}^{i*} = \{m^{i*}_{j_1}, m^{i*}_{j_2}\}$ is preferred to any alternative $\hat{\mathbf{m}}$ if and only if:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2})\right]\left[w_{d1}(\nu^{i*}_{d1} + \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} + \hat{\nu}^i_{d2}) - 2[w_{d1}\nu^i_{d1} + w_{d2}\nu^i_{d2} - (b^i_d - b^{j_d}_d)]\right] \geq 0$$

But since i has information about θ_1 only, posterior about θ_2 are all equal to the prior—i.e. $E(\theta_2|S^i_1, \mathbf{m}^{\text{-}i}_j) = \nu^i_{d2} = \nu^{i*}_{d2} = \hat{\nu}^i_{d2}$. Moreover, the strategy space when i has only one signal is degenerated, such that he can only reveal it or lie. Revealing S^i_1 is thus IC iff:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1})\right]\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) - 2(b^{j_d}_d - b^i_d)\right] \geq 0$$

It is straightforward to note that the above expression becomes:

$$\begin{aligned} \text{For } \tilde{S}^i_1 = 0: \quad & 2(b^i_d - b^{j_d}_d) \leq \frac{w_{d1}}{(k^{j_d}_1 + 3)} \\ \text{For } \tilde{S}^i_1 = 1: \quad & -2(b^i_d - b^{j_d}_d) \leq \frac{w_{d1}}{(k^{j_d}_1 + 3)} \end{aligned}$$

Which together imply equation (22).

Now, the vector $\mathfrak{B}^j_r(\mathbf{b}^j, \mathbf{k}^j)$ results from comparing equations (10) and (22). That is, assuming j_d decides over y_d only and $k^{j_d}_r = \{k^{j_d}_1, k^{j_d}_2\}$ are i's equilibrium conjectures about other agents revealing information to j_d, $\mathfrak{B}^{\mathrm{D}j}_r(\mathbf{b}^j, \mathbf{k}^j)$ can be defined as:

$$\mathfrak{B}^{j_d}_r = \left\{x : b^{j_d}_d \pm x \in \frac{1}{2} \times \left(\left|\frac{w_{d1}}{(k^{j_d}_1 + 3)} - \frac{w_{d2}}{(k^{j_d}_2 + 3)}\right|, \frac{w_{dr}}{(k^{j_d}_r + 3)}\right]\right\}$$

Under centralization, the vector $\mathfrak{B}^{\mathrm{C}}_r(\mathbf{b}^{\mathrm{C}}, \mathbf{k}^{\mathrm{C}})$ results from comparing equations (12) and (21). Denote by k^{C}_r agent i's equilibrium conjectures about other agents revealing information about $\theta_r = \{\theta_1, \theta_2\}$ to the principal under centralization, and $k^{\mathrm{C}}_{\text{-}r} \neq k^{\mathrm{C}}_r$. Then, $\mathfrak{B}^{\mathrm{C}}_r(\mathbf{k}^{\mathrm{C}})$ is defined as:

$$\mathfrak{B}^{\mathrm{C}}_r = \left\{x : |x| \in \frac{(w_{1r})^2 + (w_{2r})^2}{2} \times \left(\left[\frac{1}{(k^{\mathrm{C}}_r + 3)} - \frac{\rho_r}{(k^{\mathrm{C}}_{\text{-}r} + 3)}\right], \frac{1}{(k^{\mathrm{C}}_r + 3)}\right]\right\}$$

□

Agents' equilibrium strategies (endogenous information acquisition).

In this subsection I combine the results of Lemma 2 and Lemma 3 to characterize agent i's equilibrium information acquisition and message strategies. I start with the case of centralization ($j_1 = j_2 = P$) and then proceed to the case of delegated decisions. The following result summarizes the intuitions developed in the previous discussion, presenting the equilibrium acquisition and message strategies for a typical agent under centralization.

Proposition A4 (Equilibrium under Centralization)**.** *In the principal-optimal equilibrium under centralization,* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*, agent i only acquires signals that are cost-effective and communication is incentive compatible. In particular, i's equilibrium strategies are given by:*

Acquiring and revealing both signals: *if and only if conditions* (2) *and* (21) *hold for both signals, and* (14) *hold.*

Acquiring both signals and playing a dimensional non-separable strategy: *if condition* (19) *hold for both signals and* (21) *does not at all, in the following cases:*

- *Revealing both signals for realizations* $\tilde{\mathbf{S}} = \{\{(0,0)\};\{(1,1)\}\}$ *and sending the babbling message otherwise, if* (15) *holds;*

- *Revealing both signals for realizations* $\tilde{\mathbf{S}} = \{\{(0,1)\};\{(1,0)\}\}$ *and sending the babbling message otherwise, if* (16) *holds.*

Acquiring and revealing one signal only. *Agent* i *acquires and reveals* S_1^i *if* (21) *and* (2) *hold with respect to* θ_1 *and one of the following is true:*

- *Revealing* S_2^i *is not IC —i.e.* (21) *does not hold for* θ_2*; or*

- *Acquiring* S_2^i *is not CE —i.e.* (2) *does not hold for* θ_2*; or*

- *Acquiring* S_2^i *is CE and revealing it is IC, but revealing both signals is not IC —i.e.*(12) *and* (2) *hold for both signals, but* (14) *does not and* $\frac{(w_1)^2+(1-w_1)^2}{(k_1^*+2)(k_1^*+3)} \geq \frac{(w_2)^2+(1-w_2)^2}{(k_2^*+2)(k_2^*+3)}$

Acquiring no signal, *if only if any of the statements below is true:*

- *No signal is CE to acquire —i.e. condition* (2) *does not holds for any signal; and/or*

- *No signals is IC to reveal —i.e. condition* (21) *does not hold for any signal, nor* (14) *holds.*

Dimensional non-separable message strategies can arise under centralization. As in the pure communication game, these strategies take the form of full revelation for some realizations and babbling for the rest. Because any of these involves acquiring both signals and revealing them half of the time, they arise when costs are sufficiently low and only if revealing one signal is not IC.

Proposition A5 (Equilibrium under Delegation)**.** *When the organizational structure involves more than one decision-maker, agent* i *only acquires signals that are cost-effective and for which communication is incentive compatible. The receiver-optimal equilibrium,* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*, features* i *strategies:*

Acquiring and revealing both signals: *if and only if conditions* (10) *and* (3) *hold for both signals and at least one decision-maker and the associated decision.*

Acquiring and revealing S_1 ***only****, if acquiring this signal is both cost-effective and incentive compatible for* i *in the following cases:*

1. *Revealing* S_2 *is not IC for any decision —i.e. condition* (22) *does not hold for* S_2^i*; or*

2. *Acquiring* S_2 *is not CE for any decision —i.e. condition* (3) *does not hold for* S_2^i *for any decision; or*

3. *Both* S_1 *and* S_2 *are CE and IC, but revealing both is not IC with respect to any decision-maker —i.e. conditions* (22) *and* (3) *hold for both signals and at least one decision-maker, but* (10) *does not hold for any of them and* $\frac{(w_1)^2}{(k_1^{j1}+2)(k_1^{j1}+3)} \geq \frac{(w_2)^2}{(k_2^{j2}+2)(k_2^{j2}+3)}$ *or* $\frac{(1-w_1)^2}{(k_1^{j1}+2)(k_1^{j1}+3)} \geq \frac{(1-w_2)^2}{(k_2^{j2}+2)(k_2^{j2}+3)}$

Acquiring no signal *if only if any of the statements below are true:*

1. *Condition* (22) *does not hold for any signal and any decision, nor* (14) *hold; and/or*

2. *Condition* (3) *does not holds for any signal, any decision.*

Proof of Proposition 4

Proof. Suppose $n = 2$ and marginal costs are $C(\mathfrak{s}^i) = c \times (\#\,\mathfrak{s}^i)$. Acquiring information about θ_i is cost-effective for agent i if:

$$\begin{aligned} C(S_i^i) &\leq \frac{(w_i)^2 + (1-w_i)^2}{6(0+2)(0+3)} \\ c &\leq \frac{(w_i)^2 + (1-w_i)^2}{36} \end{aligned}$$

Then, according to Proposition A4 the receiver-optimal equilibrium has agent $i = \{1, 2\}$ acquiring and revealing information about θ_i (conditional on the other agent revealing information about $\theta_{-i} \neq \theta_i$ on-path) in three cases (see the proof of Proposition A2 for the supporting system of beliefs):

1. When (12) holds for i with respect to θ_i but not for θ_{-i};

2. When (12) holds for i with respect to both θ_1 and θ_2 but (14) does not hold. In this case, the CE condition above is not sufficient, since it must be true that information about θ_i has a larger expected influence for agent i, which according to the proof of Lemma 3 requires that:

$$\begin{aligned} \frac{w_{-i}^2 + (1-w_{-i})^2}{6(k_{-i}^{\mathrm{C}}+2)(k_{-i}^{\mathrm{C}}+3)} &\leq \frac{w_i^2 + (1-w_i)^2}{6(k_i^{\mathrm{C}}+2)(k_i^{\mathrm{C}}+3)} \\ \Leftrightarrow \frac{w_{-i}^2 + (1-w_{-i})^2}{72} &\leq \frac{w_i^2 + (1-w_i)^2}{36} \end{aligned}$$

3. When (12) holds for i with respect to both θ_1 and θ_2, and (14) also holds. In this case, the only way to specialization is that information about θ_i is more 'profitable' for i (in the sense of the equation above) and acquiring a second signal is too costly; that is:

$$\begin{aligned} C(S_{-i}^i) &> \frac{(w_{-i})^2 + (1-w_{-i})^2}{6(1+2)(1+3)} \\ c &> \frac{(w_{-i})^2 + (1-w_{-i})^2}{72} \end{aligned}$$

□

Proof of Proposition 5

Proof. Recall that an agent acquires information about a state if it is cost-effective (Lemma 3) and he is willing to reveal that information at the communication stage (Lemma 2). Before proceeding to the proof, I present a re-interpretation of the communication IC constraints (21) and (22) corresponding to centralization and delegation, respectively.

A re-interpretation of the communication IC constraints. I first characterize the "maximal incentives" to reveal information about θ_1 and θ_2 under each organizational structure. Let D1 denote the decision-maker of y_1 under delegation.

- Under delegation of y_1: $\lambda_1^{\mathrm{D1}} \equiv \{\mathbf{z} \in \mathbb{R}^2 : w_1\, z_1 = 0\}$ and $\lambda_2^{\mathrm{D1}} \equiv \{\mathbf{z} \in \mathbb{R}^2 : (1-w_2)\, z_1 = 0\}$
- Under centralization: $\lambda_1^{\mathrm{C}} \equiv \{\mathbf{z} \in \mathbb{R}^2 : w_1\, z_1 + (1-w_1)\, z_2 = 0\}$ and $\lambda_2^{\mathrm{C}} \equiv \{\mathbf{z} \in \mathbb{R}^2 : (1-w_2)\, z_1 + w_2\, z_2 = 0\}$

Note that both loci λ_1^{D1} and λ_2^{D1} coincide with the vertical axis, since revealing information under delegation of y_1 depends only on the decision-specific conflict of interest between i and D1. Under centralization, however, maximal incentives to reveal any single signal depend on how interdependence aggregates preferences; hence, λ_1^{C} and λ_2^{C} are rotated away from the vertical and horizontal axes.

Secondly, the conflict of interest between agent i and a given decision-maker is represented as the smaller distance between $\mathbf{b}^i$, and the λ associated to the decision-maker. After some linear algebra, this conflict of interest can be expressed as the distance between $\mathbf{b}^i$ and its projection onto the corresponding λ. Using $b_r^i = \{b_1^i, b_2^i\}$ to denote the bias associated to the dimension for which state $\theta_r = \{\theta_1, \theta_2\}$ is salient and $b_{\text{-}r}^i$ to denote the non-salient state, it is true that:

- $||\mathbf{b}^i - \text{Proj}_{\lambda_r^{\text{D1}}} \mathbf{b}^i|| = |b_1^i - b_1^j|$
- $||\mathbf{b}^i - \text{Proj}_{\lambda_r^{\text{C}}} \mathbf{b}^i|| = \frac{|(w_r\, b_r^i + (1-w_r)\, b_{-r}^i)|}{(w_r^2 + (1-w_r)^2)^{\frac{1}{2}}}$

Given two vectors $\mathbf{c}$ and $\mathbf{d}$, $\text{Proj}_{\mathbf{d}}\, \mathbf{c} = (\mathbf{c} \cdot \hat{\mathbf{d}})\, \hat{\mathbf{d}}$ denotes the projection of $\mathbf{c}$ onto $\mathbf{d}$, where $\hat{\mathbf{d}} = \frac{\mathbf{d}}{||\mathbf{d}||}$. Note that each of the above expressions is proportional to the left-hand side of the communication IC constraints for $\theta_r = \{\theta_1, \theta_2\}$ under centralization and delegation: (21) and (22), respectively.

I now derive the (ex-ante) expected number of agents revealing information about any given state under each organizational structure: $E(k_r^{\text{C}})$ and $E(k_r^{\text{D1}})$. Because of the uniform distribution of biases, this expectation is proportional to the area covered by the associated communication IC constraint.

Expected number of agents revealing information. I only consider incentives associated to y_1 since those of y_2 are similar. Suppose that $\mathbf{b}^i \sim U[-\mathfrak{b}, \mathfrak{b}]^2$, with $\mathfrak{b} \geq \frac{w_1}{6}$.[27] Also suppose that under delegation the principal can choose the decision-maker's bias—i.e. she chooses $\mathbf{b}^j$ from the set $[-\mathfrak{b}, -\mathfrak{b}]^2$. Because biases' distributions are independent and centred in $(0,0)$, the principal strictly prefers to appoint a decision-maker with $b_1^{DM1} = E(b_1) = 0$. To see this, note that the amount of information D1 expects to receive is proportional to the area associated to the IC constraint (22) (shown below). Hence, choosing a D1 with $b^{\text{D1}} \neq 0$ will aggregate weakly less information and lead to a biased decision from the principal's perspective.

The uniform distribution of biases implies that the probability of finding an agent with bias in any subset $\mathcal{A} \in [-\mathfrak{b}, \mathfrak{b}]^2$ is:

$$\Pr(b \in \mathcal{A}) = \int_{\mathcal{A}} f(b)\, db = \int_{\mathcal{A}} \frac{1}{(2\mathfrak{b})^2}\, db = \frac{\text{area } \mathcal{A}}{\text{total area}}$$

Let $\mathfrak{B}_1^{\text{D1}}(k_1)$ and $\mathfrak{B}_1^{\text{C}}(k_1)$ denote the set of biases for which it is incentive compatible to reveal information about θ_1 to the decision-maker of y_1 under delegation and centralization, respectively. The probability of finding an agent who is willing to reveal information about θ_1 relates to the communication IC constraints and, in each case, can be expressed as:

- Under delegation of y_1:

$$\Pr\left(b \in \mathfrak{B}_1^{\text{D1}}(k_1)\right) = \int_{\mathfrak{B}_1^{\text{D1}}(k_1)} f(b)\, db = \frac{1}{\mathfrak{b}} \cdot \frac{w_1}{2(k_1^{\text{D1}} + 3)} \tag{23}$$

- Under centralization:

$$\Pr\left(b \in \mathfrak{B}_1^{\text{C}}(k_1)\right) = \int_{\mathfrak{B}_1^{\text{C}}(k_1)} f(b)\, db = \frac{1}{\mathfrak{b}} \cdot \frac{(w_1^2 + (1-w_1)^2)}{2\, w_1\, (k_1^{\text{C}} + 3)} \tag{24}$$

[27] This guarantees maximal information transmission (in expectation) under delegation of y_1, simplifying calculations.

for $w_1 \in [0.542, 1]$. Otherwise,

$$\Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(k_1)\right) \geq \frac{1}{(4\mathfrak{b})} \cdot \frac{(w_1^2 + (1-w_1)^2)^{\frac{1}{2}}}{(k_1^{\mathrm{C}}+3)} \left[\frac{(w_1^2 + (1-w_1)^2)^{\frac{1}{2}}}{w_1} + \left| \frac{1}{2\mathfrak{b}(k_1^{\mathrm{C}}+3)} - \frac{2(1-w_1)}{(w_1^2 + (1-w_1)^2)^{\frac{1}{2}}} \right| \right] \tag{25}$$

Under delegation, the probability of finding an agent whose bias satisfies the IC constraint for fully revealing information about θ_1 is the area of a rectangle of base $\left[2 \cdot \frac{w_1}{2(k^{\mathrm{D}1}+3)}\right]$ and height $2\mathfrak{b}$, multiplied by the total area $\frac{1}{(2\mathfrak{b})^2}$.

Under centralization, communication IC constraints are information-specific: the relevant conflict of interest depends on how the information being revealed affects decisions. The loci λ_1^{C} represents the set of biases for which the conflict of interest associated to revealing information about θ_1 is zero and, hence, governs incentives for communication of that piece of information. In section 4. of the online appendix, I show such probability is equal to the area of a rectangle of base $\left[2\,\frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{2\,(k_1^{\mathrm{C}}+3)}\right]$ and height $\left[2\,\mathfrak{b}\,\frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{w_1}\right]$, times the total area $\frac{1}{(2\mathfrak{b})^2}$. Depending on w_1, the IC constraint may intersect two of the corners of the biases' support, making expression (24) overestimate the area covered by it. In such cases, a lower bound for the probability is given by a rectangle of base $\left[2 \cdot \frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{2(k_1^{\mathrm{C}}+3)}\right]$ and height $\left[||\lambda_1^{\mathrm{C}}|| + ||\mathrm{Proj}_{\lambda_1}\tilde{\mathbf{b}}||\right]$; where $\tilde{\mathbf{b}} = \left\{b : w_1 b_1 + (1-w_1) b_2 = \frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{2(k_1^{\mathrm{C}}+3)}\right\}$. When $w_1 = 0.5$, this rectangle gives the exact area of the IC constraint.

It follows that the expected number of agents who find incentive compatible to acquire and reveal information about θ_1 under delegation is:

$$E(k_1^{\mathrm{D}1}) = \sum_{\kappa=0}^{K_1^{\mathrm{D}1}} \kappa \Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(\kappa)\right)$$

And, similarly, for centralization:

$$E(k_1^{\mathrm{C}}) = \sum_{\kappa=0}^{K_1^{\mathrm{C}}} \kappa \Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right)$$

Where $K_1^{\mathrm{D}1}$ and K_1^{C} denote the maximum number of agents for whom revealing information about θ_1 is cost effective under delegation of y_1 and centralization (Proposition 2). Note that (7) entertained the possibility that $K_1^{\mathrm{D}1} = K_1^{\mathrm{C}}$ since one can find profile of preferences such that all agents would find IC to reveal information about θ_1 to both decision-makers. The probability of finding such an agent is the intersection of two IC constraints associated to communication of S_1^i: one corresponding to decision-maker of y_1 and the other to decision-maker of y_2. Recall that $\lambda_1^{\mathrm{D}1}$ coincides with the vertical axis, while $\lambda_1^{\mathrm{D}2}$ with the horizontal axis. Furthermore, conditional on finding an agent willing to reveal S_1^i to both decision-makers, the probability of finding on more of such agents decreases relative to that associated to agents revealing to D1 only.

Formally, $\Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(k_1^{\mathrm{D}1}) \cap \mathfrak{B}_1^{\mathrm{D}2}(k_1^{\mathrm{D}2})\right) = \frac{w_1(1-w_1)}{\mathfrak{b}^2(k_1^{\mathrm{D}1}+3)(k_1^{\mathrm{D}2}+3)}$. Denoting R the ratio between the probability of an agent revealing to both decision-makers and that for revealing at least to j_1,[28] it is easy

[28]That is, $R = \frac{\Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(k_1^{\mathrm{D}1}) \cap \mathfrak{B}_1^{\mathrm{D}2}(k_1^{\mathrm{D}2})\right)}{\Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(k_1^{\mathrm{D}1})\right)} = \frac{2(1-w_1)}{\mathfrak{b}(k_1^{\mathrm{D}2}+3)}$.

to check that $\frac{\partial R}{\partial k_1^{\mathrm{D}1}} = 0$ and $\frac{\partial R}{\partial k_1^{\mathrm{D}2}} < 0$. Hence, the effective upper bound imposed by cost effectiveness under delegation in this context is:

$$K_1^{\mathrm{D}1} = \left\lfloor \left[\frac{1}{4} + \frac{(w_1)^2}{6\, C(S_1)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{26}$$

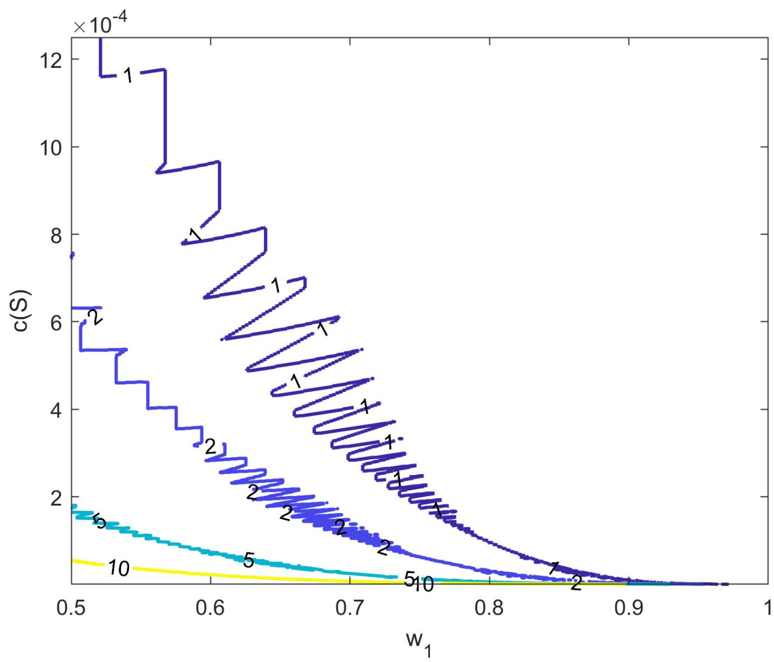
The difference between the expected amount of information is then given by:

$$\begin{aligned} E(k_1^{\mathrm{C}}) - E(k_1^{\mathrm{D}1}) &= \sum_{\kappa=0}^{K_1^{\mathrm{C}}} \kappa \Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right) - \sum_{\kappa=0}^{K_1^{\mathrm{D}1}} \kappa \Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(\kappa)\right) \\ &= \sum_{\kappa=0}^{K_1^{\mathrm{D}1}} \kappa \left[\Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right) - \Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(\kappa)\right) \right] + \sum_{\kappa=K_1^{\mathrm{D}1}+1}^{K_1^{\mathrm{C}}} \kappa \Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right) \end{aligned}$$

Any meaningful analysis of the differences in information aggregated by the organizational structure will consider costs for which some information is indeed aggregated. Hence, I assume that $C(S_1) \leq \frac{(w_1)^2 + (1-w_1)^2}{72}$ which guarantees that $K_1^{\mathrm{C}} \geq 1$.

Note that the expression above equals zero for $w_1 = 1$, since $K_1^{\mathrm{C}} = K_1^{\mathrm{D}1}$ and (23) equals (24). For all other values of w_1 there are two possible scenarios. First, for $w_1 \in [0.542, 1)$, equations (23) and (24) imply $\Pr\left(b \in \mathfrak{B}_1^{\mathrm{D}1}(\kappa)\right) < \Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right)$ for all $\kappa \geq 0$. Secondly, for $w_1 \in [0.5, 0.542)$ the difference between (25) and (23) is strictly positive; a conclusion the reader can check here. Note that in both cases the last term on the right-hand side is weakly positive, which proves the first claim of the proposition. The figure below illustrates the level curves of the differences.

Figure 4: Contour for $|E(k_1^{\mathrm{C}}) - E(k_1^{\mathrm{D}1})|$ for all w_1 and relevant $C(S_1)$.



Regarding the second claim of Proposition 5, note that $w_1 = w_2$ implies $\Pr\left(b \in \mathfrak{B}_1^{\mathrm{C}}(\kappa)\right) =$

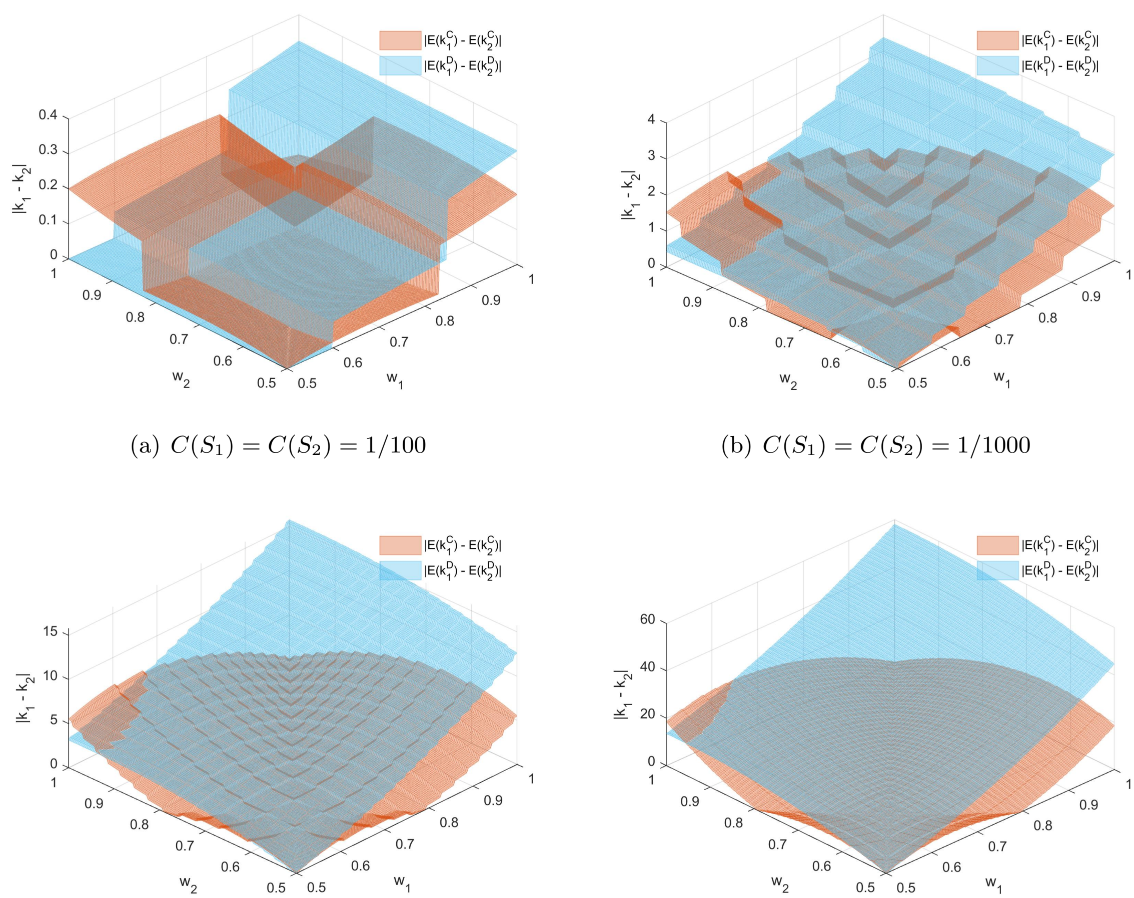
$\Pr\left(b \in \mathfrak{B}_2^{\mathrm{C}}(\kappa)\right)$ and $K_1^{\mathrm{C}} = K_2^{\mathrm{C}}$. As a consequence, $|E(k_1^{\mathrm{C}}) - E(k_2^{\mathrm{C}})| = 0$ for all $w_1 = w_2 \in [0.5, 1]$. Under delegation, however, the difference in the expected number of agents revealing information about each state is:

$$|E(k_1^{\mathrm{D1}}) - E(k_2^{\mathrm{D1}})| = \left| \frac{w_1}{2} \sum_{\kappa=0}^{K_1^{\mathrm{D1}}} \frac{\kappa}{(\kappa+3)} - \frac{(1-w_2)}{2} \sum_{\kappa=0}^{K_2^{\mathrm{D1}}} \frac{\kappa}{(\kappa+3)} \right| \tag{27}$$

Assuming that $C(S_1) = C(S_2) \geq \underline{c} \equiv \frac{(w_1)^2}{72}$, which guarantees $K_1^{\mathrm{D1}} \geq 1$. Then, a quick look at equations (26) and (7) suffices to see that (27) equals zero for $w_1 = w_2 = 0.5$ only. As a result, $|E(k_1^{\mathrm{C}}) - E(k_2^{\mathrm{C}})| < |E(k_1^{\mathrm{D1}}) - E(k_2^{\mathrm{D1}})|$ for all $w_1 = w_2 \in (0.5, 1]$.

Figure 5 depicts simulations of $|E(k_1^{\mathrm{C}}) - E(k_2^{\mathrm{C}})|$ and $|E(k_1^{\mathrm{D1}}) - E(k_2^{\mathrm{D1}})|$ for different costs of information. It shows that $|E(k_1^{\mathrm{C}}) - E(k_2^{\mathrm{C}})| < |E(k_1^{\mathrm{D1}}) - E(k_2^{\mathrm{D1}})|$ holds for larger set of w_1, w_2, and that set is decreasing in the costs of information. The MatLab code for the simulation can be found in the online appendix. $\square$

Figure 5: Differences $|E(k_1^{\mathrm{C}}) - E(k_2^{\mathrm{C}})|$ (in orange) and $|E(k_1^{\mathrm{D1}}) - E(k_2^{\mathrm{D1}})|$ (in blue).



(a) $C(S_1) = C(S_2) = 1/100$

(b) $C(S_1) = C(S_2) = 1/1000$

(c) $C(S_1) = C(S_2) = 1/10000$

(d) $C(S_1) = C(S_2) = 1/100000$

Online Appendix: Discussion and additional proofs

1. Communication IC Constraints

Proof of Proposition A1 (Proposition 1 in Habermacher, 2022).

A message strategy for i is incentive compatible, given the other players' equilibrium strategies $\mathbf{m}^{-i*}$, if for each possible deviation $\mathbf{m}^{i'}$ it is true that:

$$E\left[U_i(\gamma^i, \mathbf{b}^i, \boldsymbol{\theta})\middle|\mathbf{S}^i, \mathbf{m}^{i*}, \mathbf{m}^{-i*}\right] - E\left[U_i(\gamma^i, \mathbf{b}^i, \boldsymbol{\theta})\middle|\mathbf{S}^i, \mathbf{m}^{i'}, \mathbf{m}^{-i*}\right] \geq 0$$

Which is equivalent to:

$$-\int_{\Theta_1}\int_{\Theta_2}\sum_{d=1}^{2}\left[\left(y_d^*(\mathbf{m}^{i*}, \mathbf{m}^{-\mathbf{i}*}) - \delta_d - b_d^i\right)^2 - \left(y_d'(\mathbf{m}^{i'}, \mathbf{m}^{-\mathbf{i}*}) - \delta_d - b_d^i\right)^2\right] dF(\theta_1, \mathbf{m}^{-\mathbf{i}*}|S_1^i)\, dF(\theta_2, \mathbf{m}^{-\mathbf{i}*}|S_2^i) \geq 0$$

Using the identity $(a^2 - b^2) = (a + b)(a - b)$, the term in square brackets above can be re-arranged into $(y_d^* + y' - 2\delta_d - 2b_d^i)(y^* - y_d')$, given the receiver's optimal action and the definition of δ_d yields:

$$-\int_{\Theta_1}\int_{\Theta_2}\sum_{d=1}^{2}\left[\sum_{r=1}^{2}\left(w_{d,r}\left(E(\theta_r|\mathbf{m}^{i*}) + E(\theta_r|\mathbf{m}^{i'})\right) - 2\sum_{r=1} w_{d,r}\,\theta_r - 2b_d^i\right)\times\right.$$
$$\left.\times\left[\sum_{r=1}^{2} w_{d,r}\left(E(\theta_r|\mathbf{m}^{i*}) - E(\theta_r|\mathbf{m}^{i'})\right)\right]\right] dF(\theta_1, \mathbf{m}^{-i*}|S_1^i)\, dF(\theta_2, \mathbf{m}^{-i*}|S_2^i) \geq 0$$

Let $\nu_r = \nu_r^i(S_r^i, \mathbf{m}^{-i*}) = E(\theta_r|S_r^i, \mathbf{m}^{-i*})$, $\nu_r^* = \nu_r^{i*}(S_r^i, \mathbf{m}^{i*}, \mathbf{m}^{-i*}) = E(\theta_r|\mathbf{m}^{i*}, \mathbf{m}^{-i*})$, and $\nu_r' = \nu_r^{i'}(\mathbf{m}^{i'}, \mathbf{m}^{-i*}) = E(\theta_r|\mathbf{m}^{i'}, \mathbf{m}^{-i*})$ be sender i's expectations about the receiver's posterior beliefs about θ_r under his information, his equilibrium message strategy, and the deviation under consideration, respectively. In addition, let $\Delta_r = \Delta_r^i(\mathbf{m}^{i*}, \mathbf{m}^{i'}, \mathbf{m}^{-i*}) = E(\theta_r|\mathbf{m}^{i'}) - E(\theta_r|\mathbf{m}^{i*})$ denote the difference in the induced posterior beliefs i's expects to achieve under the deviation $\mathbf{m}^{i'}$.

Now, note that $dF(\theta_r, \mathbf{m}^{-i}|S_r^i) = f(\theta_r|S_r^i, \mathbf{m}^{-i})\, P(\mathbf{m}^{-i}|S_r^i)\, d\theta_r$. Also, given that the equilibrium message strategies for players others then i are independent of i's actual signal realizations, the expression $P(\mathbf{m}^{-i}|S_r^i)$ can be taken out the corresponding integral. Therefore, the above expression becomes:

$$-\int_{\Theta_1}\int_{\Theta_2}\sum_{d=1}^{2}\left[\sum_{r=1}^{2}\left(w_{d,r}\left(\nu_r^* + \nu_r'\right) - 2\sum_{r=1} w_{d,r}\,\theta_r - 2b_d^i\right)\left[\sum_{r=1}^{2} w_{d,r}\,(-1)\,\Delta_r\right]\right] f(\theta_1, |S_1^i)\, f(\theta_R|S_2^i)\, d\theta_1\, d\theta_2 \geq 0$$

Noting that $\int_{\Theta_r} \theta_r f(\theta_r|S_r^i, \mathbf{m}^i) d\theta_r = E(\theta_r|S_r^i, \mathbf{m}^{-i*}) = \nu_r$ for all $r = \{1, ..., R\}$, I get:

$$\sum_{d=1}^{2}\left[\sum_{r=1}^{2} w_{d,r}\left(\nu_r^* + \nu_r' - 2\nu_r\right) - 2\, b_d^i\right]\left(\sum_{r=1}^{2} w_{d,r}\,\Delta_r\right) \geq 0 \tag{28}$$

For those states for which i reveals his information, the expectation induced on the equilibrium path $E(\theta_t|m_r^{i*}, \mathbf{m}^{-i*}) = \nu_t^*$ equals i's own expectation about that state $E(\theta_t|S_t^i, \mathbf{m}^{-i*}) = \nu_t$; while the deviation induces $E(\theta_t|m_t^{i'} = 1 - S_t^i, \mathbf{m}^{-i*})$. For states in $T^i \setminus \{\tau\}$, then $\nu_r^* = \nu_r' = E(\theta_r|\mathbf{m}^{-i*})$; while, for states i does not have information $\nu_r^* = \nu_r' = \nu_r$. As a consequence, for states other than τ it is true that $\Delta_r = 0$, such that the IC constraint above:

$$\sum_{d=1}^{2}\left[\sum_{t\in\tau} w_{d,t}(\nu'_t-\nu^*_t)+\sum_{r\in T^i\setminus\{t\}} w_{d,r}\left(2E(\theta_r|\mathbf{m}^{\text{-}i*})-2E(\theta_t|S^i_t,\mathbf{m}^{\text{-}i*})\right)-2\,b^i_d\right]\left(\sum_{t\in\tau} w_{d,t}\,\Delta_t\right)\geq 0$$

$$\Leftrightarrow \sum_{d=1}^{2}\left[\sum_{t\in\tau} w_{d,t}\Delta_t-2\sum_{r\in T^i\setminus\{t\}} w_{d,r}\pi_r-2\,b^i_d\right]\left(\sum_{t\in\tau} w_{d,t}\,\Delta_t\right)\geq 0$$

Which leads to equation (9). □

2. Covert Information Acquisition

Suppose information acquisition decisions are private information of each agent. I focus on the centralization case, restricting the analysis to pure strategies at the information acquisition and communication stages. Following Argenziano et al. (2016), I show that focusing on equilibria in which messages do not convey information about the acquisition decision is without loss. As a result, messages sent at the communication stage do not convey information about decisions on information acquisition. I also show that any deviation at the information acquisition stage results in a deviation (from truth-telling) at the communication stage (Lemma OA.2).

In the covert game, there are two relevant deviations associated to information acquisition. First, when agent i deviates by *acquiring fewer signals* than on the equilibrium path, he saves on information costs but induces beliefs with larger variance. Consider an equilibrium in which he acquires information about both states. If, instead, he deviates to acquiring information about θ_1 only, his message associated to θ_2 will not depend on his information and, thus, will induce wrong beliefs for half of the possible signal realizations. At the off-path communication stage i will announce the most favourable of the possible realizations of S^i_2, so the deviation at the information acquisition stage corresponds to the case in which he lies about θ_2. As a consequence, incentive compatibility requires that the utility gains associated to lying towards his bias *and* saving on information costs are lower than the expected utility losses from inducing a larger-than-expected variance. Not surprisingly, incentive compatibility constraints in the covert game are more restrictive than in the overt game.

The second deviation consists of *acquiring more signals* than on the equilibrium path. Consider an equilibrium in which i acquires and reveals information about θ_1 only, and the deviation in which he also acquires information about θ_2. Because i cannot transmit information about θ_2 on-path, the expected utility gains from this deviation must be associated to lying on S^i_1 for some realizations of S^i_2. In particular, he lies when his information about θ_1 is unfavourable and that about θ_2 is favourable. This deviation will be profitable if the costs of acquiring S^i_2 are sufficiently low. The result below shows the set of parameter under which acquiring and revealing information is incentive compatible.

Proposition OA.1. *Let $(\mathfrak{s}^*,\mathbf{m}^*,\mathbf{y}^*)$ characterize an equilibrium in the covert game under centralization, and let $\mathbf{k}^{\text{C}}=\{k^{\text{C}}_1,k^{\text{C}}_2\}$ be agent i's equilibrium conjecture about other agents truthfully revealing information. Denote by $\mathfrak{B}^i_r=\mathfrak{B}_r\left(C(S^i_1),C(S^i_2),\mathbf{k}^{\text{C}}\right)\subseteq\mathbb{R}^2\cup\{\emptyset\}$ the set of biases for which acquiring and revealing information about θ_1 is incentive compatible for agent i. Then, $\mathfrak{B}^i_r\neq\emptyset$ if and only if*

$$\frac{w_1^2+(1-w_1)^2}{2(k^{\text{C}}_1+3)^2}>\max\left\{C(S^i_1)\,;\,\frac{w_1(1-w_2)+w_2(1-w_1)}{2(k^{\text{C}}_1+3)(k^{\text{C}}_2+3)}-2C(S^i_2)\right\}$$

Proof of Proposition OA.1.
I first present two important results and then derive the IC constraint. Since the decision-maker does not observe agents' acquisition decisions, a Perfect Bayesian Equilibrium must also specify the

decision-maker's beliefs about agents' investments in information. I focus on pure strategy equilibria at the acquisition stage. In principle, an agent may try to convey information about which signals he acquired by means of his cheap talk message to the decision-maker. I use result from Argenziano et al. (2016) to restrict attention to equilibria in which agents do not signal how much information each has acquired, which is without loss of generality.

Lemma OA.1 (Argenziano et al., 2016)**.** *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which an agent follows a pure strategy in the choice of information can be supported in a Perfect Bayesian Equilibrium in which the decision-maker's beliefs about his information acquisition decision do not vary with the agent's message.*

There would be two classes of deviations available to agents if the decision-maker's beliefs about information acquisition decisions could be affected by the choice of messages. First, an agent could acquire an off-path amount of information but still send the message corresponding to the equilibrium amount of information. Secondly, the agent could acquire an off-path amount information and send a message corresponding to an off-path information acquisition choice, which in turn may not be true. The lemma says that any equilibrium outcome under the second class of deviations can be supported as an equilibrium in which the agent cannot change the decision-maker's beliefs about his information acquisition decision.

Now, when an agent has acquired an off-path amount of information, he can choose among the messages from the equilibrium strategy at the communication stage. The result below shows any deviation at the information acquisition stage implies a deviation at the communication stage.

Lemma OA.2. *When agent i acquires fewer signals than what is expected on the equilibrium path, the messages used under the deviation are a strict subset of the equilibrium messages available. When i acquires more signals than expected on-path, he uses the additional information to deviate from truth-telling for some signal realizations.*

When i acquires fewer signals off-path, he will not be able to condition his message on the information that has not been observed. As a consequence, the set of messages effectively used under the deviation are a strict subset of the equilibrium set of messages, which implies that the set of beliefs induced under the deviation is a strict subset of the set of beliefs induced in equilibrium. On the other hand, when i acquires more signals off-path, he cannot transmit the additional information with the equilibrium message strategy (there is no way of signalling he acquired more information). Now, given the additional costs incurred off-path, a profitable deviation implies i must be obtaining some utility gains with respect to the equilibrium communication; in particular, he induces beliefs according that better suit his preferences under some signals realizations.

I now derive the IC constraints for information acquisition. Let $(\mathfrak{s}^*, \mathbf{m}^*(\mathfrak{s}^*), \mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)))$ be the equilibrium information acquisition decisions, message strategies, and decisions (respectively). Then, agent i's IC constraint at the information acquisition stage must consider any possible deviation $\hat{\mathfrak{s}}^i$ and the corresponding message strategy $\hat{m}^i(\hat{\mathfrak{s}}^i)$; that is,

$$E\Bigg[\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(m^{i*},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})f(\theta_2|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})+$$
$$+\int_0^1\int_0^1\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(\hat{m}^{i},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})f(\theta_2|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})\Bigg]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^{i}) \qquad (29)$$

Because deviations at the acquisition stage do not affect the set of influential messages (Lemma OA.1) and because any of such deviations imply a deviation at the communication stage (Lemma

OA.2), the above expression can be solved by computing the expectation over all possible signals realizations and the corresponding messages on- and off-path. In particular, the utility gains from deviations will be given by the realizations in which the messages on- and off- path are different. Formally, let $\tilde{\mathbf{S}}^i$ represent i's type,[29] which is independent of how much of that information he decides to observe (determined by $\mathfrak{s}^i$). Hence, *before deciding on information acquisition* and given the equilibrium under play, agent i evaluates the utility gains from all acquisition strategies and the corresponding messages he expects to send conditional on each possible pair of signal realizations. Equation (29) then becomes:

$$\sum_{\tilde{\mathbf{S}} \in \mathcal{S}} \Pr(\tilde{\mathbf{S}}^i) \times \int_0^1 \int_0^1 - \sum_{y_d}^{\{y_1, y_2\}} \left[\left(y_d \left(m^i(\mathfrak{s}^{i*}, \tilde{\mathbf{S}}^i) \right) - \delta_d - b_d^i \right)^2 - \left(y_d \left(m^i(\hat{\mathfrak{s}}^i, \tilde{\mathbf{S}}^i) \right) - \delta_d - b_d^i \right)^2 \right] \times$$
$$\times f(\theta_1 | \tilde{S}_1^i, \mathbf{m}^{\text{-}i*}) f(\theta_2 | \tilde{S}_2^i, \mathbf{m}^{\text{-}i*}) \geq C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)$$

Now, I proceed to analyze deviations from different equilibrium acquisition strategies.

Agent i acquires both signals in equilibrium ($\mathfrak{s}^{i*} = \mathbf{S}^i$)

Let denote by $\nu_r^{i*}(\tilde{\mathbf{S}}^i) = E\left(\theta_r | m^i(\mathfrak{s}^{i*}, \tilde{\mathbf{S}}^i), \mathbf{m}^{\text{-}i*}\right)$ the beliefs about $\theta_r = \{\theta_1, \theta_2\}$ induced by i under the equilibrium information acquisition strategies and the message corresponding to the realizations given by $\tilde{\mathbf{S}} \in \mathcal{S}$. Equivalently, denote by $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i) = E\left(\theta_r | m^i(\hat{\mathfrak{s}}^i, \tilde{\mathbf{S}}^i), \mathbf{m}^{\text{-}i*}\right)$ be the beliefs induced under the deviation at the information acquisition stage (for the same signals realizations). For the sake of exposition, let index the weights in terms of the state and the decision it is associated with. Let $w_1 = w_{11}$, $(1 - w_1) = w_{21}$, $w_2 = w_{22}$,and $(1 - w_2) = w_{12}$; that is, the first sub-index corresponds to the decision it affects, while the second sub-index to the state it refers to. Then, the IC constraint at the acquisition stage for agent i becomes:

$$\sum_{\tilde{\mathbf{S}}^i \in \mathcal{S}} \Pr(\tilde{\mathbf{S}}^i) \Bigg[- \sum_{y_d = \{y_1, y_2\}} \left[w_{d1} \left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_1^i(\tilde{\mathbf{S}}^i) \right) + w_{d2} \left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_2^i(\tilde{\mathbf{S}}^i) \right) \right] \times$$
$$\left[-w_{d1} \left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_1^i(\tilde{\mathbf{S}}^i) \right) - w_{d2} \left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_2^i(\tilde{\mathbf{S}}^i) \right) - 2b_d^i \right] \Bigg] \geq C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)$$

First consider the deviation in which i only acquires information about θ_1; that is, $\hat{\mathfrak{s}}^i = \{S_1^i\}$. It is straightforward to note that this deviation *per se* does not imply any difference in induced beliefs with respect to θ_1, formally $\nu_1^{i*}(\tilde{\mathbf{S}}^i) = \hat{\nu}_1^i(\tilde{\mathbf{S}}^i)$ for all $\tilde{\mathbf{S}}^i \in \mathcal{S}$. Now, i's message associated with S_2^i does not depend on the signal's realization, but depends on $\mathbf{b}^i$ and may also depend on S_1^i.

Let consider the case in which $\hat{m}^i = \{\tilde{S}_1^i, 1\}$, i.e. i truthfully reveals his information about θ_1 and always sends the message $\hat{m}_2^i = \{1\}$ θ_2. Then, $\hat{\nu}_1^i(\tilde{\mathbf{S}}^i) = \frac{(k_2+4)}{2(k_2+3)}$ and it is different from $\nu_1^{i*}(\tilde{\mathbf{S}}^i)$ only when $\tilde{S}_2^i = \{0\}$ which, in turn, happens for $\tilde{\mathbf{S}}^i = \{(0,0); (1,0)\}$. The IC constraint in such a case is:

$$\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) \left[\sum_{y_d} \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} \right] \right] \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i \right] \right] \right] +$$
$$\Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) \left[\sum_{y_d} \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} \right] \right] \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i \right] \right] \right] \geq C(S_2^i)$$

[29] The realizations of the two pieces of information available to him.

Given that $\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) = \Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) = 1/4$, the IC constraint becomes.

$$\frac{1}{(k_2+3)}\left[\frac{(w_{12}^2+w_{22}^2)}{2(k_2+3)} - \beta_2^i\right] \geq C(S_2^i)$$

That is, the expected utility gains of inducing the correct beliefs about θ_2 should be greater than the extra utility from saving in the costs of becoming informed about that state. It is easy to show that the case of $\hat{m}^i = \{\tilde{S}_1^i, 0\}$ has the sign of β_2^i reversed, for which *not acquiring* signal $S_r^i = \{S_1^i, S_2^i\}$ is incentive compatible if:

$$\frac{1}{(k_r+3)}\left[\frac{(w_{1r}^2+w_{2r}^2)}{2(k_r+3)} - |\beta_r^i|\right] \geq C(S_r^i) \tag{30}$$

For the deviation involving no information acquisition, $\hat{\mathfrak{s}}^i = \{\emptyset\}$, incentive compatibility depends on the message, among the equilibrium ones, i decides to announce at the communication stage. On the one hand, when the message he uses is $\hat{m}^i = \{(0,0),(1,1)\}$, the beliefs induced under the deviation will coincide with those induced under the equilibrium strategy when $\tilde{\mathbf{S}}^i = (1,1)$. Note that there will be also a 'partial' coincidence for other realizations. Put it more formally, $\nu_r^{i*}(1,1) = \hat{\nu}_r^i(1,1)$ for $\theta_r = \{\theta_1, \theta_2\}$, whereas the partial coincidence is given by $\nu_1^{i*}(1,0) = \hat{\nu}_1^i(1,1)$, and $\nu_2^{i*}(0,1) = \hat{\nu}_r^i(1,1)$. Following the characterization of equilibrium communication under centralization, the IC constraint becomes:

$$\begin{aligned}&\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)} + \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.\\ &\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2+w_{22}^2)}{2(k_2+3)} + \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)(k_2+3)}\end{aligned} \tag{31}$$

Which basically is a more strict version of the IC constraint for full revelation when signals coincide (under centralization).

Similarly, when the deviation involves announcing $\hat{m}^i = \{(0,1),(1,0)\}$ the IC constraint becomes:

$$\begin{aligned}&\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)} - \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.\\ &\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2+w_{22}^2)}{2(k_2+3)} - \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)(k_2+3)}\end{aligned} \tag{32}$$

Note that the difference between (31) and (32) is in the sign of the second term in square brackets.

Agent i acquires one signal on-path ($\mathfrak{s}^{i*} = \{S_1^i\}$).

When i acquires only one signal on-path, his assessment of the consequences of any deviation still depends on each possible pair of signal realizations. This fact becomes particularly relevant for deviations involving acquisition of more signals. Note that it is necessary to distinguish between the induced beliefs on- and off-path, and the actual information i has access to. Thus, in addition to the previously defined $\nu_r^{i*}(\tilde{\mathbf{S}}^i)$ and $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i)$, I now denote by $\nu_r^i(\tilde{\mathbf{S}}^i) = E\left(\theta_r|\tilde{\mathbf{S}}^i, \mathbf{m}^{-i*}\right)$ the beliefs about θ_r that would result from the decision-maker observing the signals available to agent i (independent of his information acquisition strategy). Then, i's IC constraint at the information acquisition stage becomes:

$$\sum_{\tilde{\mathbf{S}}^i\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\Bigg[-\sum_{y_d=\{y_1,y_2\}}\Big[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)\Big]\times$$
$$\Big[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)-2\nu_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)-2\nu_2^i(\tilde{\mathbf{S}}^i)\right)-2b_d^i\Big]\Bigg]\geq C(S_1^i)-C(\hat{\mathfrak{s}}^i)$$

When i considers not acquiring any signals and decides to announce $\hat{m}_1^i=\{1\}$, there are two cases in which he induces incorrect beliefs as compared to the equilibrium: $\tilde{\mathbf{S}}=\{(0,0);(0,1)\}$. The ex-ante expected utility losses of such strategy depends on the signal realizations, as can be noted in the expression for the IC constraint below:

$$\Pr\left(\tilde{\mathbf{S}}^i=(0,0)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{2(k_1+3)}\right)\right]\times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}+\frac{(k_1+4)}{2(k_1+3)}-\frac{2(k_1+2)}{2(k_1+3)}\right)+w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+2)}{2(k_2+3)}\right)-2b_d^i\right]\Bigg]+$$
$$\Pr\left(\tilde{\mathbf{S}}^i=(0,1)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{2(k_1+3)}\right)\right]\times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}+\frac{(k_1+4)}{2(k_1+3)}-\frac{2(k_1+2)}{2(k_1+3)}\right)+w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+4)}{2(k_2+3)}\right)-2b_d^i\right]\Bigg]\geq C(S_1^i)$$

Which, after some algebra gives:

$$\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)}-\beta_1^i\right]\geq C(S_1^i) \tag{33}$$

As before, the generic IC constraint involves the absolute value of β_r^i.

Deviations involving the acquisition of more information have the issue that i cannot signal this deviation to the decision-maker. Agent i then uses the additional information to identify situations (i.e. signal realizations) under which it is profitable to lie to the decision-maker. Such deviations are related to the credibility loss, because i would like to induce beliefs about the signal he is not expected to acquire on-path by means of messages on the signal he is believed on-path.

As analyzed in the communication game, the credibility loss takes place when signals do not coincide, $\tilde{\mathbf{S}}^i=\{(0,1);(1,0)\}$, so any deviation at the communication stage will take place in one of these cases. Moreover, given that β_1^i is typically not zero, i's incentives to lie will always be in a single direction, that is either when $\tilde{\mathbf{S}}^i=(0,1)$ or when $\tilde{\mathbf{S}}^i=(1,0)$ but not in both. The IC constraint for the deviation of acquiring both signals and announcing $\hat{m}_1^i=0$ when $\tilde{\mathbf{S}}^i=(1,0)$ will be given by:

$$-\Pr\left(\tilde{\mathbf{S}}^i=(1,0)\right)\sum_{y_d}\Bigg[\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)}-\frac{(k_1+2)}{2(k_1+3)}\right)\right]\times$$
$$\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)}+\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{(k_1+3)}\right)+w_{d2}\left(1-\frac{(k_2+2)}{(k_2+3)}\right)-2b_d^i\right]\Bigg]\geq C(S_2^i)$$

Which yields:

$$\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)^2}-\frac{(w_{11}w_{21}+w_{12}w_{22})}{2(k_1+3)(k_2+3)}-\beta_1^i\right]\geq -2C(S_2^i) \tag{34}$$

Which is equivalent to say that the cost of acquiring the second signal is too large with respect to the utility gain from deviating under ambiguous information.

Incentive compatibility then depends on $|\beta_1^i|$ being within the limits imposed by equations (21), (33), and (34). Note that equation (33) implies (21), meaning that if i is willing to acquire $\tilde{S}_1^i$ instead of acquiring no signal, then he will certainly reveal it. Incentive compatibility thus is captured by equations (33), and (34), which lead to:

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)} \leq \min\left\{\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - C(S_1^i)\,;\, \frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1+3)(k_2+3)} + 2C(S_2^i)\right\} \tag{35}$$

Now, let define $\mathfrak{B}_1\left(C(S_1), C(S_2), \mathbf{k}^{\mathrm{C}}\right) = \{\mathbf{b} : \mathbf{b}\, \text{satisfies equation (35)}\}$. Then, the LHS in equation (35) is weakly positive and, thus, $\mathfrak{B}_1 \neq \emptyset$ only if the RHS is strictly positive; that is,

$$\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} > \max\left\{C(S_1^i)\,;\, \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)} - 2C(S_2^i)\right\} \tag{36}$$

Furthermore, it is easy to check that the LHS above is increasing in $C(S_1)$ and decreasing in $C(S_2)$, which proves the last part of the proposition. □

Acquiring information about θ_1 in the covert game is incentive compatible for i if and only if the utility gains are sufficiently large. As in the overt game, these utility gains must compensate for the cost of acquiring the corresponding signal, $C(S_1^i)$. Unlike the overt game, however, the utility gains from increasing the precision of the principal's beliefs must compensate for the utility gains associated with having ambiguous information (rightmost term inside the curly brackets). In other words, if acquiring information about θ_2 is cheap for i, he will acquire it and lie to the principal whenever the associated signal favours his interests and S_1^i goes against them. Incentive compatibility then requires that the cost of the signal associated with the state is low and the cost of that associated with the other state is sufficiently high.

The relationship between incentive compatibility and the costs of the different signals is shown in the last two statements of Proposition OA.1. Because of the first type of deviations—acquiring fewer signals— i's incentives to acquire and reveal information about θ_1 decrease as the cost of S_1^i increases; saving on the cost of the signal becomes more profitable for i. The second type of deviations—acquiring more signals—leads to an increase in credibility when the cost of S_2^i increases. This effect stems from the credibility loss due to ambiguous information.

3. More than one binary signal per state

Throughout the paper I assumed each agent's information consists of one binary signal associated with each state, two in total. Here I discuss relaxing this assumption based on recent developments in the literature of strategic communication. First, I show how communication incentives would be affected if *a single agent* observes more than one signal associated to each state. I do this based on Förster (2021), which studies a uni-dimensional decision problem in which an agent observes binary signals associated to a *a single* state. I then argue that, under the notion of informational interdependence used in this paper, incentives for communication of *perfectly informed* specialists are characterized by similar measures of conflicts of interest.

Förster (2021) studies a sender's incentives for communication to a receiver in charge of one decision, when the former observes $\kappa \geq 1$ binary signals that are independent conditional on the state $\theta \in \Theta = [0, 1]$. In the most informative equilibrium, the sender's message strategy is influential if his bias is below a threshold $\bar{b}(\kappa)$, which involves full revelation of his information for sufficiently low biases, $b \leq \underline{b}(\kappa) < \bar{b}(\kappa)$. Interestingly, the threshold for influential message strategies ($\bar{b}$) is increasing in the sender's information (κ); while the threshold for fully revealing messages ($\underline{b}$) is decreasing in κ.

My focus on a single binary signals associated to each state is then a conservative estimation of communication incentives. In other words, as the number of binary signals available to an agent increases, information transmission with the principal will (weakly) improve until a threshold is reached. This threshold marks the amount of information the agent is willing to fully reveal, and depends on his bias (see also Fischer and Stocken, 2001; Ivanov, 2010). Beyond that threshold, further increasing the agent's information makes equilibrium communication converge (non-monotonically) to Crawford and Sobel (1982) (Proposition 5 in Förster, 2021).

Now, what happens when agents observe more than one binary signal in the presence of informational interdependence? In a companion paper (Habermacher, 2022), I analyze the case of two specialists, each of whom is perfectly informed about a single (different) state and observes no information about the other. I show that message strategies consist of increasing partitions of the state space. Individual IC constraints are isomorphic to those in Crawford and Sobel (1982), where sender i's bias is represented by $\frac{\beta_1^i}{w_1^2+(1-w_1)^2}$ and $\frac{\beta_2^i}{w_2^2+(1-w_2)^2}$. These IC constraints could be interpreted as the maximum bias for which an agent reveals *some* information about one state.

An additional question relates to asymmetries on agents' information. In the paper I showed that individual incentives to reveal information depend on how much information the decision-maker is expected to have in equilibrium.[30] If one agent, A^1, observes more than one binary signal in my framework, his incentives will follow a more complex form of partitional communication equilibria (as in Förster, 2021). If agents other than A^1 still observe binary signals about each state, their incentives depend on conjectures about the information the decision-maker receives in equilibrium, which includes A^1's equilibrium message strategy. Indeed, the fact that incentives for full revelation are decreasing in the number of signals implies the principal weakly prefers to delegate any decision to A^1 rather than another agent with the same conflict of interest, even if the second agent is willing to fully reveal his information to A^1.[31] This suggests that, *ceteris paribus*, the principal prefers to delegate decisions to more informed agents. Deepening these intuitions is a subject for future work.

4. Probability of $b \in \mathfrak{B}_1^c$

Figure 6 depicts the II quadrant of the two-dimensional bias space, with the communication IC constraint associated to full revelation S_1^i only, when $w_1 = 0.542$ and $k_1^{\mathrm{C}} = 0$. Biases are uniformly distributed in $[-\mathfrak{b}, \mathfrak{b}]^2$, so the probability of finding an agent willing to reveal information about θ_1 is proportional to the grey area. I show that this area equals that of a rectangle with base $\left[2\, \frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{2\,(k_1^{\mathrm{C}}+3)}\right]$ and height $\left[2\,\mathfrak{b}\, \frac{(w_1^2+(1-w_1)^2)^{\frac{1}{2}}}{w_1}\right]$, times the total area $\frac{1}{(2\mathfrak{b})^2}$.

First, note that λ_1 denotes the maximal incentives to reveal information about θ_1. The IC constraint for communication of S_1^i can then be expressed as

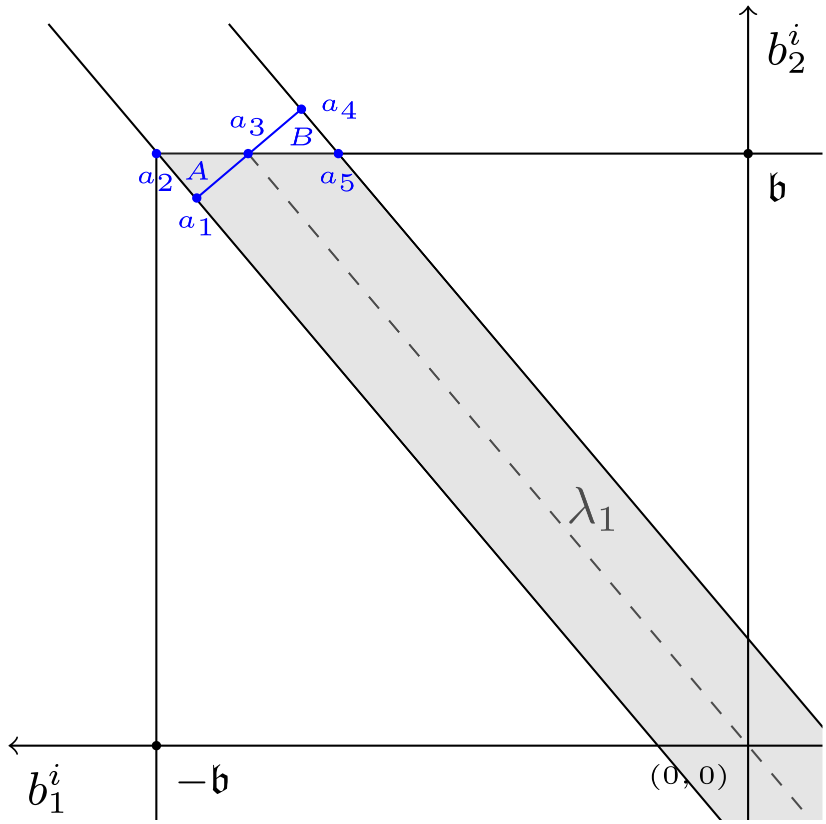
$$||\mathbf{b}^i - \mathrm{Proj}_{\lambda_1}\mathbf{b}^i|| \leq \frac{[(w_1)^2 + (1-w_1)^2]^{\frac{1}{2}}}{2(k_1^{\mathrm{C}}+3)}$$

Where $||\mathbf{b}^i - \mathrm{Proj}_{\lambda_1}\mathbf{b}^i||$ represents perpendicular distance between $\mathbf{b}^i$ and λ_1. Hence, $||\mathbf{b}^i - \mathrm{Proj}_{\lambda_1}\mathbf{b}^i|| =$

[30] In Krishna and Morgan (2001b), for example, two senders are perfectly informed and individual incentives for communication depend on the other sender's bias because it predicts how much information he reveals on-path.

[31] If the conflict of interest between A^1 and A^2 is greater than $|\underline{b}(\kappa_1)|$, communication is less than fully revealing.

Figure 6: Probability of $b \in \mathfrak{B}_1^c$ from the communication IC constraint



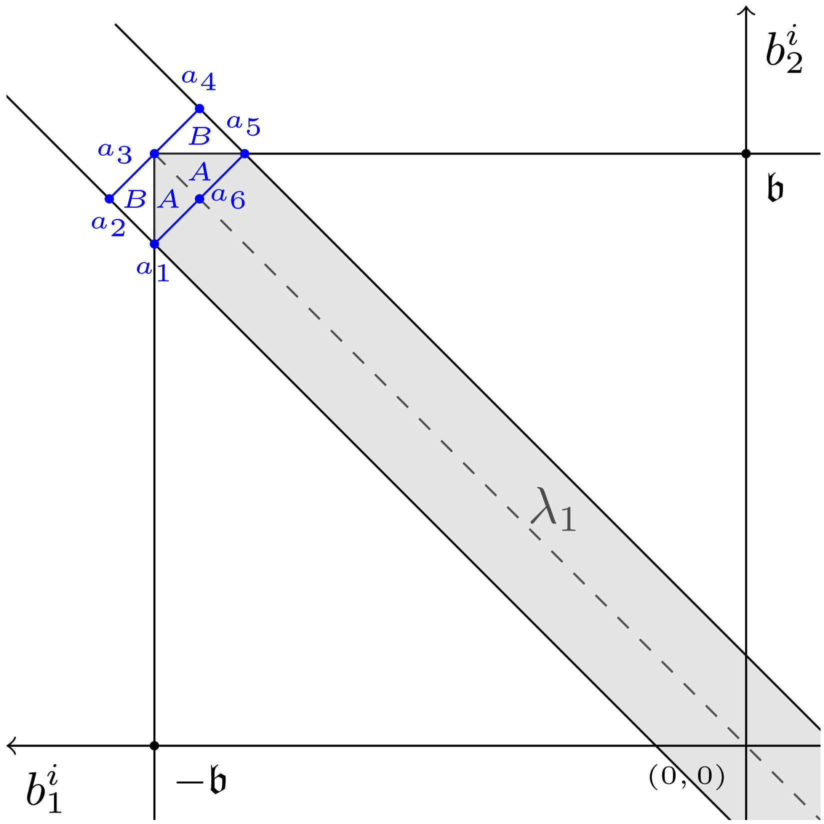
$\frac{[(w_1)^1+(1-w_1)^2]^{\frac{1}{2}}}{2(k_1^c+3)}$ constitutes the distance between any point in λ_1 and the border of the IC constraint in Figure 6, including that at the extremes of the locus. As a consequence, the distance between a_1 and a_4 equals $2\,\frac{[(w_1)^1+(1-w_1)^2]^{\frac{1}{2}}}{2(k_1^c+3)}$.

Note also that the triangles A and B are equal because the segment $\overline{a_1 a_4}$ is a rotation of $\overline{a_2 a_5}$, which share the same midpoint with respect to the boundaries of the IC constraint. This implies that the area in grey is equivalent to that of a rectangle with base equal to the module of $\overline{a_1 a_4}$, and height equal to two times the hypotenuse of the triangle of height $\mathfrak{b}$ and length $-\mathfrak{b}\frac{(1-w_1)}{w_1}$ (which I derive from the definition of λ_1). Some extra algebra and the multiplication by $\frac{1}{(2\mathfrak{b})^2}$ leads to expression (24). In addition, note that the same argument applies for all $w_1 \geq 0.542$ because the IC constraint rotates clockwise and λ_1 coincides with the vertical axis when $w_1 = 1$.

Now, for $w_1 \in (0.542, 0.5]$ I derive an upper bound for the probability. This upper bound occurs when $w_1 = 0.5$, which is depicted in Figure 7.First, let $\mathbf{b}_{\mathfrak{b}} = \left(-\mathfrak{b}\frac{(1-w_1)}{w_1}, \mathfrak{b}\right)$ represent the diagonal from $(0,0)$ to the intersection of λ_1 and the boundaries of the support for the biases (a_3 in the figure). Also, let $\tilde{\mathbf{b}} = \left\{\mathbf{b} : w_1 b_1 + (1-w_1) b_2 = \frac{[(w_1)^2+(1-w_1)^2]^{\frac{1}{2}}}{2(k_1+3)}; \text{ and } b_2 = \mathfrak{b}\right\}$ represent the boundary of the IC constraint which has $b_2 = \mathfrak{b}$ (a_5 in the figure). Then, because the areas of triangles A, B, C, and D are equal, the area within the boundaries of the IC constraint is equal to that of a rectangle of base $2\,\mathfrak{b}$ and height $||\mathbf{b}_{\mathfrak{b}}|| + ||\text{Proj}_{\lambda_1}\tilde{\mathbf{b}}||$ (the last term represented by a_6 in the figure).

It is not too difficult to show that $||\mathbf{b}_{\mathfrak{b}}|| = \mathfrak{b}\frac{[(w_1)^2+(1-w_1)^2]^{\frac{1}{2}}}{w_1}$, while $||\text{Proj}_{\lambda_1}\tilde{\mathbf{b}}|| = \left|\frac{1}{2(k_1+3)} - \frac{2\mathfrak{b}(1-w_1)}{[(w_1)^2+(1-w_1)^2]^{\frac{1}{2}}}\right|$. Then, computing the probability relative to the total area yields the right-hand-side of equation (25). This is an upper bound for $w_1 \in (0.5, 0.542)$ because the area of C [D, resp.] goes to zero [increases] as λ_1 rotates towards the vertical axis; while the areas of A and B remain equal.

Figure 7: Probability of $b \in \mathfrak{B}_1^c$ from the communication IC constraint



5. MatLab code for Figure 5 ($C(S) = 1/50$)

```
%% ASII -- Expected investment in information when biases are unknown
clear all
clc

% PARAMETERS
ww=150;
bb=200;
c=1/50;

w=linspace(0.5,1,ww);
W=(w).^2+(1-w).^2;

K_C=max(floor((((1/4)+(W./(6.*c))).^(1/2))-(5/2))+1,0);
```

```
K_D1=max(floor((((1/4)+((w.^2)./(6.*c))).^(1/2))-(5/2))+1,0);
K_D2=max(floor((((1/4)+(((1-w).^2)./(6.*c))).^(1/2))-(5/2))+1,0);

%%%% EQUILIBRIUM EQUATIONS
%%% Expected number of agents under centralization
EK_C=zeros(1,ww);
for i=1:ww
EE=zeros(1,K_C(1,i)+1);
for j=0:K_C(1,i)
EE(1,j+1)=j.*W(1,i)./(2.*w(1,i).*(j+3));
end
EK_C(1,i)=sum(EE);
end

%%% Expected number of agents revealing information about the salient state
EK_D1=zeros(1,ww);
for i=1:ww
EE=zeros(1,K_D1(1,i)+1);
for j=0:K_D1(1,i)
EE(1,j+1)=j.*w(1,i)./(2.*(j+3));
end
EK_D1(1,i)=sum(EE);
end

%%% Expected number of agents revealing information about the non-salient state
EK_D2=zeros(1,ww);
for i=1:ww
EE=zeros(1,K_D2(1,i)+1);
for j=0:K_D2(1,i)
EE(1,j+1)=j.*(1-w(1,i))./(2.*(j+3));
end
EK_D2(1,i)=sum(EE);
end

% FIGURE
% Differences in relative investment under centralization and delegation

f1= figure;
surf(repmat(w',1,ww),repmat(w,ww,1),abs(repmat(EK_C',1,ww)-repmat(EK_C,ww,1)),...
'FaceAlpha',0.35,'EdgeAlpha',0.5,'EdgeColor','#D95319','FaceColor','#D95319');
hold on
surf(repmat(w',1,ww),repmat(w,ww,1),abs(repmat(EK_D1',1,ww)-repmat(EK_D2,ww,1)),...
'FaceAlpha',0.35,'EdgeAlpha',0.5,'EdgeColor','#4DBEEE','FaceColor','#4DBEEE');
hold off
xlabel('w_1');
ylabel('w_2');
zlabel('|k_1 - k_2|');
legend({'|E(k^{C}_1) - E(k^{C}_2)|','|E(k^{D}_1) - E(k^{D}_2)|'},...
```

```
'Box','off','Location','northeast');
```