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DOCUMENTO DE TRABAJO N° 150

Junio de 2022

Citar como:

Casas, Agustin y Daniel M. Kselman (2022). Incentivizing Brokers in Clientelist Parties. *Documento de trabajo RedNIE N°150.*

Incentivizing Brokers in Clientelist Parties.*

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May 31, 2022

Abstract

Local *brokers* are essential in the implementation of clientelist politics, but their efforts on parties' behalf are not fully observable. A growing literature studies how parties address this agency problem, highlighting two distinct reward schemes: allocating promotions or prizes based on observed vote shares, or doing so based on inferred effort allocations. This paper develops a formal model to examine the conditions under which one or the other of these reward schemes is optimal for minimizing brokers' rent-seeking. Intuitively, the effort-based reward mechanism is optimal when broker effort is inferred with relative precision. Less intuitively, the vote-based mechanism will tend to be optimal when a party's supporters are *evenly distributed* across regions, and when the prize β adopts *intermediate* values, which together lead to high levels of inter-broker competition. When brokers must compete with one another over valued prizes, parties can often minimize rent-seeking without directly monitoring broker effort.

*Many thanks to Michael Becher, Zuheir Desai, Herbert Kitschelt, Susan Stokes, and Mariela Szwarcberg Daby for feedback on past versions.

1 Introduction

Clientelist parties rely on local intermediaries, commonly labeled *brokers*, to identify 'targetable' subsets of voters (Finan and Schecter 2012), distribute material resources (Stokes 2005), provide constituency service (Auerbach and Thachil 2018), and monitor voter behavior (Stokes et al. 2013). Given their pervasiveness in clientelist politics, the question of how parties incentivize broker performance becomes relevant. The central challenge of incentivizing brokers is that their efforts on behalf of the party are not directly observed (Larreguy et al. 2016; Casas 2018). This informational challenge leads to agency costs as, instead of using the party's resources to mobilize and persuade voters, brokers may shirk and keep some for their own private rents (Keefer and Vlaicu 2008; Stokes et al. 2013; Zarazaga 2014).

An emerging literature studies how parties deal with this agency problem. Some papers argue that parties make inferences about brokers' unobserved actions based on *signals*, and in turn choose rewards and sanctions based on these inferences.[1] On the other hand, a distinct set of papers argues that parties reward brokers based on absolute vote outcomes, making no attempt at inferring broker effort/performance.[2] To date, no formal theoretic model has addressed the comparative performance of distinct reward schemes. We address this challenge with a model of inter-broker competition where the principal (the party) – who observes vote outcomes but not the brokers' effort – allocates an indivisible prize β to one of two brokers. Two

[1] In some cases the emphasis is on monitoring broker 'capacity/performance' (e.g. Larreguy et al. 2017; Bowles et al. 2020; Gingerich 2020), while in others it is on monitoring broker 'effort' (Szwarcberg 2012; 2014). In all of these papers, parties reward brokers based on inferences about unobservables gleaned from more or less noisy signals. These signals do not need to be based on indirect evidence only (turnout, vote shares, rallies attendance). There may be more direct options such as soliciting information from local voters (Auerbach and Tachil 2018).

[2] Parties may use "broker-primaries" to allocate valuable administrative positions based on competitive voting contests. In Ghana, the polling station level position of Branch Executive is sometimes allocated in precinct-level primary elections (Brierly and Nathan 2021). Auerbach (2020) shows that heads of local development committees in Indian slums were often originally chosen in open voting processes. Gottlieb (2017) demonstrates that party posts in the Southern Casamance region of Senegal are generally selected in broad, village-wide voting contests. Toral (2022) demonstrates that bureaucratic posts are sometimes allocated in community-wide elections in Brazil.

A slightly different version of this mechanism resembles what Brierly and Nathan (2021) label 'piece-rate compensation' - the direct exchange of votes for payments (pg. 5) in Ghana. A similar 'votes-for-cash' mechanism is also often found in Brazil (Gingerich 2014; Novaes 2018), Colombia (Holland and Palmer Rubin 2015), South Africa (de Kadt and Larreguy 2018), Peru (Muñoz 2014), and India (Auerbach 2020). What unifies these distinct systems is that parties use absolute vote outcomes to make deterministic allocations of rewards, with no explicit attempt to infer or monitor broker effort.

institutional designs for incentivizing brokers are considered, corresponding to those reviewed above: rewarding (inferred) effort or rewarding (observed) votes. As an intuitive baseline, the effort-based reward mechanism is optimal as long as broker performance is inferred with relative precision. Less intuitively, the vote-based mechanism will tend to be optimal when a parties' supporters are *evenly distributed* across regions, and when the prize β adopts *intermediate* values, leading to high levels of inter-broker competition. When this competition is stiff enough, parties can often minimize rent-seeking without having to directly monitor broker behavior. In contrast, when regional inequalities and shabby prizes undermine inter-broker competition, effort-monitoring will be necessary. The paper thus also speaks to the expanding empirical evidence on how inter-broker competition (or the lack thereof) conditions clientelist politics (Auerbach 2016; Gottlieb 2017; Camp 2017; Novaes 2018; Auerbach and Thachil 2020).

2 The model

An electoral district has two regions $i \in \{1, 2\}$ with voting population normalized to 1. There is one **broker** from the incumbent party P in each region, labeled broker i, and there is a non-strategic opposition party O (i.e. a residual claimant of votes).[3] Each broker working for P must allocate a dollar between *private rents* (r_i) and *clientelism* (c_i), which influences their regional vote $v_i(c_i)$.

The party P allocates a prize β to one of the two brokers.[4] Let $p_i(c_1, c_2)$ be broker i's probability of receiving a prize β from P, given c_1 and c_2. As demonstrated below, the functional form of $p_i(c_1, c_2)$ depends on whether the political parties use a vote- or effort-based reward rule to allocate β, i.e., how P incentivizes the brokers. Let $f(r_i)$ be an increasing and concave function. For all $i \in \{1, 2\}$,

[3] Appendix E outlines an extension of the model to with a strategic opposition party and brokers. As discussed in the Conclusion, this extension is interesting, but should not have first order effects on the current paper's results and is beyond our current scope.

[4] These modelling choices imply two assumptions: a.) the reward β cannot be divided, and b.) it is allocated regardless of the local electoral outcome. As such, it connects with the literature on contests, as in Corchon and Serena (2018), Denter (2020), and Crutzen et al. (2020). In Appendix E, we discuss extensions of the model which relax these assumptions, and explain why the core results of our paper may carry over into these more complex environments.

broker utility can thus be written as:

$$u_i(r_i, c_i) = f(r_i) + p_i(c_1, c_2)\beta \ . \tag{1}$$

Each **voter** j in region i has a partisan bias for party P denoted σ_{ji}, distributed uniformly over the support $[\bar{\sigma}_i - 1, \bar{\sigma}_i]$. Without loss of generality we assume $\bar{\sigma}_i \in [0, 1]$ and $\bar{\sigma}_1 > \bar{\sigma}_2 \geq 0$. For simplicity, the clientelist effort c_i affects voters' utility linearly, and equally for all j.[5] We can thus write voter j's utility for P as $u_{ji} = \sigma_{ji} + c_i$. We assume full turnout hence voter j in i votes for P instead of O if $u_{ji} > \mu$, where μ captures a general preference for the opposition party. Without loss of generality let $\mu < \bar{\sigma}_2$.[6]

This is a game of incomplete information: brokers have full information, but the party observes neither how brokers spend their budget (r_i, c_i) nor the regions' preference for the incumbent, $\bar{\sigma}_i$.[7]

We begin by solving for equilibrium broker effort under two distinct reward systems. In the **vote-based** system, the broker of the region where P got the largest share of voters is awarded β, *regardless* of whether or not they exerted the highest level of effort c_i. In the **effort-based** system, the party uses vote outcomes as imperfect signals of local brokers' effort c_i, and allocates β to the broker who is inferred to have exerted the highest effort. The timing is as follows: first, given the reward system, brokers distribute handouts c_i. Second, the voters vote. Third, one of the two brokers is allocated the prize β according to the reward rule in place. Based on the broker-level results, in Section 3 we assess party P's optimal organizational choice under the assumption that P maximizes votes $v_1(c_1) + v_2(c_2)$ which – as we show below – are a linear function of clientelism, $c_1 + c_2$.

[5] We thus assume that brokers are symmetric in their capacity to generate support, which results in a parsimonious approach for a paper on moral hazard. While not included for reasons of space, our initial investigations suggest that, while in the aggregate clientelism may be scaled up or down, the current paper's comparative statics should go through in a model with capacity differences.

[6] The assumptions of linearity (density and the effect of c_i on the utility) imply that effort has a linear effect on votes. This is not at odds with the literature (see for instance Larreguy et al, 2016) but it is important to highlight that allowing for non-linearities may result in different results.

[7] We thus assume that: a.) information cannot be credibly conveyed upward to the party; and b.) $\bar{\sigma}_1$ ($\bar{\sigma}_2$) is observable to broker 2 (broker 1). The model would be similar but more complicated if brokers had better (but imperfect) information about neighboring regions than the national party.

2.1 Equilibrium

Given effort allocations c_1 and c_2, and recalling that σ_{ji} is uniformly distributed, regional vote shares in the stage 2 election $v_i(c_i)$ can be written as:[8]

$$v_i(c_i) \;=\; \begin{cases} \overline{\sigma}_i + c_i - \mu & \text{if} \quad \mu < \overline{\sigma}_i + c_i < 1 + \mu \\ 1 & \text{if} \quad \overline{\sigma}_i + c_i \geq 1 + \mu \end{cases} . \tag{2}$$

The value $\overline{\sigma}_i + c_i - \mu$ is the percentage of voters whose utility for P surpasses μ. We denote with $\ell_i = \overline{\sigma}_i - \mu$ the percentage of *loyalists* in district i: that is, all voters in i who would vote for P even when $c_i = 0$. Since $\bar{\sigma}_1 > \bar{\sigma}_2$, region 1 has more loyalists than region 2; and since brokers only benefit from c_i insofar as it produces votes, any $c_i > (1 - \ell_i)$ is weakly-dominated, and $1 - \ell_i$ is the *de facto* max effort.

Vote-Based Reward Rule

Under this reward scheme, β goes to the broker who mobilizes the *highest regional vote share* in the stage 2 election. In turn, i's probability of receiving β is:

$$p_i(c_1, c_2) = \begin{cases} 1 & \text{if } v_i(c_i) > v_{-i}(c_{-i}) \\ \frac{1}{2} & \text{if } v_i(c_i) = v_{-i}(c_{-i}) \\ 0 & \text{if } v_i(c_i) < v_{-i}(c_{-i}) \end{cases} , \tag{3}$$

The probability $p_i(c_1, c_2)$ will be weakly increasing in c_i and weakly decreasing in c_{-i}. Note that broker 1 may win β even if $c_1 < c_2$, due to the fact that she has more regional loyalists than broker 2.

Define c_1^* and c_2^* as *Pure Strategy Nash equilibria* (PSNE) effort choices and $c_V^* = c_1^* + c_2^*$ as the *Total Effort*, where V stands for vote-based. Proposition 1 solves the vote-based model for the case in which agent utility for rents is captured by a quadratic cost function[9] $f(r_i) = 1 - c_i^2$.

Proposition 1 *If $\bar{\sigma}_1 > \bar{\sigma}_2$ and $f(r_i) = 1 - c_i^2$ then:*

[8] Define $\sigma_{s,i} = \mu - c_i$ as the partisan bias of the *swing voter* in i, who is perfectly indifferent between P and O. Since σ_{ji} is distributed uniformly with a support set of width 1, we can write the % of voters in i who vote for P (i.e. for whom $\sigma_{i,j} + c_i \geq \mu$) as $\bar{\sigma}_i - (\mu - c_i) = \bar{\sigma}_i + c_i - \mu$. **QED**

[9] In Appendix A we solve the vote-based rewards model for any increasing concave function.

1. *If* $\beta \leq (\ell_1 - \ell_2)^2 \equiv \beta_V^L$ *then* $c_1^* = c_2^* = 0$ *is the game's unique PSNE.*

2. *If* $\beta \geq 2(1 - \ell_2)^2 \equiv \beta_V^H$ *then* $c_i^* = 1 - \ell_i$ *is the game's unique PSNE.*

3. *If* $\beta_V^L < \beta < \beta_V^H$ *then there exists a unique Mixed-Strategy Nash Equilibrium (MSNE), fully characterized in Appendix B.*

The thresholds (β_V^L, β_V^H) define when the corner solution with with fully shirking brokers $c_1^* = c_2^* = 0$ or the corner solution with brokers exerting maximum effort $c_i^* = 1 - \ell_i$ are the game's PSNE. Not surprisingly, the equilibrium outcome tends towards shirking when brokers place low value on the prize $\beta \leq \beta_V^L$, and towards max-effort high valuations of the prize, $\beta \geq \beta_V^H$. Hence, the higher the thresholds β_V^L and β_V^H, the lower equilibrium effort. The following Corollary shows the responsiveness of the thresholds to the distribution of loyalists:

Corollary 1 ***Political Geography:*** *(i)* β_V^L *is increasing in* ℓ_1 *and decreasing in* ℓ_2*, and (ii)* β_V^H *is increasing in* ℓ_2*.*

From (i) in Corollary 1, the vote-based system tends to generate high levels of effort when the regions are fairly homogeneous. This is due to the fact that, in districts where $\ell_1 - \ell_2$ is small, *the brokers' contest will be competitive*: firstly, because the 'disadvantaged' broker from region 2 has greater incentive to deviate from a shirking outcome; and secondly because both brokers have less the incentive to deviate from the maximum effort outcome $c_i = (1 - \ell_i)$. On the other hand, when ℓ_1 is significantly higher ℓ_2, there is a shirking equilibrium in which brokers make no effort ($c_i^* = 0$) but broker 1 wins β nonetheless. This identifies an important risk associated with the vote-based rule: that the local imbalance between 'strong' and 'weak' brokers eliminates competitive incentives to work for the party.

At intermediate values of β, i.e., in $[\beta_V^L, \beta_V^H]$, broker 2 is sufficiently motivated to deviate from the shirking strategy and choose the amount just necessary to overtake broker 1's vote share $(\ell_1 - \ell_2 + \epsilon)$. As a result, only MSNE exist, which we fully characterize in Appendix B. Using these MSNE (warranted by Dasgupta and Maskin, 1986) we can calculate the *expected clientelist effort* as $E(c_V^*) = E(c_1^*) + E(c_2^*)$. In line with the results in pure strategies, with some minimal heterogeneity between regions, effort increases with β, ℓ_2 and $-\ell_1$.[10] Full equilibrium for the vote-based system are thus summarized in the following example and figure.

Example 1: Define $c^*_V(\beta)$ as the function mapping values of β into total equilibrium effort. The following figures present the equilibrium under vote-based reward schemes by plotting $c^*_V(\beta)$. Figure 1a serves as a baseline in which $\ell_1 = .7$ and $\ell_2 = .2$. Figures 1b and 1c then study how $c^*_V(\beta)$ changes with increases in ℓ_2 and ℓ_1 respectively. Per the MSNE characterized in Appendix B, there is a discontinuity at the value $\hat{\beta} = \beta^H_V - \beta^L_V$.

(See Figure 1 on next page)

By Proposition 1, the Full-Shirking and Max-Effort outcomes emerge when $\beta < \beta^L_V$ and $\beta > \beta^H_V$ respectively. With the higher ℓ_2 in Figure 1b, the range of β over which $c^*_1 = c^*_2 = 0$ shrinks (Corollary 1), the range of β over which $c^*_i = 1 - \ell_i$ widens (Corollary 1), and the MSNE generate higher expected effort than in Figure 1a.[a] With the higher value of ℓ_1 in Figure 1c, the range of β over which $c^*_1 = c^*_2 = 0$ widens (Corollary 1) and the MSNE generate lower expected effort than in Figure 1a.

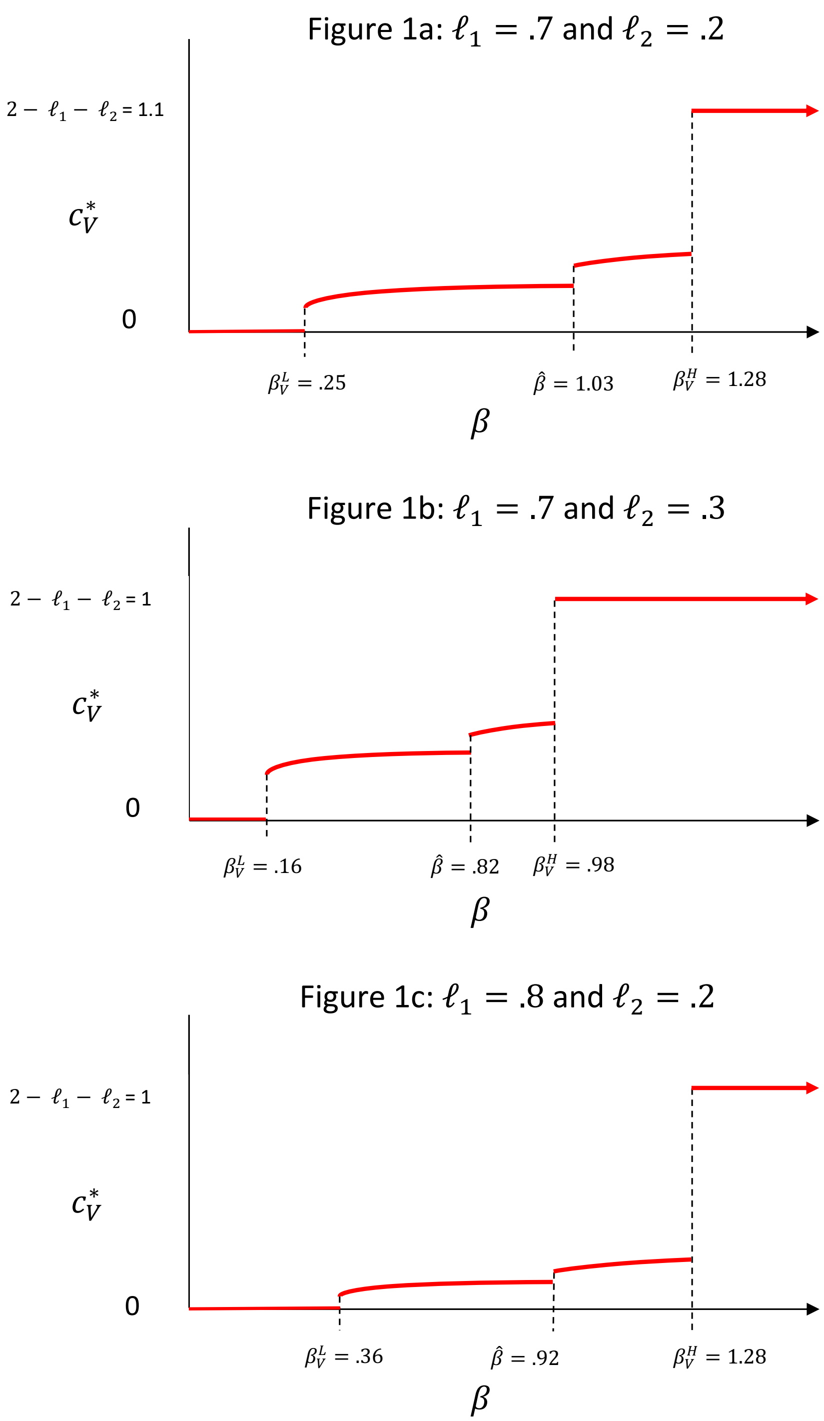
[a] For example, if $\beta = .5$ then $E(c^*_i) = .24$ in 1a and $E(c^*_i) = .276$ in 1b.

Effort-Based Reward Rule

In contrast to the vote-based system, political parties may rather assign β to the broker who exerted the highest effort level. In doing so they may eliminate the risk of rewarding a shirking broker from an advantageous region. The challenge is that the national party does not directly observe effort allocations (c_1, c_2). Suppose that, for the party, region 1's advantage $(\bar{\sigma}_1 - \bar{\sigma}_2)$ is a random variable $\tilde{\sigma}$ distributed uniformly over the support $[\delta - e, \delta + e]$, where δ capture the national party's expectation about region 1's advantage over region 2, and e captures the uncertainty about its magnitude, with $e < \delta < 1$. While $\{c_1, c_2\}$ and $(\bar{\sigma}_1, \bar{\sigma}_2)$ are brokers' private information, the distribution of $\tilde{\sigma}$ is common knowledge to all actors.

[10] This comparative static is summarized in Appendix B.1 as a corollary.

Figure 1: Equilibrium Effort in Vote-Based Systems



With this noise structure in mind, party leaders can use observed vote outcomes extract a signal about brokers' relative effort levels. Intuitively, when the votes in region 1 exceed the votes of region 2 by more (less) than expected difference δ, the party will be more likely to allocate β to broker 1 (broker 2). Specifically, define the probability that broker 1 exerts more effort than broker 2 as $\Pr(c_1 > c_2)$. Note, from (2) we can rewrite brokers' effort allocation as $c_i = v_i(\cdot) - \bar{\sigma}_i + \mu$. Thus, $\Pr(c_1 > c_2)$ can be calculated as follows:

$$\Pr(c_1 > c_2) = \Pr_1(\tilde{\sigma} < v_1(\cdot) - v_2(\cdot)) = \frac{c_1 - c_2 + \bar{\sigma}_1 - \bar{\sigma}_2 - (\delta - e)}{2e}. \tag{4}$$

We assume that P directly translates their beliefs into reward probabilities, assigning β to broker i with probability $\Pr(c_i > c_{-i})$, which is why we call this institution – for simplicity – the 'effort-based' reward rule. In turn, brokers interiorize this allocation mechanism in their effort decisions: given $\{c_1, c_2\}$, i's probability of receiving β will be $p_i(c_1, c_2) = \Pr(c_i > c_{-i})$ as derived in (4), which increases in c_i and decreases in c_{-i}. Local brokers will then maximize their utility substituting (4) into (1). Proposition 2 characterizes equilibria when $f(r_i) = 1 - c_i^2$:

Proposition 2 *Let $f(r_i) = 1 - c_i^2$, then:*[11]

- *if $\beta < 4e(1 - \ell_1) \equiv \beta_E^L$ then $c_i^* = \frac{\beta}{4e}$ is the unique PSNE.*
- *if $\beta > 4e(1 - \ell_2) \equiv \beta_E^H$ then $c_i^* = 1 - \ell_i$ is the unique PSNE.*
- *if $\beta \in [\beta_E^L, \beta_E^H]$ then $c_1^* = 1 - \ell_1$ and $c_2^* = \frac{\beta}{4e}$ is the unique PSNE.*

As before, Proposition 2 defines two key thresholds β_E^L and β_E^H, where E denotes 'effort-based' rewards. Note first that there is no corner equilibria with shirking: even at low levels of $\beta < \beta_E^L$, the effort-based system induces brokers to choose the interior solution $c_i^* = \frac{\beta}{4e}$. Indeed, this capacity to mitigate shirking even with 'cheap' prizes comprises a central advantage of the effort-based systems (Section 3). Similar to the vote-based system, total equilibrium effort $c_E^* = c_1^* + c_2^*$ is (weakly) increasing in β: at low levels of β both brokers choose the interior solution; as β surpasses β_E^L agent 1 shifts to max-effort; and agent 2 switches to max-effort as β surpasses β_E^H. Consider another Corollary (proof in Appendix C).

[11] The proof for any increasing concave function $f(r_i)$ is in Appendix C.

Corollary 2 ***Uncertainty and Partisanship:*** *Aggregate clientelist effort c_E^* is weakly decreasing in e, ℓ_1 and ℓ_2, ceteris paribus.*

If national leaders are uncertain about local partisanship (greater e), this weakens the link between effort c_i and $p_i(\cdot)$, which in turn disincentivizes effort. Similarly with regional supporters: increases in ℓ_i, holding e and ℓ_{-i} constant, lead to (weakly) lower equilibrium effort levels over the entire range of β.

Example 2: Unlike the vote-based system, where the total effort function $c_V(\beta)^*$ was characterized by 'jumps', the function $c_E(\beta)^*$ is continuous.[a]

(Figure 2 on next page)

As demonstrated in figure 2a, holding constant $\ell_1 = .7$ and $\ell_2 = .2$, increasing uncertainty from $e = .5$ to $e' = .75$ drags total effort lower by increasing the thresholds at which brokers 1 and then 2 transition to Max-Effort. In Figure 2b, changes in ℓ_1 do not affect $c_E(\beta)^*$ at low levels of β; but at higher levels of β, increasing ℓ_1 drags down the Max-Effort level by reducing the overall 'slack' of available voters in region 1, thus limiting vote-seeking incentives. A similar pattern emerges in Figure 2c w/r/t the parameter ℓ_2.

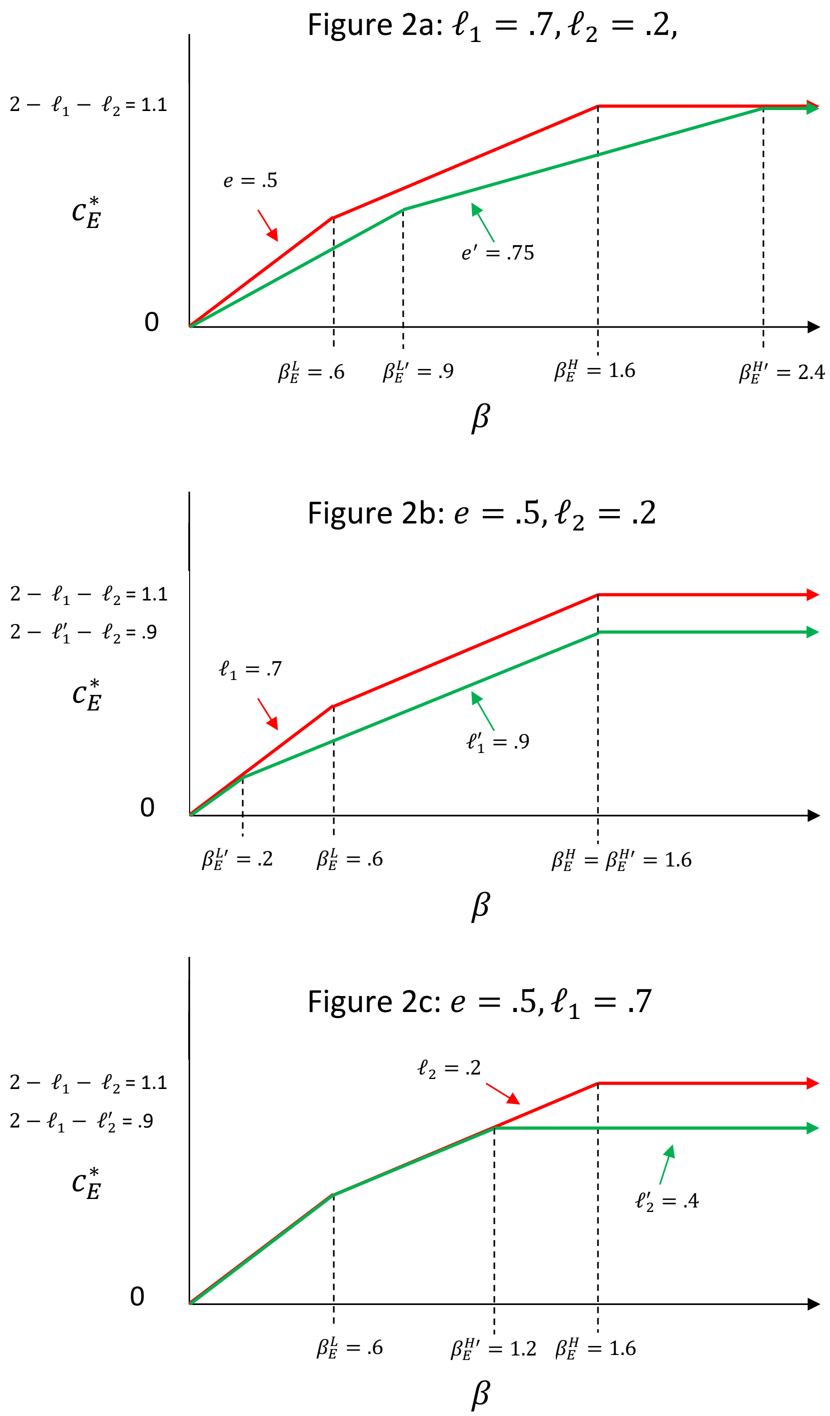
[a] When $\beta = \beta_V^L$, the values $c_E^* = \frac{\beta}{2e}$ and $c_E^* = (1 - \ell_1) + \frac{\beta}{4e}$ are equal; and when $\beta = \beta_V^H$ the values c_E^* and $2 - \ell_1 - \ell_2$ are equal.

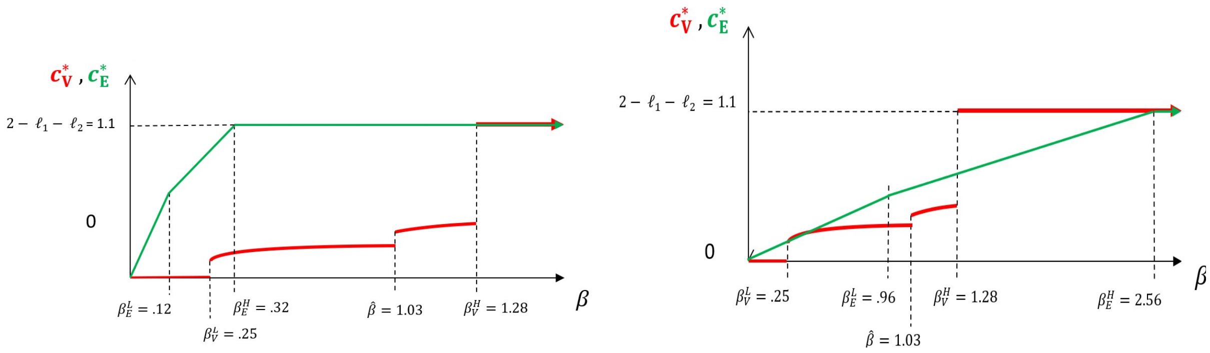
3 Institutional Comparative Statics

Having solved for equilibrium effort under different reward systems, we can now solve for the (vote-maximizing) party P's optimal choice between the two systems. We say that a system *weakly dominates* the other if, for a given set of values on $\{\ell_1, \ell_2, e\}$, it generates (weakly) higher levels of total equilibrium effort **for all** values of β.[12] The following Theorem presents *sufficient* conditions for weak dominance of effort-based reward systems (proof in Appendix D).

[12] If $\beta \to \infty$ both Brokers will choose $c_i^* = 1 - \ell_i^*$ irrespective of organizational structure. As such, one structure will only ever be 'weakly' superior to one another.

Figure 2: Equilibrium Effort under the 'Effort-Based' Rule





(a) Low uncertainty: $e = 0.1$ (b) High uncertainty: $e = 0.8$

Figure 3: Uncertainty and the Distribution of Party Loyalists. The red line represents the expected clientelist effort under vote-based rules, while the green line is for effort-based system. Without loss of generality $\ell_1 + \ell_2 = 0.9$.

Theorem 1 *Let $\overline{\sigma}_1 > \overline{\sigma}_2$ and $f(r_i) = 1 - c_i^2$. If $e < min[\frac{1-\ell_2}{2}, \frac{(\ell_1-\ell_2)^2}{4(1-\ell_1)}]$ then $\beta^j_E < \beta^j_V$, for $j \in \{H, L\}$, and so $c^*_E(\beta) \geq c^*_V(\beta)\ \forall \beta$.*

If the condition on e in Theorem 1 is met, this indicates that the max-effort corner equilibrium is reached at a lower level of β under the effort-based rule; and that under the effort-based system, the brokers move to their intermediate range of equilibria "before" the vote-based one. Larger values of e push both β^L_E and β^H_E up, and make weak dominance of the effort-based system less likely. As well, the effort-based system will tend to be Weakly Dominant region 1 has many more loyalists than region 2 (i.e. when $\ell_1 - \ell_2$ is large), and vice versa when the two regions are fairly homogeneous (i.e. $\ell_1 - \ell_2$ is small).

What happens when the sufficient conditions in Theorem 1 are not met? Consider, for example, the comparative statics when $e > \frac{1-\ell_2}{2}$ (i.e. when $\beta^H_V < \beta^H_E$).

Proposition 3 *If $f(r_i) = 1 - c_i^2$ and $e > \frac{1-\ell_2}{2}$ then: **(i)** if $\beta < \min\{\beta^L_V, \beta^L_E\}$ then $C^*_E > C^*_V$; **(ii)** if $\beta \in [\min\{\beta^L_V, \beta^L_E\}, \beta^H_V]$ then $C^*_V > C^*_E$ iff e (ℓ_1) is sufficiently large (small); **(iii)** if $\beta \in [\beta^H_V, \beta^H_E]$ then $C^*_V > C^*_E$; **(iv)** if $\beta \geq \beta^H_E$ then $C^*_E = C^*_V$.*

Condition (i) in Proposition 3 tells us that, in all cases, if β is small enough the effort-based rule is optimal. Indeed, for precisely this reason the vote-based system never meets the weak dominance criterion: when brokers care little about the prize, it is *always* best to use an effort-based rule. Conditions (ii) and (iii) tell us that, as β adopts intermediate values, the vote-based system becomes optimal; and that this range of optimality increases when information is poor (high e) and broker primaries are competitive (low ℓ_1). By condition (iv), at sufficiently high levels of β the systems become indistinguishable (both generate max effort).

Figure 3 presents the implications of Theorem 1 and Proposition 3 graphically. In both panels of the figure, we set $\ell_1 = .7$ and $\ell_2 = .2$. Moving from panel a) to b) uncertainty increases from $e = .1$ to $e = .8$. In Figure 3a, $\beta_E^H < \beta_V^H$ and $\beta_E^L < \beta_V^L$, and the effort-based system is Weakly Dominant. In Figure 3b, both $\beta_E^L > \beta_V^L$ and $\beta_E^H > \beta_V^H$, and the conditions in Proposition 3 apply: while the effort-based system is still optimal at very low values of β, at intermediate values $\beta_V^H < \beta < \beta_E^H$ the vote-based system is often optimal; and the same is true for a small range of values between β_V^L and β_E^L. In sum, when e is small enough, rewarding effort is weakly dominant. However, as e increases, the optimal organizational structure will increasingly depend on the level of β and the distance between ℓ_1 and ℓ_2. While the effort-based system will always be optimal for low values of β, when there is stiff competition between brokers, i.e. when ℓ_1 and ℓ_2 are not far apart and β takes on intermediate values, the vote-based system becomes optimal.

4 Implications, Extensions and Applications

In this section we discuss extensions and applications of the model that still can be fit within its testable implications. The most immediate theoretical extension will be to include a strategic opposition party and brokers. As formally addressed in Appendix E, even with inter-party competition, only the effort-based system should ever satisfy weak dominance; and the vote-based system still tends to be optimal when e is large, $\ell_1 - \ell_2$ is small, and β is intermediate. Furthermore, just as competitiveness between brokers from *the same* party favors the vote-based system, competitiveness between brokers from *competing* parties should favor the vote-based system. Put simply:

Testable Implication 1 *High competition between local agents, whether from the same party and/or from competing parties, should reduce the need to monitor their effort, and increase the incentive to use 'broker primaries'.*

One could also consider the model predictions to study the sub-national variance in clientelist practices across districts, regions, and time. For example, a series of papers has used the relative size and/or number of polling stations in a district as a proxy for parties' local 'information', and in turn their ability to monitor broker effort and/or performance (Larreguy 2013; Larreguy et al. 2016; Bowles et al. 2020).[13] This idea is summarized as follows:

Testable Implication 2 *We would expect to see reward mechanisms based on effort inferences where there are many polling stations per capita; and 'broker primaries' where a paucity of polling stations makes performance/effort signals noisier.*

Given that different reward-schemes are likely to be embodied in specific organizational structures, our findings may contribute to a broader understanding of party organizational choice. For example, recent work on brokers in Ghana has identified the importance of decentralized organizations, where reward decisions are controlled by local branches and primary elections determine party candidates (Ichino and Nathan 2012, 2013; Brierly and Nathan 2021). Using decentralized nomination or promotion rules will often serve as a credible commitment to allocating β to the regional vote winner, *even if* that vote winner is perceived *ex post* to have exerted less effort than her competitor. Our theory thus has implications for the relative performance of decentralized as opposed to centralized organizations, which could be assessed at the cross-national level following efforts by Kitschelt and Kselman (2013) or Yildirim and Kitschelt (2020).

Similarly, our framework could partly help understanding the dynamic implications of these rules, i.e, as a commitment to lower clientelism. As stated by Holland and Palmer-Rubin (2015, pg. 1188), "At the macrohistorical level, the path from clientelism to programmatic politics likely depends on the type of brokerage relationship that characterizes a political system." Recent work has demonstrated the important role that brokerage patterns play in moving parties from 'vote-buying' to 'relational' clientelism (Nichter 2019). Ichino and Nathan (2021) argue that the

[13] In a similar vein, our model makes predictions about the optimal reward mechanisms for different types of brokers. For example, it is more challenging to consistently monitor 'non-party' brokers than it is to monitor 'organizational' brokers with long-standing ties to specific organizations (Holland and Palmer-Rubin 2015). Thus, in districts and/or political parties where 'non-party' ('organizational') brokers are prominent, we should observe vote-based (effort-based) reward schemes.

use of primary elections to allocate organizational positions in Ghana has actually reduced the value of clientelism itself, facilitating a move towards and more diverse and programmatically oriented electorate. In explaining why parties choose particular organization rules and reward structures, we thus seek to better understand a process which itself effects a country's broader political development and modernization.

5 Conclusion

This paper addresses one of the main challenges of political parties: that of incentivizing local agents. We identify conditions under which vote- as opposed to effort-based reward allocation mechanisms serve to minimize broker rent-seeking. When parties have reasonably accurate information, they can minimize rent-seeking by allocating prizes to brokers based on their effort allocations. In contrast, when a.) this information is highly 'noisy', and b.) competition between brokers is stiff enough, parties can minimize rent-seeking using 'broker primaries' which do not require direct effort-monitoring. In identifying the situations in which party resources are directly consumed by brokers rather than transferred to voters in the form of clientelist effort, the paper helps identify the conditions under which clientelism can be seen as a more or less nefarious political practice.

Works Cited

- Auerbach, Adam Michael. 2020. Demanding Development: The Politics of Public Goods Provision in India's Urban Slums. New York: Cambridge University Press

-Auerbach, Adam Michael, and Tariq Thachil. "How clients select brokers: Competition and choice in India's slums." The American Political Science Review 112, no. 4 (2018): 775-791.

-Bowles, J., Larreguy, H., Liu, S. (2020). How weakly institutionalized parties monitor brokers in developing democracies: Evidence from postconflict Liberia. American Journal of Political Science, 64(4), 952-967.

-Brierley, Sarah, and Noah L. Nathan. 2021 "Motivating the Machine: Which Brokers Do Parties Pay?. Journal of Politics, Forthcoming.

-Casas, Agustin. 2018. "Distributive Politics with Vote and Turnout Buying." *American Political Science Review*, Vol. 112(4): 1111-1119.

-Camp, Edwin. 2017. “Cultivating Effective Brokers: A Party Leader’s Dilemma.” *British Journal of Political Science*, Vol. 47(3), 521–543.

-Corchón, Luis C., and Marco Serena. ”Contest theory.” In Handbook of Game Theory and Industrial Organization, Volume II. Edward Elgar Publishing, 2018.

-Crutzen, Benoît SY, Sabine Flamand, and Nicolas Sahuguet. ”A model of a team contest, with an application to incentives under list proportional representation.” Journal of Public Economics 182 (2020): 104109.

-Dasgupta, Partha and Maskin, Eric. ”The existence of equilibrium in discontinuous economic games, I: Theory”. Review of Economic Studies, volume 53, 1, p1-p26, 1986.

-Denter, Philipp. ”Campaign contests.” European Economic Review 127 (2020): 103459.

- De Kadt, D., Larreguy, H. A. (2018). Agents of the regime? Traditional leaders and electoral politics in South Africa. The Journal of Politics, 80(2), 382-399.

-Finan, Federico and Laura Schechter. 2012. “Vote Buying and Reciprocity.” *Econometrica*, Vol. 80(2): 863-81.

-Gingerich, Daniel W. ”Buying Power: Electoral Strategy before the Secret Vote.” American Political Science Review 114, no. 4 (2020): 1086-1102.

- Gottlieb J. Explaining Variation in Broker Strategies: A Lab-in-the-Field Experiment in Senegal. Comparative Political Studies. 2017;50(11):1556-1592.

-Holland, Alisha C. and Brian Palmer-Rubin. 2015. “Beyond the Machine: Clientelist Brokers and Interest Organizations in Latin America.” *Comparative Political Studies*, Vol. 48(9), 1186–1223.

-Ichino, Nahomi and Noah L. Nathan. 2013. “Do Primaries Improve Electoral Performance? Clientelism and Intra-Party Conflict in Ghana.” *American Journal of Political Science*, Vol. 57(2): 428–441.

-Keefer, Philip and Razvan Vlaicu. 2008. “Credibility, Clientelism and Democracy.” *Journal of Law, Economics, and Organization*, 24(2).

-Kitschelt, Herbert, and Daniel Kselman. 2013. “Economic Development, Democratic Experience, and Political Parties’ Linkage Strategies.” *Comnparative Political Studies*, Vol. 46(11): 1453-84.

-Larreguy, Horacio, John Marshall, Pablo Querubin. 2016. “Parties, Brokers, and Voter Mobilization: How Turnout Buying Depends on Parties’ Ability to Monitor Brokers.” *American Political Science Review*: Vol. 110(1), 160-79. -Larreguy, H., Montiel Olea,

C.E. and Querubin, P. (2017), Political Brokers: Partisans or Agents? Evidence from the Mexican Teachers’ Union. American Journal of Political Science, 61: 877-891.

-Muñoz, P. (2014). An Informational Theory of Campaign Clientelism: The Case of Peru. Comparative Politics, 47(1), 79–98.

-Nichter, Simeon. (2019). *Votes for Survival: Relational Clientelism in Latin America.* Cambridge University Press.

-Novaes, Lucas M. ”Disloyal brokers and weak parties.” American Journal of Political Science 62, no. 1 (2018): 84-98.

-Ravanilla, N., Haim, D., Hicken, A. (2021). Brokers, social networks, reciprocity, and clientelism. American Journal of Political Science.

-Stokes, Susan C. 2005. “Perverse Accountability: A Formal Model of Machine Politics with Evidence from Argentina.” *American Political Science Review*, Vol. 99(3): 315-25.

-Stokes, Susan C., Thad Dunning, Marcelo Nazareno and Valeria Brusco. 2013. *Brokers, Voters, and Clientelism: The Puzzle of Distributive Politics.* Cambridge: Cambridge University Press.

-Szwarcberg, Mariela. 2014. "Political Parties and Rallies in Latin America." *Party Politics*, Vol. (20:3): 456-66.

-Toral, Guillermo. "How patronage delivers: Political appointments, bureaucratic accountability, and service delivery in Brazil." Forthcoming at the *American Journal of Political Science*.

-Yıldırım, K., Kitschelt, H. (2020). Analytical perspectives on varieties of clientelism. Democratization, 27(1), 20-43.

-Zarazaga, Rodrigo. 2014. "Brokers beyond clientelism: a new perspective on brokerage through the Argentine case." *Latin American Politics and Society*, Vol. 56(3): 23–45.

Appendix (to be published online)

In this appendix the brokers are called agents, due to the principal-agent nature of the game.

Effort- vs vote-based rules

Since effort is non verifiable, parties who want to maximize votes can only "write" contracts based on the observed votes in each region. We analyze two contracts where the party allocates the rewards after observing realized outcomes, i.e., votes. Broadly speaking, the probability of agent 1 obtaining the reward is $P(v_1 - v_2 > x)$, where x depends on the type of contract. In the vote-based one, $x = 0$ and so whoever obtains the most votes, regardless of effort, wins the prize. There is no aggregate uncertainty, since vote shares v_1 and v_2 can be observed, and so there is no actual signalling. In contrast, in the effort-based contract $x = \tilde{\sigma}$ is a random variable with positive mean. Thus, the party allocates the reward to broker 1 with positive probability when the difference in votes obtained is larger than a threshold, which itself is function of the expected advantage. Put simply, after observing votes, the party can infer who made the most effort, probabilistically. Thus, using $P(v_1 - v_2 > \tilde{\sigma})$ to allocate rewards is mathematically identical to $P(c_1 - c_2 > 0)$. This is why we call the second contract "effort-based", even though effort is inferred from votes. I.e.,

$$P[c_1 - c_2 > 0] = P[(v_1 + \mu - \sigma_1) - (v_2 + \mu - \sigma_2) > 0] = P[v_1 - v_2 > \sigma_1 - \sigma_2] = P[v_1 - v_2 > \tilde{\sigma}]$$

A Vote-based Parties, Pure Strategy Equilibria

Here we solve the general model for any increasing concave function $f(r_i)$. The extension to the case of $f(r_i) = 1 - c_i^2$, the functional form used in the text, is then straightforward. We begin with a Lemma.

Lemma A1 *If either agent chooses positive effort (i.e. if $c_1 > 0$ and/or $c_2 > 0$), the strategy vector can only be a pure strategy Nash Equilibrium if $v(c_1) = v(c_2) = 1$.*

Proof: If $c_1, c_2 > 0$ but $v_1(c_1) \neq v_2(c_2)$, then one of the two agents chooses $c_i > 0$ without winning β, and would have the incentive to either drop constituency effort to 0, or raise it just high enough to secure β. If $c_1, c_2 > 0$ but $v_1(c_1) = v_2(c_2) < 1$ then by construction $c_1 < c_2$, and agent 1 could increase c_1 infinitesimally so as to just surpass agent 2's regional total, and thus win β. If $c_1 > 0$ and $c_2 = 0$ then agent 1 has the incentive to lower her effort allocation to $c_1 = 0$. Finally, agent 2's best response to the effort $c_1 = 0$ is to choose $c_2 = 0$ if $\beta < f(1 - \ell_1 - \ell_2)$ and $c_2 = \ell_1 - \ell_2 + \epsilon$ ($\epsilon \to 0$) if $\beta > f(1 - \ell_1 - \ell_2)$. If agent 2 chooses $c_2 = \ell_1 - \ell_2 + \epsilon$, then agent 1 could raise here effort infinitesimally to $c_1 = 2\epsilon$ and win β. **QED**

Proposition A1 *If $\bar{\sigma}_1 > \bar{\sigma}_2$ and $f(r_i)$ is increasing and concave, then:*

- $c_1^* = c_2^* = 0$ *is the unique PSNE if an only if* $\beta \leq [f(1) - f(1 + \ell_2 - \ell_1)]$.

- $c_i^* = 1 - \ell_i$ *is the unique PSNE if and only if* $\beta \geq 2 \cdot [f(1) - f(\ell_2)]$.
- *If* $[f(1) - f(1+\ell_2-\ell_1)] < \beta < 2 \cdot [f(1) - f(\ell_2)]$ *then there is a mixed-strategy PSNE such that* $0 < \mathrm{E}(c_i^*) < (1 - \ell_i)$.

Proof of Proposition A1

Bullet 1: At the bottom corner outcome $c_1 = c_2 = 0$ agent 1 wins β receives the maximum possible utility $f(1) + \beta$, while agent 2 receives $f(1)$. Agent 2's optimal deviation would be $c_2 = \ell_1 - \ell_2 + \epsilon$ ($\epsilon \to 0$), the value of effort necessary to just barely push her vote share above that of agent 1, making $r_i = 1 - (\ell_1 - \ell_2) = 1 + \ell_2 - \ell_1$. This deviation is optimal if $\beta > [f(1) - f(1 + \ell_2 - \ell_1)]$. In turn, $c_1 = 0$ and $c_2 = 0$ will be a PSNE as long $\beta \leq [f(1) - f(1+\ell_2-\ell_1)]$. If $\beta = [f(1) - f(1+\ell_2-\ell_1)]$ it is an PSNE in weakly-dominant strategies.

Bullet 2: At the strategy vector $\{1 - \ell_1, 1 - \ell_2\}$, agent i wins the nomination with probability $\frac{1}{2}$ and receives utility $f(1 - (1 - \ell_i)) + \frac{\beta}{2} = f(\ell_i) + \frac{\beta}{2}$. The optimal deviation from this outcome would be to devote their entire unit of effort to r_i and receive utility $f(1)$. Since $\ell_1 > \ell_2$, if agent 2 prefers the outcome $f(\ell_2) + \frac{\beta}{2}$ to the outcome $f(1)$, then agent 1 also prefers $f(\ell_1) + \frac{\beta}{2}$ to $f(1)$. In turn, the strategy vector $\{1 - \ell_1, 1 - \ell_2\}$ will be a PSNE as long $f(\ell_2) + \frac{\beta}{2} \geq f(1)$ which can be rewritten as $\beta \geq 2 \cdot [f(1) - f(\ell_2)]$. If $\beta = 2 \cdot [f(1) - f(\ell_2)]$ it is an PSNE in *weakly-dominant* strategies.

Given that $\ell_1 > \ell_2$, it is straightforward to see that $2[f(1)-f(\ell_2)] > [f(1)-f(1+\ell_2-\ell_1)]$, i.e. that the conditions for PSNE in which $c_i^* = 1 - \ell_i$ and those in which $c_i^* = 0$ are mutually exclusive. Combined with Lemma 1, this establishes that PSNE in Bullets 1 and 2 are unique.

Bullet 3: If $[f(1) - f(1 + \ell_2 - \ell_1)] < \beta < 2 \cdot [f(1) - f(\ell_2)]$ then neither the strategy vector $\{1-\ell_1, 1-\ell_2\}$ nor $\{0, 0\}$ is a PSNE. Furthermore, by Lemma A1 we know that no other strategy vector can be a pure strategy PSNE. Dasgupta and Maskin (1986a, 1986b) show sufficient continuity conditions for existence of MSNE in games with discontinuous payoffs. The continuity requirements in Theorem 5 of Dasgupta and Maskin (1986) are met in our game: (i) the sum of utility functions is upper semi-continuous in c_i for all i and (ii) each player utility is bounded and weakly lower semi-continuous. Note that $\sum U_i = f(r_1) + f(r_2) + \beta$, which is continuous. Moreover, let $\bar{c}_i$ for all i be the points of discontinuity such that $\bar{c}_i : v(c_i) = v(c_{-i})$. Then, $\lim u(c_1, \bar{c}_2)$ when $c_1 \lim \bar{c}_1$ by the right, is greater than $u(\bar{c}_1, \bar{c}_2)$. Thus, it is right lower semi-continuous.[A1] Furthermore, we know that choosing $c_i > 1 - \ell_i$ with positive probability is a strictly dominated strategy. In turn, the maximum value of *expected effort* $\mathrm{E}(c_i^*)$ will occur when agent i plays $c_i = 1 - \ell_i$ with probability 1, and the expected effort $\mathrm{E}(c_i^*)$ at any mixed strategy PSNE will be less than $1 - \ell_i$. **QED**

[A1]In other words, Definition 6 in DM holds for $\lambda = 0$, as per their notation.

B Vote-based Parties, Mixed Strategy Nash Equilibrium (MSNE)

In principle, since Dasgupta and Maskin (1986a,1986b) guarantees the existence of MSNE, both Agents could mix over the whole support. However, we can focus on MSNE in which Agents' put a positive mass in 0 and the "corner". The latter is either $1-\ell_i$, the valuation of the prize ($\sqrt{\beta}$) or, for agent 2, $\ell_1 - \ell_2$. All other actions are strictly dominated. Hence, unless otherwise stated the mix is unique. Note that, following tie-breaking makes the analysis more realistic and simple: if Agents receive equal vote shares, and one Agent chooses positive effort while the other Agent chooses 0 effort, the Agent who spends a positive amount of money wins the contest.[A2] As in the main body of the text, we assume the quadratic cost function $f(r_i) = 1 - c_i^2$. Recall from the text that $\beta_V^L = (\ell_1 - \ell_2)^2$ and $\beta_V^H = 2(1-\ell_2)^2$, and define $\hat{\beta} = \beta_V^H - \beta_V^L$ where $\beta_V^L < \hat{\beta} < \beta_V^H$.

Lemma A2 *In equilibrium, broker 1 only plays* 0 *and/or* $\max\{\sqrt{\beta}, (1-\ell_1)\}$*, while broker 2 puts a positive mass on* 0*,* $\ell_1 - \ell_2$ *and/or* $\max\{\sqrt{\beta}, (1-\ell_2)\}$.

Proof of Lemma A2

Upper bound is $\max\{\sqrt{\beta}, (1-\ell_i)\}$**.** The value of the prize is β and the cost of effort is c_i^2, hence brokers never allocate effort greater than $\sqrt{\beta}$. Additionally, with the vote-based rule all strategies $c_i > 1 - \ell_i$ are strictly dominated by $1 - \ell_i$ because they all result in 100% vote share.

Lower bound is 0. (i) If broker 1 wins playing any $c_1 = x > 0$ while broker 2 plays $c_2 = y$, then any other strategy $c_i = \max\{0, (x + y - (\ell_1 - \ell_2)/2$ strictly dominates x. In particular if $x+y < (\ell_1 - \ell_2)$, then 0 dominates x. (ii) If broker 1 loses playing $c_1 = x > 0$ while broker 2 plays $c_2 = y$, then $c_1 = 0$ strictly dominates x. Similar reasoning applies to broker 2.

Thus, broker 1 does not put any positive mass in $(0, \max\{\sqrt{\beta}, (1-\ell_1)\})$. Similarly, 0 and $\max\{\sqrt{\beta}, (1-\ell_i)\}$ are not strictly dominated for player 2. It remains to see whether $c_2 = \ell_1 - \ell_2$ is strictly dominated for player 2. It turns out that for $c_1 = 0$, it may be a best response, hence it is not strictly dominated. **QED**

Now, in Lemma A3 we show that there is a region in which $1 - \ell_2$ is strictly dominated and so, instead of mixing between 0 and $1 - \ell_2$, player 2 mixes between 0 and $\ell_1 - \ell_2$. Then, in Proposition A2 we characterize the MSNE when both players mix between 0 and $1 - \ell_i$. Instead, in Proposition A3 we characterize the equilibrium for when player 2 mixes between 0 and $\ell_1 - \ell_2$. Player 1 would mix between 0 and $\max\{(1-\ell_1)^2, \sqrt{\beta}\}$.

Lemma A3 *Let* $\beta \in [\beta_V^L, \beta_V^H]$ *and let Agent 1 play* 0 *and* $1 - \ell_1$ *with probability* p *and* $1 - p$*, respectively. There exists* $\hat{\beta}$ *such that for all* $\beta \leq \hat{\beta}$*, the action* $1 - \ell_2$ *is strictly dominated for Agent 2.*

[A2] In equilibrium, this would imply that Agent 2 beats Agent 1 when $c_1 = 0$ and $c_2 = \ell_1 - \ell_2$.

Proof *Let players put a non-negative mass in two strategies, either 0 or the corner, where we let $\ell_1 - \ell_2$ be a corner for agent 2. Let $p = \Pr[c_1 = 0]$. Let p_1 and p_2 be such that $u_2(0|p)$ equals $u_2(\ell_1 - \ell_2|p)$ and $u_2(1 - \ell_2|p)$, respectively. Then, for $p_1 < p_2$, $1 - \ell_2$ is strictly dominated. I.e., for $\beta < 2(1 - \ell_2)^2 - (\ell_1 - \ell_2)^2 = \beta_V^H - \beta_V^L$.* ***QED***

Proposition A2 (Mixed Strategy Profile 1) *If $\bar{\sigma}_1 > \bar{\sigma}_2$, $f(r_i) = 1 - c_i^2$, and $\beta \in [\hat{\beta}, \beta_V^H]$, then there is a MSNE in which Agent 1 plays $c_1 = 0$ with probability $p = \{\frac{2(1-\ell_2)^2}{\beta} - 1\}$ and $c_1 = 1 - \ell_1$ with probability $1 - p$; while Agent 2 plays $c_2 = 0$ with probability $q = \{1 - \frac{2(1-\ell_1)^2}{\beta}\}$ and $c_2 = 1 - \ell_2$ with probability $1 - q$. Moreover, this equilibrium is unique.*

Proof of Proposition A2

Proof *Let Agent 1 mix over the corner outcomes with $p = \Pr(c_1 = 0)$ and $(1 - p) = \Pr(c_1 = 1 - \ell_1)$, and define $Eu_2(c_2|p)$ as the expected value Agent 2 receives from playing c_2 given some p.*

Broker 2 mixes between 0 and $1 - \ell_2$. *In this MSNE Agent 1 must choose $\bar{p}$ such that player 2 is indifferent between playing $1 - \ell_2$ and 0:*

$$Eu_2(0|\bar{p})) = Eu_2(1 - \ell_2|\bar{p}) \implies \tag{A1}$$

$$1 = \bar{p}\left[\beta + 1 - (1 - \ell_2)^2\right] + (1 - \bar{p})\left[\frac{\beta}{2} + 1 - (1 - \ell_2)^2\right] \implies \tag{A2}$$

$$\bar{p} = \left\{\frac{2(1 - \ell_2)^2}{\beta} - 1\right\}. \tag{A3}$$

Similarly, let Agent 2 mix over the corner outcomes with $q = \Pr(c_2 = 0)$ and $(1 - q) = \Pr(c_2 = 1 - \ell_2)$. Define $\bar{q}$ such that $Eu_1(0|\bar{q})) = Eu_1(1 - \ell_1|\bar{q})$, then $\bar{q} = 1 - \frac{2(1-\ell_1)^2}{\beta}$ (algebra omitted).

Having established the value of $\bar{p}$ and $\bar{q}$ we now need to confirm that they are bounded between 0 and 1 over the range $\beta \in [\hat{\beta}, \beta_V^H]$. This is true for $\bar{p}$ if only if $\beta < 2(1 - \ell_2)^2$ and $\beta > (1 - \ell_2)^2$, respectively; and for $\bar{q}$ if and only if $\beta > 2(1 - \ell_1)^2$. Since by definition $\beta < \beta_V^H$ for the MSNE to exist, if $\beta > max\{(\ell_1 - \ell_2)^2, 2(1 - \ell_1)^2\}$ this suffices to ensure that both mixing probabilities are between 0 and 1 for all $\beta > \hat{\beta}$.

The equilibrium above is unique among those where each broker plays 0 and $1 - \ell_i$ with a positive mass. But for all $\beta > \tilde{\beta}$, broker 2 may instead allocate a positive mass to $\ell_1 - \ell_2$ and $1 - \ell_2$ when $\ell_1 - \ell_2$ dominates 0. To show that there is no MSNE that satisfies that requirement, we proceed by obtaining the mix and we show that it cannot hold for $\beta > \tilde{\beta}$.

Broker 2 mixes between $\ell_1 - \ell_2$ and $1 - \ell_2$. *In this MSNE Agent 1 must choose $\bar{p}$ such that player 2 is indifferent between playing $1 - \ell_2$ and $\ell_1 - \ell_2$:*

$$Eu_2(\ell_1 - \ell_2|\bar{p})) = Eu_2(1 - \ell_2|\bar{p}) \implies \tag{A4}$$

Then, $p = \frac{\beta - 2[(1-\ell_2)^2 - (\ell_1-\ell_2)^2]}{\beta}$*, which is always between* 0 *and* 1 *in this region. Similarly, let Agent 2 mix over the corner outcomes with* $q = \Pr(c_2 = \ell_1 - \ell_2)$ *and* $(1-q) = \Pr(c_2 = 1-\ell_2)$*. Define* $\bar{q}$ *such that* $Eu_1(0|\bar{q})) = Eu_1(1-\ell_1|\bar{q})$*, then* $q = \frac{2(1-\ell_1)^2}{\beta} - 1$*. This probability is between* 0 *and* 1 *if and only if* $\beta \in [(1-\ell_1)^2, 2(1-\ell_1)^2]$*, which is always lower than* $\tilde{\beta}$*. Hence, there is no MSNE like this, and the unique equilibrium in mixed strategies for* $\beta_v^H > \beta > \tilde{\beta}$ *is the one found above.* ***QED***

Proposition A3 (Mixed Strategy Profile 2) *If* $\bar{\sigma}_1 > \bar{\sigma}_2$*,* $f(r_i) = 1 - c_i^2$*, and* $\beta \in [\beta_V^L, \hat{\beta}]$*, then there exists a MSNE in which*

- *for* $\beta \in [\max\{\beta_V^L, (1-\ell_1)^2\}, \hat{\beta}]$ *Agent 1 plays* $c_1 = 0$ *with probability* $p = \{\frac{2(1-\ell_2)^2}{\beta} - 1\}$ *and* $c_1 = 1-\ell_1$ *with probability* $1-p$*; while Agent 2 plays* $c_2 = 0$ *with probability* $q = \{1 - \frac{2(1-\ell_1)^2}{\beta}\}$ *and* $c_2 = 1-\ell_2$ *with probability* $1-q$*. Notice that* $\max\{\beta_V^L, (1-\ell_1)^2\} = \beta_V^L$ *iff* $\bar{\sigma}_1 > \frac{1+\bar{\sigma}_2+\mu}{2}$*.*

- *for* $\beta_V^L \le \beta < (1-\ell_1)^2$*, then* $c_1 = 0$ *with probability* $p = \frac{(\ell_1-\ell_2)^2}{\beta}$ *and* $c_1 = \sqrt{\beta}$ *with probability* $1-p$*, while* $q = 0$*.*

Proof of Proposition A3

Proof *Let Agent 1 mix over the corner outcomes with* $p = \Pr(c_1 = 0)$ *and* $(1-p) = \Pr(c_1 = 1-\ell_1)$*, and define* $Eu_2(c_2|p)$ *as the expected value Agent 2 receives from playing* c_2 *given some* p*. In a MSNE Agent 1 must choose* $\bar{p}$ *such that player 2 is indifferent between playing* $\ell_1 - \ell_2$ *and* 0*:*

$$Eu_2(0|\bar{p})) = Eu_2(\ell_1 - \ell_2|\bar{p}) \implies \tag{A5}$$

$$1 = \bar{p}\{\beta + 1 - (\ell_1-\ell_2)^2\} + (1-\bar{p})\{1 - (\ell_1-\ell_2)^2\} \implies \tag{A6}$$

$$\bar{p} = \frac{(\ell_1-\ell_2)^2}{\beta}. \tag{A7}$$

Similarly, let Agent 2 mix over the corner outcomes with $q = \Pr(c_2 = 0)$ *and* $(1-q) = \Pr(c_2 = \ell_2 - \ell_2)$*. Define* $\bar{q}$ *such that* $Eu_1(0|\bar{q})) = Eu_1(1-\ell_1|\bar{q})$*, then* $\bar{q} = 1 - \frac{(1-\ell_1)^2}{\beta}$ *(algebra omitted).*

Having established the value of $\bar{p}$ *and* $\bar{q}$ *we now need to confirm that they are bounded between* 0 *and* 1 *over the range* $\beta \in [\beta_V^L, \hat{\beta}]$*. The mixing probabilities are bounded between 0 and 1 iff* $\beta > max\{(\ell_1-\ell_2)^2, (1-\ell_1)^2\}$*, which is always the case in* $[\beta_V^L, \hat{\beta}]$ *for* $\beta_V^L = (\ell_1-\ell_2)^2 > (1-\ell_1)^2\}$*, which in turn holds if* $1 + \ell_2 < 2\ell_1$*. This implies that district 1 and 2 are sufficiently different, i.e.,* $\frac{1+\ell_2}{2} < \ell_1$*, which also implies* $\implies \bar{\sigma}_1 > \frac{1+\bar{\sigma}_2+\mu}{2}$*. Similarly, when* $\beta < (1-\ell_1)^2$*, the maximum that player 1 is willing to spend is* $\sqrt{\beta}$*. Hence we find* $p : Eu_2(0|p) = Eu_2(\ell_1-\ell_2|p)$ *and* $q : Eu_1(0|q) = Eu_1(\sqrt{\beta}|q)$*.* ***QED***

B.1 Comparative statics: Total clientelism with vote-based rules.

On page 5-6 of the paper, we discuss how the total clientelism under the MSNE in the vote-based system changes with its parameters, i.e., $E(c^*_V; \ell_1, \ell_2, \beta)$. Here we show the formal proofs.

Corollary 1 *Let* $\ell_1 > \frac{1+\ell_2}{2}$*, then* $sign(\frac{\partial E(c^*_V)}{\partial \beta}) = sign(\frac{\partial E(c^*_V)}{\partial \ell_2}) = -sign(\frac{\partial E(c^*_V)}{\partial \ell_1}) > 0$

Before the proof, note that $\ell_1 > \frac{1+\ell_2}{2}$ is a necessart condition. Also, for easy of exposition, we show the expected values of aggregate effort of c^*_V.

$$E(c^*_V) = \begin{cases} (1-\bar{p})(1-\ell_1) + (1-\bar{q})(1-\ell_2) & \text{if } \beta \in [\hat{\beta}, \beta^H_V] \\ (1-\bar{p})\min\{\sqrt{\beta}, (1-\ell_1)\} + (1-\bar{q})(\ell_1-\ell_2) & \text{if } \beta \in [\beta^L_V, \hat{\beta}) \end{cases}, \tag{A8}$$

For simplicity, we assume $\sqrt{\beta} > (1-\ell_1)$ and – substituting the corresponding p^* and q^* from Propositions A2 and A3– we obtain the following expressions.

$$E(c^*_V) = \begin{cases} 2(1-\ell_1) - \frac{2}{\beta}[(1-\ell_1)(1-\ell_2)(\ell_1-\ell_2)] & \text{if } \beta \in [\hat{\beta}, \beta^H_V] \\ (1-\ell_1) + \frac{1}{\beta}[(1-\ell_1)(\ell_1-\ell_2)(1+\ell_2-2\ell_1)] & \text{if } \beta \in [\beta^L_V, \hat{\beta}) \end{cases}, \tag{A9}$$

Proof of Corollary 1

Proof *i) From equation A9, it is apparent when* $\frac{\partial E(c^*_V)}{\partial \beta} > 0$*. For large* β*, it is always the case, while for lower* β*,* $1+\ell_2-2\ell_1 < 0$ *is needed.*

ii) Similarly, $\frac{\partial E(c^*_V)}{\partial \ell_1} < 0$ *if* $1+2\ell_2-3\ell_1 < 0$*. For large* β *the derivative of Equation A9 w/r/t* ℓ_1 *is:*

$$\frac{\partial E(c^*_V)}{\partial \ell_1} = -2 - \frac{2}{\beta}\left[(1-\ell_1)(1-\ell_2) - (\ell_1-\ell_2)(1-\ell_2)\right]. \tag{A10}$$

Rearranging, we see that $\frac{\partial E(c^*_V)}{\partial \ell_1} < 0$ *iff:*

$$\beta < -\left[(1-\ell_1)(1-\ell_2) - (\ell_1-\ell_2)(1-\ell_2)\right]. \tag{A11}$$

The right-hand side is increasing in ℓ_1*, i.e. it is most likely to be satisfied when* $\ell_1 = 1$*. Substituting* $\ell_1 = 1$ *yields* $\beta < (1-\ell_2)^2$*, which contradicts the baseline condition* $\beta > \hat{\beta} = 2(1-\ell_2)^2 - (\ell_1-\ell_2)^2$*, i.e. in the range* $\beta \in [\hat{\beta}, \beta^H_V]$ *the derivative* $\frac{\partial E(c^*_V)}{\partial \ell_1}$ *will never be greater than* 0.[A3]

Via identical reasoning, it is possible to show that in the range $\beta \in [\beta^L_V, \hat{\beta}]$ *the derivative of A9 wrto* ℓ_1 *is smaller than 0 iff*

$$\beta > \left[(1-2\ell_1+\ell_2)^2 - 2(1-\ell_1)(\ell_1-\ell_2)\right]. \tag{A12}$$

In the case in which $\beta > (1-\ell_1)^2$*, we need to show that*

[A3] It is trivial to show that $(1-\ell_2)^2 < 2(1-\ell_2)^2 - (\ell_1-\ell_2)^2$, which means it is impossible for both $\beta < (1-\ell)^2$ and $\beta > 2(1-\ell_2)^2 - (\ell_1-\ell_2)^2$ to be true.

$$\begin{aligned} (1-\ell_1)^2 > &\ (1-2\ell_1+\ell_2)^2 - 2(1-\ell_1)(\ell_1-\ell_2) \\ (1-\ell_1)[1-\ell_1+2(\ell_1-\ell_2)] > &\ (1-2\ell_1+\ell_2)^2 \\ (1-\ell_1)[1+\ell_1-2\ell_2] > &\ (1-2\ell_1+\ell_2)^2. \end{aligned}$$

Since $1-2\ell_1+\ell_2$ is smaller than $1+\ell_1-2\ell_2$ and $1-\ell_1$, the equation always holds.

In the case in which $\beta \leq (1-\ell_1)^2$, the MSNE is different and the comparative statics holds for $\ell_1 > \frac{1+2\ell_2}{3}$ (algebra not shown).

iii) For large β, the second term in A9 is negative, and as ℓ_2 increases this term increases (i.e. it becomes 'less negative'). Since ℓ_2 does not appear in the first term, this is sufficient to establish that the derivative $\frac{\partial E(c_V^*)}{\partial \ell_2} > 0$ over the range $\beta \in [\hat{\beta}, \beta_V^H]$. For low β The derivative of Equation A9 w/r/t ℓ_2 is:

$$\frac{\partial E(c_V^*)}{\partial \ell_2} = \frac{1}{\beta}\left[(1-\ell_1)(\ell_1-\ell_2) - (1+\ell_2-2\ell_1)(1-\ell_1)\right]. \tag{A13}$$

The derivative is positive iff $3\ell_1 - 2\ell_2 - 1 > 0$. Similarly, if $\beta < (1-\ell_1)^2$, then the aggregate effort $[1 - \frac{(\ell_1-\ell_2)^2}{\beta}]\sqrt{\beta} + (\ell_1 - \ell_2)$ increases in ℓ_2 if $3\ell_1 - 2\ell_2 - 1 > 0$ (algebra not shown).

QED

C Equilibria in effort-based Parties

Here we derive the PSNE for any concave function $f(r_i)$. The extension to the case studied in the text $f(r_i) = 1 - c_i^2$ follows directly from the general result. Define the following implicit level of vote-seeking effort $\tilde{d}_i$:

$$f'(1-\tilde{c}_i) = \frac{\beta}{2e}. \tag{A14}$$

This effort level characterizes interior solution of the agents' maximization problem.

Proposition A4 *If $\overline{\sigma}_1 > \overline{\sigma}_2$ and $f(r_i)$ is concave and increasing, then:*

1. *If $\tilde{c}_i \in (0, 1-\ell_i)$ $\forall i$ then $c_i^* = \tilde{c}_i$ is the unique PSNE.*
2. *If $\tilde{c}_i \geq 1-\ell_i$ $\forall i$, then $c_i^* = 1-\ell_i$ is the unique PSNE.*
3. *If $\tilde{c}_i \in (0, 1-\ell_2)$ but $\tilde{c}_i \geq 1-\ell_1$, then $c_1^* = 1-\ell_1$ and $c_2^* = \tilde{c}_i$ is the unique PSNE.*
4. *If $\tilde{c}_i \leq 0$, then $c_i^* = 0$ is the unique PSNE.*

Proof of Proposition A4 Let the Langrangian for agent i be

$$\mathcal{L}_i = f(1-c_i) + p_i(c_1, c_2)\beta + \bar{\lambda}(c_i - (1-\ell_i)) - \underline{\lambda}c_i,$$

where $\bar{\lambda}$ and $\underline{\lambda}$ are the Lagrangian multipliers associated with the upper and lower bounds of c_i. The probability of winning changes linearly with c_i and the feasibility constraints in the Lagrangian are linear. Thus, the second order conditions are always met if $f(r)$ is concave. Hence, it suffices to look at the first order conditions for each agent to obtain the candidates for equilibria:

$$\frac{\partial \mathcal{L}_i}{\partial c_i} = -f'(1-c_i) + \frac{\partial p_i(c_1,c_2)}{\partial c_i}\beta + \bar{\lambda} - \underline{\lambda} = 0.$$

Recall that $\frac{\partial p_i(c_1,c_2)}{\partial c_i} = \frac{1}{2e}$. The corner equilibria arise when $\bar{\lambda} \geq 0$ and $\underline{\lambda} = 0$, and the contrary. The first case, if it exists, results in

$$\frac{\partial p_i(c_1,c_2)}{\partial c_i}\beta = \frac{\beta}{2e} \geq f'(1-c_i),$$

in which case $c_i^* = 1 - \ell_i$. The opposite case results in $c_i^* = 0$. The interior equilibrium, $\bar{\lambda} = \underline{\lambda} = 0$, is implicitly defined by

$$\frac{\partial p_i(c_1,c_2)}{\partial c_i}\beta = \frac{\beta}{2e} = f'(1-c_i). \quad \textbf{QED}$$

Proof of Corollary 2

Proof of Result 1

Hold ℓ_1 and ℓ_2 constant and assume $e^{'} > e$. Over the range $\beta \in [0, 4e(1-\ell_1)]$ the PSNE for e and $e^{'}$ imply $c_E^* = \frac{\beta}{2e}$ and $c_E^* = \frac{\beta}{2e^{'}}$ respectively, and the latter is smaller then the former. Over the range $\beta \in [4e(1-\ell_1), 4e^{'}(1-\ell_1)]$, the PSNE for e and $e^{'}$ imply $c_E^* = \frac{\beta}{4e} + (1-\ell_1)$ and $c_E^* = \frac{\beta}{2e^{'}}$ respectively, and the latter is smaller then the former. Over the range $\beta \in [4e'(1-\ell_1), 4e(1-\ell_2]$, the PSNE for e and $e^{'}$ imply $c_E^* = \frac{\beta}{4e} + (1-\ell_1)$ and $c_E^* = \frac{\beta}{4e^{'}} + (1-\ell_1)$ respectively, and the latter is smaller then the former. Over the range $\beta \in [4e(1-\ell_2), 4e^{'}(1-\ell_2)]$, the PSNE for e and $e^{'}$ imply $c_E^* = 2-\ell_1-\ell_2$ and $c_E^* = \frac{\beta}{4e^{'}} + (1-\ell_1)$ respectively, and the latter is smaller then the former. For any $\beta > 4e^{'}(1-\ell_2)$ the PSNE is the upper corner $c_E^* = 2-\ell_1-\ell_2$ for both e and $e^{'}$. **QED**

Proof of Result 2

Hold e and ℓ_2 constant and assume $\ell_1^{'} > \ell_1$. Over the range $\beta \in [0, 4e(1-\ell_1^{'})]$ the PSNE for ℓ_1 and $\ell_1^{'}$ are both $c_E^* = \frac{\beta}{2e}$. Over the range $\beta \in [4e(1-\ell_1^{'}), 4e(1-\ell_1)]$ the PSNE for ℓ_1 and $\ell_1^{'}$ are $c_E^* = \frac{\beta}{2e}$ and $c_E^* = \frac{\beta}{4e} + (1-\ell_1^{'})$ respectively, and the latter is smaller then the former. Over the range $\beta \in [4e(1-\ell_1), 4e(1-\ell_2)]$ the PSNE for ℓ_1 and $\ell_1^{'}$ are $c_E^* = \frac{\beta}{4e} + (1-\ell_1)$ and $c_E^* = \frac{\beta}{4e} + (1-\ell_1^{'})$ respectively, and the latter is smaller then the former. Over the range $\beta > 4e(1-\ell_2)$ the PSNE for ℓ_1 and $\ell_1^{'}$ are $c_E^* = 2-\ell_1-\ell_2$ and $c_E^* = 2-\ell_1^{'}-\ell_2$ respectively, and the latter is smaller then the former. **QED**

Proof of Result 3

Hold e and ℓ_1 constant and assume $\ell_2^{'} > \ell_2$. Over the range $\beta \in [0, 4e(1-\ell_1)]$ the PSNE for ℓ_2 and $\ell_2^{'}$ are both $c_E^* = \frac{\beta}{2e}$. Over the range $\beta \in [4e(1-\ell_1), 4e(1-\ell_2^{'})]$ the PSNE for ℓ_2 and $\ell_2^{'}$ are both $c_E^* = \frac{\beta}{4e} + (1-\ell_1)$. Over the range $\beta \in [4e(1-\ell_2^{'}), 4e(1-\ell_2)]$ the PSNE for ℓ_2 and $\ell_2^{'}$ are $c_E^* = \frac{\beta}{4e} + (1-\ell_1)$ and $c_E^* = 1-\ell_1-\ell_2^{'})$ respectively, and the latter is smaller then the former. Over the range $\beta > 4e(1-\ell_2)$ the PSNE for ℓ_2 and $\ell_2^{'}$ are $c_E^* = 2-\ell_1-\ell_2$ and $c_E^* = 2-\ell_1-\ell_2^{'}$ respectively, and the latter is smaller then the former. **QED**

D Institutional Comparative Statics

Proof of Theorem 1

Assume that $\bar{\sigma}_1 > \bar{\sigma}_2$ and that $f(r_i) = 1 - c_i^2$. In addition, given some level of ℓ_1, ℓ_2, and e, assume that $\beta_E^L < \beta_V^L < \beta_E^H < \beta_V^H$.[A4] Then, over the range $\beta \in [0, \beta_V^L)]$, $c_V^* = 0$ while $c_E^* > 0$. Over the range $\beta \in [\beta_V^L, \beta_E^H]$, the aggregate effort of the Vote-based game is characterized by either (A8) or (A10), while the aggregate effort of the effort-based game is $c_E^* = (1-\ell_1) + \frac{\beta}{4e}$.

The right-hand side of Equation (A8) is smaller than $(1+\ell_1) + \frac{\beta}{4e}$ iff $\frac{\beta}{4e} > (1-\ell_1) - \frac{2}{\beta}[(1-\ell_1)(1-\ell_2)(\ell_1-\ell_2)]$. Since the right-hand side of this inequality increases in ℓ_2, the hardest case for the inequality to hold is $\ell_2 = \ell_1 - \epsilon$ ($\epsilon \to 0$). Substituting $\ell_2 = \ell_1$ yields $\frac{\beta}{4e} > (1-\ell_1)$ which can be rewritten $\beta > 4e(1-\ell_1) \equiv \beta_E^L$, which is true by construction since $\beta_E^L < \beta_V^L$.

Similarly, the right-hand side of Equation (A10) is lower than $(1-\ell_1) + \frac{\beta}{4e}$ iff $\frac{\beta}{4e} > \frac{1}{\beta}[(1-\ell_1)(\ell_1-\ell_2)(1+\ell_2-2\ell_1)]$. By assumption $\bar{\sigma}_1 > \frac{1+\bar{\sigma}_2+\mu}{2} \implies 2\ell_1 > 1+\ell_2$, which means that right-hand side of the inequality is negative, which means that the inequality holds.

If the inequality above does not hold, tthe maximum (degenerated) expected clientelism cannot be larger than $\bar{C_V} = (1-\ell_1)+(\ell_1-\ell_2)$ (if they were to play with probability close to 1 the strategies with greater spending in Prop A3). Hence we need look for the sufficient condition such that

Suppose the above condition was more restrictive than condition 2 in Theorem 1? I.e., $\frac{\ell_1-\ell_2}{4} < \frac{(\ell_1-\ell_2)^2}{4(1-\ell_1}$. The inequality simplifies to $\ell_1 > (1+\ell_2)/2$, but then we know that $C_E > C_V$ even if $e < \frac{\ell_1-\ell_2}{4}$ is not true.

QED

Proof of Proposition 3 Note that for $e > \frac{1-\ell_2}{2}$ then $\beta_E^H > \beta_V^H$.

i) For all $\beta < \beta_V^L$, $C_V^* = 0$. But for all β, $C_E^* \neq 0$. Thus $C_E^* > C_V^* = 0$.

iii) For all $\beta > \beta_V^H$, $C_V^* = 1-\ell_1-\ell_2$ and for $\beta > \beta_E^H$ $C_E^* 1-\ell_1-\ell_2$, then for $\beta \in [\beta_V^H, \beta_E^H]$ then $C_V^* > C_E^*$. And,

iv) And for $\beta > \beta_E^H$, $C_V^* = C_E^*$

It remains to show the intermediate case.

ii) For $\beta \in [\min\{\beta_V^L, \beta_E^L\}, \beta_V^H]$, C_E^* is decreasing in e and C_V^* in ℓ_1. Thus, for continuity, there exists (ℓ_1, e) such that ℓ_1 is sufficiently small and/or e is sufficiently large for $C_V^* > C_E^*$

QED

[A4] The proof is identical and simpler in the case that $\beta_E^H < \beta_V^L$.

E Extensions

E.1 Prizes conditional on winning the district.

If instead of assuming that the brokers' performance and outcomes determine the winner of the prize, regardless of whether the incumbent party wins the election, an interesting avenue of research is assume "conditional prizes". I.e., if the party wins the district (i.e., $v_1 + v_2 > 1$), then there is a prize to be awarded. The new feature of this model is that besides competing for a prize, the brokers – in a sense – cooperate to produce "jointly" a public good, i.e., winning the election.

Conjecture A1 *Let $\sigma_1 > \sigma_2$ and $f(c)$ be concave. Then*

- *Let the incumbent party be a dominant party. That is, $\ell_1 + \ell_2 \geq 1$. Regardless of the brokers' action, the party wins the election and the equilibrium does not change from the one presented in the paper.*
- *Let the incumbent party not be a dominant party. That is, $\ell_1 + \ell_2 < 1$. Unless $f(1) - f(\ell_1 + \ell_2) \leq \beta < f(1) - f(2\ell_2)$, the equilibirum clientelism from proposition 1 leads to $c_1 + c_2 + \ell_1 + \ell_2 \geq 1$, then the equilibrium does not change from the one presented in the paper. Otherwise, there is an additional equilibrium where $c_2 = 0$ and broker 1 makes the effort to just win the election, i.e., $c_1 = 1 - \ell_1 - \ell_2$.*

Since $V(0,0) < \frac{1}{2}$, if both agents choose $c_i = 0$ their party loses the election and both receive utility $f(1)$. Agent 1 would only consider an unilateral deviation that makes the party win, i.e., to choose $c_1 = 1 - (\ell_1 + \ell_2)$, the constituency effort necessary to raise their party's vote share to win the election. Agent 1 prefers this deviation to choosing $c_1 = 0$ if $\beta + f(\ell_1 + \ell_2) > f(1)$, which can be rewritten as $\beta > [f(1) - f(\ell_1 + \ell_2)]$. Furthermore, Agent 1's condition for deviation from the strategy vector $\{0, 0\}$ is equal to or less constraining than that of agent 2 (i.e. agent 1 is "weakly" more likely than agent 2 to deviate).[A5] With the quadratic cost, the condition on β to ensure that a player has incentives to spend more to assure the win of the party (and himself) is then $\beta > f(1) - f(\ell_1 + \ell_2) = 1 - (1 - [1 - (\ell_1 + \ell_2)]^2) = [1 - (\ell_1 + \ell_2)]^2$.

The conditional prize proposition, in the effort-based setting would look as follows:

Conjecture A2 *Let split the proposition into two cases:*

- *When the party is either dominant or the interior equilibrium is enough to win the election (i.e., $\frac{\beta}{4e} + \frac{\beta}{4e} + \ell_1 + \ell_2 \geq 1$), then the equilibrium is as before.*
- *Otherwise, there is a broker who may have the incentive to increase its spending to make the party just win the election, like above.*

The new feature of this extension would be that both brokers are likely to collaborate in the joint production of the public good. That is, suppose that one broker has an incentive

[A5] If $\ell_1 < \frac{1}{2}$ the same unilateral deviation condition applies to agent 2; while if $\ell_1 \geq \frac{1}{2}$ agent 2 would need to devote even more effort to c_2 in order to win the seat, as she would need not to only raise the party's vote share to $\frac{1}{2}$, but also to surpass agent 1's loyalist share ℓ_1.

to "overprovide" clientelism, such as to win the election. Because effort is not observed, the party now awards the prize with positive probability for each broker, which increases the incentives for provision (of the broker who did not increase his provision originally). In this region, brokers' efforts would be strategic complements.

E.2 Opposition brokers with conditional prizes

More generally, the current model can be seen as the reduced form of a larger game that includes a conditional prizes and a strategic opposition party with its own brokers. The current model's results would then serve as inputs into the model with inter-party competition. The following paragraphs, which are reproduced in Appendix E, provide a sketch of the results to demonstrate that: a.) they should not change the current paper's core implications, and b.) they represent a promising avenue for future research.

Suppose each region is exactly as before, i.e., it has a share of loyalists and non-loyalists (up-for-grab) voters, but we now add both brokers and loyalists for the opposition party. The votes of a party in a given region are $c_i + \sigma_i - \mu_i$. In the paper $\mu_i = \mu$, but with opposition brokers now μ_i depends on the rival brokers' effort. Thus, μ_1 and μ_2 may differ. In turn, the challenge of inferring brokers' effort now becomes more complicated, since observable vote shares are a function not only of one's own brokers' effort, but of the effort of opposition brokers.

Assuming conditional prizes, some of the additional complexity of adding opposition brokers disappears. This is due to the fact that, with conditional prizes, it is often the case that only one of the two sets of brokers exerts effort in equilibrium; which in turn allows the party that wins the prize to allocate it exactly as they do in the current paper. To begin, we note in **core** districts where one of two competing parties more or less guaranteed to win the election (and thus β), brokers from the losing party will exert no effort, as their party has no chance of winning the prize β. On the other hand, within the dominant party, which receives β with certainty, the intra-broker dynamics will be identical to that modeled in the current paper (where by assumption one of the two brokers wins β). This is identical to the extension above to a conditional prize model in which the brokers' party is 'Dominant' in the district.

Things become slightly more complicated in **swing** districts where both parties have some chance of winning. Under the effort-based reward system, if one of the two parties has more reliable information, that party's brokers will exert higher equilibrium effort than the competing parties' brokers. In turn, via their impact in intra-party competition, informational advantages will also affect inter-party competition, leading better-informed parties to win elections. On other hand, if both parties have similar levels of information, swing districts will be characterized by high effort exertion by brokers in both parties.

In swing districts under the vote-based system, if one party's internal contest is competitive (because ℓ_1 and ℓ_2 are not far apart) while the other's is not, then the district will end up tilting toward the party with competitive brokerage. In turn, intra-party competition will also affect inter-party competition, leading parties with more competitive brokerage contests to win elections. On the other hand, if both parties' internal contests are competitive, then under very reasonable conditions (sufficiently high β) all brokers from both parties will invest in local effort, although equilibria will be in mixed-strategies rather pure-strategies.[A6]

Thus, the basic mechanisms identified in the current paper will carry over to a model with opposition party brokers and election-contingent prizes. Better information will increase equilibrium effort level under the effort-based rule, while intra-party competition will increase equilibrium effort levels under the vote-based rule. Rather than changing the current paper's results, this more general model would use them as inputs to study how these dynamics affect inter-party competition; and in turn how parties may strategically choose their organizational rules so as to maximize their inter-party advantage.

While very interesting for the next stage of this project, these questions are beyond the current paper's scope; and the current paper's empirical and substantive relevance should not be undermined, but rather reinforced, by this theoretical extension.

E.3 Opposition brokers without conditional prizes

What if we assume that both parties award a prize β, regardless of the electoral outcome? At first glance, the type of equilibria in the vote-based system would not change as it is determined by the comparison of votes between regions, taking everything else as constant, including the opposition party's efforts. Hence, the primary effect of adding opposition is to ***re-scale*** the problem in the vote-based system.[A7]

In contrast, the effort-based system becomes more complex when both parties allocate β. In the original setting the μ_i's are cancelled out (see equation 4), but with opposition brokers the level of μ_1 and μ_2 are endogenously determined and carried into the calculus of who exerts more effort. Each brokers' effort will thus be noisier, given the need to infer also infer the effort of opposition party brokers. We conjecture that: a.) this will reduce the comparative advantage of effort-based systems as compared to vote-based systems; b.) however,neither the paper's core comparative statics nor the weak dominance result (Theorem 1) will be altered.

The vote-based systems tend to perform better when $\ell_1 - \ell_2$ is low and when β is intermediate; while effort-based systems will perform better when e is low. Regarding Weak Dominance, due to the fact that parties still have some minimal information about the noise structure, there will still be a range of very low values of *beta* where effort-based systems generate positive effort and vote-based systems generate '0' effort, which is core to Weak Dominance proof. However, the vote-based systems will be optimal over a wider range of parameter values, due to the increased complexity of inferring effort from electoral signals.

[A6]To see this, note that at the max-effort outcome where all brokers from both parties invest in high levels of effort, those brokers from the party which loses the election will have the incentive to defect.

[A7]Depending on the oppositions' level of effort, the incumbent's brokers may hit the upper corner for smaller or larger values of β, which induces the re-scaling.

Indeed, we find this to be an additional and interesting observable implication of the model, especially given that there seems to be real-world variance in the extent to which brokers face competition from opposition party brokers. While clientelism very often takes place in the contexts where single parties are 'local' monopolists, examples such as Camp (2017), Finan and Schecter (2012), and Auerbach (2020) demonstrate that the level inter-party competition between brokers from distinct parties may exhibit some variance across precincts and districts. Though this is an interesting additional implication, this extension should not have a first order impact on the current paper's results, and as such we leave it (as well as those presented in Appendix E) for a follow-up paper.

E.4 Divisible prizes

Regarding the vote-based reward system, extension to a divisible reward changes the conditions for deviation from the 'full-shirking' outcome. In the 'winner-take-all' framework, small deviations by broker 2 from the full-shirking vector will not yield any benefits; rather, in order to benefit from a deviation, broker 2 must exert sufficient effort to overtake broker 1's natural advantage. On the other hand, if the party divides the prize β proportionally according to vote shares, small deviations by broker 2 from the full-shirking vector will lead to small increases in utility, since they increase her regional vote-share. To model this, consider the following amendment to broker payoff functions:

$$u_i(r_i, c_i) = f(r_i) + \frac{v_i(c_i)}{v_i(c_i) + v_{\sim i}(c_{\sim i})}. \tag{A15}$$

In the utility function from the text, each broker receives β in its 'entirety' with some probability. In contrast, here each broker receives a portion of *beta* proportional to their share of the total vote shares received by both brokers (if both receive equal vote shares they split β in half, and so on). We conjecture that the results would be qualitatitvely similar, i.e. if β is large, both brokers exert max effort and push their vote shares to $v_i(c_i) = 1$; if β is small, both brokers will exert 0 effort (full-shirking); and if β is intermediate we get an interior solution in which one or both of the brokers exerts intermediate effort.