

Is There a Stable Relationship between Unemployment and Future Inflation?¹

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Is There a Stable Relationship between Unemployment and Future Inflation?*

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Abstract

The empirical literature on the stability of the Phillips curve has largely ignored the bias that endogenous monetary policy imparts on estimated Phillips curve coefficients. We argue that this omission has important implications. When policy is endogenous, estimation based on aggregate data can be uninformative as to the existence of a stable relationship between unemployment and future inflation. But we also argue that regional data can be used to identify the structural relationship between unemployment and inflation. Using city-level and state-level data from 1977 to 2017, we show that both the reduced form and the structural parameters of the Phillips curve are, to a substantial degree, quite stable over time.

Keywords: Endogenous monetary policy; Stability of the Phillips curve.

JEL classifications: E52, E58.

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1 Introduction

We revisit the empirical debate on the stability of the Phillips curve over time, using data from the United States. The main innovation is the use of state-level data for that purpose. There are two principal reasons for this strategy. The first is that if a central bank responds to shocks with the purpose of maintaining inflation close to some target, aggregate data may be largely uninformative as to the existence of a stable relationship between unemployment and future inflation. The second is that as monetary policy responds to aggregate shocks only, state-level shocks can be used to identify the key parameters.

The notion that endogenous policy may introduce an estimation bias is an old one and has been applied in many contexts, including in models with Phillips curves. We revisit this point in a very simple model in which a Phillips curve relationship is assumed to be true. We also assume that the central bank optimally sets monetary policy so as to fully stabilize inflation and show that model-generated aggregate data alone cannot be used to identify the Phillips curve featured by the model. More generally, if the central bank has a dual mandate, identification is possible, but if the policy rule is misspecified, the estimates of the Phillips curve will be biased.

To motivate the empirical exercises that are the core of the paper, we use the same model to show how regional data can be used to identify the relationship between unemployment and future inflation. The main insight is that as monetary policy reacts only to aggregate shocks, region-specific variation can be used to uncover the true relationship between inflation and unemployment.¹ We use this last property to reassess the empirical debate over the existence of a stable Phillips curve, which has dominated the monetary policy literature over the last decades. The analysis with state-level data provides strong support to the notion that the relationship between inflation and unemployment has remained quite stable since

¹We thank Narayana Kocherlakota for raising this question to us during a 2012 policy briefing at the Minneapolis Fed.

the ‘70s in the United States.

The empirical analysis is done in two complementary ways. First, in Section 3 we study reduced form relationships between inflation and unemployment. We address the literature that, as in [Atkeson and Ohanian \(2001\)](#), has criticized Phillips curve models that use reduced forms. We first document that, as is well known, the estimated reduced form parameter using aggregate data does exhibit substantial variation over time. We then show that when using state-level data, as suggested by the theory, the estimate of the reduced form coefficient is remarkably stable over time. This is so, even though we compare the period of high and unstable inflation (1977–1985) with the subsequent decades, in which inflation was much lower and stable.

Second, in Section 4, we present the estimation results of a standard New Keynesian model with Calvo-type frictions in the setting of nominal prices and wages. We show that the estimated Calvo parameters for prices using state-level data are strikingly stable over time. Again, this is so even though there is substantial variation in inflation and monetary policy across periods. The analysis does detect a small statistical instability in the wage Calvo parameter. We do argue, however, that when translated to either the slope of the Phillips curve or the implied frequency of wage changes, the difference is of little economic significance. The estimates based on aggregate data, however, are sensitive to the sample period and the assumptions regarding the monetary policy rule.

Our results imply a value of about seven to eight months for the average duration of price contracts and an average duration of between five and seven months for wage contracts, both of which are in line with the micro evidence on nominal frictions, as we discuss in Section 4.

The paper is organized as follows. Section 2 provides background about the Phillips curve and discusses some key papers in the literature. In Section 3, we first show in a simple theory how endogenous monetary policy can blur the true

structural relationship in the aggregate. We also show how this is not the case for the regional data, since regional variation can be used to identify the true structural parameters. We then run the regressions implied by the theory, using data from 27 metropolitan statistical areas (MSAs) in the United States from 1976 to 2018. As we show, the regressions are remarkably consistent with the notion of a reduced form Phillips curve that has remained stable over time. In Section 4, we estimate a full New Keynesian model separately on state and aggregate data. We find that the estimates of the structural parameters that govern the frequency of price and wage adjustments are found to be quite stable over time when using state-level data, echoing the reduced form findings. On the contrary, the estimates using aggregate data vary widely over different policy regimes.

2 Background on the Phillips Curve and Related Literature

The notion that a statistical relationship between inflation and unemployment implied a trade-off that could be exploited by monetary policy was forcefully contested on theoretical grounds by the path-breaking work of Lucas (1972). His analysis of the interaction between the reduced form Phillips curve parameters estimated using statistical analysis and the policy rule adopted by the central bank was a central example in his famous critique of econometric policy evaluation methodology (Lucas, 1976). The “stagflation,” or joint increase of unemployment and inflation, that the United States and many other developed countries experienced in the years following Lucas’s work gave the theory a solid empirical backing and implied the death of the Phillips curve in its simplest original form.

By the end of the ‘60s, a reincarnation of the Phillips curve adopted the NAIRU hypothesis, which shared with Lucas’s model the notion that departures from full

neutrality of money could only last for a short time.² This feature made the models compatible, at least qualitatively, with the stagflation experience of the late ‘70s. But NAIRU-type Phillips curve models departed from the stronger notion in [Lucas \(1972\)](#) that any systematic attempt to affect the allocation of resources would be futile. They thereby provided a rationale for an active monetary policy to stabilize the economy. As these models lack microfoundations, the reasons why the full monetary-neutrality property exhibited by [Lucas \(1972\)](#) did not hold could not be studied and evaluated. This unsatisfactory feature gave rise to the development of the New Keynesian family of models that have been widely adopted in the monetary policy literature and in research divisions of central banks. By making explicit the assumptions regarding the nature of the non-neutrality of money, these models could be estimated and their structural assumptions challenged with data.

As an example, consider one of the most popular forms to introduce non-neutrality in an otherwise neoclassical model, proposed by [Calvo \(1983\)](#). The key assumption is that the ability to change a price (or a wage) is not available in every period; rather, agents can change prices only with some exogenously specified probability typically called “the Calvo parameter.” Anyone who has ever participated in a transaction knows that assumption to be absurd. However, as the intellectual founders of the New Keynesian literature have argued, the assumption may well approximate aggregate behavior if the underlying policy regime does not “change too much.”³ The exact meaning of “too much” is, of course, a quantitative issue. Addressing it belongs to the agenda pursued in this paper.

Alongside these theoretical developments, the hypothesis of an exploitable Phillips curve continues to be controversial. For example, [Atkeson and Ohanian \(2001\)](#) (henceforth AO) show that the empirical relationship between current aggregate unemployment and inflation growth is highly unstable over the period 1960–2000

²NAIRU stands for the Non-Accelerating Inflation Rate of Unemployment. Details are spelled out in [Friedman \(1968\)](#).

³See [Woodford \(2003\)](#), p. 141 and 142.

in the United States. They forcefully argue this point by showing that a naive prediction rule for inflation that simply uses past inflation is systematically better than empirical Phillips curves at forecasting inflation. A natural interpretation of their results follows from the observation that the period covered by the analysis includes changes in the policy regime. Thus, the corresponding shift in parameters is evidence that the relationship is not structural, an unavoidable corollary of the Lucas’s critique. As mentioned above, even the most extreme defender of the New Keynesian paradigm would agree with the notion that the Calvo parameter is not invariant to *any* policy regime change. The quantitative question we pursue is whether the Calvo parameters can be safely assumed to be policy invariant – and therefore not subject to the Lucas critique – *given* the policy regime changes actually experienced by the US in the postwar era. The evidence provided in this paper points towards a positive answer to that question.

Recently, the stability of the Phillips curve relationship has again been put into question. The “flattening” of the Phillips curve has been debated at length, fed by the strong changes in unemployment rates in the United States during the 2008–2009 recession and the subsequent recovery, with little sign of inflation rates responding to those movements. A series of papers addressing this issue followed the policy debate.⁴

These criticisms exhibit two main characteristics. First, aggregate data are used in the analysis.⁵ This is problematic since, as mentioned above, a bias arises when monetary policy endogenously responds to shocks, as preceding literature discussed in detail below has forcefully argued. Second, these criticisms are based, albeit most of the time implicitly, on the behavior of reduced form parameters over time, which

⁴See [Krugman \(2015\)](#); [Blanchard \(2016\)](#); and, for a recent survey of the literature, [Hooper, Mishkin and Sufi \(2019\)](#).

⁵[Beraja, Hurst and Ospina \(2019\)](#) and [Jones, Midrigan and Philippon \(2018\)](#) use state- and aggregate-level data together as part of their identification procedure; however those papers were not speaking to the issue we address – namely, the stability of Calvo price and wage parameters over time. This paper also uses information on prices at the MSA level in estimation.

makes addressing the identification problem hard.⁶ The paper of AO represents a concrete example, and its virtue is that it is explicit regarding the nature of the exercise. But arguing that the stagflation of the ‘70s represents evidence of an unstable Phillips curve, as many do, also entails a reduced form discussion, and so does arguing that the “missing” deflation in 2009 and 2010 and the subsequent “missing inflation” represent evidence of a flattening of the Phillips curve. So, while many times we will directly compare our results with a particular interpretation of AO, it should be understood that our results speak to a broader literature that evaluates the stability of the Phillips curve in its structural form as well.

Our empirical exploration using state-level data is consistent with the notion that the slopes of price and wage Phillips curves in a standard New Keynesian model are roughly invariant to the policy regimes experienced in the United States since 1977, the first year for which we have data. And it is consistent with the notion that reduced form regressions of future inflation on current unemployment are also stable across sub-periods.

These results suggest an alternative interpretation of the data used by proponents of the “shifting Phillips curve”: the changes over time in the correlation between unemployment and inflation observed in aggregate data are the results of changes in the policy followed by the Federal Reserve over the period. Thus, the stability of inflation from 2008 onwards is the result of monetary policy’s response to the state of the economy, with the purpose of maintaining stable inflation. In addition, the evidence in AO is compatible with a change in the policy rule that started somewhere in the ‘80s. And the stagflation of the ‘70s is the result of a monetary policy that made inflation persistently higher, at a time in which the economy was undergoing a recession.⁷ This rather brief account of the recent history of US monetary policy evolved in an economy where the frequency of price and wage changes remained quite stable over time – at least, so says our state-level analysis.

⁶There are a few exceptions, such as [Coibion and Gorodnichenko \(2015\)](#).

⁷See [Gao, Kulish and Nicolini \(2020\)](#) for an interpretation along these lines.

As mentioned above, the notion that endogenous policy makes identification of structural parameters problematic dates at least to the work of [Samuelson and Solow \(1960\)](#) and [Kareken and Solow \(1963\)](#). It has since then been applied in several contexts by [Brainard and Tobin \(1968\)](#), [Goldfeld and Blinder \(1972\)](#), [Worswick \(1969\)](#), [Peston \(1972\)](#), and [Goodhart \(1989\)](#). [Mishkin \(2007\)](#); [Carlstrom, Fuerst and Paustian \(2009\)](#); and [Edge and Gurkaynak \(2010\)](#) specifically apply it to a monetary policy model with a Phillips curve. These papers show that if policy reacts to the state of the economy, the relationship in the aggregate data can be blurred by the policy rule. A full analysis that encompasses our discussion as a particular case is developed in [Haldane and Quah \(1999\)](#). They assume that the central bank has a dual mandate and optimally chooses policy and show that the estimated relationship is a function of the relative weight that the central bank puts on inflation. We find it useful to reproduce the simpler, particular case of a single inflation mandate in here. We do so in order to illustrate, in a very transparent fashion, the pervasive effect of endogenous policy on the ability to identify the underlying parameters and also to provide an alternative interpretation of the analysis in [Atkeson and Ohanian \(2001\)](#).

[Nakamura and Steinsson \(2014\)](#) used regional data to identify the fiscal multiplier. We borrow their idea and apply it to a Phillips curve model. This strategy, spelled out in the working paper version of this paper (see [Fitzgerald and Nicolini, 2014](#)) has since been followed by [Kiley \(2015\)](#), [Babb and Detmeister \(2017\)](#), [Leduc and Wilson \(2017\)](#), and more recently by [Hooper, Mishkin and Sufi \(2019\)](#) and [McLeay and Tenreyro \(2020\)](#).

3 Reduced Form Analysis

In this section, we use a reduced form representation to guide some simple regression analysis. The main reason to do so is that a sizeable share of the literature addressing

the stability of the Phillips curve has framed the discussion in reduced form terms, as discussed in detail in Section 2.

Consider an economy composed of a continuum of geographically separated regions that potentially exhibit price frictions. All regions use the same unit of account and face the same monetary policy. Let $\pi_t(s), u_t(s)$ represent regional inflation and unemployment for region s . Assume also that the equilibrium solution in each region can be characterized by the following dynamic system:

$$\pi_{t+1}(s) = b\pi_t(s) + cu_t(s) + di_t + eX_t(s) + \varepsilon_{t+1}^\pi(s) + \xi_{t+1}^\pi \quad (1)$$

$$u_{t+1}(s) = b'\pi_t(s) + c'u_t(s) + d'i_t + e'X_t(s) + \varepsilon_{t+1}^u(s) + \xi_{t+1}^u, \quad (2)$$

where $\varepsilon_t^j(s)$ and ξ_t^j , for $j = u, \pi$, are the regional and aggregate shocks; i_t is the interest rate determined by monetary policy, to be discussed below; and $X_t(s)$ is a vector that allows for the inclusion of control variables in the regression analysis that follows. We call the dynamic system defined by (1) and (2) the reduced form of some structural model. The vector $X_t(s)$ is introduced to allow for control variables in the regression analysis that follows. To simplify the algebra, we now set $X_t(s) = 0$ for all t, s .

We assume that the underlying structural model is such that all shocks have zero unconditional means and regional shocks are independent of the aggregate shock. The terms di_t and $d'i_t$ describe the effect of monetary policy on the system. The timing indicates that the monetary authority decides on policy before observing the $t + 1$ shocks.

For simplicity of exposition, we assume that all regions have the same size.⁸

⁸This assumption is innocuous but simplifies the algebra that follows.

Therefore, we can define aggregates as

$$\begin{aligned}\pi_{t+1} &= \int_0^1 \pi_{t+1}(s) \, ds \\ u_{t+1} &= \int_0^1 u_{t+1}(s) \, ds.\end{aligned}$$

We obtain the following relationship between the aggregate variables:

$$\pi_{t+1} = b\pi_t + cu_t + di_t + \xi_{t+1}^\pi \tag{3}$$

$$u_{t+1} = b'u_t + c'u_t + d'i_t + \xi_{t+1}^u. \tag{4}$$

The focus of this section is the ability to identify and estimate the parameters of the reduced form equations (3) and (4).

A particular example of a structural model that delivers a reduced form like the one described above will be discussed in the next section, where we also estimate its structural parameters. But the system defined by (3) and (4) is compatible with many other models. In particular, as we show in Appendix A, this reduced form is also consistent with a simple old Keynesian model essentially identical to the one presented in Taylor (1999) and discussed in Cochrane (2011). As we show there, under this interpretation, the coefficient c in (3) can be associated with the slope of a NAIRU Phillips curve.

The stability over time of parameter c in equation (3), particularly across different monetary policy regimes, has been the focus of much discussion in the literature. In particular, the natural interpretation of the analysis in Atkeson and Ohanian (2001) is that the estimate of c obtained using aggregate data is unstable over time. We now address this issue.

3.1 Exogenous Policy

To fix ideas, assume first that the monetary authority follows an exogenous constant interest rate policy. Then, taking differences in (3), equilibrium inflation evolves as

$$\pi_{t+1} - \pi_t = b(\pi_t - \pi_{t-1}) + c(u_t - u_{t-1}) + (\xi_{t+1}^\pi - \xi_t^\pi). \quad (5)$$

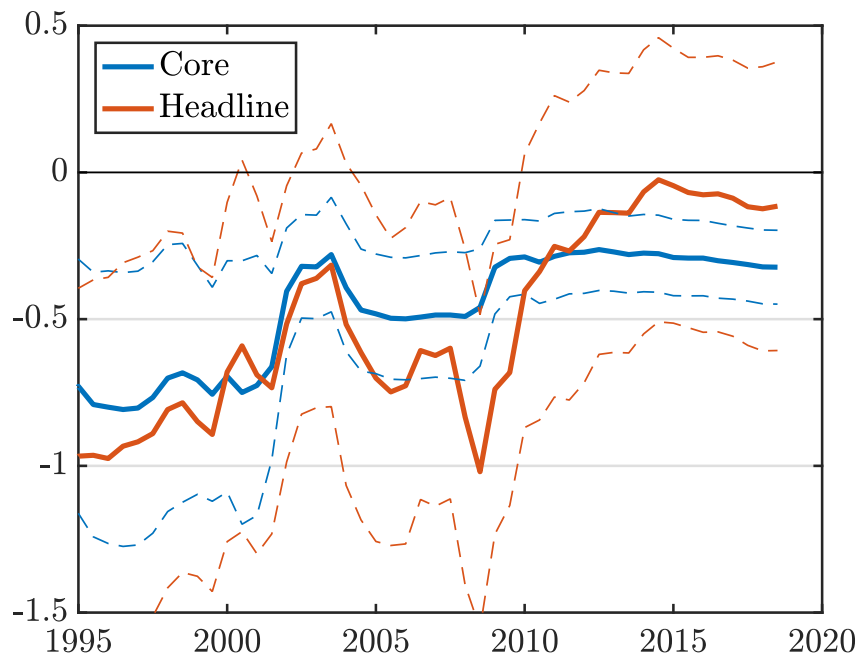
Under this policy, standard econometric techniques should suffice to identify the parameter c .

Figure 1 shows the rolling coefficient for c that results in estimating an equation (5) using inflation and unemployment data for the United States from 1975 to 2017. We estimate that equation using both headline and core inflation, which explains why we have two solid lines in the figure. Specifically, for each of the two measures of inflation, we first estimate the coefficient c in equation (5) using semiannual data from the first semester of 1975 to the second semester of 1995.⁹ The resulting point estimate is then plotted in Figure 1 as the value for the second semester of 1995. We then repeated the estimation, but using data starting and ending one semester after; plotted the point estimate for the first semester of 1996; and reproduced the steps moving forward. Each point in the series thus represents the point estimate of c for a sample size that starts 20 years before and ends at that point. The dotted lines represent 90% confidence intervals.

The figure makes clear how the point estimate for c depends on the sample period. For instance, when we use headline inflation, the first estimate is very close to -1 , but it decreases over time to become zero by the end of the sample. A similar but less drastic change is apparent for the estimates using core inflation. The picture explains why using a Phillips curve like (5) estimated using aggregate data would perform poorly as an out-of-sample forecasting device. This explains the exercise in

⁹We use semiannual data because the frequency for which we have regional data is semiannual. We also used a few controls, as explained in Appendix B. The results without controls, also reported in Appendix B, are very similar.

Figure 1: Coefficient from Rolling 20-Year Regression, Aggregate Level



Atkeson and Ohanian (2001).

To the extent that policy is exogenous, Figure 1 offers evidence that is inconsistent with a stable value for c in this model. But our take is different: as policy is not exogenous, the evidence provided in Figure 1 is in itself uninformative regarding the value of the reduced form parameter c . We address this issue next.

3.2 Endogenous Policy

We now assume the central bank has a mandate to stabilize inflation. We also assume the central bank knows the model economy. Specifically, it solves the following policy problem:

$$\min_{i_t} \frac{1}{2} E_t [\pi_{t+1} - \pi_{t+1}^*]^2,$$

given π_t , u_t , and the solution for aggregate inflation (3). The target for inflation is given by π_{t+1}^* and is part of the policy rule. The objective function is defined as the time- t expectation of the deviation of next period inflation relative to the target. Implicit in this way of writing the problem is the assumption that the central bank

chooses policy before observing time $t + 1$ shocks.

As shown in the Appendix, the optimal policy rule¹⁰ is

$$i_t^{Opt} = \frac{1}{d} [\pi_{t+1}^* - (b\pi_t + cu_t + E_t\xi_{t+1}^\pi)], \quad (6)$$

so the equilibrium value for inflation is given by

$$\pi_{t+1} = \pi_{t+1}^* + \xi_{t+1}^\pi - E_t\xi_{t+1}^\pi. \quad (7)$$

Inflation in equilibrium therefore equals the target plus a forecasting error that, by definition, is orthogonal to any variable in the central bank's information set at time t . In particular, inflation is independent of all the model parameters. This is the consequence of a central bank that knows the model of the economy and uses it to design policy so as to stabilize a specific target.¹¹ A direct implication of this observation is that if the central bank's only objective is to stabilize inflation and it uses a model that describes the economy well, the behavior of inflation in equilibrium is completely uninformative regarding the underlying model that determines inflation. It should be obvious by now that this property is independent of the model that determines inflation, as long as the central bank knows it.

The behavior of equilibrium inflation depends on the behavior of the target, π_{t+1}^* , which is not necessarily observable. To gain further insight, we next consider two specifications. Consider first the case of a constant inflation target, so $\pi_t^* = \pi^*$ for all t . Then, taking differences in (7),

$$\pi_{t+1} - \pi_t = (\xi_{t+1}^\pi - E_t\xi_{t+1}^\pi) - (\xi_t^\pi - E_{t-1}\xi_t^\pi),$$

so current unemployment would be related to the change in inflation to the extent

¹⁰We show in Appendix A that with this policy rule, there is a unique solution. See also [Cochrane \(2011\)](#) for a discussion of determinacy in models of this type.

¹¹As mentioned in the Introduction, this insight is not new. The simple case we discuss in what follows is a particular case of the analysis in [Haldane and Quah \(1999\)](#).

that the forecast error $(\xi_t^\pi - E_{t-1}\xi_t^\pi)$ affects unemployment u_t . But if an estimate of the change in inflation that is different from zero is obtained, it is unrelated to the direct effect of unemployment on future inflation, or c .

Assume next that

$$\begin{aligned}\pi_t^* &= \pi_{t-1}, \text{ if } \pi_{t-1} \in [\pi_{\min}, \pi^{\max}] \\ \pi_t^* &= \pi^{\max}, \text{ if } \pi_{t-1} > \pi^{\max} \\ \pi_t^* &= \pi_{\min}, \text{ if } \pi_{t-1} < \pi_{\min}.\end{aligned}\tag{8}$$

This case corresponds to a central bank that establishes a range for the target and, to the extent that current inflation is within the bands, wants to keep inflation equal to the previous period. As long as the target remains within the band, $\pi_{t+1}^* = \pi_t$, then

$$\pi_{t+1} - \pi_t = \xi_{t+1}^\pi - E_t \xi_{t+1}^\pi,$$

so inflation follows a random walk. In this case, current unemployment—or, for that matter, any variable in the information set at time t —should not help predict inflation growth. In this case, no forecasting rule for inflation could beat a random walk. As shown in Appendix A, the reduced form (3) and (4) are consistent with a simple NAIRU-type model. Therefore, such a model, coupled with the assumption that the central bank stabilizes inflation around a target as defined in (8), generates equilibrium observations that are fully consistent with the result that a random walk is good predictor for inflation, as in AO. The example also rationalizes the difficulty the literature encountered in its attempts at developing trustworthy forecasting models for inflation, as explained in [Stock and Watson \(2009\)](#). In the next section we explain why state-level data can be used to tackle the endogeneity problem.

3.3 State-Level Data Regressions

We now show how to estimate the reduced form parameters exploiting the fact that regional variables' deviations from the national average will not be correlated with policy.

We first replace the optimal policy (6) into the solution for inflation in each region (1) and obtain

$$\pi_{t+1}(s) = \pi_{t+1}^* + b(\pi_t(s) - \pi_t) + c(u_t(s) - u_t) + \varepsilon_{t+1}^\pi(s) + \xi_{t+1}^\pi - E_t \xi_{t+1}^\pi. \quad (9)$$

Notice that by exploiting state-level deviations from the national average, the effect of policy does not enter the solution.

In order to estimate equation (9), we need to take a stand on the evolution over time of the target for inflation. In what follows, we consider an agnostic specification. Thus, we define a time dummy and run

$$\pi_{t+1}(s) = D_t + b(\pi_t(s) - \pi_t) + c(u_t(s) - u_t) + \varepsilon_{t+1}^\pi(s) + (\xi_{t+1} - E_t \xi_{t+1}). \quad (10)$$

The time dummy is naturally interpreted as an estimate of the inflation target for each period.¹²

3.4 Results

In this section, we show the results using CPI inflation and unemployment data for 27 metropolitan statistical areas in the United States. For many MSAs and periods, the lowest frequency for the data is semiannual, so we used that frequency to construct the database. The price data for MSAs are available only as non-seasonally adjusted, so we compute yearly changes. In our regressions we define

¹²In the working paper version of this paper (Fitzgerald and Nicolini, 2014), we discuss more specific assumptions that lead to alternative formulations for the regression. We also compare the results of those regressions with this agnostic strategy.

$u_t(s)$ as the period t unemployment rate for MSA s and $\pi_{t+1}(s)$ as the inflation rate over the following year (i.e., $CPI_{t+2}(s)/CPI_t(s)$). We use headline as a measure of inflation, for which we have data since 1977.¹³

There are a few issues that we need to address in order to clarify the way we will interpret the estimated parameters of equation (10). Our first interpretation will be based on our use of system (3) and (4) as representing purely a reduced form of an unspecified structural model. As such, the estimates provide information only on such a reduced form and lack any additional interpretation. For that purpose, a simple OLS regression suffices, and the only relevant question is if the estimate of the coefficient c is stable over time.

A second possibility is to interpret the system (3) and (4) as a reduced form of a NAIRU (old) Keynesian model. Under that interpretation, the coefficient c approximates the estimate of the slope of the NAIRU Phillips curve, as we show in Appendix A. However, for the OLS estimator to be unbiased, it is necessary that unemployment be uncorrelated with the shock, $\varepsilon_{t+1}^\pi(s) + \xi_{t+1}^\pi - E_t \xi_{t+1}^\pi$. The second component, being a forecast error, presents no difficulty. However, if the region-specific shock is autocorrelated over time, there will be a bias. In that case, it may be important to use instrumental variables. To this end, we will also report two-stage least-squares (2SLS) results in what follows. We have no natural instrument, but since the problem arises only if the regional shocks are autocorrelated, using lagged values of the unemployment rate would naturally reduce the bias. Thus, we use lagged values of the unemployment rate in the first stage. As further justification for this interpretation, one can analyze the estimates of the autocorrelation of the errors. We do so in the working paper version of this paper (Fitzgerald and Nicolini, 2014), where we show that there is no strong evidence of autocorrelation being a major issue in our preferred specification.

We interpret the variables $u_t(s)$ and u_t as deviations from the natural rate of

¹³Appendix C describes this dataset in detail.

Table 1: Regressions with Headline Inflation

	1977-2018	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018
A. Headline Inflation, OLS, without Controls						
c	-0.28** (0.04)	-0.31** (0.11)	-0.41** (0.11)	-0.31** (0.05)	-0.24** (0.06)	-0.21** (0.07)
Overall R^2	0.88	0.83	0.69	0.45	0.70	0.51
Obs	2059	381	288	492	536	362
B. Headline Inflation, 2SLS, without Controls						
c	-0.27** (0.04)	-0.39** (0.12)	-0.29* (0.15)	-0.46** (0.13)	-0.21** (0.08)	-0.24** (0.08)
Overall R^2	0.88	0.79	0.71	0.39	0.70	0.51
Obs	2055	377	288	492	536	362
C. Headline Inflation, 2SLS, with Controls						
c	-0.33** (0.05)	-0.50** (0.19)	-0.45** (0.14)	-0.45** (0.10)	-0.28** (0.10)	-0.28* (0.13)
Overall R^2	0.88	0.76	0.65	0.40	0.70	0.54
Obs	1933	327	288	484	532	362

Standard errors in parentheses

* significance at 5% level, ** significance at 1% level

unemployment. To allow for the possibility that the natural rate of unemployment differs across MSAs, we introduce a region fixed effect in the regressions. To control for potential heteroscedasticity, we compute the statistical tests using standard errors that are clustered at the MSA level. All tests results are uniformly stronger if we do not cluster the errors. Finally, in some specifications, we use a series of regional controls that may correlate with shocks affecting local economic conditions, like inflation expectations and government expenditures or temperature and precipitations, as well as lagged values of both inflation and unemployment. A detailed explanation of the controls used is in Appendix B.

Table 1 provides estimates for the coefficient c in regression (10). Results are reported for OLS and 2SLS without and with controls.¹⁴ We present results for

¹⁴We report the estimates for all other parameters in Appendix B.

the whole period first and then for five sub-periods. The first sub-period is chosen to contain the years of rising inflation and the Volcker stabilization. The second sub-period contains the rest of the decade until 1990. We take these two to be the ones with policy regimes that differ from the rest of the sample.

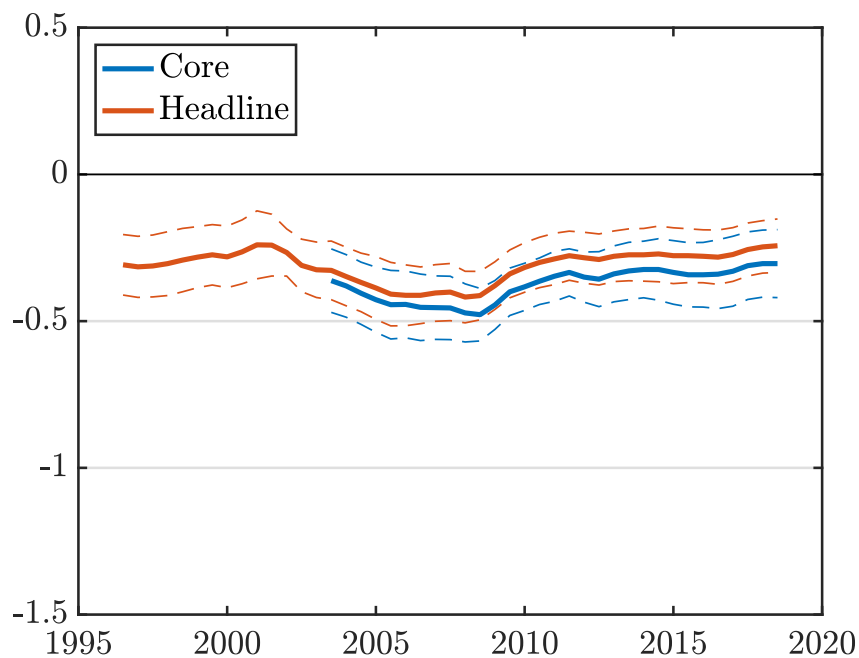
The results are striking. The point estimate for c using the whole period is close to -0.3 for the three specifications and very precisely estimated. In addition, the point estimate is similar for all the sub-periods and are all statistically significant. In fact, for all specifications and almost all sub-periods, the point estimate is within one standard deviation of -0.3 . In Appendix B, we show the estimates of the inflation target (the time dummy). The results confirm the obvious: the first two sub-periods correspond to inflation target behavior that differs from the rest of the sample. We also show that even stronger results are obtained if one uses core inflation, rather than headline – with the caveat that we have data starting only in 1985.

As further evidence of the stability of the estimated coefficient, we show in Figure 2 an exercise like the one presented in Figure 1, but using state-level data to run the rolling regressions, rather than aggregate data. In this case, it takes two pictures (figures 1 and 2) to be worth a thousand words.

In the working paper version of this paper (Fitzgerald and Nicolini, 2014) and its appendix, we performed several additional exercises. We first explored the possibility that results would be driven by a few MSAs so that other geographic issues could affect the results. We also checked if the overlapping nature of our data is important. We finally explored the extent to which autocorrelation of the errors could be an issue, given the lack of a natural instrument in our 2SLS specification. In there, we showed our results to be very robust to all these concerns.

These results can be thought of as consistent with an old Keynesian structural model; they thereby relate to the criticism of Atkeson and Ohanian and others. But they can be interpreted as reduced form regressions from the perspective of current structural New Keynesian models. One may therefore wonder the extent to which

Figure 2: Coefficient from Rolling 20-Year Regression, State Level



the results of this section speak to the stability of the frequency of price and wage adjustment in structural New Keynesian models.

This is a natural question to raise, since the coefficients of reduced form solutions are functions of the parameters of the corresponding structural model. Thus, we now estimate a simplified version of the the state-level structural model of [Jones, Midrigan and Philippon \(2018\)](#).

4 Structural Model

We now move beyond simple linear reduced forms and estimate an economy with Calvo-type rigidities in prices and wages. We use our estimation results to evaluate the stability of the parameters over time. As discussed in [Section 2](#), the assumptions in Calvo are not to be understood as invariant to any policy regime change. The question we address is whether those parameters have been stable across the monetary regime changes that have prevailed in the United States since 1977, the first year for which we have state-level data.

We employ the simplest framework, which forms the basis of numerous models in the literature. Thus, we use as a starting point the standard three-equation New Keynesian model. In adapting that model to a series of geographically separated units in which local shocks can move local pricing and employment decisions that are different than those for the country as a whole, we do need to extend that basic popular model to allow for tradable and non-tradable goods. This is the only deviation from the standard textbook example of the New Keynesian model with price and wage frictions. We make the model more precise below.

4.1 Model Description

The economy consists of a continuum of ex ante identical islands. These islands form a monetary union and trade with one another. Consumers on each island derive utility from the consumption of a final good and from leisure:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t(s) \left[\log(c_t(s)) - \frac{\eta_t^n(s)}{1+\nu} n_t(s)^{1+\nu} \right],$$

where s indexes the island, $c_t(s)$ is consumption, $n_t(s)$ is labor supplied, $\beta_t(s)$ is a preference shock, and $\eta_t^n(s)$ is a labor disutility shock. The structure of the shock processes is described below.

The final good $y_t(s)$ is assembled using inputs of non-traded goods $y_t^N(s)$ and traded goods $y_t^M(s, j)$ imported from island j :

$$y_t(s) = \left(\omega^{\frac{1}{\sigma}} y_t^N(s)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} \left(\int_0^1 y_t^M(s, j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa-1}{\kappa-1} \frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where ω determines the share of non-traded goods, σ is the elasticity of substitution between non-traded and traded goods, and κ is the elasticity of substitution across varieties of traded goods. Letting $p_t^N(s)$ and $p_t^M(s)$ denote the inputs' corresponding

prices, the price of the final good on an island is

$$p_t(s) = \left(\omega p_t^N(s)^{1-\sigma} + (1-\omega) \left(\int_0^1 p_t^M(j)^{1-\kappa} dj \right)^{\frac{1-\sigma}{1-\kappa}} \right)^{\frac{1}{1-\sigma}}. \quad (11)$$

Notice that in the particular case of $\omega = 0$, there are only traded goods and the consumption basket in each location is the same as in the aggregate, in which case inflation in each state is the same as in the aggregate and the model collapses to the simple textbook three-equation model. Thus, the only innovation of our model is to allow for non-traded goods at the state level, which in turns explains why inflation at the regional level may differ from the aggregate.

The production technologies we use are standard in both the monetary and the trade literatures. In particular, we model non-traded goods and traded export goods $y_t^X(s)$ on each island as CES composites of varieties k of differentiated intermediate inputs with an elasticity of substitution ϑ :

$$\begin{aligned} y_t^N(s) &= \left(\int_0^1 y_t^N(s, k)^{\frac{\vartheta-1}{\vartheta}} dk \right)^{\frac{\vartheta}{\vartheta-1}} \\ y_t^X(s) &= \left(\int_0^1 y_t^X(s, k)^{\frac{\vartheta-1}{\vartheta}} dk \right)^{\frac{\vartheta}{\vartheta-1}}. \end{aligned}$$

The production of the varieties of non-traded goods and the varieties of traded exports on each island is linear in labor:

$$\begin{aligned} y_t^N(s, k) &= z_t^N(s) n_t^N(s, k) \\ y_t^X(s, k) &= z_t^X(s) n_t^X(s, k), \end{aligned}$$

where $z_t^N(s)$ and $z_t^X(s)$ are productivity shocks.

Nominal frictions affect this economy in a standard way. Individual producers of tradable and non-tradable intermediate goods are subject to Calvo price adjustment frictions—parameterized by λ_p , the probability that a firm cannot reset its price in a

given period—and individual households supply differentiated varieties of labor that are subject to Calvo wage adjustment frictions—parameterized by λ_w , the probability that a labor variety cannot reset its wage in a given period. Labor is immobile across states and is aggregated using a CES aggregator with an elasticity of substitution across labor varieties of ψ . The optimal price and wage control problems thus give rise to linearized Phillips curves in price and wage inflation.

At the aggregate level, monetary policy is set using a Taylor rule when the ZLB does not bind, with the nominal interest rate i_t responding to its own lag with weight α_r ; deviations of aggregate inflation π_t with weight α_π ; deviations of output y_t from y_t^F , the level of output that prevails in the absence of nominal rigidities, with weight α_y ; and the growth rate of the output gap with weight α_x :

$$1 + i_t = (1 + i_{t-1})^{\alpha_r} \left[(1 + \bar{i}) \pi_t^{\alpha_\pi} \left(\frac{y_t}{y_t^F} \right)^{\alpha_y} \right]^{1-\alpha_r} \left(\frac{y_t}{y_{t-1}} / \frac{y_t^F}{y_{t-1}^F} \right)^{\alpha_x} \exp(\varepsilon_t^i),$$

The following shocks drive fluctuations in the model. At the state level, we have shocks to the rate of time preference of individual households, to the household’s disutility from work, to productivity, and to non-tradable productivity.¹⁵ At the aggregate level we also have shocks to the rate of time preference of individual households, labor disutility, and aggregate productivity, in addition to shocks to the interest rate rule ε_t^i and the aggregate price Phillips curve (via standard markup shocks).¹⁶

The model in [Jones, Midrigan and Philippon \(2018\)](#) has households that also derive utility from the consumption of housing goods, which must be used as collateral for household borrowing. These features allow them to capture better the relative state-level data around the Great Recession described in [Mian and Sufi \(2011, 2014\)](#) and [Jones, Midrigan and Philippon \(2018\)](#). In [Appendix E.3](#), we show our results

¹⁵In robustness exercises, we also allow for shocks to the household’s preference for housing and the loan-to-value borrowing constraint (or credit shocks).

¹⁶[Appendix D](#) contains a full description of the model.

to be very robust to adding these realistic features to the estimation.

4.2 Estimation Strategy

We use Bayesian methods, as is common in the literature. Our estimation on state-level data for 51 states over the period 1977 to 2017, however, is not standard: inflation data do not exist for around half of the 51 states in our panel. And the inflation series that are available are observed at only a biannual frequency, whereas the remaining state-level observables are observed annually. So, to rely on as much data as possible, we estimate the state-level model on an unbalanced mixed-frequency panel. To the best of our knowledge, the use of an unbalanced mixed-frequency panel in the estimation of a structural model is new in the literature. We describe the estimation in more detail below.

Approach To capture the period of zero nominal interest rates, we use a piecewise linear approximation as proposed in [Jones \(2017\)](#) and [Kulish, Morley and Robinson \(2017\)](#). Under this approximation, the reduced form solution of our model has a time-varying VAR representation:

$$\mathbf{x}_t = \mathbf{J}_t + \mathbf{Q}_t \mathbf{x}_{t-1} + \mathbf{G}_t \epsilon_t,$$

where \mathbf{x}_t collects the state and aggregate endogenous variables and ϵ_t collects the state and aggregate shocks. The time-varying coefficient matrices \mathbf{J}_t , \mathbf{Q}_t , and \mathbf{G}_t , arise because of the non-linearities induced by the ZLB. In the particular case of $\omega = 1$, the vector \mathbf{x}_t includes the current values for the aggregate shocks as well as inflation – which is the same across states – the output gap – which may be different across states, owing to local shocks and the immobile labor force – and the nominal interest rate.

Following [Jones, Midrigan and Philippon \(2018\)](#), we separate the state-level variables from the aggregate variables. We decompose the vector of variables for

each island s , expressed in log-deviations from the steady state as $\mathbf{x}_t(s)$, into a component due to state s 's dependence on its own history $\mathbf{x}_{t-1}(s)$ and its own shocks $\epsilon_t(s)$ and a component encoding the state-level dependence on aggregate variables:

$$\mathbf{x}_t(s) = \underbrace{\mathbf{Q}\mathbf{x}_{t-1}(s) + \mathbf{G}\epsilon_t(s)}_{\text{state-level component}} + \underbrace{\tilde{\mathbf{J}}_t + \tilde{\mathbf{Q}}_t\mathbf{x}_{t-1}^* + \tilde{\mathbf{G}}_t\epsilon_t^*}_{\text{aggregate component}}. \quad (12)$$

The coefficient matrices that appear in the aggregate component, $\tilde{\mathbf{J}}_t$, $\tilde{\mathbf{Q}}_t$, and $\tilde{\mathbf{G}}_t$, are time-varying because of the non-linearities induced by the ZLB. The vector \mathbf{x}_t^* which contains the aggregate variables evolves as:

$$\mathbf{x}_t^* = \mathbf{J}_t^* + \mathbf{Q}_t^*\mathbf{x}_{t-1}^* + \mathbf{G}_t^*\epsilon_t^*. \quad (13)$$

Here, ϵ_t^* are the aggregate shocks. Given this structure of our model, letting $\bar{\mathbf{x}}_t^* = \int \mathbf{x}_t(s)ds$ denote the economy-wide average of the island-level variables, the deviation of island-level variables from their economy-wide averages, $\hat{\mathbf{x}}_t(s) = \mathbf{x}_t(s) - \bar{\mathbf{x}}_t^*$, is a time-invariant function of island-level variables alone:

$$\hat{\mathbf{x}}_t(s) = \mathbf{Q}\hat{\mathbf{x}}_{t-1}(s) + \mathbf{G}\epsilon_t(s), \quad (14)$$

where we use the assumption that island-level shocks have zero mean in the aggregate, that is, $\int \epsilon_t(s)ds = 0$. We make explicit also that a key assumption we make in (12) in order to arrive at (14) is that the parameters across states are the same (that is, that the coefficient matrices \mathbf{Q} and \mathbf{G} for the state-level components are not state-specific).

The use of deviations of state-level observables from aggregates in estimation is crucial for our study. This is because by removing the dependence of state-level outcomes on aggregate variables, the nominal interest rate drops out from the reduced form just as it did in the reduced form analysis of Section 3.3 that led to

specification (10). Equation (14) therefore circumvents, as (10) did, the problem of having to rely on aggregate data to estimate the Phillips curve in the presence of endogenous and possibly time-varying policy at the aggregate level.¹⁷ This argument mirrors the one made in the reduced form analysis in Section 3.3, where subtracting aggregate optimal policy from the solution for state-level inflation removes aggregate quantities from its solution.¹⁸

In the particular case in which consumption is composed only of tradable goods ($\omega = 1$), the final goods price (11) – and therefore inflation – is the same in every state, and the deviation from the aggregate is equal to zero in every state. In this case, even with local state shocks moving the output gap, a representation like (14) would fail to identify the Calvo price parameter, as there would be no relative variation in state-level inflation data.

Practically, the use of equations (13) and (14) to estimate the model involves first expressing each state’s observable variable as a deviation from its aggregate counterpart by subtracting time effects for each year and each variable. It also involves subtracting a state-specific fixed effect and time trend for each observable, since in the model, all islands are ex ante identical.

We estimate the model using state-level data, following the strategy just described. With the purpose of comparing results, we also estimate the model using aggregate data. In doing so, we jointly estimate the structural parameters and the policy rule.

In all cases, we use Bayesian methods to estimate the model’s structural parameters.¹⁹ To construct the posterior distribution, as the island-level shocks in (14)

¹⁷Another advantage of representation (14) is that we can overcome the curse of dimensionality associated with all 51 states’ dependence on the time-varying aggregate structure, which would otherwise make our estimation computationally infeasible. This was the focus in [Jones, Midrigan and Philippon \(2018\)](#).

¹⁸More formal arguments can be found in the literature. As mentioned in Section 2, [Haldane and Quah \(1999\)](#) were the first to show that endogenous policy leads to biases in estimating New Keynesian models. A simple and very elegant argument is presented in [McLeay and Tenreyro \(2020\)](#).

¹⁹We estimate λ_p , λ_w , α_r , α_p , α_x , α_y , and the persistence and standard deviations of the

are independent and do not affect aggregate outcomes, we can write the likelihood of the model as the product of each individual state’s likelihood, computed from (14). When we estimate the model using aggregate data, we use equation (13) to compute the aggregate likelihood. For the prior distributions for the model’s structural parameters, we follow standard practice and use the same priors [Smets and Wouters \(2007\)](#) use for the Calvo parameters λ_p and λ_w . We use this procedure for both the state-level data and the aggregate data estimations.²⁰

As we want to illustrate the role that changing policy regimes may have on the estimated values of the Calvo parameters using aggregate data, we do not wish to take a strong stand on the priors for the Taylor rule parameters. For this reason, in the estimates we report, we use uniform priors for α_r , α_p , α_x , and α_y . In Appendix E.3, we show that results are similar if we instead used the priors of [Smets and Wouters \(2007\)](#) for the Taylor rule parameters.

Data We use a panel of employment, nominal output, wages, and inflation in the cross section of 51 US states from 1977 to 2017.²¹

We use aggregate data from 1977 to 2015 on employment, output, wages, inflation, and the Fed Funds rate.²² We construct these data in a similar way to the state-level data. We also use sequence of expected durations of the ZLB between 2009 and 2015 from the Blue Chip Financial Forecasts survey from 2009 to 2010 and the New York Federal Reserve’s Survey of Primary Dealers from 2011 to 2015 (see [Kulish, Morley and Robinson, 2017](#)).

autoregressive exogenous processes. See Appendix E for the full estimation results.

²⁰As it turns out, assumptions regarding prior distributions of the Calvo parameters can be quite important in standard aggregate-level estimation. On the other hand, estimates using state-level data are found robust to the assumed priors. See [Jones, Kulish and Nicolini \(2020\)](#), who discuss in detail the role of priors in the estimation of New Keynesian models with aggregate and state-level data.

²¹See Appendix C for details of data availability across states and time and how we construct our series. We extend the set of observables to household debt and house prices in robustness checks.

²²We extend the sample to 1965 onwards in robustness exercises reported in Appendix E. We also extend the set of observables to include household debt and house prices in robustness checks.

Mixed frequency/observation As mentioned above, our data is such that inflation data do not exist for around half of the 51 states in our panel, and the inflation series is biannual, while other state-level observables are annual. An innovation of our analysis is to extend the estimation of the structural model to this unbalanced panel. To do this, let N be the size of the model’s state-space, and define by \mathbf{z}_t^s the $(\hat{N}_t^s \times 1)$ vector of state s ’s observable variables at time t . Note that the dimension of state s ’s observable vector is changing over time with the availability of data. We map each state’s \mathbf{z}_t^s to the $(N \times 1)$ vector of model variables $\hat{\mathbf{x}}_t^s$ by the $(\hat{N}_t^s \times N)$ matrix \mathbf{H}_t^s :

$$\mathbf{z}_t^s = \mathbf{H}_t^s \hat{\mathbf{x}}_t^s.$$

Thus, to allow for estimation using different frequencies and observables, the differences across states and time are encoded in the matrix \mathbf{H}_t^s , so that forecast errors are computed only for the data series available at each point in time.²³

To illustrate the procedure with an example, consider an estimation using an unbalanced panel dataset consisting of two regions labeled A and B and two observables, inflation and the output gap (which, for simplicity, also define the state space; that is, $N = 2$ in the dimension of $\hat{\mathbf{x}}_t^s$). With two observables, \hat{N}_t^s can be 0, 1, or 2, depending on data availability.

Assume the following structure for the panel: from period t , the output gap is observed every two periods for both regions, while inflation is observed every period, but only for region A . Defining $\mathbf{z}_t = \left[(\mathbf{z}_t^A)' \quad (\mathbf{z}_t^B)' \right]'$ as the vector of observable variables, the panel’s structure implies that \mathbf{z}_t is of dimension $\hat{N}_t^A + \hat{N}_t^B = 2 + 1$ in period t and has dimension $\hat{N}_{t+1}^A + \hat{N}_{t+1}^B = 1 + 0$ in period $t + 1$. To map these to

²³We describe the full Kalman filter in Appendix D.

the state vector, the coefficient matrices for region A are

$$\mathbf{H}_t^A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{H}_{t+1}^A = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and the coefficient matrices for region B are

$$\mathbf{H}_t^B = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

and \mathbf{H}_{t+1}^B is of zero dimension. Notice that in period $t + 1$, region B exits the set of observable variables that are used to compute forecast errors and the model's likelihood with the Kalman filter.

To the best of our knowledge, by using this procedure, ours is the first paper to show how to bring an unbalanced panel dataset to the estimation of a structural macro model, which could prove useful in other contexts and applications. More generally, this flexible approach opens up more possibilities of how to bring regional-level data to identify key parameters of macro models, building on the work of Nakamura and Steinsson (2014); Beraja, Hurst and Ospina (2019); and Jones, Midrigan and Philippon (2018).

4.3 Estimation Results

The key objects of the estimated structural model that we focus on are the two Calvo parameters. We thus discuss our results regarding λ_p and λ_w first. This formal statistical analysis allows us to discuss the extent to which the parameters of interest are statistically stable over time. However, in order to get a sense of the extent to which any statistical difference brings about relevant economic differences, we also discuss the implications of our results regarding two transformations of the Calvo parameters. The first is to convert the Calvo parameters into slopes of the corresponding price and wage Phillips curves. This is important, since those slopes

Table 2: Posterior Distributions, Relative State Data Only

Parameter	1977 to 2017			1977 to 1998			1999 to 2017		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.60	0.59	0.61	0.57	0.55	0.60	0.62	0.60	0.64
λ_w	0.43	0.41	0.44	0.55	0.52	0.58	0.40	0.38	0.42

are the relevant objects governing the dynamics of the system. The second is to convert the Calvo parameters into frequency of price changes by firms and wage changes by unions in the model. This not only provides us with an alternative metric but also allows us to compare our implied estimates with the micro estimates found in the literature.

In light of the previous discussion, we first report in Table 2 the posterior distributions of the Calvo parameters λ_p and λ_w estimated using state-level data only. The remaining structural parameters for all estimations are reported in Appendix E, including all prior specifications. The first panel of Table 2 reports the results of the estimation for the entire sample, 1977 to 2017. We find that the Calvo parameter for prices is 0.60 at the posterior mode, and the Calvo parameter for wages is 0.43 at the posterior mode. The posterior distributions for both parameters are very tight around their respective modes, with 90% of the mass concentrated in barely 3 basis points.

The second and third panels of Table 2 report the results for two sub-samples, the first covering the 1977 to 1998 period and the second covering the 1999 to 2017 period.²⁴ As the table makes clear, the estimates for the Calvo price parameter are remarkably close to each other and to the estimate for the overall sample. Both of them are also tightly estimated, with a 90% probability interval of 4 and 5 basis points. The estimates for the Calvo wage parameter present some signs of instability.

²⁴The natural way would be to split the sample equally, choosing 1997 as the break year. However, we will check the robustness of the estimates to a model that additionally uses household debt during the buildup and subsequent bust around the financial crisis, as emphasized in Jones, Midrigan and Philippon (2018). As the debt data at the state level start in 1999, we chose to start the second sub-sample in that year.

The estimate for the second sub-sample is very close to the estimate for the overall sample and also very precisely estimated – a 90% probability interval of 4 basis points. However, the estimate for the first sub-sample (0.55) is higher than the estimate for the overall sample (0.43), with a probability interval of 6 basis points.

Table 3 shows the Calvo parameters of the same model estimated of aggregate data alone. We also report the estimated Taylor rule parameters. In estimating the model with aggregate data, there is no reason to restrict the estimation to a start date in 1977. However, in order to make a comparison of the results with the ones in Table 2, we use the exact same periods as in there. We explore and report a larger sample period for the aggregate data estimation below.

Before turning to the discussion of the estimated Calvo parameters, notice that the estimated coefficients of the Taylor rule vary substantially across the two sub-periods. How these different policy regimes may affect the estimates is discussed below.

Regarding the values for the Calvo parameters over the full sample, note first that the difference with the ones estimated using state-level data is striking: the mode of the Calvo price parameter is 0.92 (compared with 0.60 in Table 2), while for the Calvo wage parameter, the mode is 0.84 (compared with 0.43 in Table 2).²⁵

The sample size of the aggregate data is substantially shorter than the size of the panel used in the state-level analysis. In spite of that, the Calvo price parameter is quite precisely estimated, with a 90% probability band of 4 basis points. The case of the wage Calvo parameter is slightly less precise, with a corresponding value of 8 basis points. In comparing the differences between the estimates of the two different sub-samples we see differences (8 basis points for the Calvo price and 7 basis points for the Calvo wage parameter), but they are orders of magnitude smaller than those for the Calvo wage parameter in using state-level data (15 basis points).

²⁵The finding that wages are more flexible at the state level compared with the aggregate-level has already been pointed out in [Beraja, Hurst and Ospina \(2019\)](#) and in [Jones, Midrigan and Philippon \(2018\)](#). We find that observation applies also to prices.

Table 3: Posterior Distributions, Aggregate Data Only

Parameter	1977 to 2015			1977 to 1998			1999 to 2015		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
<i>Calvo Parameters</i>									
λ_p	0.92	0.90	0.94	0.85	0.79	0.89	0.93	0.90	0.95
λ_w	0.84	0.80	0.88	0.91	0.87	0.93	0.84	0.80	0.89
<i>Taylor Rule Parameters</i>									
α_r	0.81	0.73	0.85	0.63	0.38	0.78	0.81	0.71	0.86
α_p	2.35	1.98	3.03	2.02	1.62	3.38	1.35	1.05	2.59
α_x	0.46	0.37	0.65	1.72	0.99	1.96	0.17	0.13	0.25
α_y	0.26	0.21	0.39	0.05	0.01	0.23	0.26	0.21	0.35

These rather small differences in the estimated Calvo parameters across the two sub-periods using aggregate data mask much larger differences in the implied slopes of the Phillips curves, which have been the elasticities focused on in the literature (see the discussion in Section 2). Just as in standard New Keynesian models, the slope of the Phillips curve in our model is a non-linear function of the Calvo parameter. Indeed, the relationship between the Calvo parameter and the implied coefficient in the slope of the respective Phillips curve is given by

$$\text{slope}_k = \frac{(1 - \beta\lambda_k)(1 - \lambda_k)}{\lambda_k}, \quad k \in \{p, w\}. \quad (15)$$

A quick inspection of (15) reveals that a change in λ_k from 0.9 to 0.95, say, implies a more drastic change in the Phillips curve slope than a change in λ_k from, say, 0.6 to 0.65.

With this non-linearity in mind, we map the implied Calvo price and wage estimates to the slopes of the Phillips curves in Table 4 to get a sense of what our estimates for the Calvo price and wage parameters imply for the slopes of their respective Phillips curves.²⁶ As expected, the implied slopes vary considerably de-

²⁶The slope of the curves may involve other parameters from preferences or technology. But the

Table 4: Implied Slopes of Phillips Curve at Baseline Estimates

	1977 to 2015	1977 to 1998	1999 to 2015
A. State-Level Estimates			
Prices [*]	0.276	0.317	0.237
Wages [†]	0.814	0.363	0.892
B. Aggregate-Level Estimates			
	1977 to 2017	1977 to 1998	1999 to 2017
Prices [*]	0.008	0.030	0.006
Wages [†]	0.035	0.011	0.031

*: Price Phillips curve slope is $(1 - \beta\lambda_p)(1 - \lambda_p)/\lambda_p$

†: Wage Phillips curve slope is $(1 - \beta\lambda_w)(1 - \lambda_w)/\lambda_w$

pending on whether we use the state-level estimates or the aggregate ones. Our state-level estimate for the whole sample of λ_p implies a slope of 0.28. The aggregate estimates give a much flatter slope, closer to 0.01, consistent with New Keynesian models estimated with aggregate data in the literature. Incidentally, the slope of the price Phillips curve implied by our whole sample estimate of the Calvo price parameter, very close to 0.3, is statistically indistinguishable from the estimate of the preferred specification in [McLeay and Tenreyro \(2020\)](#). The point estimate they report is 0.379 with a standard deviation of 0.052 (see column 4 of Table 3 on page 273). They also use MSA-level data, but they use a limited information approach, a somewhat different sample, and different observables than we do.

But the key finding we want to emphasize is how the estimates of the implied slope of the Phillips curves change across sub-periods. As expected from the previous discussion, there are no relevant differences across subperiods in the estimation of the slopes for the price Calvo parameters using state-level data. But there are major differences using aggregate data. For the case of the wage Phillips curve, there are detectable differences in the implied slope using the state-level estimates. But the

term (15) is typically found in the formulas for the slopes (see [Galí, 2008](#)).

differences relative to the estimated slope using the whole sample are larger when using aggregate data.

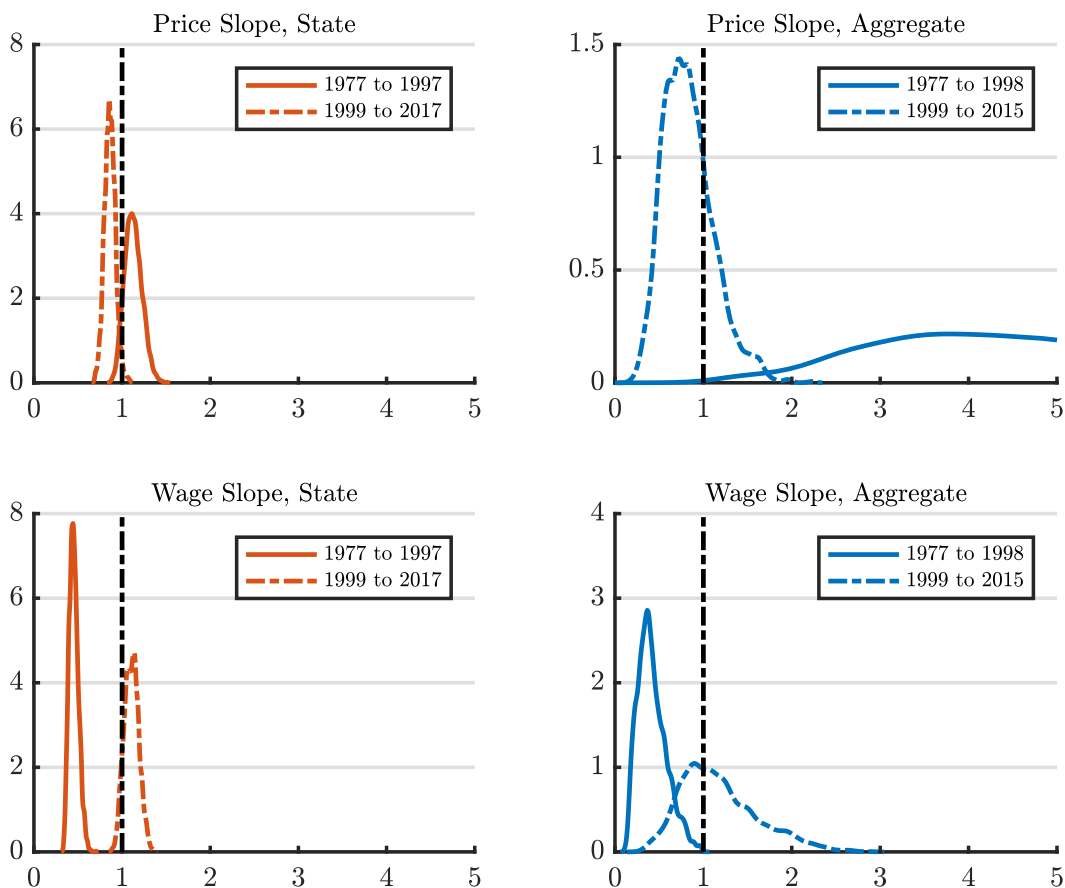
This is most apparent in Figure 3, which plots the posterior distribution of the slopes implied by the posterior distribution of Calvo parameters for two subsamples, but they are normalized to the full sample mode to aid the comparison. The distribution of Phillips curve slopes is not only significantly wider using the estimates coming from aggregate data but also significantly different across periods.

In the case of the wage slope estimated of state-level data (bottom left panel of in Figure 3), although the distributions suggest statistically different slopes across periods, the difference is small and of little economic significance.

To see this in a different metric, note that the Calvo parameters governing nominal rigidities in our model have a precise interpretation: the timing of price and wage adjustments are time dependent, with an average contract duration of $1/(1 - \lambda_k)$, $k \in \{p, w\}$. Thus, at the mode, these different slopes in the wage Phillips curve correspond to a frequency of wage adjustment of 2 quarters for the 1977 to 1998 sample and 1.7 quarters for the 1999 to 2017 sample. For the comparable estimates on aggregate data, the frequency of wage adjustment is around 10 quarters for the 1977 to 1998 sample but 6.2 quarters for the 1999 to 2017 sample. In Table 5, we present a full analysis of the mapping between Calvo parameters and frequency of price and wage changes for our estimates in Tables 2 and 3.

Table 5 highlights the close match between our state-level estimates and existing micro evidence on the frequency of price and wage changes. Because of the importance of price stickiness for aggregate dynamics, a large literature has developed that uses micro evidence to shed light on the frequency of price and wage adjustments and thus λ_p and λ_w . Our estimates are surprisingly close to those reported in these studies. For instance, Nakamura and Steinsson (2008) find average price durations of about 7 to 9 months, while our range of estimates of between 0.55 and 0.64 for the Calvo price parameter λ_p over the subsamples implies average durations

Figure 3: Distributions of Phillips Curve Slopes



Notes: Each sub period posterior distribution of slopes is normalized by the mode of the full sample slope.

between $6\frac{2}{3}$ to $8\frac{1}{3}$ months. For wages, [Bihan, Montornes and Heckel \(2012\)](#) find that the mean duration of a wage spell is just over 2 quarters or 6 months, using firm-level data from France. Our range of estimates, depending on the sample, of between 0.38 and 0.58 for the Calvo wage parameter λ_w implies an average duration of a wage contract of about 1.6 quarters (or just under 5 months) to 2.4 quarters (about 7 months).

The large differences in the distributions of the slope that emerge when relying on aggregate data reflect changes in the monetary policy regime, according to our interpretation of the results presented so far. These differences are therefore

Table 5: Average Contract Durations Implied by Calvo Parameters

	State-Level			Aggregate-Level		
	Mode	5%	95%	Mode	5%	95%
A. Price Contracts (Quarters)						
Full Sample	2.5	2.4	2.6	12.4	9.7	17.5
1977 to 1998	2.4	2.2	2.5	6.5	4.7	9.2
1999 to 2017	2.6	2.5	2.7	14.3	10.4	19.0
B. Wage Contracts (Quarters)						
Full Sample	1.7	1.7	1.8	6.4	4.9	8.3
1977 to 1998	2.2	2.1	2.4	10.8	7.6	14.5
1999 to 2017	1.7	1.6	1.7	6.2	4.9	9.0

consistent with the evidence provided in Section 3: while the reduced form parameter on state-level data was invariant to the sub-periods used for the estimation, the slopes implied by the estimates using aggregate data that depends on the policy rule changed over time. The structural estimation, however, allows us to move beyond those qualitative statements and evaluate the quantitative relevance of the key conceptual point raised by [Haldane and Quah \(1999\)](#): that endogenous changes in the policy regime blur the ability to estimate the structural parameters using aggregate data.

In order to do so, we show the results obtained from two exercises. In the first, we use the fact that the estimated Taylor rule parameters α_r , α_p , α_x , and α_y vary widely across the two sub-samples, as shown in Table 3. For instance, we find that the weight on the growth rate of potential output is highest in the first sub-sample of 1977 to 1998, while the weight on inflation deviations is smallest over the second sub-sample (which includes the zero lower bound period).

With this fact in mind, we repeat the estimation using aggregate data only over the full sample of 1977 to 2015, comparable with the first panel in Table 3. But rather than jointly estimating the Taylor rule, we fix its parameters at the

sub-sample estimates from Table 3. Thus, we estimate the Calvo parameters for the whole sample but fix the Taylor rule at the values estimated for the 1977 to 1998 sub-sample, as reported in the second panel of Table 3 (that is, $\alpha_r = 0.85$, $\alpha_p = 3.03$, $\alpha_x = 0.65$, and $\alpha_y = 0.39$). Then, we repeat the same estimation but fix the parameters of the Taylor rule at the values estimated for the 1999 to 2015 subsample (that is, $\alpha_r = 0.81$, $\alpha_p = 1.35$, $\alpha_x = 0.17$, and $\alpha_y = 0.26$).

These results are in Panel A of Table 6. The first column reports the estimated Calvo parameters when the Taylor rule is estimated for the full sample. These are the same as the ones reported in the first column of Table 3. We added them to aid the comparison. To avoid clutter, we also chose not to report the confidence intervals as they are similar to what was reported so far and the full results can be found in the Appendix. The second column reports the estimates when the Taylor rule is fixed at the estimated values of the first sub-period. The third column reports the estimates when fixing the Taylor rule parameters at the estimated values of the second sub-period.

In our second and final exercise, we repeat the estimation using aggregate data, but without restricting the sample period to coincide with the state-level data. The motivation to do so is the presumption that the period of increasing inflation and subsequent stabilization that the US experienced starting in the mid '60s and ending in the mid '80s was a different policy regime than the one that followed after the Volcker stabilization. That presumption leads us to estimate the model for the whole 1965-2017 period as well as for the sub-periods that are obtained by dividing the sample in 1985, much in the spirit of the results reported in Table 3, but without restricting the estimation to be over the same sample period than with the state-level data exercise. The results are reported in Panel B of Table 6. The bottom panel shows the estimated values for the policy rule and confirms the presumption of large variations across sub-periods.

Again, there is substantial variation over sub-periods in the estimated values

Table 6: Mode of Posterior Distributions, Interaction With Policy Rules

A. Aggregate Data Only, Fixed Taylor Rule Parameters			
Parameter	1977 to 2015 [*]	1977 to 2015 [†]	1977 to 2015 [‡]
λ_p	0.92	0.89	0.92
λ_w	0.83	0.78	0.83
B. Aggregate Data Only, Policy Regime Periods			
Parameter	1965 to 2015 [§]	1965 to 1985 [§]	1986 to 2015 [§]
<i>Calvo Parameters</i>			
λ_p	0.86	0.72	0.93
λ_w	0.90	0.91	0.87
<i>Taylor Rule Parameters</i>			
α_r	0.93	0.95	0.86
α_p	4.02	4.48	2.42
α_x	0.46	0.55	0.21
α_y	0.77	0.82	0.27

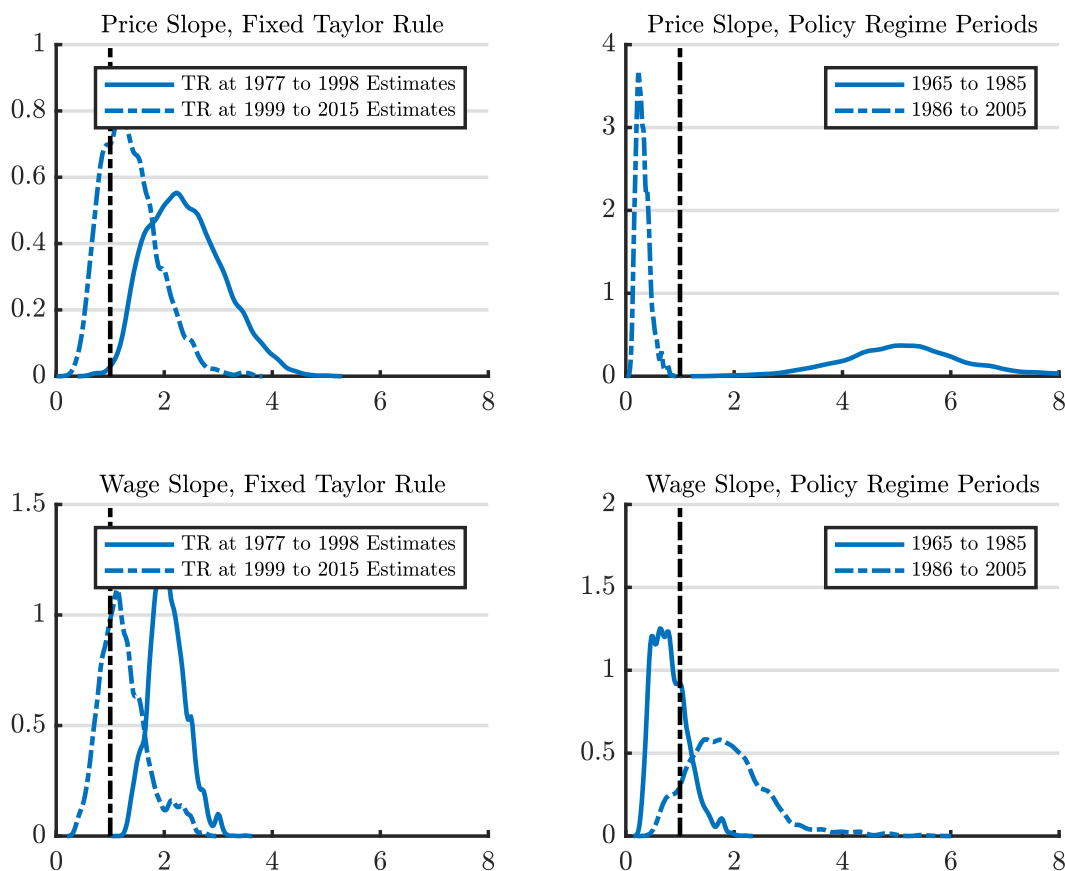
^{*}: Estimated Taylor Rule with uniform priors

[†]: Taylor Rule parameters fixed at 1977 to 1998 estimates (see Table 3)

[‡]: Taylor Rule parameters fixed at 1999 to 2015 estimates (see Table 3)

[§]: No credit or house price series and no credit or housing preference shocks

Figure 4: Distributions of Phillips Curve Slopes, Interaction with Policy



Notes: Each sub-period posterior distribution of slopes is normalized by the mode of the full sample slope.

for the Calvo parameters. The implications for the estimated slopes of the corresponding Phillips curves presented in Figure 4 are even more pronounced, which is consistent with these sub-samples capturing more clear policy regime changes. This figure is comparable to Figure 3 and illustrates the wide dispersion of implied slopes over the aggregate posterior distributions of λ_p and λ_w .

5 Conclusion

The empirical literature on the stability of the Phillips curves has largely ignored the impact of endogenous monetary policy on Phillips curve regression coefficients.

As has been discussed in the literature, this omission has important implications: when policy is endogenous, regressions on aggregate data are uninformative as to the existence of a stable relationship between unemployment and future inflation. We show how regional data can be used to identify the structural relationship between unemployment and inflation. This insight guides our empirical strategy: we use city-level and state-level data from 1977 to 2017 and show that both the reduced form and the structural parameters of the Phillips curve are quite stable over time.

Our analysis implies that these parameters can be safely assumed to be invariant to policy regime changes of the magnitude observed in the United States since the mid '70s. These implications are consistent with the findings in [Alvarez, Beraja, Gonzalez-Rozada and Neumeyer \(2018\)](#), which show that a model with exogenous Calvo frictions approximates very well an estimated menu-cost model as long as inflation rates are not much higher than 10% a year.

We therefore conclude that in designing monetary policy in the United States, the assumptions that prices change on average about every $2^{1/2}$ quarters while wages change on average every 2 quarters are not subject, quantitatively, to the Lucas critique.

References

- Alvarez, Fernando, Martin Beraja, Martín Gonzalez-Rozada, and Pablo Andres Neumeyer**, “From Hyperinflation to Stable Prices: Argentina’s Evidence on Menu Cost Models,” *The Quarterly Journal of Economics*, 2018, *134* (1), 451–505.
- Atkeson, Andrew and Lee E. Ohanian**, “Are Phillips Curves Useful for Forecasting Inflation?,” *Quarterly Review*, 2001, *25* (1), 2–11.
- Babb, Nathan R. and Alan K. Detmeister**, “Nonlinearities in the Phillips Curve for the United States: Evidence Using Metropolitan Data,” Finance and Economics Discussion Series 2017-070, Board of Governors of the Federal Reserve System June 2017.
- Beraja, Martin, Erik Hurst, and Juan Ospina**, “The Aggregate Implications of Regional Business Cycles,” *Econometrica*, 2019, *87* (6), 1789–1833.
- Bihan, Herve Le, Jeremi Montornes, and Thomas Heckel**, “Sticky Wages: Evidence from Quarterly Microeconomic Data,” *American Economic Journal: Macroeconomics*, 2012, *4* (3), 1–32.
- Blanchard, Olivier**, “The Phillips Curve: Back to the ‘60s?,” *American Economic Review*, 2016, *106* (5), 31–34.
- Brainard, William C. and James Tobin**, “Pitfalls in Financial Model-Building,” Cowles Foundation Discussion Papers 244, Cowles Foundation for Research in Economics, Yale University 1968.
- Calvo, Guillermo A.**, “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 1983, *12* (3), 383–98.

- Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian**, “Inflation Persistence, Monetary Policy, and the Great Moderation,” *Journal of Money, Credit and Banking*, 2009, 41 (4), 767–786.
- Cochrane, John H.**, “Determinacy and Identification with Taylor Rules,” *Journal of Political Economy*, 2011, 119 (3), 565–615.
- Coibion, Olivier and Yuriy Gorodnichenko**, “Is the Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Disinflation,” *American Economic Journal: Macroeconomics*, 2015, 7 (1), 197–232.
- Edge, Rochelle M. and Refet S. Gürkaynak**, “How Useful Are Estimated DSGE Model Forecasts for Central Bankers?,” *Brookings Papers on Economic Activity*, 2010, 41 (2), 209–259.
- Fitzgerald, Terry and Juan Pablo Nicolini**, “Is There a Stable Relationship between Unemployment and Future Inflation? Evidence from U.S. Cities,” 2014.
- Friedman, Milton**, “The Role of Monetary Policy,” *The American Economic Review*, 1968, 58 (1), 1–17.
- Galí, Jordi**, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, 2008.
- Gao, Han, Mariano Kulish, and Juan Pablo Nicolini**, “Two Illustrations of the Quantity Theory of Money Reloaded,” 2020.
- Goldfeld, Stephen M. and Alan S. Blinder**, “Some Implications of Endogenous Stabilization Policy,” *Brookings Papers on Economic Activity*, 1972, 3 (3), 585–644.
- Goodhart, C. A. E.**, *Money, Information and Uncertainty*, second edition ed., Macmillan International Higher Education, 1989.

- Haldane, Andrew and Danny Quah**, “UK Phillips curves and Monetary Policy,” *Journal of Monetary Economics*, 1999, 44 (2), 259–278.
- Hamilton, James D.**, *Time Series Analysis*, Princeton University Press, 1994.
- Hooper, Peter, Frederic S Mishkin, and Amir Sufi**, “Prospects for Inflation in a High Pressure Economy: Is the Phillips Curve Dead or is It Just Hibernating?,” Working Paper 25792, National Bureau of Economic Research 2019.
- Jones, Callum**, “Unanticipated Shocks and Forward Guidance at the Zero Lower Bound,” 2017.
- , **Mariano Kulish, and Juan Pablo Nicolini**, “The Role of Priors in the Estimation of New Keynesian Models with Aggregate and State-Level Data,” 2020.
- , **Virgiliu Midrigan, and Thomas Philippon**, “Household Leverage and the Recession,” Working Paper 2018.
- Kareken, John and Robert M. Solow**, *Stabilization Policies*, Englewood Cliffs, New Jersey: Prentice-Hall, 1963.
- Kiley, Michael T.**, “What Can the Data Tell Us About the Equilibrium Real Interest Rate?,” Finance and Economics Discussion Series 2015-77, Board of Governors of the Federal Reserve System 2015.
- Krugman, Paul**, “Anchors Away (Slightly Wonkish),” *The Conscience of a Liberal*, New York Times December 4 2015.
- Kulish, Mariano and Adrian Pagan**, “Estimation and Solution of Models with Expectations and Structural Changes,” *Journal of Applied Econometrics*, 2017, 32 (2), 255–274.
- , **James Morley, and Tim Robinson**, “Estimating DSGE Models with Zero Interest Rate Policy,” *Journal of Monetary Economics*, 2017, 88, 35–49.

- Leduc, Sylvain and Daniel J. Wilson**, “Has the Wage Phillips Curve Gone Dormant?,” 2017.
- Lucas, Robert E.**, “Econometric Policy Evaluation: A Critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1976, *1*, 19–46.
- Lucas, Robert E. Jr.**, “Expectations and the Neutrality of Money,” *Journal of Economic Theory*, 1972, *4* (2), 103–124.
- McLeay, Michael and Silvana Tenreyro**, “Optimal Inflation and the Identification of the Phillips Curve,” *NBER Macroeconomics Annual 2019*, 2020, *34*, 199–255.
- Mian, Atif and Amir Sufi**, “House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis,” *American Economic Review*, 2011, *101* (5), 2132–2156.
- **and** – , “What Explains the 2007-2009 Drop in Employment?,” *Quarterly Journal of Economics*, 2014, *82* (6), 2197–2223.
- Mishkin, Frederic S.**, “Inflation Dynamics,” *International Finance*, 2007, *10* (3), 317–334.
- Nakamura, Emi and Jon Steinsson**, “Five Facts about Prices: A Reevaluation of Menu Cost Models,” *The Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.
- **and Jón Steinsson**, “Fiscal Stimulus in a Monetary Union: Evidence from US Regions,” *American Economic Review*, 2014, *104* (3), 753–792.
- Peston, Maurice**, “The Correlation between Targets and Instruments,” *Economica*, 1972, *39* (156), 427–431.
- Samuelson, Paul A. and Robert M. Solow**, “Analytical Aspects of Anti-Inflation Policy,” *The American Economic Review*, 1960, *50* (2), 177–194.

Smets, Frank and Rafael Wouters, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 2007, 97 (3), 586–606.

Stock, James and M. W. Watson, “Forecasting in Dynamic Factor Models Subject to Structural Instability,” in Jennifer Castle and Neil Shepard, eds., *The Methodology and Practice of Econometrics: A Festschrift in Honor of David F. Hendry*, Oxford University Press, 2009, pp. 1–57.

Taylor, John B., “A Historical Analysis of Monetary Policy Rules,” in John B. Taylor, ed., *Monetary Policy Rules*, University of Chicago Press, 1999, pp. 319–348.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

Worswick, G. D. N., “Fiscal Policy and Stabilization in Britain,” *Journal of Money, Credit and Banking*, 1969, 1 (3), 474–495.

Online Appendix

Not for Publication

A An Old Keynesian Model

In this appendix, we present a slightly modified version of the model used by Taylor (1999) and discussed in Cochrane (2011). We show that model to deliver a reduce form like the one analyzed in Section 3. The model specifies a NAIRU-type Phillips curve, where the growth rate of inflation holds a negative linear relationship with the difference between the current unemployment rate and a constant level (known as the natural rate of unemployment). Thus, we write

$$\pi_t - \pi_{t-1} = -\gamma(u_{t-1} - u) - \varepsilon(u_t - u) + e_t^\pi,$$

where π_t is the inflation rate; u_t is the unemployment rate; γ, u , and ε are positive parameters, and e_t^π is a shock. This is the same equation used by Taylor (1999), except that he assumes $\varepsilon = 0$.²⁷ This assumption implies that the policy rate has no immediate effect on the inflation rate. By letting $\varepsilon > 0$, albeit it is small, we allow for that immediate effect.

The second equation establishes a negative linear relationship between unemployment and the difference between the policy interest rate and the inflation rate, so we write

$$u_t = \sigma(i_t - \pi_t - r) + e_t^u,$$

where σ, r are positive parameters and e_t^u is a shock.²⁸

In what follows, we interpret the unemployment rate as deviations from its steady state level u , or, equivalently, we set $u = 0$.

Using the second in the first, we have

$$\pi_t = \pi_{t-1} - \gamma u_{t-1} - \varepsilon(\sigma(i_t - \pi_t - r) + e_t^u) + e_t^\pi$$

or

$$\pi_t = \frac{\pi_{t-1} - \gamma u_{t-1} + \sigma \varepsilon r - \sigma \varepsilon i_t - \varepsilon e_t^u + e_t^\pi}{(1 - \varepsilon \sigma)},$$

whereas using the first in the second, we have

$$u_t(1 - \varepsilon \sigma) = \sigma(i_t - \pi_{t-1} + \gamma u_{t-1} - e_t^\pi - r) + e_t^u$$

²⁷Taylor's model is expressed in terms of output deviations instead of unemployment deviations. Our specification implies a negative linear relationship between output deviations and unemployment deviations.

²⁸To the extent that the term in parentheses on the right-hand side of this equation aims at capturing movements in the real interest rate as deviations from r (presumably its steady state value), the fact that π_t rather than $E_t \pi_{t+1}$ is in this equation may appear surprising. However, as we show below, this equation—with a reinterpretation of the parameters—will arise exactly as the solution in any case, as long as ε is zero. Given the lack of microfoundations, this reinterpretation seems innocuous to us.

$$u_t = -\frac{\sigma}{(1-\sigma\varepsilon)}\pi_{t-1} + \frac{\sigma\gamma}{(1-\sigma\varepsilon)}u_{t-1} + \frac{\sigma i_t - \sigma r - \sigma e_t^\pi + e_t^u}{(1-\sigma\varepsilon)}.$$

Thus, we can write the system as

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{1}{(1-\varepsilon\sigma)} & -\frac{\gamma}{(1-\varepsilon\sigma)} \\ -\frac{\sigma}{(1-\sigma\varepsilon)} & \frac{\sigma\gamma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)} \\ \frac{\sigma}{(1-\varepsilon\sigma)} \end{bmatrix} (i_t - r) + \begin{bmatrix} -\frac{\varepsilon}{(1-\varepsilon\sigma)} & \frac{1}{(1-\varepsilon\sigma)} \\ \frac{1}{(1-\sigma\varepsilon)} & -\frac{\sigma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix}.$$

Recall that we had assumed that $\varepsilon > 0$, albeit it is small. Thus, the coefficient of unemployment in the inflation equation is close to $-\gamma$, which is the slope of the NAIRU Phillips curve.

A.1 The Interest Rate Rule

If we assume, as [Taylor \(1999\)](#) and [Cochrane \(2011\)](#) do, that

$$i_t = r + \phi_\pi \pi_t + \phi_y y_t,$$

then the solution is

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \\ \frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \frac{1}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} \begin{bmatrix} -\varepsilon & 1 \\ 1 & \sigma(\phi_\pi-1) \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix}.$$

The two roots are given by

$$\begin{aligned} \lambda_1 \lambda_2 &= 0 \\ \lambda_1 + \lambda_2 &= \frac{(1+\sigma\phi_u) - (\phi_\pi-1)\sigma\gamma}{(1+\sigma\phi_u) + \sigma(\phi_\pi-1)\varepsilon}, \end{aligned}$$

so one root is zero, and the other is given by

$$\frac{(1+\sigma\phi_u) - (\phi_\pi-1)\sigma\gamma}{(1+\sigma\phi_u) + \sigma(\phi_\pi-1)\varepsilon},$$

which is less than one as long as $\phi_\pi > 1$, as described in [Taylor \(1999\)](#). Therefore, the system has a unique bounded solution.

A.2 Characterizing the Optimal Policy Rule

Recall that the solution is given by

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{1}{(1-\varepsilon\sigma)} & -\frac{\gamma}{(1-\varepsilon\sigma)} \\ -\frac{\sigma}{(1-\sigma\varepsilon)} & \frac{\sigma\gamma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)} \\ \frac{\sigma}{(1-\varepsilon\sigma)} \end{bmatrix} (i_t - r) + \begin{bmatrix} -\frac{\varepsilon}{(1-\varepsilon\sigma)} & \frac{1}{(1-\varepsilon\sigma)} \\ \frac{1}{(1-\sigma\varepsilon)} & -\frac{\sigma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix},$$

so, in the notation of the paper,

$$\pi_{t+1} = a + b\pi_t + cu_t + di_t + \xi_t^\pi,$$

so

$$b = \frac{1}{(1-\varepsilon\sigma)}, c = -\frac{\gamma}{(1-\varepsilon\sigma)}, d = -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)},$$

and the optimal policy is

$$i_t^{Opt} = \frac{1}{d} [\pi_{t+1}^* - (a + b\pi_t + cu_t + E_t\xi_{t+1}^\pi)],$$

so

$$i_t^{Opt} = \frac{1}{-\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)}} \left[\pi_{t+1}^* - \left(a + \frac{1}{(1-\varepsilon\sigma)}\pi_t + -\frac{\gamma}{(1-\varepsilon\sigma)}u_t + E_t\xi_{t+1}^\pi \right) \right]$$

or

$$i_t^{Opt} = \left[-\frac{(1-\varepsilon\sigma)}{\sigma\varepsilon}\pi_{t+1}^* + \frac{(1-\varepsilon\sigma)}{\sigma\varepsilon}a + \frac{1}{\sigma\varepsilon}\pi_t - \frac{\gamma}{\sigma\varepsilon}u_t - E_t\xi_{t+1}^\pi \right].$$

Thus, as long as $\sigma\varepsilon < 1$, which will hold for small values of ε , the conditions for a unique stable solution are satisfied.

A.3 The Reduced Form Parameter versus the Structural Form Parameter

The solution of the model is given by

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \\ \frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \frac{1}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} \begin{bmatrix} -\varepsilon & 1 \\ 1 & \sigma(\phi_\pi-1) \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix},$$

so we can write the solution for inflation as

$$\pi_t = \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\pi_{t-1} - \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma u_{t-1} + \frac{e_t^\pi - \varepsilon e_t^u}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}.$$

Thus, the reduced form parameter $\hat{\gamma}$ is equal to

$$\hat{\gamma} = \frac{(1 + \sigma\phi_u)}{(1 + \sigma\phi_u) + \sigma(\phi_\pi - 1)\varepsilon} \gamma = \frac{1}{1 + \varepsilon \frac{\sigma(\phi_\pi - 1)}{(1 + \sigma\phi_u)}} \gamma,$$

so it is lower than the structural parameter γ but arbitrarily close when ε is close to zero.

B Regression Specifications

In this section, we present more details on the empirical specifications presented in Section 3. In Section B.1, we provide the list of variables that are used as controls in Table 1 and their sources. In Section B.2, we present the regressions models we adopt in the main text. Section B.3 shows additional results in the reduced form analysis.

B.1 Control Variables

- ***rgdp***: state-level real GDP growth relative to national average in the same period, trend term of HP filtered series with smoothing parameter 400.
Source: Bureau of Economic Analysis (BEA) <https://apps.bea.gov/regional/downloadzip.cfm>, all MSAs since 1960.
- ***temp***: MSA-level temperature relative to regional average in 1960 and 2018.
Source: National Centers for Environmental Information (NCEI), <https://www.ncdc.noaa.gov/cag/>, all MSAs since 1960, except for Kansas City since 1972 and Honolulu since 1965.
- ***prec***: MSA-level precipitation relative to regional average in 1960 and 2018.
Source of variable ***prec*** is the same as ***temp***.
- ***infExp***: division-level inflation expectation relative to national average in the same period.
Source: Survey of Consumers from the University of Michigan, https://data.sca.isr.umich.edu/sda-public/cgi-bin/hsda?harc_sda+sca, all divisions since 1978.
- ***bartik***: interaction of regional exposure variable (combining regional industrial employment composition and government expenditure shipment by industry) with a measure of the growth rate of real aggregate federal government consumption.
Source: constructed following [McLeay and Tenreyro \(2020\)](#).

The variable x in region i in period t is denoted by x_t^i , we further define its cross-sectional deviation from US average Δx_t^i and its deviation from 1960–2018 regional average $\Delta^R x_t^i$ as

$$\begin{aligned}\Delta x_t^i &= x_t^i - X_t^{US} \\ \Delta^R x_t^i &= x_t^i - \frac{1}{N} \sum_{t=1960}^{2018} x_t^i\end{aligned}$$

B.2 Regression Specifications

- We specify the OLS regression models without controls in the following form:

$$\pi_{t+1}^i = b\Delta u_t^i + c\Delta\pi_t^i + \sum_s \mathbb{I}\{t = s\}\alpha_s.$$

- For the 2SLS without control regression models, we use Δu_{t-1}^i as instruments for the first stage.
- For the regression models with controls (both OLS and 2SLS), we extend the models without controls to include $\Delta^R temp_t^i$, $\Delta^R prec_t^i$, $\Delta infExp_t^i$, Δu_{t-2}^i , $\Delta\pi_{t-1}^i$, $\Delta\pi_{t-2}^i$, and $bartik_t$ as explanatory variables. However, we can use the $bartik_t$ starting in 1990 only, owing to the availability of data. We show that when adding this control to the sub-samples from 1990 onwards, the results do not change for headline or core inflation.

B.3 Full Reduced Form Results

In this subsection, we present a complete set of results corresponding to the regression analysis of Section 3. First, we show the estimated value for the time dummy, which corresponds to the estimate of the inflation target. We report the results using both core and headline inflation in Figure 5.

In Tables 7 to 9, we report complete results for the regressions using headline inflation, including OLS and 2SLS, with and without controls. We also show, in Table 10, results including the $bartik_t$ variable as a control, which we have only since the late 80s. As we show, the results barely change when including that additional control for the period in which we have data. We report only the case of 2SLS with controls, but the results are also robust for the other specifications and also when we use core inflation rather than headline. We then present the results for OLS and 2SLS with and without controls when using core in tables 11 to 13.

Figure 5: Model Estimation of Inflation Target

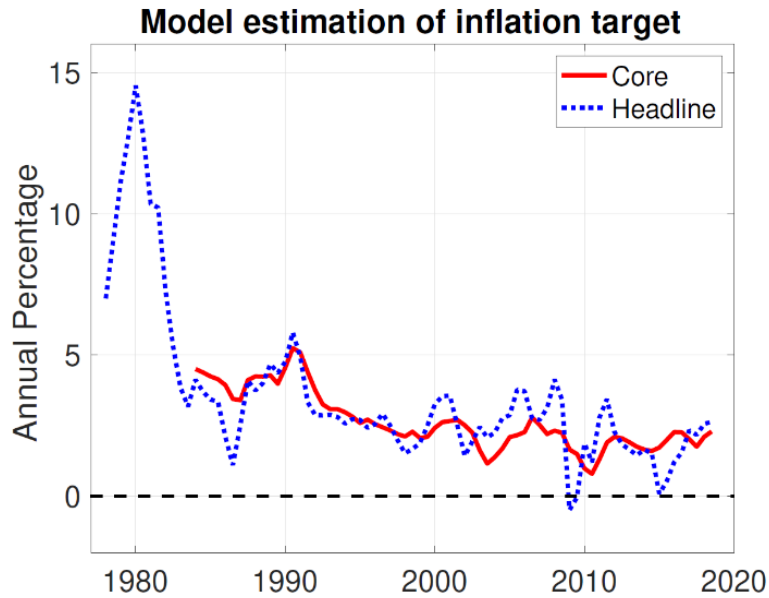


Table 7: Headline without Controls

Model	Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
OLS	c	-0.31** (0.11)	-0.41** (0.11)	-0.31** (0.05)	-0.24** (0.06)	-0.21** (0.07)	-0.28** (0.03)	-0.28** (0.04)
	b	-0.17** (0.06)	0.00 (0.06)	0.21** (0.06)	0.05 (0.06)	0.04 (0.07)	0.22** (0.04)	0.10** (0.05)
	Overall R^2	0.83	0.69	0.45	0.70	0.51	0.72	0.88
2SLS	c	-0.39** (0.12)	-0.29* (0.15)	-0.46** (0.13)	-0.21** (0.08)	-0.24** (0.08)	-0.24** (0.03)	-0.27** (0.04)
	b	-0.18** (0.06)	0.05 (0.09)	0.14 (0.09)	0.06 (0.06)	0.03 (0.07)	0.24** (0.05)	0.10** (0.05)
	Overall R^2	0.79	0.71	0.39	0.70	0.51	0.74	0.88
Observations		377	288	492	536	362	1678	2055

* significant at 5% level, ** significant at 1% level

Table 8: Headline OLS with Controls

Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
c	-0.19 (0.18)	-0.43** (0.09)	-0.14* (0.07)	-0.26** (0.07)	-0.17 (0.10)	-0.28** (0.04)	-0.27** (0.07)
b	-0.30** (0.10)	-0.01 (0.08)	0.19** (0.06)	0.06 (0.06)	0.02 (0.07)	0.23** (0.04)	0.08 (0.04)
$e(infExp)$	-0.30** (0.10)	-0.01 (0.08)	0.19** (0.06)	0.06 (0.06)	0.02 (0.07)	0.23** (0.04)	0.08 (0.04)
$e(prec)$	-0.20 (0.13)	0.00 (0.06)	0.03 (0.04)	-0.01 (0.05)	0.07* (0.03)	0.05 (0.03)	0.00 (0.03)
$e(temp)$	0.19** (0.07)	0.01 (0.06)	-0.02 (0.03)	-0.09* (0.04)	0.07 (0.04)	0.00 (0.01)	0.00 (0.02)
$e(u(-1))$	-0.21 (0.15)	-0.02 (0.12)	-0.27* (0.12)	-0.02 (0.09)	-0.10 (0.11)	-0.07 (0.05)	-0.06 (0.06)
$e(u(-2))$	-0.04 (0.15)	0.02 (0.07)	0.09 (0.11)	0.03 (0.12)	0.12 (0.08)	0.12* (0.05)	0.08 (0.05)
$e(\pi(-1))$	-0.22* (0.11)	-0.23** (0.08)	0.06 (0.05)	-0.09 (0.06)	-0.03 (0.06)	0.03 (0.03)	0.05 (0.04)
$e(\pi(-2))$	-0.23** (0.06)	-0.11 (0.08)	-0.09 (0.05)	-0.02 (0.06)	0.09 (0.06)	-0.01 (0.03)	0.00 (0.02)
Overall R^2	0.80	0.64	0.46	0.70	0.54	0.73	0.88
Observations	327	288	484	532	362	1666	1993

* significant at 5% level, ** significant at 1% level

Table 9: Headline 2SLS with Controls

Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
c	-0.50** (0.19)	-0.45** (0.14)	-0.45** (0.10)	-0.28** (0.10)	-0.28* (0.13)	-0.35** (0.05)	-0.33** (0.05)
b	-0.29** (0.09)	-0.02 (0.09)	0.14 (0.08)	0.06 (0.06)	0.01 (0.08)	0.22** (0.04)	0.08 (0.04)
$e(infExp)$	-0.04 (0.33)	-0.07 (0.18)	0.17 (0.17)	0.11 (0.25)	0.08 (0.23)	0.16 (0.11)	-0.13 (0.15)
$e(prec)$	-0.20 (0.13)	0.00 (0.06)	0.04 (0.04)	-0.01 (0.05)	0.08** (0.03)	0.05 (0.03)	0.00 (0.03)
$e(temp)$	0.20** (0.08)	0.01 (0.06)	-0.02 (0.03)	-0.09* (0.04)	0.08* (0.03)	0.00 (0.01)	0.00 (0.02)
$e(u(-2))$	-0.06 (0.14)	0.02 (0.06)	0.00 (0.10)	0.02 (0.11)	0.09 (0.07)	0.10** (0.04)	0.07 (0.04)
$e(\pi(-1))$	-0.20* (0.10)	-0.22** (0.08)	0.07 (0.06)	-0.09 (0.06)	-0.03 (0.06)	0.03 (0.03)	0.05 (0.04)
$e(\pi(-2))$	-0.21** (0.07)	-0.10 (0.08)	-0.09 (0.05)	-0.02 (0.05)	0.09 (0.06)	-0.01 (0.03)	0.00 (0.02)
Overall R^2	0.76	0.65	0.40	0.70	0.54	0.74	0.88
Observations	327	288	484	532	362	1666	1993

* significant at 5% level, ** significant at 1% level

C Data

C.1 Description of Data for Reduced Form Exercises

This appendix describes our data sources and calculations for the reduced form exercises. We analyze semiannual CPI inflation and unemployment data for the United States and for 27 metropolitan statistical areas (MSAs). All semiannual data for unemployment and CPI price indices are computed as the arithmetic average of monthly data for the first and second half of each year. Inflation and price data for MSAs are available only as non seasonally adjusted, so all the data are not seasonally adjusted.

C.1.1 Inflation Data

The Bureau of Labor Statistics (BLS) publishes CPI data for 27 MSAs. The BLS publishes semiannual data for 13 MSAs and higher frequency data (monthly or bimonthly) for the other 14 MSAs. We use semiannual data to obtain the largest possible sample. Headline CPI is available back to 1941 for 23 MSAs, with data for the remaining MSAs starting in 1977, 1987, 1997, and 2002. Core CPI is available back to 1982 for 24 MSAs, with data for the remaining MSAs starting in 1987, 1997, and 2002. When semiannual data are not available as a published series, we compute

Table 10: Headline 2SLS with Controls Including Bartik Variable

Coefficient	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.47** (0.11)	-0.29** (0.10)	-0.28* (0.13)	-0.36** (0.06)
b	0.17* (0.08)	0.06 (0.06)	0.00 (0.08)	0.20** (0.04)
$e(infExp)$	0.13 (0.16)	0.11 (0.25)	0.09 (0.24)	0.14 (0.11)
$e(prec)$	0.06 (0.03)	-0.01 (0.05)	0.07** (0.03)	0.04 (0.03)
$e(temp)$	-0.02 (0.03)	-0.09* (0.05)	0.07* (0.03)	0.00 (0.01)
$e(bartik)$	-1.09 (5.60)	3.59 (5.52)	2.41 (2.98)	-0.37 (2.87)
$e(u(-2))$	0.04 (0.09)	0.02 (0.11)	0.09 (0.07)	0.11* (0.05)
$e(\pi(-1))$	0.07 (0.05)	-0.10 (0.06)	-0.03 (0.06)	0.02 (0.04)
$e(\pi(-2))$	-0.06 (0.05)	-0.02 (0.05)	0.09 (0.06)	0.01 (0.02)
Overall R^2	0.55	0.69	0.54	0.70
Observations	532	532	362	1426

* significant at 5% level, ** significant at 1% level

Table 11: Core – without Controls

Model	Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
OLS	c	-0.47** (0.10)	-0.33** (0.05)	-0.34** (0.07)	-0.29** (0.09)	-0.32** (0.04)
	b	0.09 (0.06)	0.23** (0.08)	0.10 (0.06)	0.06 (0.08)	0.26** (0.04)
	Overall R^2	0.41	0.36	0.34	0.10	0.61
2SLS	c	-0.34** (0.15)	-0.41** (0.13)	-0.25** (0.08)	-0.27** (0.10)	-0.24** (0.04)
	b	0.13 (0.11)	0.20 (0.10)	0.12* (0.06)	0.07 (0.07)	0.30** (0.04)
	Overall R^2	0.42	0.34	0.36	0.13	0.63
Observations		288	492	536	362	1678

* significant at 5% level, ** significant at 1% level

Table 12: Core OLS with Controls

Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.44** (0.09)	-0.21* (0.08)	-0.37** (0.07)	-0.30** (0.11)	-0.36** (0.05)
b	0.10 (0.09)	0.21** (0.08)	0.12* (0.06)	0.03 (0.07)	0.26** (0.03)
$e(infExp)$	0.08 (0.22)	0.12 (0.17)	0.11 (0.30)	0.26 (0.29)	0.17 (0.12)
$e(prec)$	0.00 (0.07)	0.02 (0.04)	0.03 (0.04)	0.07 (0.05)	0.05* (0.02)
$e(temp)$	0.03 (0.06)	-0.06 (0.04)	-0.08 (0.05)	0.00 (0.05)	-0.03* (0.01)
$e(u(-1))$	-0.10 (0.10)	-0.22 (0.15)	0.00 (0.11)	0.01 (0.12)	-0.06 (0.07)
$e(u(-2))$	0.13 (0.10)	0.14 (0.13)	0.04 (0.14)	0.04 (0.11)	0.16** (0.05)
$e(\pi(-1))$	-0.17 (0.09)	-0.09* (0.04)	-0.07 (0.06)	0.07 (0.08)	0.06* (0.03)
$e(\pi(-2))$	-0.08 (0.08)	-0.07 (0.05)	-0.10 (0.06)	0.01 (0.07)	-0.03 (0.02)
Overall R^2	0.37	0.41	0.34	0.13	0.63
Observations	260	484	532	362	1638

* significant at 5% level, ** significant at 1% level

Table 13: Core 2SLS with Controls

Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.57** (0.15)	-0.46** (0.12)	-0.37** (0.13)	-0.29 (0.16)	-0.41** (0.06)
b	0.08 (0.09)	0.18* (0.08)	0.12* (0.06)	0.04 (0.07)	0.25** (0.03)
$e(infExp)$	0.03 (0.19)	0.13 (0.17)	0.11 (0.30)	0.27 (0.29)	0.16 (0.13)
$e(prec)$	0.00 (0.07)	0.03 (0.04)	0.03 (0.04)	0.07 (0.05)	0.06* (0.02)
$e(temp)$	0.04 (0.06)	-0.06 (0.04)	-0.08 (0.05)	0.00 (0.05)	-0.03* (0.01)
$e(u(-2))$	0.09 (0.11)	0.07 (0.11)	0.04 (0.13)	0.04 (0.09)	0.14** (0.04)
$e(\pi(-1))$	-0.17 (0.09)	0.10* (0.05)	-0.07 (0.06)	0.07 (0.08)	0.06* (0.03)
$e(\pi(-2))$	-0.08 (0.08)	-0.07 (0.05)	-0.10 (0.06)	0.01 (0.07)	-0.03 (0.02)
Overall R^2	0.37	0.38	0.35	0.15	0.64
Observations	260	484	532	362	1638

* significant at 5% level, ** significant at 1% level

the semiannual average following BLS methodology: first, interpolate the missing monthly indices using a geometric mean of values in adjacent months; second, calculate the arithmetic average of the monthly data in the first and second half of each year.

C.1.2 Unemployment Data

The BLS publishes a monthly unemployment rate, not seasonally adjusted, for each of the 27 MSAs with corresponding CPI price indices. Published BLS data are available back to 1990. The BLS has unpublished unemployment data back to 1976, but these data are not consistent with the published data because of changes in the MSA geographic definitions and other factors. However, the BLS also has unemployment and labor force data by county, going back to 1976. We used the county-level data to construct a geographically consistent definition of MSAs, going back to 1976. The constructed unemployment and labor force series overlap very closely with the published data in the post-1990 period. We combine our pre-1990 constructed unemployment rates with the published data to obtain unemployment rate series back to 1976. The lack of readily accessible unemployment data before 1976 is a limiting factor for our analysis.

C.2 Description of Data for Structural Model Estimation

C.2.1 State Level

We use the MSA-level inflation data, described above, and map the 27 MSA regions into 20 states with the mapping in Table 14. For states which contain multiple MSA regions (for example, Cincinnati and Cleveland are both in Ohio), we select only the data of one of the MSA regions.

Table 14: MSA to State Mapping

State	MSA
AK	Anchorage
AZ	Phoenix
CA	Los Angeles
CO	Denver
FL	Miami
GA	Atlanta
HI	Honolulu
IL	Chicago
KS	Kansas City
MA	Tampa
MD	Baltimore
MI	Detroit
MO	St. Louis
NY	New York
OH	Cincinnati
OR	Portland
PA	Philadelphia
TX	Dallas
WA	Seattle
WI	Minneapolis

For the other state-level data series, we use state-level data on employment, output, and compensation. The observed state data are annual. To construct the data, we first take each state's series relative to its initial value, compute the deviation of each state's observation from the state mean, regress that series on time dummies, weighted by the state's relative population, and work with the residuals. We then take out a linear trend from the resulting series, for each subsample studied.

Main estimations Here, we provide more details on each series.

- Output: We use state-level data on Gross Domestic Product in current dollars. (BEA SAGDP2S). The data are available for download at the [BEA website](#).

- Employment: We use state-level data on total employment from the [BEA annual table SA4](#). In our empirical analysis, we scale this measure of employment by each state's population.
- Labor Compensation: We use state-level data on compensation of employees from the [BEA annual table SA6N](#).
- Wages: To construct our wages series, we divide total labor compensation by the number of employed individuals, using the two series described above.
- Population: We use state-level data on population from the [BEA annual table SA1-3](#).

Robustness Exercises In robustness exercises, we use the following data series.

- Income: We use state-level data on personal income from the [BEA annual table SA4](#).
- Household Debt: We use data from the FRBNY Consumer Credit Panel [Q4 State statistics by year](#). Our measures of debt include auto loans, credit card debt, mortgage debt and student loans. This database also provides information on the number of individuals with credit scores in each state, which we use to express the debt data in per capita terms. We then construct a debt-to-income series by dividing this measure of per capita debt by per capita income, using the data described above on income and population from the BEA.
- House Prices: We used data on the not seasonally adjusted house price index available on the [FHFA website](#).
- Consumption: For the robustness exercise with consumption, we use state-level data on total personal consumption expenditures by state from the BEA, net of housing. The data are available for download at the [BEA website](#).

C.2.2 Aggregate Level

At the aggregate level, we use the GDP deflator for inflation, employment, output, wages, the Fed Funds rates, and ZLB durations from NY Federal Reserve Survey Data. The codes for each raw data series are as follows:

- Gross Domestic Product: Implicit Price Deflator (GDPDEF).
- Gross Domestic Product: (GDP).
- Cumulated nonfarm business section compensation (PRS85006062) minus employment growth (PRS85006012) and deflated by the GDP deflator.
- Total employment net of construction, over the civilian noninstitutional population.

In robustness exercises, we use:

- Household Debt from FRED (code CMDEBT) deflated by PCE deflator, and expressed relative to income (from the BEA Table 2.1).
- House Prices from Case-Logic.
- Personal Consumption Expenditures (BEA Table 2.4.5U). Current, \$. We subtract housing from consumption.

Fed Funds rate: the interest rate is the Federal Funds Rate, taken from the Federal Reserve Economic Database.

ZLB Durations: we follow the approach of [Kulich, Morley and Robinson \(2017\)](#) and use the ZLB durations extracted from the New York Federal Reserve Survey of Primary Dealers, conducted eight times a year from 2011Q1 onwards.²⁹ We take the mode of the distribution implied by these surveys. Before 2011, we use responses from the Blue Chip Financial Forecasts survey.

D Structural Model

The model description follows [Jones, Midrigan and Philippon \(2018\)](#). We describe the model with the full operative credit channel. But we note that absent this credit channel and the tradeable production structure, the model would reduce to the familiar 3-equation New Keynesian model.

D.1 Full Model with Credit Channel

Household problem The economy consists of a continuum of ex ante identical islands of measure 1 that belong to a trading bloc in a monetary union. Consumers on each island derive utility from the consumption of a final good, leisure, and housing. Let s index an individual island and $p_t(s)$ denote the price of the final consumption good. Individual households on each island belong to labor unions that sell differentiated varieties of labor. We assume perfect risk-sharing across households belonging to different labor unions on a given island. Labor is immobile across islands and the housing stock on each island is in fixed supply. The problem of a household that belongs to labor union ι is to

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t-1} \beta_j(s) \right) \left[\int_0^1 v_{it}(s) \log(c_{it}(s)) \, di + \eta_t^h(s) \log(h_t(s)) - \frac{\eta_t^n(s)}{1+\nu} n_{t,\iota}(s)^{1+\nu} \right] \quad (16)$$

where $h_t(s)$ is the amount of housing the household owns, $n_{t,\iota}(s)$ is the amount of labor it supplies, and $c_{it}(s)$ is the consumption of an individual member i . The term $v_{it}(s) \geq 1$ represents a taste

²⁹See the website [here](#). For example, in the survey conducted on January 18 2011, one of the questions asked was: “Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase” (Question 2b). Responses were given in terms of a probability distribution across future quarters.

shifter, an i.i.d random variable drawn from a Pareto distribution:

$$\Pr(v_{it}(s) \leq v) = F(v) = 1 - v^{-\alpha}. \quad (17)$$

Here, $\alpha > 1$ determines the amount of uncertainty about v . A lower α implies more uncertainty. The terms $\eta_t^h(s)$ and $\eta_t^n(s)$ affect the preference for housing and the disutility from work, while $\beta_t(s)$ is the household's one-period-ahead discount factor. We assume that each of these preference shifters have an island-specific component and an aggregate component, all of which follow AR(1) processes with independent Gaussian innovations. The household's budget constraint is:

$$p_t(s)x_t(s) + e_t(s)(h_{t+1}(s) - h_t(s)) = w_t(\iota, s)n_t(\iota, s) + q_t l_t(s) - b_t(s) + (1 + \gamma q_t)a_t(s) + T_t(\iota, s), \quad (18)$$

where $x_t(s)$ are transfers made to individual members in the goods market, $e_t(s)$ is the price of housing, $w_t(\iota, s)$ is the wage rate, and $T_t(\iota, s)$ collects the profits households earn from their ownership of intermediate goods firms, transfers from the government aimed at correcting the steady state markup distortion, and the transfers stemming from the risk-sharing arrangement.³⁰ We let $a_t(s)$ denote the amount of coupon payments the household is entitled to receive in period t , $b_t(s)$ the amount it must repay, and q_t the economy-wide price of the securities described below. Thus, $q_t a_t(s)$ represents the household's total asset holdings (savings), while $q_t b_t(s)$ represents its outstanding debt. We describe a household's holdings of the security by recording the amount of coupon payments b_t that the household has to make period t . Letting $l_t(s)$ denote the amount of securities the household sells in period t , the date $t + 1$ coupon payments are

$$b_{t+1}(s) = \sum_{i=0}^{\infty} \gamma^i l_{t-i}(s) = l_t(s) + \gamma b_t(s). \quad (19)$$

The household also faces a liquidity constraint limiting the consumption of an individual member to be below the amount of real balances the member holds:

$$p_t(s)c_{it}(s) \leq p_t(s)x_t(s). \quad (20)$$

The household also faces a borrowing constraint

$$q_t l_t(s) \leq m_t(s)e_t(s)h_{t+1}(s). \quad (21)$$

The law of motion for a household's assets is given by

$$q_t a_{t+1}(s) = p_t(s) \left(x_t(s) - \int_0^1 c_{it}(s) di \right). \quad (22)$$

³⁰We assume that households on island s exclusively own firms on that particular island.

There are no barriers to capital flows, so all islands trade securities at a common price q_t . The credit limit $m_t(s)$ evolves as the product of an island-specific and aggregate component, both of which are AR(1) processes with Gaussian disturbances.

At this point, we note that as $\alpha \rightarrow \infty$, $v_{it}(s) \rightarrow 1$. In this case, there is no idiosyncratic uncertainty. There is no meaningful role for the liquidity constraints and, since housing is separable in the utility function and exogenously fixed, there is no role for credit, and the economy collapses to the standard 3-equation New Keynesian model (see [Jones, Midrigan and Philippon, 2018](#), for details and a discussion of this point).

Final goods producers Final goods producers on island s produce $y_t(s)$ units of the final good using $y_t^N(s)$ units of non-tradable goods produced locally and $y_t^M(s, j)$ units of tradable goods produced on island j and imported to island s :

$$y_t(s) = \left(\omega^{\frac{1}{\sigma}} y_t^N(s)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} \left(\int_0^1 y_t^M(s, j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa-1}{\kappa-1} \frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (23)$$

where ω determines the share of non-traded goods, σ is the elasticity of substitution between traded and non-traded goods and κ is the elasticity of substitution between varieties of the traded goods produced on different islands. Letting $p_t^N(s)$ and $p_t^M(s)$ denote the prices of these goods on island s , the final goods price on an island is

$$p_t(s) = \left(\omega p_t^N(s)^{1-\sigma} + (1-\omega) \left(\int_0^1 p_t^M(j)^{1-\kappa} dj \right)^{\frac{1-\sigma}{1-\kappa}} \right)^{\frac{1}{1-\sigma}}. \quad (24)$$

The demand for non-tradable intermediate goods produced on an island is

$$y_t^N(s) = \omega \left(\frac{p_t^N(s)}{p_t(s)} \right)^{-\sigma} y_t(s), \quad (25)$$

while demand for an island's tradable exports $y_t^X(s)$ is an aggregate of what all other islands purchase:

$$y_t^X(s) = (1-\omega) p_t^M(s)^{-\kappa} \left(\int_0^1 p_t^M(j)^{1-\kappa} dj \right)^{\frac{\kappa-\sigma}{1-\kappa}} \left(\int_0^1 p_t(j)^{\sigma} y_t(j) dj \right). \quad (26)$$

Intermediate goods producers Traded and non-traded goods on each island are themselves CES composites of varieties of differentiated intermediate inputs with an elasticity of substitution ϑ . The demand for an individual variety k for non-tradeable goods (for example) are

$$y_t^N(s, k) = (p_t^N(s, k)/p_t^N(s))^{-\vartheta} y_t^N(s).$$

Individual producers of intermediate goods are subject to Calvo price adjustment frictions. Let

λ_p denote the probability that a firm does not reset its price in a given period. A firm that resets its price maximizes the present discounted flow of profits weighted by the probability that the price it chooses at t will still be in effect at any particular date. As was the case earlier, the production function is linear in labor, but it is now subject to sector-specific productivity disturbances $z_t^N(s)$ and $z_t^X(s)$, so that

$$y_t^j(s, k) = z_t^j(s) n_t^j(s, k), \text{ for } j \in \{N, X\}$$

so that the unit cost of production is simply $w_t(s)/z_t^j(s)$ in both sectors.

For example, a traded intermediate goods firm that resets its price solves

$$\max_{p_t^{X^*}(s)} \sum_{j=0}^{\infty} \left(\lambda_p^j \prod_{i=0}^{j-1} \beta_{t+i}(s) \right) \mu_{t+j}(s) \left(p_t^{X^*}(s) - \tau_p \frac{w_{t+j}(s)}{z_{t+j}^X(s)} \right) \left(\frac{p_t^{X^*}(s)}{p_{t+j}^X(s)} \right)^{-\vartheta} y_{t+j}^X(s), \quad (27)$$

where $\mu_{t+j}(s)$ is the shadow value of wealth of the representative household on island s – that is, the multiplier on the flow budget constraint (18) – and $\tau_p = \frac{\vartheta-1}{\vartheta}$ is a tax the government levies to eliminate the steady state markup distortion. This tax is rebated lump sum to households on island s . The composite price of traded exports or non-traded goods is then a weighted average of the prices of individual differentiated intermediates. For example, the price of export goods is

$$p_t^X(s) = \left[(1 - \lambda_p) p_t^{X^*}(s)^{1-\vartheta} + \lambda_p p_{t-1}^X(s)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}. \quad (28)$$

Wage setting We assume that individual households are organized in unions that supply differentiated varieties of labor. The total amount of labor services available in production is

$$n_t(s) = \left(\int_0^1 n_t(\ell, s)^{\frac{\psi-1}{\psi}} d\ell \right)^{\frac{\psi}{\psi-1}}, \quad (29)$$

where ψ is the elasticity of substitution between labor varieties. Demand for an individual union's labor given its wage $w_t(\ell, s)$ is therefore $n_t(\ell, s) = (w_t(\ell, s)/w_t(s))^{-\psi} n_t(s)$. The problem of a union that resets its wage is to choose a new wage $w_t^*(s)$ to

$$\max_{w_t^*(s)} \sum_{j=0}^{\infty} \left(\lambda_w^j \prod_{i=0}^{j-1} \beta_{t+i}(s) \right) \left[\tau_w \mu_{t+j}(s) w_t^*(s) \left(\frac{w_t^*(s)}{w_{t+j}(s)} \right)^{-\psi} n_{t+j}(s) - \frac{\eta_{t+j}^n(s)}{1+\nu} \left(\left(\frac{w_t^*(s)}{w_{t+j}(s)} \right)^{-\psi} n_{t+j}(s) \right)^{1+\nu} \right], \quad (30)$$

where λ_w is the probability that a given union leaves its wage unchanged and $\tau_w = \frac{\psi-1}{\psi}$ is a labor income subsidy aimed at correcting the steady state markup distortion. The composite wage at which labor services are sold to producers is

$$w_t(s) = \left[(1 - \lambda_w) w_t^*(s)^{1-\psi} + \lambda_w w_{t-1}(s)^{1-\psi} \right]^{\frac{1}{1-\psi}}. \quad (31)$$

D.2 Monetary Policy

Let $y_t = \int_0^1 p_t(s)y_t(s)/p_t ds$ be total real output in this economy, where $p_t = \int_0^1 p_t(s)ds$ is the aggregate price index. Let $\pi_t = p_t/p_{t-1}$ denote the rate of inflation and

$$1 + i_t = \mathbb{E}_t R_{t+1} \quad (32)$$

be the expected nominal return on the long-term security, which we refer to as the nominal interest rate. Aggregation over the pricing choices of individual producers implies, up to a first-order approximation,

$$\log(\pi_t/\bar{\pi}) = \bar{\beta}\mathbb{E}_t \log(\pi_{t+1}/\bar{\pi}) + \frac{(1 - \lambda_p)(1 - \lambda_p\bar{\beta})}{\lambda_p} (\log(w_t) - \log(z_t)) + \theta_t,$$

where we add an AR(1) disturbance θ_t to individual firms' desired markups, $\bar{\beta}$ is the steady state discount factor, and $\bar{\pi}$ is the steady-state level of inflation.

We assume that monetary policy is characterized by a Taylor rule when the ZLB does not bind:

$$1 + i_t = (1 + i_{t-1})^{\alpha_r} \left[(1 + \bar{\imath}) \pi_t^{\alpha_\pi} \left(\frac{y_t}{y_t^*} \right)^{\alpha_y} \right]^{1 - \alpha_r} \left(\frac{y_t/y_t^*}{y_{t-1}/y_{t-1}^*} \right)^{\alpha_x} \exp(\varepsilon_t^r),$$

where ε_t^r is a monetary policy shock; α_r determines the persistence; and α_π ; α_y ; and α_x determine the extent to which monetary policy responds to inflation, deviations of output from its flexible price level y_t^* , and the growth rate of the output gap, respectively. We assume that $\bar{\imath}$ is set to a level that ensures a steady state level of inflation of $\bar{\pi}$. When the ZLB binds, then

$$i_t = 0.$$

The interest rate may be at zero either because aggregate shocks cause the ZLB to bind, or because the Fed commits to keeping i_t at 0 for a longer time period than implied by the constraint. We thus implicitly assume that the Fed can manipulate expectations of how the path of interest rates evolves, as in Eggertsson and Woodford (2003) and Werning (2015). In our estimation we use survey data from the New York Federal Reserve to discipline the expected duration of the zero interest rate regime during the 2009 to 2015 period.

Since we assume that an individual island is of measure zero, monetary policy does not react to island-specific disturbances. The monetary union is closed so aggregate savings must equal aggregate debt:

$$\int_0^1 a_{t+1}(s)ds = \int_0^1 b_{t+1}(s)ds. \quad (33)$$

D.3 Likelihood Function

We use Bayesian likelihood methods to estimate the parameters of the island economy and the shocks. We use a panel dataset across states for the state-level estimation, and aggregate data and the ZLB for the aggregate-level estimation. We formulate the state-space of the model so as to separate our estimation into a collection of regional components to make it computationally feasible.

We discuss the likelihood function of the state/regional component and then the likelihood function of the aggregate component.

D.3.1 Likelihood of the State Component

We use Bayesian methods. We first log-linearize the model. The log-linearized model has the state space representation

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t \quad (34)$$

$$z_t = \mathbf{H}_t x_t. \quad (35)$$

The state vector is x_t . The error is distributed $\varepsilon_t \sim N(0, \Omega)$, where Ω is the covariance matrix of ε_t . We assume no observation error of the data z_t .

Denote by ϑ the vector of parameters to be estimated. Denote by $\mathcal{Z} = \{z_\tau\}_{\tau=1}^T$ the sequence of $N_z \times 1$ vectors of observable variables, combined over states. By Bayes law, the posterior $\mathcal{P}(\vartheta | \mathcal{Z})$ satisfies

$$\mathcal{P}(\vartheta | \mathcal{Z}) \propto L(\mathcal{Z} | \vartheta) \times \mathcal{P}(\vartheta).$$

With Gaussian errors ε_t , the likelihood function $L(\mathcal{Z} | \vartheta)$ is computed using the sequence of structural matrices and the Kalman filter, described below:

$$\log L(\mathcal{Z} | \vartheta) = - \left(\frac{N_z T}{2} \right) \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log \det \mathbf{S}_t - \frac{1}{2} \sum_{t=1}^T \tilde{y}_t^\top (\mathbf{S}_t)^{-1} \tilde{y}_t,$$

where \tilde{y}_t is the vector of forecast errors and \mathbf{S}_t is its associated covariance matrix.

D.3.2 Kalman Filter

The Kalman filter recursion is given by the following equations. The state of the system is $(\hat{x}_t, \mathbf{P}_{t-1})$. In the predict step, the structural matrices \mathbf{J} , \mathbf{Q} and \mathbf{G} are used to compute a forecast of the state

$\hat{x}_{t|t-1}$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as

$$\begin{aligned}\hat{x}_{t|t-1} &= \mathbf{J} + \mathbf{Q}\hat{x}_t \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}\mathbf{P}_{t-1}\mathbf{Q}^\top + \mathbf{G}\mathbf{\Omega}\mathbf{G}^\top.\end{aligned}\tag{36}$$

We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors \tilde{y}_t and their associated covariance matrix \mathbf{S}_t as

$$\begin{aligned}\tilde{y}_t &= z_t - \mathbf{H}_t\hat{x}_{t|t-1} \\ \mathbf{S}_t &= \mathbf{H}_t\mathbf{P}_{t|t-1}\mathbf{H}_t^\top.\end{aligned}$$

The Kalman gain matrix is given by

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}_t^\top\mathbf{S}_t^{-1}.$$

With \tilde{y}_t , \mathbf{S}_t and \mathbf{K}_t in hand, the optimal filtered update of the state x_t is

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t\tilde{y}_t,$$

and for its associated covariance matrix,

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and \mathbf{P}_0 determined from their unconditional moments and is computed until the final time period T of data. We can show that the stationary \mathbf{P}_0 has the expression

$$\text{vec}(\mathbf{P}_0) = (\mathbf{I} - \mathbf{Q} \otimes \mathbf{Q})^{-1} \text{vec}(\mathbf{G}\mathbf{\Omega}\mathbf{G}^\top)\tag{37}$$

D.3.3 Kalman Smoother

With the estimates of the parameters on a sample up to time period T , the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information. With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, \mathbf{K}_t , and \mathbf{S}_t in hand from the Kalman filter, the vector $x_{t|T}$ is computed by

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1}r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion

$$r_{t|T} = \mathbf{H}_t^\top\mathbf{S}_t^{-1} (z_t - \mathbf{H}_t\hat{x}_{t|t-1}) + (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by e_t , we can compute

$$e_t = \mathbf{G}\varepsilon_t = \mathbf{G}r_{t|T}.$$

D.3.4 Block Structure

The regional component of the model has a block structure separated by state. For example, consider two states so that the log-linearized state-space representation is

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}^1 \\ \mathbf{J}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{Q}^1 & 0 \\ 0 & \mathbf{Q}^2 \end{bmatrix} \begin{bmatrix} x_{t-1}^1 \\ x_{t-1}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{G}^1 & 0 \\ 0 & \mathbf{G}^2 \end{bmatrix} \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix}$$

Under this block structure, the forecast covariance matrix $P_{t|t-1}$ also has a block structure. This is clear from the expressions (36) and (37).

The block structure is also helpful for computational reasons. The log-likelihood becomes a weighted sum of state-by-state log-likelihood functions. To show this: because $\mathbf{P}_{t|t-1}$ has a block structure, so does \mathbf{S}_t . And because \mathbf{S}_t has a block structure

$$\log \det \mathbf{S}_t = \log \prod_j \det \mathbf{S}_t^j = \sum_j \log \det \mathbf{S}_t^j.$$

Also, because \mathbf{S}_t has a block structure, so does its inverse, so that the last term in the log-likelihood can also be separated by state. The log-likelihood is then

$$\log L(\mathcal{Z} | \vartheta) = \sum_s \log L^s(\mathcal{Z}^s | \vartheta).$$

D.4 Likelihood of the Aggregate Component

D.4.1 Solution with Zero Lower Bound

We write the model that approximates the ZLB in the following way. Under the ZLB, the economy has time variation in the evolution of the model's structural parameters \mathbf{A}_t , \mathbf{B}_t , \mathbf{C}_t , \mathbf{D}_t , and \mathbf{F}_t , where

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t \epsilon_t.$$

For example, if the ZLB binds, the equation describing the Taylor rule becomes $i_t = 0$, changing the structural matrices \mathbf{A}_t , and so on. With time-varying structural matrices, the solution we seek is the time-varying VAR representation:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \epsilon_t, \tag{38}$$

where \mathbf{J}_t , \mathbf{Q}_t and \mathbf{G}_t are conformable matrices that are functions of the evolution of beliefs about the time-varying structural matrices \mathbf{A}_t , \mathbf{B}_t , \mathbf{C}_t , \mathbf{D}_t , and \mathbf{F}_t (Kulish and Pagan, 2017). These matrices satisfy the recursion

$$\begin{aligned}\mathbf{Q}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{B}_t \\ \mathbf{J}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} (\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}) \\ \mathbf{G}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{E}_t,\end{aligned}$$

where the final structures \mathbf{Q}_T and \mathbf{J}_T are known and computed from the time invariant structure above under the terminal period's structural parameters—that is, the no-ZLB case.

Given a sequence of ZLB durations, the state-space of the model is

$$\begin{aligned}x_t &= \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t \\ z_t &= \mathbf{H}_t x_t.\end{aligned}$$

The observation equation is time-varying because the nominal interest rate becomes unobserved when it is at its bound.

Denote by ϑ the vector of parameters to be estimated and by \mathbf{T} the vector of ZLB durations that are observed each period. Denote by $\mathcal{Z} = \{z_\tau\}_{\tau=1}^T$ the sequence of vectors of observable variables. With Gaussian errors, the likelihood function $L(\mathcal{Z}, \mathbf{T} \mid \vartheta)$ for the aggregate component is computed using the sequence of structural matrices associated with the sequence of ZLB durations, and the Kalman filter:

$$\log L(\mathcal{Z}, \mathbf{T} \mid \vartheta) = - \left(\frac{N_z T}{2} \right) \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log \det \mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top - \frac{1}{2} \sum_{t=1}^T \tilde{y}_t^\top \left(\mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top \right)^{-1} \tilde{y}_t.$$

D.4.2 Kalman filter

The state of the system is $(\hat{x}_t, \mathbf{P}_{t-1})$. In the predict step, the structural matrices \mathbf{J}_t , \mathbf{Q}_t , and \mathbf{G}_t are used to compute a forecast of the state $\hat{x}_{t|t-1}$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as

$$\begin{aligned}\hat{x}_{t|t-1} &= \mathbf{J}_t + \mathbf{Q}_t \hat{x}_t \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}_t \mathbf{P}_{t-1} \mathbf{Q}_t^\top + \mathbf{G}_t \Omega \mathbf{G}_t^\top.\end{aligned}$$

This formulation differs from the time-invariant Kalman filter used at the state level, because in the forecast stage, the matrices \mathbf{J}_t , \mathbf{Q}_t and \mathbf{G}_t can vary over time. We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors \tilde{y}_t

and its associated covariance matrix \mathbf{S}_t as

$$\begin{aligned}\tilde{y}_t &= z_t - \mathbf{H}_t \hat{x}_{t|t-1} \\ \mathbf{S}_t &= \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top.\end{aligned}$$

The Kalman gain matrix is given by

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With \tilde{y}_t , \mathbf{S}_t , and \mathbf{K}_t in hand, the optimal filtered update of the state x_t is

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \tilde{y}_t,$$

and for its associated covariance matrix:

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and \mathbf{P}_0 determined from their unconditional moments and is computed until the final time period T of data.

D.4.3 Kalman Smoother

With the estimates of the parameters and durations in hand at time period T , the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information (Hamilton, 1994). With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, \mathbf{K}_t and \mathbf{S}_t in hand from the Kalman filter, the vector $x_{t|T}$ is computed by

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion:

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} (z_t - \mathbf{H}_t \hat{x}_{t|t-1}) + (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by e_t , we compute:

$$e_t = \mathbf{G}_t \varepsilon_t = \mathbf{G}_t r_{t|T}.$$

D.5 Posterior Sampler

This section describes the sampler used to obtain the posterior distribution of interest. We compute the likelihood function at the state level and the aggregate level, together with the prior. The

Table 15: Calibrated Parameters

Parameter	Value	Description	Source/Target
ν	2	Inverse labor supply elasticity	
β	0.995	Quarterly discount factor	2% annual real rate
ω	0.7	Weight on non-traded goods	
σ	0.5	Elasticity traded/non-traded	
κ	4	Elasticity traded goods	Simonovska and Waugh (2014)
ψ	21	Elasticity labor aggregator	Christiano, Eichenbaum and Evans (2005)

posterior of our full model $\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z})$ satisfies

$$\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z}) \propto L(\mathcal{Z}, \mathbf{T} \mid \vartheta) \times \mathcal{P}(\vartheta).$$

We use a Markov Chain Monte Carlo procedure to sample from the posterior. It has a single block, corresponding to the parameters ϑ .³¹ The sampler at step j is initialized with the last accepted draw of the structural parameters ϑ_{j-1} .

First, start by selecting which parameters to propose new values. For those parameters, draw a new proposal ϑ_j from a proposal density centered at ϑ_{j-1} and with thick tails to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20% to 25%. The proposal ϑ_j is accepted with probability $\frac{\mathcal{P}(\vartheta_j \mid \mathbf{T}, \mathcal{Z})}{\mathcal{P}(\vartheta_{j-1} \mid \mathbf{T}, \mathcal{Z})}$. If ϑ_j is accepted, then set $\vartheta_{j-1} = \vartheta_j$.

E Additional Structural Model Estimation Results

E.1 Calibrated Parameters

Table 15 details the small set of parameters that are calibrated prior to estimation.

E.2 Full Structural Model Estimation Results

Tables 16 and 17 give the full prior and posterior distributions of the estimated structural parameters using state and aggregate data, respectively.

The parameters are the Calvo parameter on prices λ_p , the Calvo parameter on wages λ_w , the persistence of TFP shocks ρ_z , the persistence of labor disutility shocks ρ_n , the persistence of preference shocks ρ_b , the persistence of non-tradeable TFP shocks ρ_z^N , and the respective standard deviations of those four shocks. At the aggregate level, we also have the persistence of markup shocks ρ_p , the standard deviation of markup shocks σ_p , and the standard deviation of policy interest rate shocks σ_r . The Taylor rule parameters are given by α_r , α_p , α_x , and α_y .

³¹It is worth noting that as in Kulish, Morley and Robinson (2017), in addition to the structural parameters, one can estimate the expected zero lower bound durations, in which case an additional block is needed in the posterior sampler.

Table 16: Structural Estimation, State Data Only

Parameter	Prior			Posterior		
	Dist	Mean	SD	Mode	5%	95%
λ_p	B	0.5	0.1	0.602	0.586	0.614
λ_w	B	0.5	0.1	0.427	0.411	0.444
ρ_z	B	0.5	0.2	0.993	0.990	0.995
ρ_n	B	0.5	0.1	0.956	0.950	0.964
ρ_b	B	0.5	0.2	0.960	0.954	0.963
ρ_z^N	B	0.5	0.2	0.961	0.955	0.968
σ_z	IG	2.0	1.4	1.507	1.464	1.545
σ_n	IG	2.0	1.4	0.033	0.031	0.036
σ_b	IG	2.0	1.4	0.717	0.656	0.822
σ_z^N	IG	2.0	1.4	1.372	1.312	1.423

We choose the same prior as [Smets and Wouters \(2007\)](#) for the Calvo parameters. Our remaining priors are chosen to be wide/diffuse. We choose a relatively tighter prior on the persistence of labor disutility shocks at the state-level as preliminary estimations took ρ_n to a value of 1. We use uniform priors over a wide range for the parameters of the Taylor rule.

E.3 Robustness

E.3.1 Estimation with Credit Channel

Results with an active credit channel and the use of household debt and house prices as observables are shown in [Tables 18 to 22](#). The structure of the results is similar to that of the main tables reported in the text.

E.3.2 Smets and Wouters (2007) Priors on Calvo and Taylor Rule Parameters

[Table 23](#) shows the estimated structural parameters when the same priors as [Smets and Wouters \(2007\)](#) are used on the Calvo parameters and on the Taylor rule parameters. In these estimations, there is a role for the credit channel.

E.3.3 State-Level Estimation with Consumption

The results from an estimation using state-level consumption spending are given in [Table 24](#). The estimated nominal frictions are lower—in the model, nominal output equals nominal consumption, and since consumption is less volatile than output, the model estimation explains relatively more volatile prices and wages with more flexible prices. The addition of credit shocks does not change the estimated λ_p and λ_w , as for the estimation using nominal output.

Table 17: Structural Estimation, Aggregate Data Only

Parameter	Prior			Posterior		
	Dist	Mean	SD	Mode	5%	95%
λ_p	B	0.5	0.1	0.920	0.897	0.943
λ_w	B	0.5	0.1	0.844	0.796	0.880
ρ_z	B	0.5	0.2	0.959	0.939	0.975
ρ_n	B	0.5	0.2	0.065	0.032	0.176
ρ_b	B	0.5	0.2	0.859	0.840	0.879
ρ_p	B	0.5	0.2	0.905	0.858	0.945
σ_z	IG	2.0	1.4	0.586	0.531	0.644
σ_n	IG	2.0	1.4	0.097	0.067	0.206
σ_b	IG	2.0	1.4	2.755	2.365	3.256
σ_p	IG	2.0	1.4	0.389	0.287	0.523
σ_r	IG	2.0	1.4	1.485	1.290	2.086
α_r	U	0.5	0.3	0.809	0.725	0.848
α_p	U	4.5	2.6	2.351	1.983	3.031
α_x	U	1.0	0.6	0.460	0.370	0.653
α_y	U	1.0	0.6	0.260	0.210	0.391

Table 18: Posterior Distributions, Relative State Data Only, with Credit

Parameter	1977 to 2017			1977 to 1998			1999 to 2017		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.59	0.58	0.61	0.58	0.55	0.60	0.62	0.61	0.64
λ_w	0.38	0.35	0.39	0.50	0.46	0.54	0.40	0.39	0.43

Table 19: Posterior Distributions, Aggregate Data Only, with Credit

Parameter	1977 to 2015			1977 to 1998			1999 to 2015		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
<i>Calvo Parameters</i>									
λ_p	0.92	0.89	0.94	0.84	0.79	0.89	0.93	0.90	0.94
λ_w	0.83	0.81	0.94	0.90	0.87	0.93	0.84	0.79	0.88
<i>Taylor Rule Parameters</i>									
α_r	0.80	0.72	0.85	0.68	0.40	0.78	0.77	0.70	0.84
α_p	2.38	1.93	2.82	2.00	1.52	3.13	1.06	1.03	1.85
α_x	0.44	0.37	0.65	1.56	0.89	1.94	0.18	0.13	0.24
α_y	0.27	0.20	0.36	0.07	0.01	0.24	0.23	0.20	0.30

Notes: $Beta(0.5, 0.1)$ prior on Calvos. Uniform priors on Taylor Rule parameters

Table 20: Implied Slopes of Phillips Curve at Baseline Estimates, with Credit

	1977 to 2015	1977 to 1998	1999 to 2015
A. State-Level Estimates			
Prices [*]	0.279	0.306	0.228
Wages [†]	1.044	0.517	0.882
B. Aggregate-Level Estimates			
	1977 to 2017	1977 to 1998	1999 to 2017
Prices [*]	0.008	0.032	0.007
Wages [†]	0.034	0.012	0.031

^{*}: Price Phillips curve slope is $(1 - \beta\lambda_p)(1 - \lambda_p)/\lambda_p$

[†]: Wage Phillips curve slope is $(1 - \beta\lambda_w)(1 - \lambda_w)/\lambda_w$

Table 21: Posterior Distributions, Interaction with Policy Rules, with Credit

A. Aggregate Data Only, Fixed Taylor Rule Parameters									
Parameter	1977 to 2015 [*]			1977 to 2015 [†]			1977 to 2015 [‡]		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.92	0.89	0.94	0.87	0.80	0.89	0.92	0.89	0.94
λ_w	0.83	0.81	0.94	0.71	0.67	0.73	0.95	0.80	0.95
B. Aggregate Data Only, Policy Regime Periods									
Parameter	1965 to 2015			1965 to 1985 [§]			1986 to 2015 [§]		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.86	0.83	0.90	0.72	0.67	0.77	0.93	0.90	0.95
λ_w	0.90	0.87	0.93	0.91	0.88	0.94	0.87	0.83	0.90
α_r	0.93	0.90	0.95	0.95	0.86	0.96	0.86	0.81	0.91
α_p	4.02	3.30	7.29	4.48	2.81	9.43	2.42	1.85	3.62
α_x	0.46	0.40	0.59	0.55	0.44	0.79	0.21	0.15	0.27
α_y	0.77	0.46	1.13	0.82	0.34	1.72	0.27	0.20	0.38

^{*}: Estimated Taylor Rule with uniform priors

[†]: Taylor Rule parameters fixed at 1977 to 1998 estimates (see Table 3)

[‡]: Taylor Rule parameters fixed at 1999 to 2015 estimates (see Table 3)

[§]: No credit or house price series and no credit or housing preference shocks

Table 22: Implied Slopes of Phillips Curve at Aggregate Estimates

	1977 to 2015	1977 to 1998	1999 to 2015
A. Aggregate-Level Estimates, Fixed Taylor Rule			
Prices*	0.008	0.020	0.008
Wages†	0.034	0.117	0.003
B. Aggregate-Level Estimates, Policy Regime Periods			
	1965 to 2005	1965 to 1985	1986 to 2005
Prices*	0.022	0.107	0.006
Wages†	0.012	0.009	0.020

*: Price Phillips curve slope is $(1 - \beta\lambda_p)(1 - \lambda_p)/\lambda_p$

†: Wage Phillips curve slope is $(1 - \beta\lambda_w)(1 - \lambda_w)/\lambda_w$

Table 23: Aggregate-Level, Smets and Wouters (2007) Priors on Calvo and Taylor Rule

Parameter	1977 to 2015			1977 to 1998			1999 to 2015		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.92	0.89	0.94	0.87	0.82	0.91	0.92	0.90	0.94
λ_w	0.84	0.79	0.88	0.90	0.87	0.93	0.84	0.80	0.89
α_r	0.79	0.75	0.82	0.80	0.74	0.84	0.78	0.72	0.82
α_p	1.70	1.53	1.87	1.61	1.44	1.81	1.41	1.24	1.62
α_x	0.30	0.26	0.35	0.33	0.28	0.39	0.15	0.11	0.20
α_y	0.20	0.16	0.25	0.18	0.13	0.24	0.23	0.19	0.26

Table 24: Posterior of Calvo Prices λ_p and Calvo Wages λ_w

	(1)			(2)		
	Mode	10%	90%	Mode	10%	90%
λ_p	0.57	0.55	0.58	0.58	0.57	0.60
λ_w	0.33	0.31	0.35	0.34	0.33	0.36

(1): 1999 to 2015, consumption spending and no credit shocks

(2): 1999 to 2015, consumption spending