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Abstract

I examine a cheap talk game with multiple interdependent decisions, in which biased senders privately observe information about payoff-relevant states. I find that senders are willing to use open (state-specific) communication channels to strategically convey information about other states that otherwise cannot be revealed. In equilibrium, this leads to a loss of credibility that reduces the set of parameters for which communication is incentive compatible. The credibility loss associated with a sender arises when a given piece of information is relevant for both low- and high-conflict decisions. Surprisingly, when the receiver is expected to observe more of such information on path, the associated credibility loss recedes—i.e. the sender is more willing to reveal information that is only relevant for low-conflict decisions. Finally, I fully characterize the communication equilibrium in a simple version of the model, which I use as baseline to analyze the interaction between informational interdependence and preferences for coordinated decisions.

Keywords: Information Economics, Cheap Talk, Multidimensional Communication.
JEL: D21, D83.

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1 Introduction

This paper studies multidimensional communication under informational interdependence. A principal has to take action on multiple decisions and needs information about multiple states, such that information about each state affects many decisions. There are many experts who privately observe information and can send state-specific messages which are also costless and non-verifiable. To isolate the effect of interdependence, I assume preferences are additively separable on decisions. The model extends the analysis of multidimensional cheap talk (Battaglini, 2002; Levy and Razin, 2007; Ambrus and Takahashi, 2008; Deimen and Szalay, 2019) by proposing a decomposition of communication incentives. Such decomposition allows me to identify a novel determinant of communication arising from the interdependence and the nature of the senders' information.[1]

The particular environment I investigate is rooted in several real-world applications. Public policies, for instance, comprise a large number of different provisions, which typically complement each other in addressing the different issues requiring governmental intervention. Information about these issues is generally dispersed among political actors, including members of legislative committees, bureaucrats in governmental agencies, and sectoral experts among others. How much a policy-maker can trust any of such agents depends on their interests over the different provisions of the policy—i.e. legislators will try direct policies towards their supporters, bureaucrats towards their agencies, special interest groups towards their businesses. Additive separability may reflect agents who have strong interests over particular provisions of a given policy, which are invariant to the other instruments in it; examples include constituencies demanding specific local public works, or bureaucrats pushing for particular policy instruments that would be managed by the agencies they belong to. In section 4 I discuss non additively-separable preferences.

I find that incentives for truthful communication in this environment are negatively affected by what a sender knows beyond the information he is expected to reveal. Interdependence means there are preferences such that information about a given state will affect both high-conflict and low-conflict decisions. Similarly to Levy and Razin (2007), the combination of interdependence and sufficiently large conflict of interest in some dimensions will kill any possibility of communication. My framework shows this happens due to two different mechanisms. First, interdependence aggregates decision-specific biases and this can lead to communication breakdown; however, the aggregation can also result in transmission of more information as compared to decisions taken separately (e.g. by two decision-makers with similar preferences, as in 'mutual discipline in Goltsman and Pavlov, 2011). Second, and most importantly, having information mainly relevant for high-conflict decisions will affect a sender's beliefs

[1] Alonso et al. (2008, 2015); Rantakari (2008) analyze two-dimensional cheap talk communication in which senders observe a perfectly informative signal about one state but have only prior information about the other.

associated to low-conflict decisions for which truthful communication is expected. The mechanism behind this result is fundamentally different from Fischer and Stocken (2001) and Ivanov (2010)—who find that restricting a sender's access to information can improve communication with the receiver—because it is proportional to the interdependence between states. Because of such interdependence, the sender is tempted to use the communication channels open to him to strategically convey information he cannot credibly transmit otherwise. This leads to an equilibrium loss of credibility that reduces the set of biases for which communication is incentive compatible.

The presence of the credibility loss leads to another important insight: Informational congestion can improve communication. The typical result in the literature states that the more information a decision-maker is expected to receive in equilibrium, the weaker the individual incentives for communication will be. The basic intuition goes back to Austen-Smith and Riker (1987) but it has been further developed in diverse contexts (Morgan and Stocken, 2008; Galeotti et al., 2013; Moreno de Barreda, 2013; Migrow, 2021). Under informational interdependence, however, the credibility loss affecting a sender's incentives weakens as the number of other senders revealing the associated information increases. This intuition has relevant implications for the conformation of advisory boards. In particular, a decision-maker will obtain more from each of her (non-specialist) experts if they expect others to communicate on different (but related) issues—i.e. a group of experts in which each of them 'saturates' the decision-maker with information about a non-overlapping set of issues. Another interpretation of such implication relates to the effects of two-sided information in Moreno de Barreda (2013): The beneficial congestion holds when the receiver observes information associated to the credibility loss.

I then proceed to fully characterize the equilibrium in a simpler version of the model that features two decisions, two states, and two senders, each of whom observes two signals. To do that, I adapt the uniform-binary information structure in Galeotti et al. (2013), which allows me to illustrate the credibility loss and the beneficial congestion analytically. Besides, I show that equilibrium communication is not only based on messages on separate dimension but also features strategies in which a sender fully reveals his information for some signal realizations and announces the non-influential messages for the others. I also show that beneficial congestion leads to full revelation for a non-empty set of preference parameters (biases), which is increasing in the receiver's (equilibrium) information about the state associated with the credibility loss.

Finally, I extend the previous model to analyze the effects of preferences for coordinated decisions. Whether this benefits or hampers communication depends on whom such preferences belong to. When it is the receiver, her reaction to information suggesting decisions should be closer will be very strong. Senders will find deviations doing precisely that very costly, so communication incentives improve. When, on the other hand, are the senders who have preferences for coordinated decisions, they will find deviations moving decisions close to each other more profitable. Truthful communication will then be more difficult than in the baseline.[2]

Related literature. The paper contributes to the theoretical literature on multidimensional cheap talk. When senders are perfectly informed, the receiver can extract all the information by restricting the individual influence to the dimension of common interests (Battaglini, 2002).[3] In other words, the receiver (in equilibrium) can commit to ignore part of the information each sender provides because it will be provided by other senders on path. Levy and Razin (2007) show that the receiver loses such equilibrium commitment power when senders are imperfectly informed and decisions are interdependent. Senders' incentives thus depend on how information affects both decisions and his preferences over them. Sufficiently large conflict of interests in one dimension leads to communication breakdown. But interdependence can also benefit communication. In the case of two decision-makers facing one sender, Goltsman and Pavlov (2011) show decision-specific conflict of interest may compensate each other and, in these cases, public communication dominates private (decision-specific) communication (see also Farrell and Gibbons, 1989; Chakraborty and Harbaugh, 2010).[4] My paper shows that the aggregation of decision-specific preferences is only one of the mechanisms through with interdependence affects communication incentives. The other involves a sender's beliefs over the decisions he is expected to influence, and how these beliefs are affected by the information also relevant for high-conflict decisions (for which no information transmission will be credible).

Strategic communication with informational interdependence has received some recent attention in the work of Deimen and Szalay (2019). Their paper focuses on an unidimensional decision problem with two payoff-relevant states, such that principal and agent disagree about the state upon which the decision has to be calibrated. Hence, the conflict of interest decreases with the correlation between states. This feature makes state-specific communication infeasible. My model features state-independent conflicts of interest, such that the effect of interdependence on communication is non-monotonic. I find equilibria in which a sender's strategy include state-specific messages and, moreover, communication incentives may improve upon banning his access to information about some states.

The paper also brings some novel intuitions to the literature on organizational design. Strategic communication has important consequences on the organization of legislative institutions (Gilligan and Krehbiel, 1987, 1989; Krishna and Morgan, 2001a,b; Dewan et al., 2015), fiscal authority in federations (Kessler, 2014), and bureaucracies (Epstein and O'Halloran, 1994; Gailmard, 2002). By considering multiple decisions with informational interdependence, my paper provides a more thorough

[2]Note that in both cases senders are more willing to reveal information that is consistent with coordinated decisions.

[3]Battaglini (2004) shows that this results is robust to imperfect signals when states are orthogonal. Ambrus and Takahashi (2008) show that it is not robust to restricted state spaces for large conflict of interests.

[4]Note that Battaglini and Makarov (2014) find evidence about the mechanisms in Goltsman and Pavlov (2011).

understanding of the mechanisms underlying strategic communication in such complex environments, providing real-world implications for the conformation of advisory boards in these contexts.

The notion of interdependent decisions is also important among firms, but this strand of literature has mainly focused on the trade-off between coordination and adaptation (Dessein and Santos, 2006; Alonso et al., 2008, 2015; Rantakari, 2008). In such environments, incentives for communication depend on non-separability of preferences (need for coordination), senders' informational advantage (need for adaptation), and the difference in issue salience among players (biases). However, the nature of some industries leads to different trade-offs. For instance, product design in multi-product firms relies on information about consumers' preferences, and technological or design innovations over attributes, where different products have different combinations of such attributes. Some evidence from International Business points towards the important role of informational interdependence on the transmission of knowledge among sub-units of multinational corporations (see Andersson et al., 2007; Kunisch et al., 2019). My paper isolates the effects of informational interdependence on communication, showing it can reduce the amount of information decision-makers can aggregate.

Besides, I discuss non-separable preferences in section 4 based on a formal analysis in the appendix. As expected, there is a complementarity between informational and preference interdependencies that exacerbates the incentive effects of the former. The analysis shows this complementarity has its roots in the *receiver*'s preferences for coordination. But this is not the only mechanism, as senders will be less [more, resp.] willing to reveal information that contradicts [supports] *their* preferences for coordinated decisions, making truthful communication more difficult.

The rest of the paper proceeds as follows. Section 2 presents the basic set up and derive the main results. Section 3 fully characterizes the equilibrium in a more tractable environment. Finally, section 4 discusses non-separable preferences and concludes.

2 Set-up

A decision-maker (receiver) has to take actions $\mathbf{y} \in \mathbb{R}^D$ and need information in hands of I biased agents (senders). Each action, y_d, is affected by $\boldsymbol{\theta} = \{\theta_1, ..., \theta_R\}$ states of the world. The common prior over θ_r is a distribution $F_r(\theta_r)$ on $\Theta \in [0,1]$ with continuous and strictly positive density f_r, such that $F_r \perp F_t$ for all $r \neq t$. States affect the calibration of decision according to the vector $\boldsymbol{\delta}(\boldsymbol{\theta})$ of dimension D, with typical element $\delta_d = \sum_{r=1}^{R} w_{d,r}\theta_r$. Sender i's payoff is thus defined in terms of decisions, composite states, and biases as follows:

$$U^i(\mathbf{y}, \mathbf{b}^i, \boldsymbol{\gamma}^i, \boldsymbol{\delta}) = -\sum_{d=1}^{D} \gamma_d^i \, (y_d - \delta_d - b_d^i)^2$$

The vector $\mathbf{b}^i \in \mathbb{R}^D$ represents player i's bias, while $\boldsymbol{\gamma}^i \in \mathbb{R}^D$ represents the salience of the different decisions. The receiver's preferences are the same as those for senders, but I normalize $\mathbf{b}^{\mathrm{R}} = (0, 0)$ and $\boldsymbol{\gamma}^{\mathrm{R}} = (1, 1)$, such that $\mathbf{b}^i$ represents the conflict of interest with i, and $\boldsymbol{\gamma}^i$ the difference in decisions' relevance between them. In section 4 I discuss the case of preferences that are not additively-separable.

The receiver obtains information about the states through private, cheap talk communication with senders. Each sender observes signals about a subset of states, $\mathbf{S}^i \in \mathcal{S}^i = \{0, 1\}^{T^i}$ where T^i is a partition of R. Note that similar information structures has been extensively used to study information aggregation in many environments (see Austen-Smith, 1990; Morgan and Stocken, 2008; Galeotti et al., 2013; Förster, 2021; Migrow, 2021 among others). A well-know property of this information structure is the positive but decreasing marginal value of information.

Remark 1. *Let k_r denote the number of signals about state θ_r the decision-maker expects to receive on the equilibrium path, and let $MSE(k_r) \equiv E[(\theta_r - E(\theta_r | S_r^1, ..., S_r^{k_r}))^2 | k_r]$. Then, $MSE(k_r - 1) - MSE(k_r) > MSE(k_r) - MSE(k_r + 1) > 0$.*

The game proceeds as follows: first, senders privately observe their information and send simultaneous cheap talk messages to the receiver; second, the receiver decides on actions; third, payoffs realize. The message space is $\mathbf{m}^i \in \mathcal{M}^i = \mathcal{S}^i$.[5] As typical in this literature, I focus on pure-strategy Perfect Bayesian Equilibria (PBE).[6] Communication incentives will depend on the information each sender has and, in some cases, on the other sender's equilibrium message strategy. I denote by $\mathbf{m}^i(\mathbf{S}^i) \in \mathcal{M}^i$ the message strategy of sender i. A PBE of this game is characterized by a decision vector, $\mathbf{y}^* = \{y_1^*, ..., y_D^*\}$, and a collection of message strategies, $\mathbf{m}^* = \{\mathbf{m}^{1*}, ..., \mathbf{m}^{I*}\}$, such that:

- Each action taken by the receiver satisfies:

$$y_d^*(\mathbf{m}^*) = \sum_{r=1}^{R} w_{d,r}\, E(\theta_r | \mathbf{m}^*)$$

- Whereas sender i's message strategy satisfies:

$$\mathbf{m}^{i*}\left(\mathbf{S}^i\right) \in \arg\max_{\mathbf{m}^i} \left\{ -\sum_{d=1}^{D} E\left[\gamma_d^i \big(y_d(\mathbf{m}^i, \mathbf{m}^{\text{-}i*}) - \delta_d - b_d^i\big)^2 \big| \mathbf{S}^i\right] \right\}$$

In this section, I focus on message strategies that involve revealing information about one state, at most. Doing so allows me to characterize the main forces behind communication incentives for a

[5]This assumption is without loss since the type of information each sender observes is common knowledge.

[6]As discussed in Galeotti et al. (2013), this assumption is not without loss. The information provided by a sender using a mixed strategy on path is noisier than if he played a pure strategy. Other senders then face weaker informational congestion and, thus, truthful communication is incentive compatible for a larger set of parameters. The mechanism applies to the model in this paper (see Propositions 1 and 3).

typical sender. Equilibrium communication is fully characterized in section 3, for which I use a simpler environment and more structure. There I show the qualitative results in this section remain intact.

Sender i's communication incentives. Let $\mathbf{m}^{i*} \equiv \mathbf{m}^{i*}(\mathbf{S}^i)$ denote i's equilibrium message strategy which involves truthful revelation of at least one of his signals,[7] and $\mathbf{m}^{i'} \equiv \mathbf{m}^{i'}(\mathbf{S}^i)$ denote a deviation in which he misrepresents at least one of his signals. In addition let $\nu_r = \nu_r^i(S_r^i, \mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*})$ denote i's own posterior beliefs about θ_r, let $\nu_r^* = \nu_r^{i*}(S_r^i, \mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*})$ denote the belief he expect to induce on the receiver when following his equilibrium message strategy, and let $\nu_r' = \nu_r^{i'}(\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*})$ denote the belief he expects to induce under the deviation. Finally, let $\Delta_r = \Delta_r^i(\mathbf{m}^{i*}, \mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}) = \nu_r' - \nu_r^*$ and $\pi_r = \pi_r(S_r^i, \mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*}) - E(\theta_r|\mathbf{m}^{\text{-}i*})$.

Proposition 1. *Consider sender i's message strategy consisting of $m_t^{i*}(S_t^i) = \{S_t^i\}$ for $t \in \tau$ and $m_r^{i*}(S_r^i = 0) = m_r^{i*}(S_r^i = 1)$ for all $r \in T^i \setminus \{\tau\}$. Then, $\mathbf{m}^{i*}$ is incentive compatible if and only if, for all possible S_t^i and S_r^i, and associated Δ_t and π_r, given $\mathbf{m}^{i'}$ and $\mathbf{m}^{\text{-}i*}$, it is true that:*

$$\sum_{t\in\tau} \beta_t^i \, \Delta_t \leq \sum_{d=1}^{D} \frac{\gamma_d^i}{2} \Big(\sum_{t\in\tau} w_{d,t} \, \Delta_t \Big) \left[\Big(\sum_{t\in\tau} w_{d,t} \, \Delta_t \Big) - 2 \Big(\sum_{r\in T^i \setminus \{\tau\}} w_{d,r} \, \pi_r \Big) \right] \tag{1}$$

Where $\beta_t^i = \sum_{d=1}^{D} \gamma_d^i \, b_d^i \, w_{d,t}$.

Proof. All proofs are postponed to the Appendix. □

Corollary 1. *Sender i 's incentives to reveal information can be decomposed into three determinants:*

- *The* ***aggregate conflict of interest*** *associated to the information i former is expected to reveal: β_t^i for all $t \in \tau$;*
- *The* ***aggregate influence*** *that the information revealed will have on decisions: $\sum_{d=1}^{D} \gamma_d^i \, (w_{d,t} \, \Delta_t)^2$ for all $t \in \tau$; and*
- *The* ***credibility loss*** *associated to information i has but is not expected to reveal on the equilibrium path: $\Big(\sum_{t\in\tau} w_{d,t} \, \Delta_t \Big) \Big(\sum_{r\in T^i \setminus \{\tau\}} w_{d,r} \, \pi_r \Big)$.*

Proposition 1 and Corollary 1 characterize communication under dispersed information and interdependence. Truthful communication is incentive compatible depending i's preferences over decisions, on how his information affects them, and his expectations about the receiver's beliefs. The first two determinants in Corollary 1 reflect well-established results of the cheap talk literature. The notion of

[7] If $\mathbf{m}^{i*}$ involves non-influential messages about all states, no deviation off the equilibrium path would be possible since the receiver does not update beliefs upon hearing any of i's messages.

aggregate conflict of interest is present in Farrell and Gibbons (1989); Levy and Razin (2007); Goltsman and Pavlov (2011). The aggregate influence is subject to congestion effects due to the decreasing marginal value of information (Remark 1), which is a typical result in information aggregation (see Morgan and Stocken, 2008; Galeotti et al., 2013 among others).

The third determinant reflects the novel mechanism driving incentives to reveal information: The credibility loss. Despite Fischer and Stocken (2001) and Ivanov (2010) find that restricting a sender's information can improve communication, the credibility loss in the present environment arises from informational interdependence, leading to qualitatively different implications. First, note the effects of credibility loss depend on the degree to which information i is not able to convey in equilibrium would affect the decisions he influences on path. This is reflected in the interaction between Δ_t and π_r. On the one hand, Δ_t refers to the magnitude and direction in which i expects the receiver to change her beliefs over θ_t if he deviates to $m_r^{i'}$; the term $(w_{d,t}\Delta_t)$ then captures the effects such deviation would have on y_d. On the other hand, $\left(\sum_{r\in T^i\setminus\{\tau\}} w_{d,r}\,\pi_r\right)$ represents how i's information about states other than θ_t would affect the calibration of y_d, had he the possibility of revealing it. In other words, sender i is willing to use the communication channels that are open to him to convey information that is relevant but he is not able to transmit. Because the receiver anticipates such incentives, then, 'knowing about too many things' damages credibility.

Secondly, the specific mechanism affecting communication constitutes a case for specialization of experts working in complex decision-making environments. The magnitude of the harm to communication increases with the number of elements in $T^i\setminus\{\tau\}$: In the limit, there is no credibility loss when i only observes information he is willing to reveal. Finally, an additional novel implication emerges in relation to the effects of informational congestion, as shown in the following result.

Proposition 2 (Beneficial Congestion)**.** *Suppose the set $T^i\setminus\{\tau\}$ is not empty, and let $k_r\equiv k_r^*(\boldsymbol{\gamma},\mathbf{b})$ denote the number of agents other than i reporting truthfully their information about state θ_r for $r\in T^i\setminus\{\tau\}$. Then, $|\pi_r(k_r)|>|\pi_r(k_r+1)|$; therefore, the IC constraint (1) loosens as k_r increases.*

The credibility loss stems from the fact that sender i has information that cannot be credibly transmitted in equilibrium, but its effect depends on the incidence such information would have on the decisions i actually influences. For instance, there will be no harm to incentives if there were no informational interdependence—i.e. $w_{d,r}=0$ for all $d\neq r$. Proposition 4 shows that, as the equilibrium features more agents revealing information about states in $T^i\setminus\{\tau\}$, i's information about these would have a smaller influence on the decisions affected by his messages and, therefore, he is less tempted to deviate at the communication stage. Such congestion makes truthful communication (of information about θ_t for $t\in\tau$) incentive compatible for a larger set of biases.

The above argument qualifies an important implication of Proposition 1 and Corollary 1. When it is not possible to restrict a sender's expertise to the information he is willing to reveal, the decision-maker can minimize the credibility loss by hiring experts whose preferences are aligned with her in non-overlapping set of decisions. That way, the information a given expert transmits on path reduces other experts' incentives to deviate from truthful communication. This mechanism resembles that underlying the construction of fully revealing equilibria in Battaglini (2002). With perfectly informed senders, the receiver can restrict the influence of each of them to a dimension of common interest as long as his bias is not collinear to that of the others. But this mechanism may no longer be available when the state space is restricted (Ambrus and Takahashi, 2008), or when senders are imperfectly informed under interdependence (Levy and Razin, 2007). My paper shows the conditions under which the perverse incentives impeding communication in the latter case can be overcome and, thus, truthful communication recovered.

3 Full equilibrium characterization for a simpler environment

The model of this section features two decisions, two states, and two senders. Each sender observes two noisy signals, one associated to each state. The states are uniformly distributed over the interval $[0, 1]$ and are orthogonal to each other. Therefore, after receiving the senders' messages, the decision-maker updates beliefs according to a Beta-binomial process. Let $k_r^* \leq 2$ denote and equilibrium number of senders truthfully revealing their signals and ℓ_r^* the signals that equal to one, where $r = \{1, 2\}$ indexes the states. The receiver's updated expectations for θ_r in equilibrium and the associated decision she will take, $y_d^* = \{y_1^*, y_2^*\}$, given an equilibrium profile of message strategies, $\mathbf{m}^*$, are:[8]

$$E(\theta_r|k_r^*, \ell_r^*) = \frac{(\ell_r^* + 1)}{(k_r^* + 2)} \qquad\qquad y_d^* = w_{d1}\frac{(\ell_1^* + 1)}{(k_1^* + 2)} + w_{d2}\frac{(\ell_2^* + 1)}{(k_2^* + 2)}$$

From now on, let $k_r, \ell_r = \{0, 1\}$ (no superscript) denote i's *conjecture* about the other sender's truthfully reporting to the receiver.[9] For the sake of exposition, I take $w_{d,r} \geq 0$ for both decisions and states, which amounts to assume that informational interdependence is characterized by positive correlation. Because of the possibility of multiple equilibria, I focus on the receiver-optimal equilibrium as most papers in the cheap talk literature. Here it is defined as the message strategy maximizing the

[8] The conditional pdf and the receiver's posterior are, respectively:

$$f(\ell_r|\theta_r, k_r) = \frac{k_r!}{\ell_r!(k_r - \ell_r)!}\theta_r^{\ell_r}(1 - \theta_r)^{k_r - \ell_r} \qquad\qquad h(\theta_r|\ell_r, k_r) = \frac{(k_r + 1)!}{\ell_r!(k_r - \ell_r)!}\theta_r^{\ell_r}(1 - \theta_r)^{k_r - \ell_r}$$

[9] Sender i's conjecture will be correct on path, and whenever his equilibrium message strategy involves revealing the corresponding signal then $k_r^* = k_r + 1$, while $k_r^* = k_r$ otherwise.

receiver's ex-ante expected utility, since the availability of more messages does not necessarily mean the transmission of more information ex-post. Before I characterize the incentive compatibility constraints, it is useful to have the expressions for Δ_r and π_r.

Lemma 1. *Suppose* $\mathbf{y} \in \mathbb{R}^2$, $\theta_r \sim U[0,1]$ *for* $\theta_r = \{\theta_1, \theta_2\}$, *and* $S^i \in \{0,1\}^2$ *for the two senders* $i = \{1,2\}$. *Let* k_r *denote* i*'s conjecture about the number of truthful messages received by the decision-maker on the equilibrium path. The expressions for* $\Delta_r(S_r^i, \mathbf{m}^{\text{-}i*})$ *and* $\pi_r(S_r^i, \mathbf{m}^{\text{-}i*})$ *are the following:*

$$\Delta_r(S_r^i = 0, \mathbf{m}^{\text{-}i*}) = \frac{1}{(k_r+3)} \qquad \pi_r(S_r^i = 0, \mathbf{m}^{\text{-}i*}) = -\frac{1}{2(k_r+3)}$$

$$\Delta_r(S_r^i = 1, \mathbf{m}^{\text{-}i*}) = -\frac{1}{(k_r+3)} \qquad \pi_r(S_r^i = 1, \mathbf{m}^{\text{-}i*}) = \frac{1}{2(k_r+3)}$$

Note that Δ_r and π_r have opposite signs. When i observes a $S_r^i = 0$, for instance, a deviation from truthful revelation implies inducing beliefs on the receiver that are closer to one. On the other hand, if i is not influential in the equilibrium under consideration, observing $S_r^i = 0$ means *he believes* the receiver expectations should be closer to zero.

Lemma 2. *Suppose* $\mathbf{y} \in \mathbb{R}^2$, $\theta_r \sim U[0,1]$ *for* $\theta_r = \{\theta_1, \theta_2\}$, *and* $S^i \in \{0,1\}^2$ *for the two senders* $i = \{1,2\}$. *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\{S_1^i\} \in \mathbf{m}^{i*}$. *Revealing information about state* θ_1 *is incentive compatible for sender* i *if and only if:*

$$|\beta_1^i| \leq \frac{1}{2}\left[\frac{[\gamma_1^i\,(w_{1,1})^2 + \gamma_2^i\,(w_{2,1})^2]}{(k_1+3)} - \frac{(\gamma_1^i\, w_{1,1}\, w_{1,2} + \gamma_2^i\, w_{2,1}\, w_{2,2})}{(k_2+3)}\right] \tag{2}$$

Proof. Follows from Lemma 1 and equation (1). □

Incentives to reveal one signal are relatively straightforward because there is only one possible deviation. The three components of incentives described in Corollary 1 are then clear. The aggregate conflict of interest refers to the way in which state θ_1 affects both decisions, their relative importance for i, and his bias. The aggregate influence is captured by the first term in square bracket in the left-hand side of (2). Sender i's incentives are weaker when j reveals his information about θ_1, that is $k_1 = 1$. This is the 'traditional' informational congestion. The credibility loss is represented by the second term in square brackets, where the beneficial congestion effects also become clear. If sender j reveals information about θ_2 on path, i will find that revealing S_1^i is incentive compatible for a larger set of parameters. This is because his information about θ_2 is a weaker substitute for the influence θ_1 has on decisions.

Note that the credibility loss is increasing in the degree of interdependence. Suppose, for instance, that $w_{1,1} = w_{2,2} = w$ and $w_{1,2} = w_{2,1} = 1-w$. In such a case, the second term in square brackets reaches a minimum when $w = 1$ (independent decisions), and increases as w converges to 0.5. As states have similar effects on decisions, information i observes but cannot credibly transmit incentivize deviations from truthful revelation. Next, I characterize incentives to reveal both signals.

Lemma 3. *Suppose* $\mathbf{y} \in \mathbb{R}^2$*,* $\theta_r \sim U[0,1]$ *for* $\theta_r = \{\theta_1, \theta_2\}$*, and* $S^i \in \{0,1\}^2$ *for the two senders* $i = \{1,2\}$*. Consider the equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\{S_1^i, S_2^i\} \in \mathbf{m}^{i*}$*. Truthful revelation of both signals is incentive compatible if and only if all the following are true:*

- *Lying about an individual signal is not incentive compatible; that is, for* $\theta_r = \{\theta_1, \theta_2\}$*:*

$$|\beta_r^i| \leq \frac{[\gamma_1^i (w_{1,r})^2 + \gamma_2^i (w_{2,r})^2]}{2\,(k_r + 3)} \tag{3}$$

- *Lying on both signals when* $\mathbf{S}^i = \{(0,0);(1,1)\}$ *is not incentive compatible; that is:*

$$\left|\frac{\beta_1^i}{(k_1+3)} + \frac{\beta_2^i}{(k_2+3)}\right| \leq \sum_{r=\{1,2\}} \frac{[\gamma_1^i (w_{1,r})^2 + \gamma_2^i (w_{2,r})^2]}{2(k_r+3)^2} + \frac{(\gamma_1^i\, w_{1,1}w_{1,2} + \gamma_2^i\, w_{2,1}w_{2,2})}{(k_1+3)(k_2+3)} \tag{4}$$

- *Lying on both signals when* $\mathbf{S}^i = \{(0,1);(1,0)\}$ *is not incentive compatible; that is:*

$$\left|\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)}\right| \leq \sum_{r=\{1,2\}} \frac{[\gamma_1^i (w_{1,r})^2 + \gamma_2^i (w_{2,r})^2]}{2(k_r+3)^2} - \frac{(\gamma_1^i\, w_{1,1}w_{1,2} + \gamma_2^i\, w_{2,1}w_{2,2})}{(k_1+3)(k_2+3)} \tag{5}$$

First, note the similarities between equations (3) and (2): The only difference is the credibility loss is absent in the former. As described in the previous section, the credibility loss arises because a sender has incentives to use information about states he cannot reveal in equilibrium but are relevant for the decisions he influences through communication. However, when i reveals both signals in equilibrium he is conveying all the information he has and, thus, these perverse incentives disappear.

Regarding the incentives to lie on both signals simultaneously, the IC constraints show that the effects of such deviation depend on both the interdependence and the signals' realizations. Because I have assumed positive correlation, truthful revelation of $\mathbf{S}^i = \{(0,0);(1,1)\}$ moves both decisions in the same direction with respect to the prior. Revealing each signal thus reinforces the effect of revealing the other, which can be seen in the last term in the right-hand side of (4). The conflict of interest associated to this strategy now depends on a weighted average of the conflict of interests associated to each piece of information, where the weights depend on how much information the decision-maker expects to receive in equilibrium. In other words, the set of biases for which full revelation is incentive compatible changes with the number of other agents willing to reveal information in equilibrium. This of course may be a source of beneficial congestion. To see this, suppose θ_1 [θ_2, resp.] is more relevant for y_1 $[y_2]$; that is, $w_{1,1} > w_{2,1}$ and $w_{2,2} > w_{1,2}$. When sender j is expected to reveal information about θ_2 only, i's incentives to fully reveal his information will put a smaller weight on b_2^i because such message strategy has a lower influence in the second dimension. Therefore, there exists a $\mathbf{b}^i$ for which full revelation is IC for $k_2^* = 1$ but is not for $k_2^* = 0$ (see Proposition 4 in the next section).

Incentives to fully reveal signals $\mathbf{S}^i = \{(0,1);(1,0)\}$ are somewhat different because the receiver's beliefs about each state update in opposite directions. A first implication of that relates to positive correlation: i expect decisions to move in opposite direction with respect to the prior. As a consequence, the measure of the aggregate conflict of interest is different than if his signals coincide—i.e. the left-hand side of (5) will be smaller when b_1^i and b_2^i have the same sign. Secondly, the overall influence on each decision will also be smaller than if signals coincide. This means the right-hand side of (5) will typically be smaller than that of (4), making the former IC constraint tighter.

Now, the equilibrium characterization in this game is not just based on communication on separate dimensions independently. It is possible that senders credibly transmit information for some signals realizations but not for others. This is the case when, for example, i's biases are $b_1^i > 0$ and $b_2^i < 0$, and they are large but similar in magnitude. If such a sender announces $\mathbf{m}^i = \{(1,1)\}$, the receiver should believe him because his utility gains from inducing a movement of y_1 towards 1 compensate his losses from moving y_2 towards 0. But the IC constraints associated to $\mathbf{m}^i = \{\{(0,1)\};\{(1,0)\}\}$ are different because if these messages were taken at face value by the receiver, i would *always* announce $\mathbf{m}^i = \{(1,0)\}$ (recall $b_1^i > 0$ and $b_2^i < 0$). As a result, such a sender finds incentive compatible to fully reveal signals when they coincide but this is not true when they do not. I call the resulting message strategy dimensional non-separable (DNS, henceforth). In the Appendix I show that the only strategies of this class arising in the receiver-optimal equilibrium has the same structure as the example: Full revelation for two combinations of signal realizations and babbling for the other two. The lemma below characterizes communication incentives when the message strategy includes both babbling and influential messages, for the class of DNS strategies arising in equilibrium:

Lemma 4. *Suppose* $\mathbf{y} \in \mathbb{R}^2$*,* $\theta_r \sim U[0,1]$ *for* $\theta_r = \{\theta_1, \theta_2\}$*, and* $S^i \in \{0,1\}^2$ *for the two senders* $i = \{1,2\}$*. Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\mathbf{m}^{i*}$ *includes a non-influential message strategy and full revelation of some signal realizations. Then, full revelation is incentive compatible for sender* i *if and only if:*

- *Announcing the non-influential message is not incentive compatible for* $\mathbf{S}^i = \{(0,0);(1,1)\}$*:*

$$\left|\frac{\beta_1^i}{(k_1+3)} + \frac{\beta_2^i}{(k_2+3)}\right| \leq \sum_{r=\{1,2\}} \frac{[\gamma_1^i (w_{1,r})^2 + \gamma_2^i (w_{2,r})^2]}{4(k_r+3)^2} + \frac{(\gamma_1^i\, w_{1,1}w_{1,2} + \gamma_2^i\, w_{2,1}w_{2,2})}{2(k_1+3)(k_2+3)} \tag{6}$$

- *Announcing the non-influential message is not incentive compatible for* $\mathbf{S}^i = \{(0,1);(1,0)\}$*:*

$$\left|\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)}\right| \leq \sum_{r=\{1,2\}} \frac{[\gamma_1^i(w_{1,r})^2 + \gamma_2^i(w_{2,r})^2]}{4(k_r+3)^2} - \frac{(\gamma_1^i\, w_{1,1}w_{1,2} + \gamma_2^i\, w_{2,1}w_{2,2})}{2(k_1+3)(k_2+3)} \tag{7}$$

The similarities with conditions in Lemma 3 stem from the fact that the influential messages in the above strategies fully reveal the sender's information. The main differences lie on the right-hand sides; in particular, conditions (6) and (7) hold for a smaller set of biases than (4) and (5), respectively. This is due to the fact that i has a non-influential message available. Therefore, his deviation from full revelation has an effect that is halfway that of lying on both signals simultaneously; a 'less risky' deviation, attractive for a larger set of parameters. The same applies to non-influential types. Note that this class of strategies rely on the existence of non-influential messages in equilibrium; that is, the equilibrium features the pooling of types $(0,1)$ and $(1,0)$ such that there is an associated 'babbling' message. Having described incentive compatibility for the different communication strategies, I now characterize the receiver-optimal equilibrium of this game.

Proposition 3. *Suppose* $\mathbf{y} \in \mathbb{R}^2$*,* $\theta_r \sim U[0,1]$ *for* $\theta_r = \{\theta_1, \theta_2\}$*, and* $S^i \in \{0,1\}^2$ *for* $i = \{1,2\}$*. The strategy profile* $(\mathbf{y}^*, \mathbf{m}^*)$ *constitutes the receiver-optimal equilibrium when, for every possible message strategy from sender* i *to the decision-maker, then* i*:*

1. ***Reveals information about both states****, if and only if conditions in Lemma 3 hold for both signals.*

2. ***Reveals information about*** θ_r ***only****, if and only if the condition in Lemma 2 holds for* θ_r *and*

   (a) condition (2) does not hold with respect to the other state; or

   (b) condition (2) holds with respect to both states and:

$$\frac{[(w_{1,r})^2 + (w_{2,r})^2]}{(k_r+2)(k_r+3)} > \frac{[(w_{1,\text{-}r})^2 + (w_{2,\text{-}r})^2]}{(k_{\text{-}r}+2)(k_{\text{-}r}+3)}; \text{ or} \tag{8}$$

   (c) condition (4) holds and (8) holds.

3. ***Reveals information about both states when*** $\mathbf{S}^i = \{(0,0);(1,1)\}$ ***and no information otherwise****, if and only if (6) holds and*

   (a) condition (2) does not hold; or

   (b) condition (2) holds but (8) does not.

4. ***Reveals information about both states when*** $\mathbf{S}^i = \{(0,1);(1,0)\}$ ***and no information otherwise*** *if and only if condition (7) holds and (2) does not.*

5. ***Reveals no information*** *(babbling strategy) if and only if none of the previous applies.*[10]

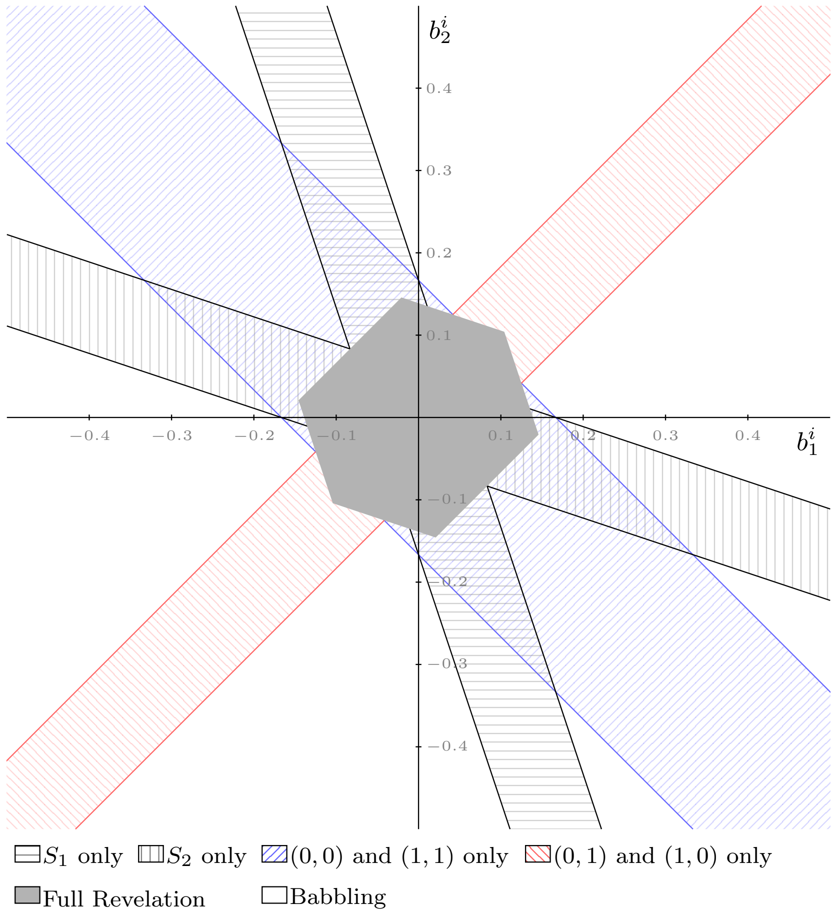
First note that the set of strategies that constitute an equilibrium is a strict subset of the strategy space. Proposition 3 implies that equilibrium communication depends on the profile of biases and the informational interdependence. The system of beliefs that supports each strategy is characterized in the appendix. Intuitively, when a message strategy involves some sort of pooling between types, there are at least two messages that induce the same actions and, upon hearing any of such messages, the receiver assigns equal probability to the types involved. Note also that the non-influential messages in items 3 and 4 involve signal realizations that perfectly compensate each other in terms of the induced beliefs (if each were taken at face value), and the receiver puts equal probability on each pair. Figure 1 illustrates the set of biases for which the different strategies arise in equilibrium.

The shapes of the correspondences reflect how the different message strategies affect decisions. The Fully Separating Equilibrium (#1 in Proposition 3) arises only if the aggregate conflict of interest between i and the receiver is sufficiently small and is represented by the solid grey area in Figure 1. Equilibria featuring revelation of information about one state only (#2) come from the IC constraints in Lemma 2, from three possible cases: When it holds with respect to one state only, when it holds for both states individually but full revelation is not IC, and when it is also IC to play the DNS message strategy that reveals both signals only when they coincide. The first case is straightforward. The second and third cases, however, feature multiple equilibria and (8) reflects the selection criterion: The receiver is ex-ante better off when i reveals information about θ_r than under any alternative equilibrium message strategy.

Dimensional non-separable message strategies (#3 and #4) are represented in the blue and red regions Figure 1. Note that the blue region consists mainly of biases in the II and IV quadrants, meaning that the b_1^i and b_2^i satisfying conditions in #4 typically have different signs. When i's biases lie on this region and his signals are $\mathbf{S}^i = \{(0,0);(1,1)\}$, full revelation moves decisions in the same direction and, hence, i's utility gains associated to one decision compensate his utility losses on the other. This interaction between interdependence, biases, and messages leads to a relatively small aggregate conflict of interest and, thus, to credible messages. On the contrary, were the receiver to believe any of the messages $\mathbf{m}^i = \{\{(0,1)\};\{(1,0)\}\}$, i would announce the message that moves decisions in the direction of his biases. A similar intuition applies for full revelation of $\mathbf{S}^i = \{(0,1);(1,0)\}$ when biases are in the red region (quadrants I and III), but with truthful messages inducing decisions to move in

[10]Formally, the message strategy in (1) is $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$; in (2) when revealing S_1^i is $\mathbf{m}^i = \Big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\Big\}$, and when revealing S_2^i is $\mathbf{m}^i = \Big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\Big\}$; in (3) is $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$, in (4) is $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$; and in (5) is $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

Figure 1: Equilibria in Proposition 3



Note: $\gamma_1^i = \gamma_2^i = 1$, $w_{1,1} = w_{2,2} = 3/4$, $w_{1,2} = w_{2,1} = 1/4$ and $k_1 = k_2 = 0$.

different directions. Below, I derive the beneficial congestion result.

Proposition 4. *Suppose $\gamma_1^i = \gamma_2^i = 1$, $w_{1,1} = w_{2,2} = w > \frac{1}{2}$, and $w_{1,2} = w_{2,1} = 1 - w$. Let k_1, k_2 be sender i's conjecture about the number of truthful messages the decision-maker receives in equilibrium, and let $k_1' > k_1$. For any $1/2 < w < 1$, there exists $\mathbf{b}^i$ such that:*

1. *Sender i plays a babbling message strategy in the equilibrium with k_1 and reveals information about θ_2 in the equilibrium with k_1'.*

2. *Sender i plays a DNS message strategy in the equilibrium with k_1 and fully reveals his information in the equilibrium with k_1', if* [11]

$$\frac{(k_2+3)}{(k_1+3)} < \min\left\{ \frac{(k_1'+3)}{(k_2+3)}\,;\, \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]} \right\} \tag{9}$$

[11] More precisely, i's equilibrium message strategies in the first case are $\mathbf{m}^{i*}(k_1,k_2) = \{\{(0,0);(1,0);(0,1);(1,1)\}\}$ and $\mathbf{m}^{i*}(k_1',k_2) = \{\{(0,0);(1,0)\};\{(1,0);(1,1)\}\}$, respectively; while in the second case they are $\mathbf{m}^{i*}(k_1,k_2) = \{\{(0,0)\};\{(1,0);(0,1)\};\{(1,1)\}\}$ and $\mathbf{m}^{i*}(k_1',k_2) = \{\{(0,0)\};\{(1,0)\};\{(1,0)\};\{(1,1)\}\}$, respectively.

As discussed in section 2, congestion of information about one state may lead some senders to reveal information about the other because it weakens the credibility loss. The first claim in Proposition 4 illustrates this case and is easily derived from (2). The second claim shows the case in which increasing information about state θ_1 the decision-maker receives in equilibrium induces a sender to fully reveal his information. To see this, consider the case in which sender i has a relatively large b_1^i and a small b_2^i, such that his preferences lie on the intersection of the blue and the vertical-lines areas in Figure 1. If sender j reveals information about θ_1 in equilibrium, i's influence from full revelation has a smaller effect on y_1. The two conditions necessary for this beneficial congestion are described in (9). First, k_1' must be sufficiently large with respect to k_1: The necessary difference between them is increasing in the magnitude of k_2. In other words, i needs the (beneficial) congestion associated to θ_1 to compensate for the (harmful) congestion on θ_2. The second condition relates to the informational interdependence. As the second term in the right-hand side of (9) shows, beneficial congestion is not possible when $w = 1$ (informational interdependence). If that were the case, the fact that the decision-maker receives more information on θ_1 does not affect i's incentives to reveal information about θ_2.

In summary, beneficial congestion only arises under informational interdependence, as presented in this paper. It has important implications for the conformation of advisory teams such as executive cabinets or legislative committees (see Austen-Smith, 1990; Dewan and Hortala-Vallve, 2011; Dewan et al., 2015; Ambrus et al., 2021), but can also be applied more broadly to organizational design (see Alonso et al., 2008, 2015; Rantakari, 2008; Deimen and Szalay, 2019).

4 Discussion and Conclusion

I studied communication of soft information about multiple states, each of which is relevant for multiple decisions. In the model, a receiver needs to aggregate information from many senders who observe noisy signals about a subset of states. Besides how the information affects the different decisions and its interaction with decision-specific preferences, I find that truthful communication also depends on how much of the information a sender cannot credibly transmit in equilibrium *is relevant* for the decisions in which his preferences are aligned with the receiver. Note that the latter requires informational interdependence, since the 'problematic information' refers to states that are also relevant for high-conflict decisions. To put it differently, a sender will be willing to use the communication channels open to him to (strategically) convey information he cannot transmit otherwise. In equilibrium, the loss of credibility restricts the set of parameters for which truthful communication is incentive compatible.

The credibility loss described above leads to beneficial congestion. Reducing the influence of the 'problematic information' means the sender has less incentives to condition influential messages (if any) on that information. In the limit, if a sender knows the receiver will be perfectly informed about the states affecting both high-conflict and low-conflict sets of decisions, his incentives to reveal information relevant for the latter are maximal; that is, the credibility loss disappears. As a practical implication, decision-makers facing the type of complexity analyzed in this paper will benefit from consulting experts in non-overlapping issues, whose preferences are aligned on the decisions these issues are important for.[12]

There are many real-world environments for which this framework would present a suitable starting point for analysis, ranging from issue linkage in international negotiations (Davis, 2004; Trager, 2011), to multidimensional policy debate (Dewan and Hortala-Vallve, 2011; Kessler, 2014; Dewan et al., 2015; Schnakenberg, 2015), and the organization of knowledge-based international corporations (Andersson et al., 2007; Kunisch et al., 2019).

Non additively-separable preferences. In the appendix, I extend the environment of section 3 to capture preference interdependence (need for coordinated decisions) in two cases. First, I analyze the case where is the *receiver* who has such preferences, which affect senders' communication incentives via two independent channels. The first channel arise because the decision-maker's reaction upon receiving information is more balanced across decision dimensions. This effect is equivalent to a higher degree of informational interdependence in the sense of Corollary 1, but is present even if decisions are independent (as in Alonso et al., 2008; Rantakari, 2008). The second channel relates to the posterior beliefs any sender expects to induce by communication. Because of her coordination preferences, the receiver will react strongly to information moving her posteriors closer to each other. A sender will then find deviations that induce such posteriors less profitable. In other words, the incentive problems created by 'contradictory information' ($\mathbf{S}^i = \{(0,1),(1,0)\}$) in the baseline model are compensated by the receiver's strong reaction to information supporting her preferences for coordinated decisions. As a consequence, senders will have additional incentives to be truthful;[13] such incentives are increasing the preferences for coordination and decreasing in the informational interdependence.

Secondly, I analyze the case in which preferences for coordinated decisions are exclusive of *senders*. Naturally, any sender will have increased incentives to reveal information moving decisions closer to each other. But, by the same mechanism, his incentives to be truthful decrease when a deviation would lead to closer decisions. It will then be more difficult to sustain influential equilibria because of such increased incentives to deviate for some signal realizations. In summary, the effects of preference interdependence on communication depend on its prevalence among senders and receivers.

[12] Despite my analysis omits senders' information on the intensive margin, Förster (2021) shows that, in a similar environment without interdependence, the threshold for full revelation of a sender's information is decreasing in his bias.

[13] There is a more subtle argument related to information that supports the receiver's preferences for coordination, which is developed in more detail in the appendix.

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Appendix

Proof of Proposition 1.

A message strategy for i is incentive compatible, given the other players' equilibrium strategies $\mathbf{m}^{\text{-}i*}$, if for each possible deviation $\mathbf{m}^{i'}$ it is true that:

$$E\left[U_i\big(\gamma^i, \mathbf{b}^i, \boldsymbol{\theta}\big)\big|\mathbf{S}^i, \mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*}\right] - E\left[U_i\big(\gamma^i, \mathbf{b}^i, \boldsymbol{\theta}\big)\big|\mathbf{S}^i, \mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}\right] \geq 0$$

Which is equivalent to:

$$-\int\limits_{\Theta_1} \dots \int\limits_{\Theta_R} \sum_{d=1}^{D} \gamma_d^i \left[\big(y_d^*(\mathbf{m}^{i*}, \mathbf{m}^{\text{-}\mathbf{i}*}) - \delta_d - b_d^i\big)^2 - \big(y_d'(\mathbf{m}^{i'}, \mathbf{m}^{\text{-}\mathbf{i}*}) - \delta_d - b_d^i\big)^2\right] dF(\theta_1, , \mathbf{m}^{\text{-}\mathbf{i}*}|S_1^i) \dots dF(\theta_R, \mathbf{m}^{\text{-}\mathbf{i}*}|S_R^i) \geq 0$$

Using the identity $(a^2 - b^2) = (a + b)(a - b)$, the term in square brackets above can be re-arranged into $(y_d^* + y' - 2\delta_d - 2b_d^i)(y^* - y_d')$, given the receiver's optimal action and the definition of δ_d yields:

$$-\int\limits_{\Theta_1} \dots \int\limits_{\Theta_R} \sum_{d=1}^{D} \gamma_d^i \left[\sum_{r=1}^{R} \Big(w_{d,r}\, \big(E(\theta_r|\mathbf{m}^{i*}) + E(\theta_r|\mathbf{m}^{i'})\big) - 2 \sum_{r=1} w_{d,r}\, \theta_r - 2b_d^i\Big) \times \right.$$

$$\left. \times \left[\sum_{r=1}^{R} w_{d,r}\, \big(E(\theta_r|\mathbf{m}^{i*}) - E(\theta_r|\mathbf{m}^{i'})\big)\right]\right] dF(\theta_1, , \mathbf{m}^{\text{-}i*}|S_1^i) \dots dF(\theta_R, \mathbf{m}^{\text{-}i*}|S_R^i) \geq 0$$

Let $\nu_r = \nu_r^i(S_r^i, \mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*})$, $\nu_r^* = \nu_r^{i*}(S_r^i, \mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*})$, and $\nu_r' = \nu_r^{i'}(\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*})$ be sender i's expectations about the receiver's posterior beliefs about θ_r under his information, his equilibrium message strategy, and the deviation under consideration, respectively. In addition, let $\Delta_r = \Delta_r^i(\mathbf{m}^{i*}, \mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{i'}) - E(\theta_r|\mathbf{m}^{i*})$ denote the difference in the induced posterior beliefs i's expects to achieve under the deviation $\mathbf{m}^{i'}$.

Now, note that $dF(\theta_r, \mathbf{m}^{\text{-}i}|S_r^i) = f(\theta_r|S_r^i, \mathbf{m}^{\text{-}i})\, P(\mathbf{m}^{\text{-}i}|S_r^i)\, d\theta_r$. Also, given that the equilibrium message strategies for players other than i are independent of i's actual signal realizations, the expression $P(\mathbf{m}^{\text{-}i}|S_r^i)$ can be taken out the corresponding integral. Therefore, the above expression becomes:

$$-\int\limits_{\Theta_1} \dots \int\limits_{\Theta_R} \sum_{d=1}^{D} \gamma_d^i \left[\sum_{r=1}^{R} \Big(w_{d,r}\, (\nu_r^* + \nu_r') - 2 \sum_{r=1}^{R} w_{d,r}\, \theta_r - 2b_d^i\Big) \left[\sum_{r=1}^{R} w_{d,r}\, (-1)\, \Delta_r\right]\right] f(\theta_1, |S_1^i) \dots f(\theta_R|S_R^i)\, d\theta_1 \dots d\theta_R \geq 0$$

Noting that $\int\limits_{\Theta_r} \theta_r f(\theta_r|S_r^i, \mathbf{m}^i) d\theta_r = E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*}) = \nu_r$ for all $r = \{1, ..., R\}$, I get:

$$\sum_{d=1}^{D} \gamma_d^i \left[\sum_{r=1}^{R} w_{d,r}\, \big(\nu_r^* + \nu_r' - 2\nu_r\big) - 2\, b_d^i\right] \left(\sum_{r=1}^{R} w_{d,r}\, \Delta_r\right) \geq 0 \tag{10}$$

For those states for which i reveals information on path, the beliefs he expect to induce $E(\theta_t|m_r^{i*}, \mathbf{m}^{\text{-}i*}) = \nu_t^*$ equals his own beliefs about that state $E(\theta_t|S_t^i, \mathbf{m}^{\text{-}i*}) = \nu_t$; while with the deviation he expects to induce $E(\theta_t|m_t^{i'} = 1 - S_t^i, \mathbf{m}^{\text{-}i*})$. For states in $T^i \setminus \{\tau\}$, then $\nu_r^* = \nu_r' = E(\theta_r|\mathbf{m}^{\text{-}i*})$, while for states he does not have information, $\nu_r^* = \nu_r' = \nu_r$. As a consequence, for states other than τ it is true that

$\Delta_r = 0$, such that the IC constraint can be split as follows:

$$\sum_{d=1}^{D} \gamma_d^i \left[\sum_{t\in\tau} w_{d,t}(\nu_t' - \nu_t^*) + \sum_{r\in T^i\setminus\{t\}} w_{d,r} \left(2E(\theta_r|\mathbf{m}^{\text{-}i*}) - 2E(\theta_t|S_t^i, \mathbf{m}^{\text{-}i*})\right) - 2\, b_d^i \right] \left(\sum_{t\in\tau} w_{d,t}\, \Delta_t \right) \geq 0$$

$$\Leftrightarrow \sum_{d=1}^{D} \gamma_d^i \left[\sum_{t\in\tau} w_{d,t}\Delta_t - 2 \sum_{r\in T^i\setminus\{t\}} w_{d,r}\pi_r - 2\, b_d^i \right] \left(\sum_{t\in\tau} w_{d,t}\, \Delta_t \right) \geq 0$$

Which leads to equation (1), noting that $\sum_{d=1}^{D} \gamma_d^i\, b_d^i \left(\sum_{t\in\tau} w_{d,t}\, \Delta_t \right) = \sum_{t\in\tau} \Delta_t \left(\sum_{d=1}^{D} \gamma_d^i\, b_d^i\, w_{d,t} \right) = \sum_{t\in\tau} \Delta_t\, \beta_t^i$. □

Proof of Proposition 4.

Suppose the set $T^i \setminus \{\tau\}$ is not empty, and let $k_r \equiv k_r^*(\boldsymbol{\gamma}, \mathbf{b})$ denote the number of agents other than i reporting truthfully information about state θ_r for $r \in T^i \setminus \{\tau\}$. Given S_r^i, Remark 1 implies $|\pi_r(k_r)| > |\pi_r(k+1)|$.

For the second claim, note that the sign of the right-hand side of (1) depends on the sign of $\left(\sum_{t\in\tau} w_{d,t}\, \Delta_t \right) w_{d,r}\, \pi_r$ only. The credibility loss associated to S_r^i as k_r increases thus becomes $\Leftrightarrow$ $\left(\sum_{t\in\tau} w_{d,t}\, \Delta_t \right) w_{d,r} \left[E(\theta_r|S_r^i, k_r) - E(\theta_r|S_r^i, k_r+1) \right]$. The second claim follows after noting that, given i's information is private, the binding IC constraint corresponds to the set of signal realizations for which the signs of Δ_t and π_r coincide. □

Proof of Lemma 1.

I first I work out the expressions for the expectations and variance. Suppose that the decision-maker holds k_r^* signals about one of the state, for $r = \{1, 2\}$. Let ℓ_r^* denote the number of such signals that equal 1; then the conditional pdf is:

$$f(\ell_r^*|\theta_r, k_r^*) = \frac{k_r^*!}{\ell_r^*!(k_r^* - \ell_r^*)!} \theta_r^{\ell_r^*} (1-\theta_r)^{k_r^* - \ell_r^*}$$

And her posterior is:

$$h(\theta_r|\ell_r^*, k_r^*) = \frac{(k_r^*+1)!}{\ell_r^*!(k_r^* - \ell_r^*)!} \theta_r^{\ell_r^*} (1-\theta_r)^{k_r^* - \ell_r^*}$$

Consequently:

$$E(\theta_r|\ell_r^*, k_r^*) = \frac{(\ell_r^*+1)}{(k_r^*+2)}$$

$$\text{Var}(\theta_r|\ell_r^*, k_r^*) = \frac{(\ell_r^*+1)(k_r^* - \ell_r^* + 1)}{(k_r^*+2)^2(k_r^*+3)}$$

For $r = \{1, 2\}$.

Now, let k_r be i's (on path) conjecture about other senders revealing information about θ_r to the decision-maker. If the equilibrium features him revealing his own signal, then the total number of agents revealing truthfully on path will be $K_r^* = k_r + 1$, while the number of ones received $\ell_r^* = \ell_r + S_r^i$. Therefore, i's conjecture about the receiver's updated beliefs when $S_r^i = 0$ is:

$$E\left(\theta_r|S_r^i = 0, \mathbf{m}^{\text{-}i}\right) = \frac{(\ell_r + 1)}{(k_r + 3)}$$

By the Law of Iterated Expectations, recalling that $f(\ell_r|\theta_r, k_r) = h(\theta_r|\ell_r, k_r)/(k_r+1)$, I get:

$$\begin{aligned} E\left(\theta_r|S_r^i=0,\mathbf{m}^{\text{-}i}\right) &= E\left[E\left(\theta_r|\ell_r, S_r^i=0,\mathbf{m}^{\text{-}i}\right)\right] \\ &= \frac{1}{(k_r+3)}\sum_{\ell_r=0}^{k_r}\frac{(\ell_r+1)}{(k_r+1)} \\ &= \frac{(k_r+2)}{2(k_r+3)} \end{aligned}$$

And now the expectation conditional on $S_r^i = 1$:

$$\begin{aligned} E\left[E\left(\ell_r,\theta_r|S_r^i=1,\mathbf{m}^{\text{-}i}\right)\right] &= \frac{1}{(k_r+3)}\sum_{\ell_r=0}^{k_r}\frac{(\ell_r+2)}{(k_r+1)} \\ &= \frac{(k_r+4)}{2(k_r+3)} \end{aligned}$$

Note that $\Delta_t(S_t^i,\mathbf{m}^{\text{-}i*}) = E(\theta_r|m_r^i = 1-S_r^i,\mathbf{m}^{\text{-}i*}) - E(\theta_r|m_r^i = S_r^i,\mathbf{m}^{\text{-}i*})$ and, then:

$$\begin{aligned} \Delta_t(S_t^i=0,\mathbf{m}^{\text{-}i*}) &= \frac{(k_t+4)}{2(k_t+3)} - \frac{(k_t+2)}{2(k_t+3)} = \frac{1}{(k_t+3)} \\ \Delta_t(S_t^i=1,\mathbf{m}^{\text{-}i*}) &= \frac{(k_t+2)}{2(k_t+3)} - \frac{(k_t+4)}{2(k_t+3)} = -\frac{1}{(k_t+3)} \end{aligned}$$

Finally, note that $\pi_t(S_t^i,\mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i,\mathbf{m}^{\text{-}i*}) - E(\theta_r|\mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i,\mathbf{m}^{\text{-}i*}) - \frac{1}{2}$ and, hence:

$$\begin{aligned} \pi_r(S_r^i=0,\mathbf{m}^{\text{-}i*}) &= \frac{(k_r+2)}{2(k_r+3)} - \frac{1}{2} = -\frac{1}{2(k_r+3)} \\ \pi_r(S_r^i=1,\mathbf{m}^{\text{-}i*}) &= \frac{(k_r+4)}{2(k_r+3)} - \frac{1}{2} = \frac{1}{2(k_r+3)} \end{aligned}$$

□

Proof of Lemma 3.

Consider the equilibrium in which sender i reveals all the information he has; that is $\mathbf{m}^{i*} = \{S_1^i, S_2^i\}$. In this case, there are two types of deviations available: lying on one of the signals ($\mathbf{m}^{i'} = \{1-S_1^i\,,\, S_2^i\}$ when lying on θ_1), or lying on both signals ($\mathbf{m}^{i'} = \{1-S_1^i\,,\, 1-S_2^i\}$). Note that, as i is revealing all the information he has, π_r is absent from the IC constraint.

First, incentives to reveal information about any state individually means the deviation consists of lying on one signal (say, S_2^i) and, thus, equation (1) becomes:

$$\begin{aligned} \beta_1^i\,\Delta_1 + \beta_2^i\,\Delta_2 &\le \frac{1}{2}\left[\gamma_1^i\left(w_{1,1}\,\Delta_1 + w_{1,2}\,\Delta_2\right)^2 + \gamma_2^i\left(w_{2,1}\,\Delta_1 + w_{2,2}\,\Delta_2\right)^2\right] \Leftrightarrow \\ \beta_2^i\,\Delta_2 &\le \frac{1}{2}\left[\gamma_1^i\,\left(w_{1,2}\,\Delta_2\right)^2 + \gamma_2^i\left(w_{2,2}\,\Delta_2\right)^2\right] \end{aligned}$$

The last step relies on the deviation being to lie on θ_2 only, so $\Delta_1 = 0$. Then, equation (3) follows from replacing each possible signal realization in Δ_2.

Now, for the incentives to lie on both signals simultaneously, i.e. $\mathbf{m}^{i'} = \{1-S_1^i\,,\, 1-S_2^i\}$, from equation (1) I get:

$$\beta_1^i\,\Delta_1 + \beta_2^i\,\Delta_2 \leq \frac{1}{2}\left[\Delta_1^2\left(\gamma_1^i w_{1,1}^2 + \gamma_2^i w_{2,1}^2\right) + \Delta_2^2\left(\gamma_1^i w_{1,2}^2 + \gamma_2^i w_{2,2}^2\right) + 2\Delta_1\Delta_2(\gamma_1^i w_{1,1}w_{1,2} + \gamma_2^i w_{2,1}w_{2,2})\right] \tag{11}$$

Which configures the IC constraints depending on signals' realizations. So using Lemma 1 for the corresponding Δ_1 and Δ_2, I get:

- for $\mathbf{S}^i = \{0,0\}$ I get

  $$\frac{\beta_1^i}{(k_1+3)} + \frac{\beta_2^i}{(k_2+3)} \leq \frac{1}{2}\left[\frac{\left(\gamma_1^i w_{1,1}^2 + \gamma_2^i w_{2,1}^2\right)}{(k_1+3)^2} + \frac{\left(\gamma_1^i w_{1,2}^2 + \gamma_2^i w_{2,2}^2\right)}{(k_2+3)^2} + 2\frac{(\gamma_1^i w_{1,1}w_{1,2} + \gamma_2^i w_{2,1}w_{2,2})}{(k_1+3)(k_2+3)}\right]$$

  Now, noting that for $\mathbf{S}^i = \{1,1\}$ the signs of both Δ_1 and Δ_2 is the opposite as in the equation above, and combining both IC constraints I obtain equation (4).

- for $\mathbf{S}^i = \{0,1\}$ I get

  $$\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)} \leq \frac{1}{2}\left[\frac{\left(\gamma_1^i w_{1,1}^2 + \gamma_2^i w_{2,1}^2\right)}{(k_1+3)^2} + \frac{\left(\gamma_1^i w_{1,2}^2 + \gamma_2^i w_{2,2}^2\right)}{(k_2+3)^2} - 2\frac{(\gamma_1^i w_{1,1}w_{1,2} + \gamma_2^i w_{2,1}w_{2,2})}{(k_1+3)(k_2+3)}\right]$$

  Note that in this case $(\Delta_1\Delta_2) < 0$, so the third term on the right-hand side remains the same for $\mathbf{S}^i = \{1,0\}$, while the signs of each tern in the left-hand side reverses. Combining the two IC constraints I get equation (5).

□

Proof of Lemma 4.

Consider the equilibrium in which sender i reveals all his information truthfully for some signals realizations, and announces a non-influential message for the rest. Note that this implies the existence of a non-influential message, which is assumed here but shown an equilibrium in the proof of Proposition 3. This implies $E(\theta_r|m_r^{i*}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*})$ and $E(\theta_r|m_r^{i'}, \mathbf{m}^{\text{-}i*}) = E(\theta_r|\mathbf{m}^{\text{-}i*})$. Also note that sender i uses all the information available to him such that, as in the previous lemma, the terms involving π_r are absent. However, one of the deviations now involves a non-influential message (besides lying on both signals). Note that deviations associated to lying on one signal are no longer available in this equilibrium, because the corresponding message now leads to the receiver ignoring i completely.[14]

From now on, the proof follows the same steps as that of Lemma 3 noting that:

$$\Delta_t = E(\theta_t|\mathbf{m}^{\text{-}i*}) - E(\theta_r|S_r^i, \mathbf{m}^{\text{-}i*})$$

Hence,

$$\Delta_t^{DNS}(S_t^i = 0, \mathbf{m}^{\text{-}i*}) = \frac{1}{2} - \frac{(k_t+2)}{2(k_t+3)} = \frac{1}{2(k_t+3)}$$
$$\Delta_t^{DNS}(S_t^i = 1, \mathbf{m}^{\text{-}i*}) = \frac{1}{2} - \frac{(k_t+4)}{2(k_t+3)} = -\frac{1}{2(k_t+3)}$$

Plugging these intro (11), and noting the symmetry of the IC constraints for the non-influential types, I get equations (6) and (7). □

[14]More precisely, the deviation to lie on θ_r in Lemma 3 involves announcing the message that has the corresponding signal changed; for instance, if a sender of type $(0,0)$ deviates to lying on θ_1, he announces $\mathbf{m}^i = \{(1,0)\}$. For DNS message strategies, such deviation means the decision-maker will not update her beliefs upon receive such message (or, alternatively, i will never utter this message because off-path is fully revealing).

Proof of Proposition 3.

This proof consists on two steps. First, I construct the equilibrium for each of the message strategies. To do that, I use the conditions that guarantee the corresponding strategy is incentive compatible, and construct consistent beliefs for the receiver. Secondly, I show that these strategies constitute the receiver-optimal equilibrium for the set of preferences it applies.

Part 1. The Fully Separating equilibrium consists of i revealing his two signals. Thus, for b_1^i and b_2^i satisfying (3), (4), and (5) with respect to both signals makes the fully separating message strategy incentive compatible for i. Now, consider the following system of beliefs:

$$\mu^*\left((0,0)|\mathbf{m}^i = \{(0,0)\}\right) = 1 \qquad \mu^*\left((1,0)|\mathbf{m}^i = \{(1,0)\}\right) = 1$$
$$\mu^*\left((0,1)|\mathbf{m}^i = \{(0,1)\}\right) = 1 \qquad \mu^*\left((1,1)|\mathbf{m}^i = \{(1,1)\}\right) = 1$$

And the following equilibrium actions:

$$y_d^*\left(\mathbf{m}^i = \{(0,0)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)} \qquad y_d^*\left(\mathbf{m}^i = \{(1,0)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)}$$
$$y_d^*\left(\mathbf{m}^i = \{(0,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)} \qquad y_d^*\left(\mathbf{m}^i = \{(1,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)}$$

These are consistent with $\mathbf{m}^{i*} = \{(0,0);(0,1);(1,0);(1,1)\}$ being an equilibrium message strategy.

Part 2. When i's preferences satisfy (2) with respect to S_1^i only, revealing that information is incentive compatible. Consider the following beliefs for the receiver:

$$\mu^*\left((0,0)|\mathbf{m}^i = \{(0,0)\}\right) = \mu^*\left((0,1)|\mathbf{m}^i = \{(0,0)\}\right) = \frac{1}{2} \qquad \mu^*\left((0,1)|\mathbf{m}^i = \{(0,1)\}\right) = \mu^*\left((0,0)|\mathbf{m}^i = \{(0,1)\}\right) = \frac{1}{2}$$
$$\mu^*\left((1,0)|\mathbf{m}^i = \{(1,0)\}\right) = \mu^*\left((1,1)|\mathbf{m}^i = \{(1,0)\}\right) = \frac{1}{2} \qquad \mu^*\left((1,1)|\mathbf{m}^i = \{(1,1)\}\right) = \mu^*\left((1,0)|\mathbf{m}^i = \{(1,1)\}\right) = \frac{1}{2}$$

These mean that, upon hearing any message the receiver infers i's information about θ_1 but cannot do so about his information about θ_2. Her optimal actions then are:

$$y_d^*\left(\mathbf{m}^i = \{(0,0)\}, \mathbf{m}^{-i}\right) = y^*\left(\mathbf{m}^i = \{(0,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$
$$y_d^*\left(\mathbf{m}^i = \{(1,0)\}, \mathbf{m}^{-i}\right) = y^*\left(\mathbf{m}^i = \{(1,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$

When $\mathbf{b}^i$ satisfies (2) but not (4) the above system of beliefs is consistent with the strategy $\mathbf{m}^{i*} = \big\{\{(0,0),(0,1)\};\{(1,0),(1,1)\}\big\}$, such that i is influential trough S_1^i only.

Part 3. The case of i revealing S_2^i only is equivalent to the previous and thus omitted.

Part 4. Note that when condition (6) holds but (2) does not, incentives to deviate from truth-telling arise for types $(0,1)$ and $(1,0)$ only. Hence, in this equilibrium types $(0,1)$ and $(1,0)$ will announce the associated babbling strategy, while for types $(0,0)$ and $(1,1)$ full revelation is incentive compatible. Let consider the following beliefs for the receiver upon having seen sender i's message $\mathbf{m}^i$, given that i's bias satisfy condition (6) but not (2).

$$\mu^*\left((0,0)|\mathbf{m}^i = \{(0,0)\}\right) = 1 \qquad \mu^*\left((1,1)|\mathbf{m}^i = \{(1,1)\}\right) = 1$$

$$\mu^*\left((0,1)|\mathbf{m}^i = \{(0,1)\}\right) = \mu^*\left((1,0)|\mathbf{m}^i = \{(0,1)\}\right) = \frac{1}{2} \qquad \mu^*\left((1,0)|\mathbf{m}^i = \{(1,0)\}\right) = \mu^*\left((0,1)|\mathbf{m}^i = (1,0)\right) = \frac{1}{2}$$

And the optimal actions by the receiver:

$$\begin{aligned}
y_d^*\left(\mathbf{m}^i = \{(0,0)\}, \mathbf{m}^{-i}\right) &= w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)} \\
y_d^*\left(\mathbf{m}^i = \{(1,1)\}, \mathbf{m}^{-i}\right) &= w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)} \\
y_d^*\left(\mathbf{m}^i = \{(1,0)\}, \mathbf{m}^{-i}\right) &= y_d^*\left(\mathbf{m}^i = \{(0,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+2)} + w_{d2}\frac{(\ell_2+1)}{(k_2+2)}
\end{aligned}$$

Meaning that upon hearing either $\mathbf{m}^i = \{(0,0)\}$ or $\mathbf{m}^i = \{(1,1)\}$ the receiver updates her beliefs about θ_1 and θ_2, but she does not when receiving any other message. From the sender's perspective, types $(0,1)$ and $(1,0)$ should prefer an equilibrium babbling strategy (mixing between these two messages)[15] rather than announcing any of the influential messages. Condition (4) guarantees that types $(0,0)$ and $(1,1)$ do not profit from 'pooling' to each other. Note that this condition is implied by (6). Now, the incentives for full revelation of types $(0,0)$ and $(1,1)$, given types $(0,1)$ and $(1,0)$ are babbling, stem from Lemma 4. Note that condition (6) guarantees types $(0,0)$ and $(1,1)$ have no incentives to 'pool' to the non-influential types under the above system of beliefs. The same conditions guarantee types $(0,1)$ and $(1,0)$ prefer to announce the corresponding non-influential message rather than announcing any of the influential ones.

Part 5. In this equilibrium types $(0,0)$ and $(1,1)$ announce the corresponding non-influential messages, while types $(0,1)$ and $(1,0)$ find incentive compatible to fully reveal their information. The proof is similar to that in Part 4, adapting the equilibrium beliefs to the messages and types that are playing each strategy.

Part 6. The babbling equilibrium is always part of the available equilibria in any cheap talk game. In Part 6 of the following section I show that, for the set of preferences that do not satisfy any of the conditions above, the unique equilibrium available is the babbling.

Characterization of the receiver-optimal equilibrium.

Part 1. Trivially, there cannot exist a more informative equilibrium given this represent the fully separating case.

Part 2. Any strategy that is more informative than revealing only one signal has at least one type playing the fully separating strategy. Consider the following message strategies:

- $\tilde{\mathbf{m}}^i = \{\{(0,0)\}\,;\{(0,1)\}\,;\{(1,0);(1,1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0,0);(0,1)\}\,;\{(1,0)\}\,;\{(1,1)\}\}$

[15] The precise characterization in pure strategies involves the message that would move decisions in the direction of i's biases. Hence, the equilibrium babbling strategy depends on his preferences. The reference to mix strategies is used here as a short-cut.

a. Condition (2) holds with respect to S_1^i only. For $\tilde{\mathbf{m}}^i$ to be an equilibrium, types $(0,0)$ and $(0,1)$ must have incentives to separate from each other, which requires condition (2) to hold for S_2^i, leading to a contradiction. The same applies to $\hat{\mathbf{m}}^i$ for separation between types $(1,0)$ and $(1,1)$.

b. Condition (2) holds for both signals. Note that, under both $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$, the decision-maker obtains information about S_2^i for two out of four possible signal realizations. If condition (2) holds for both signals, then the set of $\mathbf{b}^i$ for which strategies $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$ are incentive compatible coincide, which leads to multiple equilibria. Moreover, these strategies result in the same ex-ante expected payoff for the receiver because she obtains additional information about S_2^i than when only S_2^i is truthfully revealed. However, implementing either $\tilde{\mathbf{m}}^i$ or $\hat{\mathbf{m}}^i$ requires that the 'pooling' types play mixed strategies. In other words, there are no out-of-equilibrium (pure) strategies and associated beliefs that sustain any of these.

Part 3. Any strategy that is more informative than revealing only S_2^i must be of the form:

- $\tilde{\mathbf{m}}^i = \{\{(0,0)\}\,;\{(1,0)\}\,;\{(0,1);(1,1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0,0);(1,0)\}\,;\{(0,1)\}\,;\{(1,1)\}\}$

Then, the arguments in Part 2.a and 2.b apply.

Part 4. Given the equilibrium message strategy $\mathbf{m}^i = \{\{(0,0)\}\,;\{(1,1)\}\,;\{(0,1);(1,0)\}\}$, a more informative message strategy necessarily involves at least one additional type revealing both signals—revelation of one signal is not available due to pooling types not sharing any single realization. As a consequence, the only message strategy that is more informative than $\mathbf{m}^i$ is the fully separating one, for which condition (5) must hold; a contradiction.

Part 5. The argument in Part 4 also applies to this case.

Part 6. The equilibrium message strategy is given by $\mathbf{m}^i = \{(0,0);(0,1);(1,0);(1,1)\}$. Let first consider the partition in which a single type fully separates. Lemma 4 implies that for types $(0,0)$ and $(1,1)$ (individually) the corresponding condition leads to the equilibrium in Part 4, which rules out this possibility. The same argument applies to the cases in which either types $(0,1)$ or $(1,0)$, the receiver-optimal equilibrium for the set of preferences satisfying the corresponding condition would involve the following message strategy $\mathbf{m}^i = \{\{(0,0);(0,1)\}\,;\{(1,0)\}\,;\{(1,1)\}\}$.

Secondly, consider the cases in which one type with coincident signals and another with non-coincident signals separate and the others play babbling. By Lemma 4 whenever type $(0,0)$ separates, type $(1,1)$ has incentives to do so and the deviation is profitable for the receiver; while type $(0,1)$ has incentives to separate if and only if $(1,0)$ has.

Finally, I must rule out the message strategy in which only one realization of a given signal is revealed –i.e. the equilibrium message strategies being either $\mathbf{m}^i = \{\{(0,0);(0,1)\}\,;\{(1,0);(1,1)\}\}$ or $\mathbf{m}^i = \{\{(0,0);(1,0)\}\,;\{(0,1);(1,1)\}\}$, with only one message being influential in each. The argument is simple: if, for instance, sender i were willing to reveal only $S_1^i = 1$; then the receiver would infer $S_1^i = 0$ when i plays the babbling strategy, which implies the equilibrium in Part 1. □

Equilibrium selection. Following Galeotti et al. (2013), the ex-ante expected utility for player i in equilibrium $(\mathbf{y}, \mathbf{m})$ is given by:

$$E\left[U^i(\boldsymbol{\delta}, \mathbf{b}); \mathbf{m}\right] = -\left[\left(b_1^i\right)^2 + \left(b_2^i\right)^2\right] - \frac{[(w_{11})^2 + (w_{21})^2]}{6\,(k_1+2)} - \frac{[(w_{12})^2 + (w_{22})^2]}{6\,(k_2+2)}$$

In order to analyse the ex-ante optimality of each equilibrium in Proposition 3.6, I "break" $\mathbf{m}$ into $\mathbf{m}^i$ and $\mathbf{m}^{-i}$. Then, denoting by $\tilde{k}_r$ the equilibrium number of truthful messages for senders other than i, the expected variance of each possible message strategy for i become:

$$E\left[U_R\left(\mathbf{y}, \mathbf{m}^i = \{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\}, \mathbf{m}^{-i}\right)\right] = -\frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 3\right)} - \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 2\right)} \quad (12)$$

$$E\left[U_R\left(\mathbf{y}, \mathbf{m}^i = \{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\}, \mathbf{m}^{-i}\right)\right] = -\frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 2\right)} - \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 3\right)} \quad (13)$$

$$\begin{aligned} E\left[U_R\left(\mathbf{y}, \mathbf{m}^i = \{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\}, \mathbf{m}^{-i}\right)\right] = &-\frac{1}{2}\left[\frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 3\right)} + \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 3\right)}\right] \\ &-\frac{1}{2}\left[\frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 2\right)} + \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 2\right)}\right] \end{aligned} \quad (14)$$

Where the fact that in (12) sender i reveals S_1^i only can be seen in the different numerators of its first and second term, and the same applies to (13). Now, when i reveals $(0,0)$ and $(1,1)$ only (Part 4.c) the receiver's ex-ante expected utility weights the probability of i being one of these types, and the complementary probability of being $(0,1)$ or $(1,0)$ and not receiving any information. The following step consist in finding the conditions under which each of the above expressions are ex-ante optimal for the receiver, given the equilibrium strategies of the other senders.

1. Sender i reveals S_1^i only:
   (a) (12) $\geq$ (13):
   $$-\frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 3\right)} - \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 2\right)} + \frac{[(w_{11})^2 + (w_{21})^2]}{6\left(\tilde{k}_1 + 2\right)} + \frac{[(w_{12})^2 + (w_{22})^2]}{6\left(\tilde{k}_2 + 3\right)} \geq 0$$
   Which leads to:
   $$\frac{[(w_{11})^2 + (w_{21})^2]}{(\tilde{k}_1 + 2)(\tilde{k}_1 + 3)} \geq \frac{[(w_{12})^2 + (w_{22})^2]}{(\tilde{k}_2 + 2)(\tilde{k}_2 + 3)} \quad (15)$$
   (b) (12) $\geq$ (14):
   $$\frac{[(w_{11})^2 + (w_{21})^2]}{2}\left[\frac{1}{(\tilde{k}_1 + 2)} - \frac{1}{(\tilde{k}_1 + 3)}\right] - \frac{[(w_{12})^2 + (w_{22})^2]}{2}\left[\frac{1}{(\tilde{k}_2 + 2)} - \frac{1}{(\tilde{k}_2 + 3)}\right] \geq 0$$
   Which is straightforward to note that leads to (15).
2. Sender i reveals S_2^i only: requires that (13) $\geq$ (12) and (13) $\geq$ (14); which following the above algebra happen if and only if
   $$\frac{[(w_{11})^2 + (w_{21})^2]}{(\tilde{k}_1 + 2)(\tilde{k}_1 + 3)} \leq \frac{[(w_{12})^2 + (w_{22})^2]}{(\tilde{k}_2 + 2)(\tilde{k}_2 + 3)} \quad (16)$$
3. Sender i reveals $(0,0)$ and $(1,1)$ only: requires that (14) $\geq$ (12) and (14) $\geq$ (13); which happen if and only if,
   $$\frac{[(w_{11})^2 + (w_{21})^2]}{(\tilde{k}_1 + 2)(\tilde{k}_1 + 3)} \geq \frac{[(w_{12})^2 + (w_{22})^2]}{(\tilde{k}_2 + 2)(\tilde{k}_2 + 3)}$$

and

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)} \leq \frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)}$$

Both equations above are compatible if and only if:

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)} = \frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{17}$$

The above equation tells us that the receiver's ex-ante utility is maximal when i reveals $(0,0)$ and $(1,1)$ only, when she has similar amount of truthful messages for each state in the equilibrium being played. In addition, equation (17) implies that for i revealing S_1^i only to be the optimal ex-ante equilibrium for the receiver (15) must hold with inequality, and the same applies for S_2^i and (16). □

Observation 1. *The RHS of (5) is positive if and only if:*

$$[w_{11}(k_2+3)-w_{12}(k_1+3)]^2+[w_{22}(k_1+3)-w_{21}(k_2+3)]^2>0$$

Since $\frac{w_{11}}{(k_1+3)}$ represents sender i's influence on y_1 through revealing S_1^i, then $w_{11}(k_2+3)-w_{12}(k_1+3)=0$ implies that the influence on y_1 through S_1^i perfectly outweighs that of S_2^i. Consequently, RHS of (5) equal to zero implies sender i has no influence on any decision when he reveals both signals (and he is either type $(0,1)$ or $(1,0)$), which again depends on the number of other sender revealing truthfully their signals.

Proof of Proposition 4.

For the sake of exposition, I assume that $\gamma_1^i=\gamma_2^i=1$, $w_{1,1}=w_{2,2}=w$, and $w_{1,2}=w_{2,1}=1-w$. I show the existence of beneficial congestion associated to two message strategies. I first show the case of sender i going from 'babbling' to revealing information about θ_2 only due to congestion on θ_1. Second, I show i going from partial revelation to full revelation due to congestion on θ_2.

From Lemma 2, congestion on information about θ_1 leading sender i to reveal his information about θ_2 means the following IC constraints:

$$|\beta_2^i| > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_2+3)}-\frac{2[w(1-w)]}{(k_1+3)}\right] \qquad \text{and} \qquad |\beta_2^i| \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_2+3)}-\frac{2[w(1-w)]}{(k_1'+3)}\right]$$

It is straightforward to check that such β_2^i exists as long as $k_1'>k_1$.

Secondly, necessary conditions for congestion on θ_1 leading to full revelation mean the following:

$$|\beta_1^i| \leq \frac{w^2+(1-w)^2}{2\,(k_1'+3)} \tag{18}$$

$$|\beta_2^i| \leq \frac{w^2+(1-w)^2}{2\,(k_2+3)} \tag{19}$$

$$\left|\frac{\beta_1^i}{(k_1'+3)}-\frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] \tag{20}$$

$$\left|\frac{\beta_1^i}{(k_1+3)}-\frac{\beta_2^i}{(k_2+3)}\right| > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] \tag{21}$$

First note that LHS of both (20) and (21) depend on the signs of β_1^i and β_2^i. Because Proposition

4 is an existence result, I focus on the (easiest) case of $\text{sign}[\beta_1^i] \neq \text{sign}[\beta_2^i]$, and $k_1' = k_1 + 1$; hence:

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| = \frac{|\beta_1^i|}{(k_1'+3)} + \frac{|\beta_2^i|}{(k_2+3)}$$

Secondly, note that given the signs of β_1^i and β_2^i, (18) and (19) imply the IC constraint for revealing both signals when they coincide in equation (4). Replacing the left-hand sides of conditions (20) and (21), and after some algebra I get that a necessary condition on β_2^i such that congestion in θ_1 improves communication is:

$$\frac{(k_1+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] - \frac{|\beta_2^i|\,(k_1+3)}{(k_2+3)} <$$
$$< \frac{(k_1'+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] - \frac{|\beta_2^i|\,(k_1'+3)}{(k_2+3)}$$

Which after some algebra leads to:

$$|\beta_2^i| \leq \frac{[w^2+(1-w)^2]}{2\,(k_2+3)}\left[1 - \frac{(k_2+3)^2}{(k_1+3)(k_1'+3)}\right]$$

Note that the above implies (19), and the existence of such aggregate bias requires that its right-hand side is greater than zero, which yields:

$$(k_1+3)(k_1'+3) > (k_2+3)^2$$

Similarly, a necessary condition over β_1^i is:

$$\frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] - \frac{|\beta_1^i|}{(k_1+3)} <$$
$$< \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] - \frac{|\beta_1^i|}{(k_1'+3)}$$

Which requires that:

$$|\beta_1^i| > \frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1'+3)}\right] - \frac{2\,w(1-w)}{(k_2+3)}$$

This, together with (18) implies that such β_1^i exists if and only if:

$$\frac{(k_2+3)}{(k_1+3)} < \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

Combining the above condition with that for β_2^i, I get (9). □

Non additively-separable preferences

In order to focus on the intuitions, I divide the analysis into two parts. First, I analyze the effects of non-separability in the principal's preferences to isolate the mechanism of 'influence on decisions' while keeping senders' preferences as in the baseline model. Secondly, I focus on non-separability in the senders' payoffs to analyze the 'preference for coordination' mechanism.

To keep the analysis simple, I focus on the case of two decisions and two uniformly-distributed states, where each sender observes one binary signal associated to each state. Players' preferences take the generic functional form:

$$U^i(\mathbf{y}, \boldsymbol{\gamma}^i, \kappa^i, \boldsymbol{\theta}) = -\sum_{d=1}^{2} \left(y_d - \delta_d(\theta_1, \theta_2) - b_d^i\right)^2 - \kappa^i\,(y_1 - y_2)^2 \tag{22}$$

Where, as before, $\delta_d = w_{d,1}\,\theta_1 + w_{d,2}\,\theta_2$, $\mathbf{b}^{\mathrm{R}} = (0, 0)$. I further assume that $\boldsymbol{\gamma}^{\mathrm{R}} = \gamma^i = (1, 1)$, $w_{1,1} = w_{2,2} = w \in [0.5, 1]$, and that $w_{2,1} = w_{1,2} = 1 - w$, which yields a more intuitive interpretation of the informational interdependence.

Non-separability of the decision-maker's preferences. In this case, preferences feature $\kappa^i = 0$ for all i. The principal's optimal decision on the first dimension is characterized by the following reaction function:

$$y_1^*(\boldsymbol{\theta}, \mathbf{m}^*) = \frac{1}{(1+\kappa^{\mathrm{R}})} E(\delta_1|\mathbf{m}^*) + \frac{\kappa^{\mathrm{R}}}{(1+\kappa^{\mathrm{R}})} y_2^*(\boldsymbol{\theta}, \mathbf{m}^*)$$

Denoting $g^{\mathrm{R}} = \frac{(1+\kappa^{\mathrm{R}})}{(1+2\kappa^{\mathrm{R}})}$, this leads to the following optimal decision:

$$\begin{aligned} & y_d^* = E(\delta_d|\mathbf{m}^*)\,g^{\mathrm{R}} + E(\delta_e|\mathbf{m}^*)\,(1 - g^{\mathrm{R}}) \\ \Leftrightarrow\ & y_d^* = E(\theta_1|\mathbf{m}^*)\ [g^{\mathrm{R}}\,w_{d,1} + (1-g^{\mathrm{R}})\,w_{e,1}] + E(\theta_2|\mathbf{m}^*)\ [g^{\mathrm{R}}\,w_{d,2} + (1-g^{\mathrm{R}})\,w_{e,2}] \end{aligned}$$

For $y_d \neq y_e = \{y_1, y_2\}$. In other words, the principal's optimal decision in each dimension when she has preferences for coordination is a weighted average of the expected optimal calibration of both decisions. The weights reflect the principal's coordination motives.

Truthful communication for i is incentive compatible, provided that it leads to decisions $y_d^* = y_d(\mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*})$, if and only if for every possible deviation $\mathbf{m}^{i'}$ leading to $y_d' = y_d(\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*})$ it is true:

$$\int_{\Theta_1}\int_{\Theta_2} \left[-\sum_{d=1}^{2} \Big(y_d^* + y_d' - 2(w_{d,1}\,\theta_1 + w_{d,2}\,\theta_2 + b_d^i)\Big)\big(y_d^* - y_d'\big) \right] dF(\theta_1, \mathbf{m}^{\text{-}i*}|S_1^i)dF(\theta_2, \mathbf{m}^{\text{-}i*}|S_2^i) \geq 0$$

Now, let $W_{d,r}^{\mathrm{R}} = [g^{\mathrm{R}}\,w_{d,r} + (1-g^{\mathrm{R}})\,w_{e,r}]$; which implies that $W_{d,r}^{\mathrm{R}} - w_{d,r} = (1-g^{\mathrm{R}})\,[w_{d,r} - w_{e,r}]$. Recall that $\nu_r^* = E(\theta_r|\mathbf{m}^{i*}, \mathbf{m}^{\text{-}i*})$, $\nu_r' = E(\theta_r|\mathbf{m}^{i'}, \mathbf{m}^{\text{-}i*})$, and $\nu_r = E(\theta_r|\mathbf{S}_r^i, \mathbf{m}^{\text{-}i*})$. In addition, recall that $\Delta_r = \nu_r' - \nu_r^*$ and that $\pi_r = \nu_r - 1/2$; hence, the IC constraint above becomes:

$$\sum_{d=1}^{2} \left[\left[W_{d,1}^{\mathrm{R}}\Big(\nu_1^* + \nu_1' - 2\nu_1\Big) + W_{d,2}^{\mathrm{R}}\Big(\nu_2^* + \nu_2' - 2\nu_2\Big) - 2\,b_d^i \right] - \Big[2\,(1-g^{\mathrm{R}})(2w-1)\big(\nu_d - \nu_{\text{-}d}\big)\Big] \right] \Big(W_{d,1}^{\mathrm{R}}\,\Delta_1 + W_{d,2}^{\mathrm{R}}\,\Delta_2\Big) \geq 0 \tag{23}$$

Where $\nu_d = E(\theta_d|S_d^i)$, that is, the indices of state and decision coincide. The above IC constraint differs from that corresponding to additively separable preferences, (10), in two main aspects. First, the way in which the sender weighs the effects of his information on decisions; that is, $w_{d,r}$ is replaced by $W_{d,r}^{\mathrm{R}}$, which is a linear combination of $w_{d,r}$ and $w_{d,e}$ that represents the effect information about θ_r has on both decisions due to the coordination motives. This effect is equivalent to a higher degree of interdependence between decisions, as $\min\{w_{d,r}, w_{e,r}\} \leq W_{d,r}^{\mathrm{R}} \leq \max\{w_{d,r}, w_{e,r}\}$.

The second difference between (23) and (10) consists of the second term in the former, which after some algebra and noting that $W_{1,1}^{\mathrm{R}} - W_{2,1}^{\mathrm{R}} = w_{1,1} - w_{2,1} = 2w - 1$, becomes

$$\sum_{d=1}^{2} \big[2\,(1-g^{\mathrm{R}})(2w-1)(\nu_d - \nu_{\text{-}d})\big]\big(W_{d,1}^{\mathrm{R}}\,\Delta_1 + W_{d,2}^{\mathrm{R}}\,\Delta_2\big) = 2(1-g^{\mathrm{R}})(2w-1)^2(\nu_1 - \nu_2)(\Delta_1 - \Delta_2)$$

The sign of the expression above depends on $(\nu_1 - \nu_2)(\Delta_1 - \Delta_2)$. Recall that, by LIE:

$$\nu_r^{(0)} := E(\theta_r | S_r^i = 0, \mathbf{m}^{\text{-}i*}) = \frac{(k_r + 2)}{2(k_r + 3)} \qquad \Delta_r^{(0)} := \Delta_r(S_r^i = 0, \mathbf{m}^{\text{-}i*}) = \frac{1}{(k_r + 3)}$$

$$\nu_r^{(1)} := E(\theta_r | S_r^i = 1, \mathbf{m}^{\text{-}i*}) = \frac{(k_r + 4)}{2(k_r + 3)} \qquad \Delta_r^{(1)} := \Delta_r(S_r^i = 1, \mathbf{m}^{\text{-}i*}) = -\frac{1}{(k_r + 3)}$$

After a bit of algebra, it can be shown the expression associated to the **deviation of lying on both signals** for $\mathbf{S}^i = \{(0,0),(1,1)\}$ is

$$\big(\nu_1 - \nu_2\big)\big(\Delta_1 - \Delta_2\big) = -\frac{(k_1 - k_2)^2}{2[(k_1 + 3)(k_2 + 3)]^2} \tag{24}$$

While that associated to $\mathbf{S}^i = \{(0,1),(1,0)\}$ are

$$\big(\nu_1 - \nu_2\big)\big(\Delta_1 - \Delta_2\big) = -\frac{(k_1 + k_2 + 6)}{2[(k_1 + 3)(k_2 + 3)]^2} \tag{25}$$

For the set of **deviations involving lying in one signal only**, the expression when $\mathbf{S}^i = \{(0,1),(1,0)\}$ is

$$\big(\nu_1 - \nu_2\big)\,\Delta_1 = \big(\nu_1 - \nu_2\big)\,(-\Delta_2) = -\frac{(k_1 + k_2 + 6)}{2(k_1 + 3)^2(k_2 + 3)} \tag{26}$$

While those associated to $\mathbf{S}^i = \{(0,0),(1,1)\}$ are

$$\big(\nu_1^{(0)} - \nu_2^{(0)}\big)\,\Delta_1^{(0)} = \big(\nu_1^{(1)} - \nu_2^{(1)}\big)\,\Delta_1^{(1)} = \frac{(k_1 - k_2)}{2(k_1 + 3)^2(k_2 + 3)} \tag{27}$$

$$\big(\nu_1^{(0)} - \nu_2^{(0)}\big)\,(-\Delta_2^{(0)}) = \big(\nu_1^{(1)} - \nu_2^{(1)}\big)\,(-\Delta_2^{(1)}) = \frac{(k_2 - k_1)}{2(k_1 + 3)(k_2 + 3)^2} \tag{28}$$

When i's messages imply his signals coincide and they are taken at face value by the receiver, their effect on decisions will be strong due to her preferences for coordination. Equation (26) illustrates this, showing that any deviation involving misrepresentation of one of his signals in that direction involves a 'cost' in terms of decisions that is higher than when preferences are separable.

However, the same strong response to information validating the receiver's preferences for coordinated decisions makes i more willing to lie on one of his signals when $\mathbf{S}^i = \{(0,0),(1,1)\}$ (equations (27) and (28)). Sender i will then be more willing to deviate from truthful communication when doing so will trigger such strong response from the decision-maker. Note these incentives are increasing in the difference between k_1 and k_2, but disappear when $k_1 = k_2$; also, note that i's temptation to lie corresponds to the state the receiver has more information on path. Therefore, i's deviation incentives arise because he considers decisions should not be too close together as y_1 [y_2, resp.] should be more sensitive to θ_1 [θ_2]. Note that $\lim_{k_r \to \infty} \nu_r^{(0)} = \lim_{k_r \to \infty} \nu_r^{(1)} = \frac{1}{2}$, so i deviation responds to his intention to induce y_r closer to the prior. The fact that $2(1 - g^{\text{R}})(2w - 1)^2(\nu_1 - \nu_2)(\Delta_1 - \Delta_2)$ is increasing in w (decreasing in the degree of informational interdependence) supports the previous claim.

Non-separability in the sender's preferences. In this case $\kappa^{\mathrm{R}} = 0$. Hence, optimal decisions are the same as in the baseline model, while sender i's incentives for communication are given by:

$$\int_{\Theta_1}\int_{\Theta_2}\Bigg[\sum_{d=1}^{2} -\Big(y_d^* + y_d' - 2(w_{d,1}\,\theta_1 + w_{d,2}\,\theta_2 + b_d^i)\Big)\big(y_d^* - y_d'\big) -$$

$$- \kappa^i \left(\big(y_1^* - y_2^*\big)^2 - \big(y_1' - y_2'\big)^2\right)\Bigg]\, dF(\theta_1, \mathbf{m}^{\text{-}i*}|S_1^i) dF(\theta_2, \mathbf{m}^{\text{-}i*}|S_2^i) \geq 0$$

Which, after some algebra and recalling that $\Delta_r = \nu_r' - \nu_r^*$ I get:

$$\sum_{d=1}^{2}\left[w_{d,1}\Big(\nu_1^* + \nu_1' - 2\nu_1\Big) + w_{d,2}\Big(\nu_2^* + \nu_2' - 2\nu_2\Big) - 2\, b_d^i\right]\Big(w_{d,1}\,\Delta_1 + w_{d,2}\,\Delta_2\Big) -$$

$$- \kappa^i (2w-1)^2 \left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1' - \nu_2'\big)^2\right] \geq 0 \tag{29}$$

The first term above is the same as in equation (10). Note that the second term is binding for signal realizations and associated deviations that would induce the decision-maker's to believe the states are closer. Therefore, it is straightforward to show that for any deviation involving lying in both signals $\left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1' - \nu_2'\big)^2\right] = 0$.

Among the deviations that imply lying in one signal only, those that would induce the receiver's posteriors to be further apart will not be binding—i.e. $\left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1' - \nu_2^*\big)^2\right] = \left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1^* - \nu_2'\big)^2\right] = -\frac{1}{(k_1+3)(k_2+3)}$ when $\mathbf{S}^i = \{(0,0),(1,1)\}$. However, when i's information is $\mathbf{S}^i = \{(0,1),(1,0)\}$, then $\left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1' - \nu_2^*\big)^2\right] = \left[\big(\nu_1^* - \nu_2^*\big)^2 - \big(\nu_1^* - \nu_2'\big)^2\right] = \frac{1}{(k_1+3)(k_2+3)}$ which makes the second term in (29) smaller or equal to zero. This means that i's preferences for coordinated decisions will create additional incentives to deviate from truthful communication when his information would move decisions apart.

Finally, note that the second term in (29) is increasing in κ^i and w. The latter means that informational interdependence mitigates the perverse effect of preference interdependence on incentives.