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False-name-proof and Strategy-proof Voting Rules under Separable Preferences*

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Abstract

We consider the problem of a society that uses a voting rule to select a subset from a given set of objects (candidates, binary issues or alike). We assume that voters' preferences over subsets of objects are separable: Adding an object to a set leads to a better set if and only if the object is good (as a singleton set, the object is better than the empty set). A voting rule is strategy-proof if no voter benefits by not revealing its preferences truthfully and it is false-name-proof if no voter gains by submitting several votes under other identities. We characterize all voting rules that verify false-name-proofness, strategy-proofness, unanimity, anonymity, and neutrality as either the class of voting by quota one (all voters can be decisive for all objects) or the class of voting by full quota (all voters can veto all objects).

Keywords: False-name-proofness; Strategy-proofness; Separable Preferences.
JEL Classification: D71.

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1 Introduction

Societies take decisions by means of voting rules, mapping profiles of voters' preferences into social alternatives. But since individual preferences are private information, voters may behave strategically by not submitting their preferences truthfully. Voting rules that are immune to this kind of manipulation are called strategy-proof. Another way of behaving strategically, especially when the identities of voters cannot or are not easily verified, is by voting several times under other identities. This is in fact a real and growing phenomenon in anonymous online voting. Many social decisions processes are held online around the world by voting, particularly during the Covid-19 lock-downs. Selection processes for Massive Open Online (MOO) Courses or MOO Schools, rating systems for goods and services, Internet auctions (a popular part of Electronic Commerce), or Facebook allowing users to vote on its future terms of use (see Zuckerberg (2009)), are all examples where a voter could benefit from voting multiple times. As many institutions or websites may not have enough resources to correctly identify each voter in this kind of voting situations, repeated voting can be a highly relevant issue. An online anonymous poll on an specific political issue, run by a popular news website, although not binding, may lead to an overwhelming pressure to the government. A voting rule where agents cannot benefit by voting several times is usually known in the literature as false-name-proof (see Yokoo, Sakurai, and Matsubara (2004)).[1]

When voters preferences do not have a specific structure, results on voting rules satisfying some form of false-name-proofness are rather negative, even for random voting rules.[2] Conitzer (2008) characterizes all anonymous-proof and neutral random voting rules under strict preferences over a finite set of alternatives.[3] Each element in the class identified by

[1]Here, we shall say that a voting rule is *false-name-proof* if no agent can benefit by repeating the same vote several times, while a voting rule is *strong false-name-proof* if no voter can benefit by casting several votes (not necessarily the same). The proof of our main result shows that, in our setting with separable preferences, the classes of false-name-proof and strong false-name-proof voting rules do coincide on the family of voting rules satisfying strategy-proofness, unanimity, anonymity and neutrality (see Theorem 1 and Corollary 1).

[2]Bu (2013) shows that, under unrestricted domains of voters' preferences, if a voting rule is strategy-proof, anonymous, and population monotonic, then it is strong false-name-proof; moreover, under strict preferences, the converse also holds. Population monotonicity is a strong requirement: When new voters arrive and vote, each voter initially present shouldn't be strictly better off than it was before.

[3]A voting rule is anonymous-proof if it anonymous and satisfies strong false-name-proofness and participation (namely, all voters prefer to vote than to abstain), which all together imply strategy-proofness (see Proposition 4 in Subsection 3.2). A random voting rule is neutral if it does not depend on the name of the

Conitzer (2008) is described by a probability $p \in [0,1]$ with which an alternative is chosen with uniform probability and with probability $1-p$ a pair of alternatives is chosen with uniform probability and if all voters unanimously prefer one alternative over the other, this preferred alternative is chosen, and otherwise a fair coin is used to decide between the two. These voting rules are perceived as being very unresponsive to voters' preferences.

Nevertheless, in many applications the particular structure of the set of alternatives suggests that not all voters' preferences are conceivable. A large literature on social choice presumes that a natural restriction on the domain of voters' preferences holds, the one that is meaningful with respect to that structure. A prominent example of this kind of domain restriction is when the set of alternatives has a linear order structure relative to which single-peaked preferences can be naturally defined. Then, the median voter rule (that selects the median of the profile of voters' top alternatives relative to the linear order) is strategy-proof (see Moulin (1980)). However, the median voter rule is not false-name-proof since a voter with the lowest top can manipulate the voting rule by casting several votes for its own top. The later was shown by Todo, Iwasaki, and Yokoo (2011), who also characterize the class of all strong false-name-proof, efficient and anonymous voting rules as those that, for each set of voters N (with cardinality n), the voting rule selects the median of the n reported tops together with $n-1$ fixed ballots for a given *a priory* alternative α.[4] Some form of false-name-proofness has also been studied in other settings, often under the name of duplication as in Congar and Merlin (2012) and García-Lapresta and Martínez-Panero (2017); for instance, in social networks (see Brill, Conitzer, Freeman, and Shah (2016)), in matching problems (see Todo and Conitzer (2013)), or together with other properties (see Wagman and Conitzer (2008) and Waggoner, Xia, and Conitzer (2012)).

In this paper we consider social choice problems where societies have to choose a subset from a given set of objects (candidates, binary issues or alike), and voters have separable preferences over subsets of objects. A voter's preference is *separable* over the family of all subsets of objects if the ranking of subsets is guided by the partition separating the set of objects into the set of good objects (as singleton sets, objects that are better than the empty set) and bad objects (as singleton sets, objects that are worse than the empty set). Adding a good object to any set leads to a better set, while adding a bad object leads to a worse set. Note that the best subset (the top) of a separable preference is the union of good

alternatives.

[4]Todo, Iwasaki, and Yokoo (2011) and Todo, Okada, and Yokoo (2020) extend the analysis to the case where the set of alternatives has a tree structure.

objects, and that all additively representable preferences are separable. This is the setting considered by Barberà, Sonnenschein, and Zhou (1991), where they characterize the family of all strategy-proof, unanimous, anonymous, and neutral voting rules as the class of all voting by quota.[5] A (neutral) voting by quota for N specifies (and it can be identified with) an integer q_N between 1 and n, where $n = |N|$. Then the choice of the subset of objects made by the voting by quota q_N at a profile of separable preferences is done object-by-object as follows: Object x belongs to the chosen set if and only if the set of voters for which x is a good object has cardinality larger or equal to q_N. Hence, voting by quota can be seen as a family of qualified majority voting where the two alternatives at stake (in each voting) are whether or not x belongs to the collectively chosen subset of objects.

We want to identify here, for this setting under separable preferences, simple voting rules that are simultaneously immune to two types of manipulations: Not voting according to the true preferences (strategy-proofness) and voting many times (false-name-proofness). Our main result (Theorem 1) characterizes all voting rules that verify strategy-proofness, false-name-proofness, unanimity, anonymity, and neutrality as either the class of voting by quota one (all agents can be decisive for all objects) or the class of voting by full quota (all agents can veto all objects). Our proof of Theorem 1 shows that false-name-proofness in this characterization can be replaced by strong false-name-proofness (Corollary 1). Moreover, we show in Proposition 2 that if a voting rule is strategy-proof, false-name-proof, unanimous, anonymous, and neutral, then it satisfies participation. In Proposition 4 we establish that, in any setting and any domain of preferences, if a voting rule is strong false-name-proof and satisfies anonymity and participation, then it is strategy-proof as well. Finally, Example 2 shows that the statement of Proposition 4 does not hold if strong false-name-proofness is replaced by false-name-proofness, even under the domain of separable preferences.

The rest of the paper is organized as follows. Section 2 contains preliminaries, the definition of separable preferences, properties of voting rules, and the definition of voting by quota. Section 3 contains the statement of our main result, its proof, additional results, and the examples showing that the axioms used in Theorem 1 are independent.

[5]They characterize the larger family of all strategy-proof and unanimous voting rules as the class of voting by committees.

2 Preliminaries and definitions

2.1 Voters, alternatives, separable preferences and voting rules

We are interested in studying social choice procedures under which societies choose a subset from a given set of objects. Our aim is to identify those that are simultaneously immune to voters' manipulations by revealing non-truthful preferences and by providing additional preferences under other identities. While the first property assumes that the society is fixed, the second one requires to consider different societies. For this reason, we consider societies with a variable set of voters. Let $\mathcal{N}$ be the family of all finite and no-empty subsets of the set of positive integers $\mathbb{Z}_+$. An element $N \in \mathcal{N}$ is interpreted as a society. We denote the cardinality of N by n and refer to an element $i \in N$ as a *voter*. Each set of voters $N \in \mathcal{N}$ has to collectively choose a subset from a given finite set $K = \{1, \ldots, M\}$ of objects. Then the set of *alternatives* from which any society has to choose from is the family of all subsets of objects 2^K.

For each voter i, let P_i be voter i's preference over 2^K. We assume that P_i is a linear order and denote by $\mathcal{D}$ a generic set of strict preferences over 2^K, which we will refer to as a *domain*. A *profile* (for $N \in \mathcal{N}$) is an ordered list of preferences $P = (P_i)_{i \in N} \in \mathcal{D}^N$, one for each voter in N. By convention, we set $\mathcal{D}^{\emptyset} = \emptyset$. For $N, N' \in \mathcal{N}$ (with $N \cap N' = \emptyset$) and two profiles of preferences $P = (P_i)_{i \in N} \in \mathcal{D}^N$ and $P' = (P'_i)_{i \in N'} \in \mathcal{D}^{N'}$, we denote by (P, P') the profile $((P_i)_{i \in N}, (P'_i)_{i \in N'}) \in \mathcal{D}^{N \cup N'}$. Let $P = (P_i)_{i \in N} \in \mathcal{D}^N$ be a profile and let $i \in N$ be a voter, we denote by $P_{-i} \in \mathcal{D}^{N \setminus \{i\}}$ the profile obtained from P after deleting P_i.

Let P_i be a preference over 2^K. An object is good (respectively, bad) according to P_i if as a singleton set is strictly preferred (respectively, less preferred) to the empty set. A preference P_i is separable if the division between good and bad objects guides the ordering between some pairs of subsets: Adding a good object to any set leads to a better set, while adding a bad object to any set leads to a worse set. Formally, a preference $P_i \in \mathcal{D}$ is *separable* if for all $T \in 2^K$ and $x \notin T$,

$$T \cup \{x\} P_i T \text{ if and only if } \{x\} P_i \emptyset.$$

Let $\mathcal{S}$ be the set of all separable preferences over 2^K.

A preference $P_i \in \mathcal{D}$ is *additive* if there exists a function $u : K \cup \{\emptyset\} \to \mathbb{R}$ such that $u(\emptyset) = 0$ and for all $T, T' \in 2^K$,

$$T P_i T' \text{ if and only if } \sum_{x \in T} u(x) > \sum_{x \in T'} u(x),$$

where by convention we set $\sum_{x \in \widehat{T}} u(x) = 0$ for $\widehat{T} = \emptyset$. In this case, we say that u (additively) represents P_i. Of course, all additive preferences are separable. Let $\mathcal{A}$ be the set of all additive preferences over 2^K. Given $x \in K$, we denote by $u^x : K \cup \{\emptyset\} \to \mathbb{R}$ any function such that $u^x(x) > |u^x(y)| > u^x(\emptyset) = 0$ for all $y \in K \setminus \{x\}$, and if $x \in T$ and $x \notin T'$ then $\sum_{y \in T} u^x(y) > \sum_{y \in T'} u^x(y)$. Accordingly, if P_i is additive and u^x represents P_i then $x \in T$ and $x \notin T'$ imply $T P_i T'$. Similarly, given $y \in K$, we denote by $u_y : K \cup \{\emptyset\} \to \mathbb{R}$ any function such that $|u_y(y)| > |u_y(x)| > u_y(\emptyset) = 0 > u_y(y)$ for all $x \in K \setminus \{y\}$, and if $y \in T$ and $y \notin T'$ then $\sum_{x \in T} u_y(x) < \sum_{x \in T'} u_y(x)$. Accordingly, if P_i is additive and u_y represents P_i then $y \in T$ and $y \notin T'$ imply $T' P_i T$.

Given $P_i \in \mathcal{D}$, let $t(P_i)$ and $b(P_i)$ be respectively the best and worst subsets of 2^K according to P_i; namely, $t(P_i) P_i T$ for all $T \in 2^K \setminus t(P_i)$ and $T P_i b(P_i)$ for all $T \in 2^K \setminus b(P_i)$. It is easy to see that if $P_i \in \mathcal{S}$, then $t(P_i) = \{x \in K \mid \{x\} P_i \emptyset\}$ and $b(P_i) = \{x \in K \mid \emptyset P_i \{x\}\}$, and if $P_i \in \mathcal{A}$, then $t(P_i) = \{x \in K \mid u(x) > 0\}$ and $b(P_i) = \{x \in K \mid u(x) < 0\}$ for any u that represents P_i.

A voting rule for $N \in \mathcal{N}$ on a domain $\mathcal{D}$ selects a subset of K for each profile $P \in \mathcal{D}^N$. Namely, given a domain $\mathcal{D}$, a *voting rule for* N is a mapping $f_N : \mathcal{D}^N \to 2^K$. A *voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *(on* $\mathcal{D}$*)* is a family of voting rules, one for each $N \in \mathcal{N}$.

2.2 Properties of voting rules

We define desirable properties for voting rules. The first property states that if all voters unanimously agree that a subset of objects is the best one, then the voting rule must select it.

Definition 1 *A voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is* unanimous *if, for all* $P \in \mathcal{D}^N$ *such that* $t(P_i) = T$ *for all* $i \in N$, $f_N(P) = T$. *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* unanimous *if* f_N *is unanimous for each* $N \in \mathcal{N}$.[6]

The following two properties are very natural in online environments, and state that no voter nor alternative should receive a differential treatment.

Definition 2 *A voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is* anonymous *if, for each bijection* $\sigma : N \to N$ *and each* $P \in \mathcal{D}^N$, $f_N(P) = f_N(\sigma(P))$, *where* $\sigma(P) = (P_{\sigma(i)})_{i \in N}$. *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* anonymous *if* f_N *is anonymous for each* $N \in \mathcal{N}$.

[6]All unanimous voting rules are *onto*; namely, for each $N \in \mathcal{N}$ and each $T \in 2^K$, there exists a profile $P \in \mathcal{D}^N$ such that $f_N(P) = T$. In all our results, unanimity could be replaced by ontoness.

Definition 3 *A voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is* neutral *if, for each bijection* $\mu : K \to K$ *and each* $P \in \mathcal{D}^N$, $f_N(P) = \mu(f_N(\mu(P)))$, *where* $\mu(T)$ *and* $\mu(P) = (P'_i)_{i \in N}$ *are the subset of objects and the preference profile obtained respectively from* $T \in 2^K$ *and* $P \in \mathcal{D}^N$ *by permuting the objects according to* μ*; namely,* $\mu(T) = \{x \in K \mid x = \mu(y) \text{ for } y \in T\}$ *and, for each* $i \in N$ *and pair* $x, y \in K$, $\mu(x) P'_i \mu(y)$ *if and only if* $x P_i y$. *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* neutral *if* f_N *is neutral for each* $N \in \mathcal{N}$.

The first manipulation property is that the voting rule should give voters incentives to be truthful.

Definition 4 *A voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is* strategy-proof *if for all* $P \in \mathcal{D}^N$, $i \in N$ *and* $P'_i \in \mathcal{D}$,

$$f_N(P_i, P_{-i}) R_i f_N(P'_i, P_{-i}).$$

A voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* strategy-proof *if* f_N *is strategy-proof for each* $N \in \mathcal{N}$.

Another important requirement, specially in online voting, is that a voter should never have incentives to cast repeated votes.

Definition 5 *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* false-name-proof *if for all* $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, *all* $i \in N$ *and all* $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ *such that* $N \cap N' = \emptyset$ *and* $P'_j = P_i$ *for all* $j \in N'$,

$$f_N(P) R_i f_{N \cup N'}(P, P').$$

Conitzer (2008) refers to false-name-proof (random) voting rules as those satisfying a stronger version of our notion for non-random voting rules. Conitzer (2008)'s condition imposes stronger restrictions on the voting rule by not requiring that the additional preferences submitted by voter $i \in N$ coincide with agent i's original preference P_i.[7] For this reason, we refer here to Conitzer (2008)'s original notion as strong false-name-proofness.

Definition 6 *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* strong false-name-proof *if for all* $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, *all* $i \in N$ *and all* $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ *such that* $N \cap N' = \emptyset$,

$$f_N(P) R_i f_{N \cup N'}(P, P').$$

[7]Even more, in Conitzer (2008) it is not required that the voter in N submits its true preference at all.

Moreover, Conitzer (2008) also requires that (random) voting rules induce voters to vote. We shall show that, in our context, this participation property follows from strategy-proofness, false-name-proofness, unanimity, anonymity, and neutrality. For non-random voting rules, participation is as follows.

Definition 7 *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *satisfies* participation *if, for all* $(N, P) \in \mathcal{N} \times \mathcal{D}^N$ *such that* $n \geq 2$ *and all* $i \in N$,

$$f_N(P) R_i f_{N \setminus \{i\}}(P_{-i}).$$

Conitzer (2008) refers to a voting rule as anonymous-proof if it satisfies strong false-name-proofness, anonymity and participation. We shall show in Proposition 4 below that, in any domain of preferences, strategy-proofness follows from these three properties.

2.3 Voting by quota

Let $N \in \mathcal{N}$ be a given set of voters with cardinality n, and let $q_N \in \{1, \ldots, n\}$ be a *quota.*

Definition 8 *A voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is* voting by quota q_N *if for all* $P \in \mathcal{D}^N$ *and all* $x \in K$,

$$x \in f_N(P) \text{ if and only if } |\{i \in N \mid x \in t(P_i)\}| \geq q_N.$$

A voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is* voting by quota *if for each* $N \in \mathcal{N}$ *there is* $q_N \in \{1, \ldots, n\}$ *such that* f_N *is voting by quota* q_N.

Note that by definition, voting by quota rules are very simple; specifically, they are unanimous, anonymous, neutral, and depend only on the profile of most-preferred subsets of objects. Moreover, the selected subset of objects at each profile of preferences is obtained in a decomposable way, object-by-object. For a fixed society, Barberà, Sonnenschein, and Zhou (1991) characterize the class of voting by quota (the subclasses of anonymous and neutral voting by committees) when they operate on either the domain of separable or the domain of additive preferences. The following proposition follows from their results.

Proposition 1 (Barberà, Sonnenschein, and Zhou (1991)) *Let* $N \in \mathcal{N}$ *and* $\mathcal{D}$ *be a domain such that* $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. *Then, a voting rule* $f_N : \mathcal{D}^N \to 2^K$ *for* N *is strategy-proof, unanimous, anonymous, and neutral if and only if* f_N *is voting by quota* q_N.

Two special voting by quota that shall deserve our attention are the following.

Definition 9 *Let* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *be a voting by quota. We say that* f *is* voting by quota one *if, for each* $N \in \mathcal{N}$*,* f_N *is voting by quota one. We say that* f *is* voting by full quota *if, for each* $N \in \mathcal{N}$*,* f_N *is voting by quota* n*.*

These two voting by quota are very extreme, and each of them can be seen as the dual of the other. Voting by quota one gives to each voter i the power of imposing x (*i.e.*, i is decisive for x), while voting by full quota gives to each voter i the power to veto x, or imposing that x is not selected (*i.e.*, i is decisive for not x).

3 Results

3.1 Main result

We now state our main result characterizing the family of all strategy-proof, false-name-proof, unanimous, anonymous, and neutral voting rules (on any separable domain containing the additive domain) as the class of voting by quota one or voting by full quota.

Theorem 1 *Let* $\mathcal{D}$ *be a domain such that* $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$*. Then, a voting rule* $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ *is strategy-proof, false-name-proof, unanimous, anonymous, and neutral if and only if* f *is either voting by quota one or voting by full quota.*

Before moving on to the formal proof, we sketch its main ideas. For the 'if' part, by Barberà, Sonnenschein, and Zhou (1991), voting by quota one and voting by full quota are strategy-proof, unanimous, anonymous and neutral. On the one hand, as voting by quota one makes every voter decisive for every object, there is no need to repeat a vote in order to obtain a better subset of objects (the voter has already given support to good objects and cannot avoid that bad objects are elected). On the other hand, as voting by full quota gives every voter the power to veto any object, all bad objects are excluded in the selected subset and casting repeated ballots will not include any good object not previously included (as it has already been vetoed by another voter). For the 'only if' part the proof consists of a claim and an inductive step. The claim states that quotas must be increasingly monotonic with respect to n; otherwise, if for a superset N' of N the quota $q_{N'}$ is strictly smaller than q_N, then a voter for whom x is good, and whenever x is not selected at N, may benefit by submitting extra votes to include x at N' because $q_{N'} < q_N$. The induction step states that the quota must be always one for every N, or always n for every N. The reason is that having different quotas for different subsets of voters can be used by a voter, just by adding repeated votes, to exclude from or to include in the selected subset of objects an object x, depending on whether the voter considers x as a bad or as a good object.

We now turn to the formal proof.

Proof. $\Longleftarrow$) Assume first that $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is voting by full quota; namely, $q_N = n$ for all $N \in \mathcal{N}$. By Proposition 1, the voting rule f_N for N is strategy-proof, unanimous, anonymous, and neutral, and so is f. We show that f is strong false-name-proof, which implies that f is false-name-proof. Let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, $i \in N$, and $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ be arbitrary, and assume $N \cap N' = \emptyset$. As f_N and $f_{N \cup N'}$ are quota n and $n + n'$ respectively, $f_N(P) = \cap_{j \in N} t(P_j) \supseteq \cap_{j \in N \cup N'} t(P_j) = f_{N \cup N'}(P, P')$. Therefore, $f_{N \cup N'}(P, P') = f_N(P) \setminus T$ for some $T \subseteq t(P_i)$. By iteratively applying separability to each object in T and transitivity of P_i, we obtain that $f_N(P) R_i f_{N \cup N'}(P, P')$ and so f is strong false-name-proof. Therefore f is also false-name-proof.

Assume now that $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is voting by quota one; namely, $q_N = 1$ for all $N \in \mathcal{N}$. By Proposition 1, the voting rule f_N for N is strategy-proof, unanimous, anonymous, and neutral, and so is f. We show that f is strong false-name-proof, which implies that f is false-name-proof. Let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, $i \in N$, and $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ be arbitrary, and assume $N \cap N' = \emptyset$. As f_N and $f_{N \cup N'}$ are both quota one, $t(P_i) \subseteq f_N(P) = \cup_{j \in N} t(P_j) \subseteq \cup_{j \in N \cup N'} t(P_j) = f_{N \cup N'}(P, P')$. Therefore, $f_{N \cup N'}(P, P') = f_N(P) \sqcup B$ for some $B \subseteq b(P_i)$, where $\sqcup$ stands for the disjoint union. By iteratively applying separability to each object in B and transitivity of P_i, we obtain that $f_N(P) R_i f_{N \cup N'}(P, P')$ and so f is strong false-name-proof. Therefore f is also false-name-proof.

$\Longrightarrow$) Assume $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is a strategy-proof, false-name-proof, unanimous, anonymous, and neutral voting rule. By definition and Proposition 1, for each $N \in \mathcal{N}$, f_N is a voting by quota $q_N \in \{1, \ldots, n\}$. For object $x \in K$, set of voters N, and profile $P \in \mathcal{D}^N$, define $x_N(P) = |\{i \in N \mid x \in t(P_i)\}|$.

Claim 1 *Let $N, N' \in \mathcal{N}$ be such that $N \cap N' = \emptyset$. Then, $q_N \leq q_{N \cup N'}$.*

Proof. To obtain a contradiction, suppose that $q_N > q_{N \cup N'}$. Hence, $q_N > 1$. Consider $i \in N$, $x \in K$, and $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u^x and $x_N(P) = q_N - 1$. Observe that $x \in t(P_i)$ by definition of u^x, and $x \notin f_N(P)$ by definition of voting by quota q_N. Consider now a profile $P' \in \mathcal{D}^{N'}$ such that $P'_j = P_i$ for all $j \in N'$. Since $N \cap N' = \emptyset$, $x_{N \cup N'}(P, P') = x_N(P) + x_{N'}(P') = q_N - 1 + n' > q_{N \cup N'}$, by our hypothesis. Since $f_{N \cup N'}$ is voting by quota $q_{N\cup N'}$, $x \in f_{N\cup N'}(P,P')$ and, by definition of u^x, $f_{N\cup N'}(P,P')P_i f_N(P)$, a contradiction with false-name-proofness. ■

To proceed with the proof of Theorem 1, first observe that by the definition of voting by quota, anonymity, and Claim 1, $q_1 = 1 \leq q_2 \leq 2$ holds, where $q_1 = q_{\{i\}}$ for any $i \in \mathbb{Z}_+$ and $q_2 = q_{\{j,l\}}$ for any $j, l \in \mathbb{Z}_+$ such that $j \neq l$.

We distinguish between the two possible cases and, for each of them, we show that the statement of Theorem 1 holds by induction on n (the number of voters).

Case 1: $q_1 = q_2 = 1$. *Induction hypothesis*: Assume $q_1 = \cdots = q_n = 1$.
Consider the case $n+1$, where $n \geq 2$. We want to show that $q_{n+1} = 1$. To obtain a contradiction, suppose $q_{n+1} > q_n = 1$. Consider $i \in N$, $y \in K$ and a profile $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u_y and $y_N(P) = 1$. Observe that $y \notin t(P_i)$ by definition of u_y, and $y \in f_N(P)$ by definition of voting by quota q_N. Let $j \notin N$ and $P_j = P_i$. Then $y_{N\cup\{j\}}(P,P_j) = 1 < q_{n+1}$ and accordingly, $y \notin f_{N\cup\{j\}}(P,P_j)$, which is a contradiction with false-name-proofness because, by definition of u_y, $f_{N\cup\{j\}}(P,P_j)P_i f_N(P)$.

Case 2: $q_1 = 1 < 2 = q_2$. *Induction hypothesis*: Assume $q_t = t = |T|$ for every $T \in \mathcal{N}$ such that $1 \leq t \leq n$.
Consider the case $n+1$, where $n \geq 2$. We want to show that $q_{n+1} = n+1$. By the definition of quota, Claim 1 and the induction hypothesis we have that $n \leq q_{N\cup\{j\}} \leq n+1$ for $j \notin N$. Suppose that $q_{N\cup\{j\}} = n$. Consider $i \in N$, $x \in K$ and a profile $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u^x and $x_N(P) = q_N - 1 = n - 1$. Observe that $x \in t(P_i)$ by definition of u^x, and $x \notin f_N(P)$ by definition of voting by quota q_N. Let $j \notin N$ and $P_j = P_i$. Then, $x_{N\cup\{j\}}(P,P_j) = n - 1 + 1 = q_{N\cup\{j\}}$ and accordingly, $x \in f_{N\cup\{j\}}(P,P_j)$, which is a contradiction with false-name-proofness because, by definition of u^x, $f_{N\cup\{j\}}(P,P_j)P_i f_N(P)$.

Thus, f is either a voting by quota one or a voting by full quota. ■

From the fact that strong false-name-proofness implies false-name-proofness and the proof of Theorem 1, the following corollary holds.

Corollary 1 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N\in\mathcal{N}}$ is strategy-proof, strong false-name-proof, unanimous, anonymous, and neutral if and only if f is either voting by quota one or voting by full quota.*

3.2 Additional results

In this subsection we present additional results. We first show that voting by quota one and voting by full quota satisfy participation. Hence, participation follows from the hypothesis assumed on voting rules in Theorem 1. Namely, the following proposition holds.

Proposition 2 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and assume the voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is strategy-proof, false-name-proof, unanimous, anonymous, and neutral. Then, f satisfies participation.*

Proof. Assume that the hypothesis of Proposition 2 hold. By Theorem 1, f is either voting by quota one or voting by full quota.

Assume first that $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is voting by quota one; namely, $q_N = 1$ for all $N \in \mathcal{N}$. To show that f verifies participation, let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$ be such that $n \geq 2$ and let $i \in N$ be arbitrary. As f_N and $f_{N \setminus \{i\}}$ are both quota one, $f_N(P) = \cup_{j \in N} t(P_j)$ and $f_{N \setminus \{i\}}(P_{-i}) = \cup_{j \in N \setminus \{i\}} t(P_j)$. Therefore, $f_N(P) = f_{N \setminus \{i\}}(P_{-i}) \cup t(P_i)$ which means that $f_{N \setminus \{i\}}(P_{-i}) = f_N(P) \setminus T$ for some $T \subseteq t(P_i)$. By iteratively applying separability to each object in T and transitivity of P_i, we obtain that $f_N(P) R_i f_{N \setminus \{i\}}(P_{-i})$ and so f satisfies participation.

Assume now that $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is voting by full quota; namely, $q_N = n$ for all $N \in \mathcal{N}$. To show that f verifies participation, let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$ be such that $n \geq 2$ and let $i \in N$ be arbitrary. As f_N and $f_{N \setminus \{i\}}$ are quota n and $n-1$ respectively, $f_N(P) = \bigcap_{j \in N} t(P_j)$ and $f_{N \setminus \{i\}}(P_{-i}) = \bigcap_{j \in N \setminus \{i\}} t(P_j)$. Therefore, $f_N(P) = f_{N \setminus \{i\}}(P_{-i}) \cap t(P_i)$ which means that $f_{N \setminus \{i\}}(P_{-i}) = f(N, P) \sqcup B$ for some $B \subseteq b(P_i)$. By iteratively applying separability to each object in B and transitivity of P_i, we obtain that $f_N(P) R_i f_{N \setminus \{i\}}(P_{-i})$ and so f satisfies participation. ■

The following example shows that, in general, false-name-proofness and strategy-proofness do not imply participation if the voting rule does not verify anonymity and neutrality.

Example 1 *Let $K = \{x, y, z\}$ and let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Consider the voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ where, for each $N \in \mathcal{N}$, f_N is voting by quota one if $1 \notin N$ and f_N selects $\{x\}$ if $1 \in N$. Consider $N = \{1, 2, 3, 4\}$ and any profile of preferences $P = (P_1, P_2, P_3, P_4) \in \mathcal{S}^N$ such that $t(P_1) = \{x, y, z\}$, $t(P_2) = \{x\}$, $t(P_3) = \{y\}$ and $t(P_4) = \{z\}$. This voting rule is false-name-proof and strategy-proof but does not verify participation, as $f_{\{2,3,4\}}(P_{-1}) = \{x, y, z\}\ P_1\ \{x\} = f_{\{1,2,3,4\}}(P)$.*

For the case of a unique object, which corresponds to the general setting where there are only two alternatives (identified as $\{x\}$ and $\{\emptyset\}$), participation implies strategy-proofness.[8]

Proposition 3 *Assume $K = \{x\}$. Then, participation implies strategy-proofness.*

Proof. Assume f satisfies participation and, to obtain a contradiction, suppose f is not strategy-proof. Let $N \in \mathcal{N}$, $P \in \mathcal{D}^N$, $i \in N$ and $P'_i \in \mathcal{D}$ be such that $P'_i \neq P_i$ and

$$f_N(P'_i, P_{-i}) P_i f_N(P_i, P_{-i}). \tag{1}$$

Since $K = \{x\}$, $\mathcal{D}$ contains only two preferences and so either $\{x\}P_i\{\emptyset\}$ or $\{\emptyset\}P_i\{x\}$. Suppose that the first holds. Then,

$$\{\emptyset\}P'_i\{x\}. \tag{2}$$

Observe that with only one object, the chosen alternative is either $f_N(P) = \{x\}$ or $f_N(P) = \{\emptyset\}$. In the former case, it is clear that $f_N(P_i, P_{-i}) R_i f_N(P'_i, P_{-i})$, which contradicts (1). Therefore, we must have that $f_N(P) = \{\emptyset\}$ and, according to (1) and $\{x\}P_i\{\emptyset\}$,

$$f_N(P'_i, P_{-i}) = \{x\} P_i \{\emptyset\} = f_N(P_i, P_{-i}) \tag{3}$$

holds. By participation, $f_N(P) R_i f_{N\setminus\{i\}}(P_{-i})$, and so by (3), $f_{N\setminus\{i\}}(P_{-i}) = \{\emptyset\}$. Applying now participation to agent i with preference P'_i, $f_N(P'_i, P_{-i}) R'_i f_{N\setminus\{i\}}(P_{-i})$ holds as well. But then, by (3), we have $\{x\}R'_i\{\emptyset\}$, a contradiction with (2). The proof is analogous for the case when $\{\emptyset\}P_i\{x\}$. ∎

It is easy to see that if there are two or more objects, participation does not imply strategy-proofness: The Borda count, combined with a tie-breaking that selects one among the potentially many winners, is an example of a voting rule that satisfies participation and it is not strategy-proof.

We now show that in *any* setting (and, in particular, in *any* domain of preferences) strong false-name-proofness, anonymity and participation imply strategy-proofness.[9] Let A be any set of social alternatives, let $\mathcal{U}$ be any set of complete and transitive preferences over A, and adapt the properties of a rule $f_N : \mathcal{U}^N \to A$ for N as previously defined for our setting. Denote by R_i a generic (and possibly weak) preference of voter i over A in $\mathcal{U}$ and let $R = (R_i)_{i \in N} \in \mathcal{U}^N$ be a profile.

[8]This result can be seen as a particular case of Lemma 1 in Wagman and Conitzer (2008) when submitting an extra vote does not convey any cost.

[9]Conitzer (2008) already observes that this holds in his setting as a consequence of his characterization.

Proposition 4 *Let $\mathcal{U}$ be any domain of preferences over A and let $f = \{f_N : \mathcal{U}^N \to A\}_{N \in \mathcal{N}}$ be a strong false-name-proof voting rule that satisfies anonymity and participation. Then, f is strategy-proof.*

Proof. Fix $N \in \mathcal{N}$, $R \in \mathcal{U}^N$, $i \in N$, and let $R'_i \in \mathcal{U}$ be arbitrary. Consider any $j \notin N$ and set $R_j = R'_i$. Then,

$$\begin{array}{rcll} f_N(R_i, R_{-i}) & R_i & f_{N\cup\{j\}}(R, R_j) & \text{by strong false-name-proofness} \\ & = & f_{N\cup\{j\}}((R_{-i}, R_j), R_i) & \text{by anonymity} \\ & R_i & f_{(N\cup\{j\})\setminus\{i\}}(R_{-i}, R_j) & \text{by participation} \\ & = & f_{(N\setminus\{i\})\cup\{j\}}(R_{-i}, R_j) & \text{by anonymity} \\ & = & f_{(N\setminus\{i\})\cup\{i\}}(R_{-i}, R_j) & \text{by anonymity} \\ & = & f_N(R_{-i}, R'_i) & \text{since } R_j = R'_i. \end{array}$$

Hence, f is strategy-proof. ■

Therefore, Corollary 2 below follows from Theorem 1.

Corollary 2 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is strong false-name proof, unanimous, anonymous, neutral and verifies participation if and only if f is either a voting by quota one or a voting by full quota.*[10]

Example 2 below shows that the statement of Proposition 4 does not hold if strong false-name-proofness is replaced by false-name-proofness, even if the domain of the voting rule is restricted to be the set of separable preferences.

Example 2 *For $P_i \in \mathcal{S}$ denote by $ts(P_i)$ the most-preferred singleton set from the set of good objects or the empty set if there is none. Let $f^s = \{f^s_N : \mathcal{S}^N \to 2^K\}_{N \in \mathcal{N}}$ be the voting rule where, for each $N \in \mathcal{N}$, $f^s_N : \mathcal{S}^N \to 2^K$ is defined by setting, for each $P \in \mathcal{S}^N$, $f^s_N(P) = \cup_{i \in N} ts(P_i)$ if there exist voters i and j such that $t(P_i) \neq t(P_j)$, and $f^s_N(P) = t(P_i)$ otherwise. It is easy to check that f^s is false-name-proof and verifies participation. The following example for $N = \{1, 2\}$ and $K = \{x, y\}$ shows that f^s is neither strategy-proof nor strong false-name-proof. Consider the profile of separable preferences $(P_1, P_2, P'_1) \in \mathcal{S}^{N\cup\{3\}}$, where $\{x,y\}P_1\{x\}P_1\{y\}P_1\{\emptyset\}$, $\{x\}P_2\{\emptyset\}P_2\{x,y\}P_2\{y\}$ and $\{x,y\}P'_1\{y\}P'_1\{x\}P'_1\{\emptyset\}$. Then, by definition of f^s_N, $f^s_N(P'_1, P_2) = \{x,y\}P_i\{x\} = f^s_N(P_1, P_2)$, which means that f^s_N is not strategy-proof, so neither is f^s. Moreover, $f^s_{\{1,2,3\}}(P_1, P_2, P'_1) = \{x,y\}P_1\{x\} = f^s_{\{1,2\}}(P_1, P_2)$, which means that f^s is not strong false-name-proof.*

[10] It is easy to check that every voting by quota verifies participation.

3.3 Independence of the axioms in Theorem 1

In this subsection we show that the axioms in Theorem 1 are independent. For each of the axioms in the statement of Theorem 1 we exhibit an example of a voting rule, different from quota one or full quota, that satisfies all axioms but one.

Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ be a voting rule.

- All but STRATEGY-PROOFNESS: The rule f^s defined in Example 2.
- All but FALSE-NAME-PROOFNESS: Fix $m \in \mathbb{Z}_+$ with $m \geq 2$. For each $N \in \mathcal{N}$, let f_N be voting by quota n if $n \leq m$ and quota $n-1$ if $n > m$. It is easy to check that f satisfies all the properties but false-name-proofness. To see that f is not false-name-proof, consider the case where $m = 3$, $N = \{1, 2, 3\}$, and the profile $P = (P_1, P_2, P_3, P_4) \in \mathcal{D}^{N \cup \{4\}}$ with $t(P_i) = \{x\}$ for $i = 1, 2, 4$, $P_1 = P_4$, and $t(P_3) = \{\emptyset\}$. Then, since $f_{N\cup\{4\}}(P_1, P_2, P_3, P_4) = \{x\} P_1 f_N(P_1, P_2, P_3) = \{\emptyset\}$, f is not false-name-proof.
- All but UNANIMITY: For each $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, define $f_N(P) = K$.
- All but ANONYMITY: Fix an agent $i \in \mathbb{Z}_+$. For each $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, define $f_N(P) = t(P_i)$ if $i \in N$ and f_N is voting by quota one otherwise.
- All but NEUTRALITY: Fix an object $k \in K$. For each $N \in \mathcal{N}$, let f_N be the non-neutral voting by quota $q_N = (q_{x,N})_{x \in K}$, where $q_{k,N} = 1$ and $q_{x,N} = n$ for each $x \in K \backslash \{k\}$; namely, for each $(N, P) \in \mathcal{N} \times \mathcal{D}^N$,

  $$k \in f_N(P) \text{ if and only if } |\{i \in N \mid k \in t(P_i)\}| \geq 1$$

  and, for each $x \in K \backslash \{k\}$,

  $$x \in f_N(P) \text{ if and only if } |\{i \in N \mid x \in t(P_i)\}| = n.$$

The example of the voting rule that satisfies all properties but neutrality gives us a clue for the following characterization (that can be seen as a corollary of our main result).

Corollary 3 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^K\}_{N \in \mathcal{N}}$ is strategy-proof, false-name-proof, unanimous and anonymous if and only f is a voting by quota such that for each $x \in K$ either $q_{x,N} = q_{x,N'} = 1$ for all $N, N' \in \mathcal{N}$ or $q_{x,N} = n$ and $q_{x,N'} = n'$ for all $N, N' \in \mathcal{N}$.*

Proof. In the proof of Theorem 1, neutrality forces that the quota has to be the same for all objects. Without neutrality, we would have voting by quotas, with different quotas for different objects. False-name-proofness requires then that for each object, the quotas must be either one or n, where $n = |N|$. ■

Declaration of Interests

None

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References

S. Barberà, H. Sonnenschein, and L. Zhou. Voting by committees. *Econometrica*, 59:595–609, 1991.

M. Brill, V. Conitzer, R. Freeman, and N. Shah. False-name-proof recommendations in social networks. In *Proceedings of AAMAS*, pages 332–340, 2016.

N. Bu. Unfolding the mystery of false-name-proofness. *Economics Letters*, 120:559–561, 2013.

R. Congar and V. Merlin. A characterization of the maximin rule in the context of voting. *Theory and Decision*, 72:131–147, 2012.

V. Conitzer. Anonymity-proof voting rules. In *Proceeding of WINE*, pages 295–306, 2008.

J. L. García-Lapresta and M. Martínez-Panero. Positional voting rules generated by aggregation functions and the role of duplication. *International Journal of Intelligent Systems*, 32:926–946, 2017.

H. Moulin. On strategy-proofness and single peakedness. *Public Choice*, 35:437–455, 1980.

T. Todo and V. Conitzer. False-name-proof matching. In *Proceedings of AAMAS*, pages 311–318, 2013.

T. Todo, A. Iwasaki, and M. Yokoo. False-name-proof mechanism design without money. In *Proceedings of AAMAS*, pages 651–658, 2011.

T. Todo, N. Okada, and M. Yokoo. False-name-proof facility location on discrete structures. In *Proceedings of ECAI*, pages 227–234. 2020.

B. Waggoner, L. Xia, and V. Conitzer. Evaluating resistance to false-name manipulations in elections. In *Proceedings of AAAI*, pages 1485–1491, 2012.

L. Wagman and V. Conitzer. Optimal false-name-proof voting rules with costly voting. In *Proceedings of AAAI*, pages 190–195, 2008.

M. Yokoo, Y. Sakurai, and S. Matsubara. The effect of false-name bids in combinatorial auctions: New fraud in internet auctions. *Games and Economic Behavior*, 46:174–188, 2004.

M. Zuckerberg. Voting begins on governing the facebook site. *The Facebook Blog*, 16, 2009.