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Patent Length and Breadth as Policy Instruments: A Systematic Review of Recent Contributions to the Theory of Optimal Patent Design

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Patent Length and Breadth as Policy Instruments: A Systematic Review of Recent Contributions to the Theory of Optimal Patent Design

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Abstract

This article offers a systematic review of the last ten years of contributions to the theoretical literature on optimal patent policy, with focus on patent length and breadth as alternative tools for encouraging innovation, growth or welfare improvement. The articles were classified according to their assumptions about the interconnectedness of different innovations. Three cases have been identified: isolated innovations, cumulative innovations (which, in turn, can be divided into "research tool" and "quality ladder" cases) and complementary innovations. According to the results, optimal patent length is finite in some models, whereas it is infinite in others. A similar diversity of results was found in models featuring patent breadth: some of them suggest that it should be minimum, others conclude that an intermediate value would be optimal, and others are more prone to favour maximum breadth. Few works seemed to conclude that two or more kinds of soutions were possible. The four kinds of models exhibited, up to a certain degree, this seeming inconsistency. As a consequence, works presenting sufficient conditions for different sorts of optimal solutions have not been found within this period.

1 Introduction

The main objective of daily economic activity is human well-being. Unfortunately, the degree of success achieved by this kind of effort is limited by technological possibilities. The only way to ameliorate this limitation is by means of discovery or creation of new possibilities, and the introduction of these new products or methods into the economic system. These processes are known as *invention* and *innovation* respectively. Because of the beneficial impact of invention and innovation on human welfare, the assessment and elaboration of policies aimed at favouring the production and difussion of inventions should be a priority for academics and authorities worldwide.

In fact, Arrow (1962) has already shown that there is a tendency for under-provision of invention in competitive markets. The main reasons for this are indivisibilities, innapropriability, and uncertainty in markets for information. A wide variety of solutions has been proposed to this problem. The creation of patents, a type of intellectual property rights, may be the best one in specific asymmetric information contexts (see Wright, 1983; Weyl & Tirole, 2012; Clancy & Moschini, 2013; Rietzke & Chen, 2020), and even beyond the asymmetry assumption (Bagchi & Mukherjee, 2021). Other theoretical contributions (Spulber, 2021) and empirical evidence (Neves, Afonso, Silva, & Sochirca, 2021) also provides support to the implementation of patent systems.

Provided that a patent system is necessary to reach a certain policy objective (for example, a more efficient economic system), a new question arises: How should such a system be? The aim of this paper is to shed some light on this problem. More concretely, the objective of this paper is to provide a review of what theoretical literature has said during the last ten years about optimal patent policy. The focus here is on two dimensions of patents: their duration (*patent length*) and their scope (*patent breadth*). The problem of finding the optimal mix between patent length and breadth seems to be relevant from a theoretical point of view, which is reflected by the fact that a recent work by Denicolò and Zanchettin (2022) described it as a "non trivial problem" (see footnote in p. 4). It might be relevant from an empirical perspective too. The two main dimensions of patent system seem to be relevant in the pharmaceutical industry (Khazabi & van Quyen, 2017), a sector whose performance have been crucial for the pace of the world economy under the recent COVID-19 pandemic (Volpert & Riepe, 2021).

This review improves on previous atempts by three ways. First of all, it provides the first systematic review of the literature in the field of optimal patent design. Secondly, it offers an actualisation of previous reviews, since it covers the last ten years of contributions. Lastly, it puts emphasis on the length-breadth trade-off, providing a classification of existing models according to their conclusions in relation to that issue. Such a task has not been performed before. To sum up, this article offers an acutalised and complete picture of theoretical literature on optimal patent policy —with focus on length and breadth as alternative tools for encouraging innovation, growth or welfare improvement— by means of a systematic review of the last ten years of contributions.

The rest of the paper is organized as follows. The next section defines some basic concepts in the field of optimal patent desgin literature, which are essential to interpret what is said in the rest of the text. Section three describes the methodology employed to obtain the list of recent theoretical contributions. After that, four sections (one for every kind of patentable innovation models) describe the works and their conclusions in respect of the main issue of this review. Finally, the last section sumarises what has been done and presents the conclusions of this paper.

2 Basic concepts

A patent is an intellectual property right which protects an invention during an specified period of time. A patented invention "cannot be commercially made, used, distributed, imported or sold by others without the patent owner's consent" (WIPO, 30 August 2022). Due to the fact that patents are territorial rights, details of patent law vary from country to country, altough national legislations have been harmonised up to a point since the TRIPS Agreement, which was signed in 1995.

Technological change aimed at obtaining patents could be modelled as a three-step process (Figure 1). In the first one, potential inventors invest their resources in research and development (R&D) activities with the objective of transforming their ideas in new products or processes. In the second step, successful inventors or innovators initiate the filing process for obtaining a patent. During this phase of the process, potential patentees are supposed to give as much information as they can about their inventions to the patent granting authority. If this administrative entity declares that the invention fulfils the patentability requirements, the third step begins, and innovators are then allowed to use their patent rights with the objective to earn extraordinary profits.

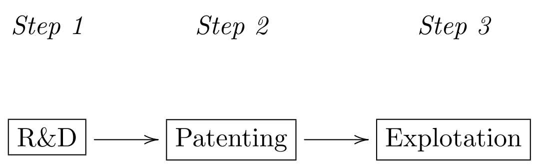

Figure 1: Steps of patentable technological change.
Source: van Dijk (1994).

For the purposes of this paper, a patent system will be defined as the set of norms governing this process in a country. Furthermore, the patent system will be understood as a multidimensional object, and some of the dimensions of this system will be identified as policy levers. Concretely, theoretical literature (see Scotchmer, 2004, ch. 4) identifies three basic dimensions of patent policy: patent length, patent breadth and patentability requirements.

Patent length is the duration of patent rights as it was set by patent law. It is also known as the "statutory length" of patents when it comes necessary to distinguish it from the "effective length" of patents (see below). By means of defining statutory patent length, policy makers can affect the duration of step 3 in Figure 1.

Patent breadth has more than one definition. It is supposed to be the scope of patent rights, but there are many ways of defining what is understood by "scope" (Novelli, 2015). For example, Nordhaus (1972) proposed defining it as the proportion of the cost reduction stemming from the innovation but not spreading to innovator's competitors. Other widespread definitions come from Gilbert and Shapiro (1990), who interpreted it as "the flow rate of profit available to the patentee while the patent is in force" (p. 106) or "the ability of the patentee to raise price" (p. 107).

Patent breadth can be interpreted as a multidimensional concept. For example, O'Donoghue, Scotchmer, and Thisse (1998) distinguish between "lagging breadth" (protection against imitations) and "leading breadth" (protection against improvements). These distinction is relevant, since a lower leading breadth means a higher probability that a new product render the incumbent's product obsolete, making its "effective" patent life shorter than its "statutory" patent length.

Patentability requirements are the criteria employed by patent offices to approve or disapprove patent applications. For example, US Patent Law requires statutory, novelty, usefulness, and non-obviousness. European Patent Law, instead, mentions statutory, novelty, inventive step, and industrial application. By means of this lever, authorities influence step 2 of Figure 1. This is different from influencing setp 3 of the process, which is what patent breadth does. It is important to highlight that patent breadth and patentability requirements are not the same (see Scotchmer, 2004, ch. 3) because it has been a source of confussion among theorists.

This paper will focus on the first two instruments: patent length and breadth. The objective is to review what has been said in theoretical literature during the last ten years about the optimal levels of patent length and breadth, assuming that the government wants to maximise efficiency, growth, R&D expenditures or another common policy goal. The next section will explain in more detail the approach employed to address that issue in this paper.

3 Methodology

This section describes the methodology employed to obtain the results. As it was stated before, what has been done is a systematic review of the literature on patent length and breadth in optimal patent design problems. The search was carried out in the Scopus repository in 29 July 2022, and the following keywords and expressions were employed: "optimal patent", "length" and "breadth". As this terminology has become standard in patent design studies (O'Donoghue, 1998), other terms for identifying the patent policy dimensions have been ignored. The period covered by this review ranges from 2012 to the date of the search. 260 results have been obtained.

Once the results have been otained, they were filtered. The first works which have been excluded were encyclopedic articles and review papers, since the objective of this review was to look for original contributions instead of compilations of previous ones. Empirical works have been excluded too, because this review aimed at analysing theoretical contributions. Articles without a mathematical model were also excluded from the analysis. Finally, articles not including patent length nor patent breadth as policy variables (instruments) have been discarded too, due to the fact that the main concern of this article is the problem of finding the optimal levels of these variables. To sum up, there have been reviewed works featuring a mathematical model where (1) there is a variable representing patent length or breadth and (2) different levels of that variable are compared in terms of an objective function.

The dependent variable of such objective function must represent a policy variable. Otherwise, the model would not have been included in this review. A common approach in optimal patent design problems consists of maximising the expected present value of the sum of producers' and consumers' surplus. In these cases, the objective variable it is usually labelled as "eficiency" or "welfare". In other contributions, the objetive variable is the rate of growth of the overall economy or the level of R&D instead. All these works were also included in the review. Let $W \in \mathbb{R}$ be the value of the dependent variable in objective function, no matter its meaning. If $l \in \mathbb{R}_+$ is the patent length, $b \in \mathbb{R}^m$ is the patent breadth, and $z \in \mathbb{R}^n$ is a vector of other policy variables (for instance, subsidies and interest rates), then the models analysed in this review have the following structure.

$$\underset{l,b,z}{\text{maximize}} \quad W(l,b,z) \tag{1a}$$

$$\text{subject to} \quad \underline{l} \leq l \leq \overline{l}, \tag{1b}$$

$$\underline{b} \leq b \leq \overline{b}, \tag{1c}$$

$$\underline{z} \leq z \leq \overline{z} \tag{1d}$$

Concerning patent length, the most interesting case arises when $\underline{l} = 0$ and $\overline{l} = \infty$. However, it is more common to find articles where $\underline{l} = \overline{l}$, which represents the case where patent length is treated as given or is not explicitly modelled. Other cases are extremely rare.

In respect of patent breadth, the most common case arises when $b \in \mathbb{R}$. In such a case, $\underline{b}$ and $\overline{b}$ are the values beyond which b has no economic interpretation. It might happen that $\underline{b} = -\infty$ and $\overline{b} = \infty$, but this case is not very common. It is also possible to find models where $\underline{b} = \overline{b}$, which belong to those scenarios where where patent breadth is treated as an exogenous parameter or is not explicitly modelled. Of course, these last cases are the least interesting among the considered ones. The extension of this way of interpreting the borders to cases where $b \in \mathbb{R}^m$ is straightforward.

While the interpretation of the constraints on patent length and breadth deserved some clarifications, the meaning of the borders $\underline{z}$ and $\overline{z}$ is not of particular interest, since the optimal levels of the variables in z are not the focus in this paper. These variables are of interest here to the extent that interaction effects with patent lenght and breadth ($\frac{\partial^2 W}{\partial l \partial z}$ and $\frac{\partial^2 W}{\partial b \partial z}$) have some relevance to the discussion of the reviewed works.

Having clarified the nature of the borders $\underline{l}$, $\overline{l}$, $\underline{b}$, and $\overline{b}$, it must be stated that models which simultaneously assume that $\underline{l} = \overline{l}$ and $\underline{b} = \overline{b}$ have been discarded, since they do not deal with the problem of finding the optimal levels of patent

l and b at all. Furthermore, models which explicitly conflate patent length and breadth in a single decision variable (for instance, $x \equiv l \cdot b$) were also discarded, since they do not aim to focus on the length-breadth trade-off.

The contributions included in this review were classified according to their conclusions in regard to the optimal levels of patent length and breadth. The main concern of this review was if the optimal values were "minimum", "intermediate" or "maximum". For example, it will be said that optimal patent length (l^*) is minimum if $l^* = \underline{l}$ or $\frac{\partial W}{\partial l} < 0$ along the feasible values of this variable. Alternatively, it is intermediate if $\underline{l} < l^* < \bar{l}$, or the function relating W and l is inverse-U shaped. Finally, it is maximum if $l^* = \bar{l}$ or $\frac{\partial W}{\partial l} > 0$ along the relevant domain of the function. The analogy with patent breadth dimensions is straightforward.

Following Hall and Harhoff (2012), the articles were classified according to their assumptions about the interconnectedness of different innovations. More concretely, three cases were identified: isolated innovations, cumulative innovations and complementary innovations. In turn, cumulative innovations can be divided into "research tool" and "quality ladder" cases. The definition of all these cases will be developed in the next sections.

4 Isolated innovations case

The simplest case consists of a setting where there is only one possible innovation or there are a few unrelated innovations. Naturally, as this was the easiest situation to analyse, this case consituted the first setting analysed in optimal patent policy literature.

4.1 Partial equilibrium models

Models with variable patent length and fixed patent breadth:

Budish, Roin, and Williams (2016) built a model with no patent races. Authorities set patent length in order to maximise social welfare. Their model is a contemporary version of Nordhaus' (1969) model. Their model leads to the conclusion that optimal patent lengths should be finite and different among industries, like in Nordhaus' (1969) model.

Eswaran and Gallini (2019) built a model with no patent races. The authors analysed the antibiotics market. Authorities set patent length and the level of the patentee's output in order to maximise welfare. They found that optimal patent length is finite.

Meng (2019) built a single-country model with no patent races. Authorities set patent lenght in order to maximise welfare. The author found that optimal length is infinite. He also suggested additional patent policy reforms, such as extending patent breadth and banning exclusive licenses, but they are not supported by his model.

Models with fixed patent length and variable patent breadth:

Denicolò and Franzoni (2012) built a model with patent races. Authorities set patent breadth in order to maximise social welfare. They concluded that patents should be strong or weak depending of the results of a ratio test, so they favours an intermediate patent breadth.

Kwon (2012a) built a model with patent races. Assuming that firms may use secrecy instead of patents to protect their innovations, authorities set patent breadth (probability of imitation or inventing around) in order to maximise investment in innovation. The author found that patent breadth has a positive effect on investment when inventors always patent, but it has a negative effect when their respective patenting propensities (probability of patenting their own inventions) are less than 1.

Mukherjee (2014) built a model with no patent races. Given that the innovator has already invested in process innovation, authorities set patent breadth (i.e. if patents are strong or weak) in order to maximise consumer surplus and total welfare. (1a) Shubik and Levitan demand, Cournot competition: If we evaluate consumer surplus, patents should be weak if they have no effect on differentiation investments, but they should be strong if weaker patents encourage investments. If we evaluate total welfare, the only difference is that optimal patents can be weak or strong in the second situation. (1b) Shubik and Levitan demand, Bertrand competition: If we evaluate consumer surplus, patents should be weak if they have no effect on differentiation investments, but they should be strong if weaker patents encourage investments. If we evaluate total welfare, the only difference is that optimal patents can be weak or strong in the second situation. (2a) Bowley demand, Cournot competition: Patents should be weak. (2b) Bowley demand, Bertrand competition: If we evaluate consumer surplus, patents should be weak if they have no effect on differentiation investments, but optimal patents might be strong if weaker patents encourages investments. These results might be sensitive to the value of certain parameters. If we evaluate total welfare, the main difference is that optimal patents can be weak or strong in the second situation.

Slivko and Theilen (2014) built a model with no patent races. Authorities set patent breadth (spillovers) in order to maximise welfare. They conclude that intermediate breadth tends to be optimal.

Wang and Mukherjee (2014) built a model with no patent races. Assuming that the innovator can license his technology to the innovator, authorities set patent breadth (understood as knowledge spillover) in order to maximise investment in innovation or total welfare. If we evaluate investments in innovation, optimal patent breadth is finite, and it might be maximum if the marginal cost is low but the required investment is high. In the absence of technology licensing, optimal patent breadth is the maximum. If we evaluate total welfare, optimal patent breadth is finite. In the absence of technology licensing, maximum patent breadth might be optimal.

Jeon (2015) built a single-country model with no patent races. Assuming infintely living patents, authorities set the degree of patent infringement (i.e., the fraction of profit that can be appropriated by the challenger, which is inversely related with patent breadth) in order to minimise the time-index of R&D investment. Simulation results suggest that optimal patent breadth might be minimum or maximum. For certain intermediate values, "submarine patents" emerge.

Marjit and Yang (2015) build a model with patent races. Authorities define whether patents are strong or weak (patent breadth) in order to maximise (the number of) innovations. If pirates are not innovators, maximum patent breadth is optimal. However, if they are not, minimum patent breadth is the best policy.

Leiva-Bertran and Turner (2017) built a model with no patent races. Authorities set patent breadth (two-part patent royalties for compulsory licenses) in order to maximise welfare. Their results might be interpreted as implying that patent breadth should be finite and that compulsory rates might be welfare-superior to a system of prizes.

Duan, Shi, and Sun (2017) built a model with patent races. There are only two firms. Assuming that social welfare is higher when imitation is not allowed, authorities set prevention in advance (the probability of successful imitation) or punishment afterwards (a fine to imitators) in order to maximise social welfare. Prevention in advance should be maximum if cost reduction by innovation is high, whereas it should be minimum if such cost reduction is low. Punishment afterwards should adopt whatever value surpassing an specific threshold level. If innovation cost is low, government should adopt prevention in advance. If it is high, it should adopt it if cost reduction is low, but government should adopt punishment afterwards if cost reduction is high. Thus, maximum patent breadth is optimal under a variety of assumptions.

Hylton and Zhang (2017) built a model with no patent races. Authorities set damages awards for patent infringement (patent breadth) in order to maximise social welfare. The author found that patent breadth should be finite.

Arai (2018) built a model of the software industry with one country and with no patent races. Authorities set the level of enforcement of patent protection in order to maximise welfare. The author found that protection should be finite (intermediate) if research cost is low, whereas any level of protection is optimal if it is high. The article also compares patents with copyright protection.

Jeon and Nishihara (2018) built a model with no patent races. Authorities set probabilistic validity of patents and penalty upon infringement, which can be interpreted as patent breadth dimensions, in order to maximise social welfare. They found that optimal values for both dimensions are finite.

Billette de Villemeur, Ruble, and Versaevel (2019) built a model with patent races. Authorities set imitation cost (patent breadth) in order to maximise welfare. They found that optimal patent breadth is finite, unless the discounting rate is big enough. In such a case, optimal patent breadth is infinite.

Jeon (2019) built a model with patent races. Authorities set patent strength and R&D subsidies in order to maximise social welfare (the sum of firm values). Under symmetric information about the quality of R&D firms, maximum patent protection is optimal. However, under asymmetric information, patent protection should be finite.

Hylton and Xu (2020) built a model with no patent races. Authorities set the maximum price allowed (patent breadth) in order to maximise welfare. They found that patent breadth should be finite so as to balance the objectives of both innovation and atitrust policies by means of what is known as a “ratio test”.

Bergin (2022) built a single-country model with patent races. The impact of non-practising entities (NPE) in the equilibrium level of investment in innovation is adressed. With wide patents, NPEs have a negative impact, whereas with narrow patents, NPEs have a positive impact. Full-blocking scenarios lead to less investment than no-blocking ones, which could be interpreted as a case in favour of minimum patent breadth. Welfare analysis is not the focus of this paper.

Models with variable patent length and breadth:

Brabazon, Silva, and O’Neill (2012) built an agent-based model. Authorities set patent length and breadth in order to maximise the rate of technological advance. They found that patent length and breadth should be minimum (no patent system at all).

Galasso, Mitchell, and Virag (2016) built a single-country model with no patent races. Authorities set patent length, patent breadth (maximum price or minimum quantity allowed) and a scheme of annual rewards (transfers to the innovator) to maximise expected discounted social surplus. They found that optimal patents have finite length and intermediate or minimum breadth.

Meng (2017) built a model and conducted a partial equilibrium analysis with no patent races. Authorities set patent lenght and breadth in order to maximise social surplus. The auhtor found that there are two main possibilities: (1) patent length should be infinite and patent breadth should be maximum, or (2) patent system should be abolished.

J. Chen, Liu, Long, and Luo (2019) built a model with no patent races. In a first problem, authorities set patent length in order to maximise social welfare. In a second one, they set patent length, prices (patent breadth) and subsidies in order to achieve the same objective. In both cases, they found that patent length should be finite. Government pricing (narrow patents) is better than monopoly pricing (wide patents) if subsidies are high (surpass a certain threshold level).

Meng and Chen (2019) built a model with no patent races. Authorities set patent system dimensions in order to maximise welfare and growth. They suggest implementing infinite patent length and finite patent breadth.

4.2 Open economy extensions

Models with fixed patent length and variable patent breadth:

Berthoumieu (2017) built a two-country model with no patent races. The Northern authority implements a production subsidy, a patent subsidy, an import tariff, and an import quota in order to maximise national welfare. The first three instruments have a positive effect on northern (effective, not statutory) patent length, whereas the last one has a negative effect. The Southern authority implements a production subsidy, an import tariff, an import quota, and a public R&D investment in order to maximise national welfare. All these instruments have a negative effect on Northern patent length. A production subsidy war increases patent length, whereas an import tariff war or a quota war decreases patent length. Assessment of welfare effects in their model lead to ambiguous results (patent breadth dimensions might assume minimum, intermediate or maximum values at the optimal solution).

Grabiszewski and Minor (2019) built a two-country model, with one firm in each country, with no patent races. Authorities set the cost of Economic Espionage (an analogy with patent breadth could be traced) for the foreign firm in order to minimise stealing or maximise domestic R&D. They found multiple equilibria and ambiguous effects. However, from the graphical analysis it follows that maximum counterespionage is R&D-maximising and espionage-minimising.

Ikeda, Tanno, and Yasaki (2021) built a two-country model with no patent races. Southern authorities set patent breadth in order to maximise consumer surplus in their country. When the cost of innovation is low, maximum patent breadth is optimal. However, when it is high, optimal patent breadth is finite (intermediate).

Models with variable patent length and breadth:

Bagchi and Roy (2012) built a model two-country model (a developed economy and an emerging one) with no patent races. There is only a firm in each country, and authorities set patent length and breadth in order to maximise national welfare. They conclude that optimal patent length and breadth tend to assume intermediate values in both countries. More interestingly, they conclude that the optimal length-breadth mix for a country depends on the laws of the other one.

4.3 Summary

	Objective Variable	Optimal Length	Optimal Breadth
Partial equilibrium models			
Budish et al. (2016)	Welfare	Finite	
Eswaran and Gallini (2019)	Welfare	Finite	
Meng (2019)	Welfare	Infinite	

Denicolò and Franzoni (2012)	Welfare		Intermediate
Kwon (2012a)	R&D Inv.		Max or Min
Mukherjee (2014)	Cons. Surplus		Max or Min
Mukherjee (2014)	Welfare		Max or Min
Slivko and Theilen (2014)	Welfare		Intermediate
Wang and Mukherjee (2014)	R&D Inv.		Max or Int
Wang and Mukherjee (2014)	Welfare		Max or Int
Jeon (2015)	R&D Inv.		Max or Min
Marjit and Yang (2015)	Innovations		Max or Min
Leiva-Bertran and Turner (2017)	Welfare		Intermediate
Duan et al. (2017)	Welfare		Maximum?
Hylton and Zhang (2017)	Welfare		Intermediate
Arai (2018)	Welfare		Max or Any
Jeon and Nishihara (2018)	Welfare		Intermediate
Billette de Villemeur et al. (2019)	Welfare		Max or Int
Jeon (2019)	Welfare		Max or Int
Hylton and Xu (2020)	Welfare		Intermediate
Bergin (2022)	R&D Inv.		Minimum
Brabazon et al. (2012)	Tech. Advance	Minimum	Minimum
Galasso et al. (2016)	Welfare	Finite	Int or Min
Meng (2017)	Welfare	Infinite or zero	Max or Min
J. Chen et al. (2019)	Welfare	Finite	Minimum
Meng and Chen (2019)	Growth	Infinite	Intermediate
Meng and Chen (2019)	Welfare	Infinite	Intermediate
Open economy extensions			
Berthoumieu (2017)	Welfare		Ambiguous
Grabiszewski and Minor (2019)	R&D Inv.		Maximum
Ikeda et al. (2021)	Cons. Surplus		Max or Int
Bagchi and Roy (2012)	Welfare	Finite	Intermediate

5 “Research tool” case

The “research tool” case can be understood as a situation where the disclosure of an innovation is a necessary condition for the development of another one. For simplicity, it is usually assumed that there is no time lag between the introduction of the first innovation and the introduction of the second one. In such a context, even if both investments are socially efficient, it do not always arise that each firm gets more revenue than the cost of its respective development. Firms might not want to invest unless they have bargained over their joint profit and signed what is known as a ’prior agreement’. In other situations, an *ex post* ’license agreement’ might provide enough incentives. As Scotchmer (1991) has pointed out, the existence of those possibilities has important implications for optimal patent design, because patent breadth sets ’threat points’ for such negotiations. As a result, optimal patent design depends critically on whether prior agreements between potential innovators are allowed or not.

5.1 Partial equilibrium models

Models with variable patent length and fixed patent breadth:

Budish, Roin, and Williams (2015) built a model with no patent races. Authorities set patent length and other policy dimensions in order to maximise social welfare. They found that infinite patent length might be optimal. They suggest other possible alternatives for improving the patent system.

Gilchrist (2016) built a model with no patent races. Assuming that patents are narrow and there is a well stablished incumbent in the market, authorities set patent length in order to maximise consumer surplus and social welfare. The effect of incumbent's remaining patent length on consumer surplus is positive, whereas the its effect on social welfare is ambiguous, depending on the difference in quality between incumbent and entrant. The more different products are, the more likely is that patent length has a posistive effect on welfare.

Models with fixed patent length and variable patent breadth:

Jeon (2016) built a model with no patent races. Assuming infinite patent length, authorities set patent breadth (probability that the first innovator wins the suit) in order to maximise social welfare. Simulation results suggest that social welfare is maximum for a finite level of patent breadth.

Y. Chen and Sappington (2018) built a model with no patent races. Given patent strength (probability of the first patent of being ruled valid in a court), authorities set the damage payment from firm 2 to firm 1 as the sum of three components: a lump-sum monetary payment, a fraction of the amount by which firm 2's operation reduces firm 1's profit, and a fraction of firm 2's profit. These three components could be interpreted as dimensions of patent breadth. The objective is to maximise expected welfare. They found that dimensions of patent breadth should assume intermediate values.

Antonelli (2019) built a single-country model with no patent races. Authorities decide levels of exclusivity for intellectual property rights against different types of users, which can be interpreted as dimensions of patent breadth, in order to maximise welfare. The author suggests implementing patents that are strong against intra-industry users, but are weak in front of users from other industries. This result can be interpreted as a case in favour of intermediate levels of patent breadth.

Models with variable patent length and breadth:

Eswaran and Gallini (2019) built a model with no patent races of the antibiotics market. Authorities set patent length, patent breadth, and a pigouvian tax in order to maximise social welfare. They found that finite (intermediate) levels of patent length and breadth, complemented by a pigouvian tax, are optimal.

5.2 Summary

	Objective Variable	Optimal Length	Optimal Breadth
Partial equilibrium models			

Budish et al. (2015)	Welfare	Infinite	
Gilchrist (2016)	Cons. surplus	Infinite	
Gilchrist (2016)	Welfare	Ambiguous	
Jeon (2016)	Welfare		Intermediate
Y. Chen and Sappington (2018)	Welfare		Intermediate
Antonelli (2019)	Welfare		Intermediate
Eswaran and Gallini (2019)	Welfare	Finite	Intermediate

6 "Quality ladder" case

The "quality ladder" case arises when *each* innovation can be thought of as a necessary condition for the development of another one. As a result, there is an infinite sequence of future products, where each one represents an improvement upon the previous one. In such a scenario, the life of a patent might come to an end either because its statutory life has expired or the patented product has been rendered obsolete by an improvement. The main concern in this kind of models is not the distribution of joint profit between innovators, since every firm will act as a "first" or a "second" innovator, depending on the current stage of the innovative process. Instead, the main question is how to to increase total profit while minimizing monopoly distortions.

6.1 Partial equilibrium models

Models with variable patent length and fixed patent breadth:

Adilov and Waldman (2013). Authorities set copyright length in order to maximise social surplus. They conclude that optimal copyright length is infinite under certain assumptions.

Models with fixed patent length and variable patent breadth:

Acemoglu and Akcigit (2012) built two models with patent races. Authorities set leading breadth in order to maximise R&D. In the partial equilibrium model, under a uniform patent policy leading breadth should be maximum. However, under a state-dependent policy, leading breadth for one-step leaders should be less than maximum.

Grossmann (2013) built a model with no patent races of the pharmaceutical market. Assuming the existence of a health insurance system, the existence of price regulations, and that all the firms invest the same amount on quality-improving R&D, authorities set patent breadth (the inverse of the number of firms) in order to maximise R&D per firm. He concluded that an intermediate level of patent breadth would be optimal.

Y. Chen, Pan, and Zhang (2014) built a model with no patent races. In a two-innovators setting, authorities set patent strength (breadth) in order to maximise innovation. They found that maximum patent breadth is optimal for high discount rates, whereas an intermediate breadth is optimal for lower rates.

Bondarev (2016) built a model with patent races. Authorities set patent breadth (level of spillovers) in order to maximise total generation of innovations. The author found that patent breadth should be industry-specific, being minimum in those industries where competitors have a similar level of productivity.

Models with variable patent length and breadth:

In a setting that resembles the work of O'Donoghue et al. (1998), Çevikarslan (2017) adopted an evolutionary agent-based modelling approach. His model assumes heterogeneous agents and multi-product firms, which compete in the R&D market and the final goods sector as well. Furthermore, consumers' preferences evolve as technology changes. He carried out a series of simulation experiments varying patent length and breadth within prespecified ranges and found that optimal patents have medium length and maximum breadth within such intervals. It is important to highlight that, for a patent in this model, having "maximum breadth" means being wide enough to protect against all nonradical innovations.

Parra (2019) incorporated in these kind of models what is known as the Arrow's (1962) *replacement effect*, namely the decrease in market leader's profits from its patented product after introducing an improvement. This assumption introduces nonstationarity in the incentives of innovators throughout patent's life. In such a scenario, the author took the speed of innovative activity as the value function of the problem and found that optimal patent length is finite. He also found that optimal patents are long and narrow in industries where innovations are costlier to develop or less frequent. On the other hand, in sectors where innovations are less costly or more frequent, optimal patents are short and wide. Moreover, if total welfare was taken as the objective function instead, optimal patents would be shorter and wider.

The fact that patentees might create entry barriers for subsequent innovators constitutes what is known as the "*litigation* effect" of strengthening patent protection. It can be found in every quality ladder model in the above-cited articles. However, there is also another effect, which is known as the "*competition* effect" of patent protection. It consists of deterring imitators from introducing their versions of the patented product. Krasteva, Sharma, and Wang (2020) showed the consequences for optimal patent policy of including this effect in a discrete-time, infinite-horizon model without patent races. Assuming finite lagging breadth and infinite leading breadth, they found that optimal patents have moderate validity and infinite length if innovators' discount rates are high or R&D costs are low. On the other hand, they have high validity and finite length in the inverse situation, i.e., when such rates are low or R&D costs are high. It must be pointed out that such policies are "optimal" in the sense that they are innovation-maximizing.

6.2 General equilibrium models

Models with variable patent length and fixed patent breadth:

Lin (2015) built a model with no patent races. Authorities set statutory patent length in order to maximise innovation or welfare. The author employed numerical methods and found that infinite patent length maximises innovation, but finite patent length maximises welfare.

Lin (2016) built a single-country model with no patent races. Authorities set patent length in order to maximise welfare. The author suggested that optimal patents have infinite length. However, he showed that a fine-tuned patent system is welfare-dominated by a system of state-dependent intertemporal bounties.

Models with fixed patent length and variable patent breadth:

Acemoglu, Gancia, and Zilibotti (2012) built a model with no patent races. Authorities set the cost of standardization (positively related with patent breadth) in order to maximise growth and welfare. To maximise growth, optimal protection should be finite (intermediate). To maximise welfare, the same level of protection is optimal if discount rate is low. When discount rate is high, optimal breadth also has an intermediate value, but it is lower.

Acemoglu and Akcigit (2012) built two models with one country and with patent races. Authorities set leading breadth in order to maximise welfare. In the general equilibrium model, simulation results suggest that maximum leading breadth is optimal under a uniform patent policy. However, under a state-dependent policy, simulations suggest that maximum leading breadth is not optimal for one or two-step leaders, although it seems to be the best policy for three or more-step leaders.

Cysne and Turchick (2012) built a model with no patent races. Authorities set the imitation rate (inversely related with patent breadth) in order to maximise growth or intertemporal utility (welfare). Maximum breadth is optimal if the objective is to maximise growth, or if it is to maximise intertemporal utiliy and consumers want to have a balanced path of consumption. In a different situation, intermediate patent breadth might be optimal.

Denicolò and Zanchettin (2012) built a single-country model with patent races. Assuming infinite patent length, that every innovation is patentable, and that no innovation infringes on previous ones, authorities set lagging patent breadth (maximum price cap allowed) and leading patent breadth (licensing fees) in order to maximise growth. Assuming minimum leading patent breadth, maximum-lagging-breadth patents maximise growth when lagging patent breadth is uniform. They also maximise growth when it is state-dependent instead, but patent breadth may have no effect on growth in such case. Assuming maximum lagging breadth, optimal leading breadth may be maximum or minimum, since its effect on growth is ambiguous (it depends on the parameters of the model).

Chu, Pan, and Sun (2012) built a model with no patent races. Given infinite patent length, authorities set patent breadth in order to maximise growth. In an inelastic labour supply setting, or in a labour-driven innovation scenario, maximum patent breadth is optimal. However, in a capital-driven innovation setting with elastic labour supply, finite breadth patents are optimal.

Chu and Furukawa (2013) built a model with no patent races. Assuming infinite patent length, authority sets patent breadth (the division of profit between basic and applied researchers) and patentability requirements (for basic researchers) in order to maximise growth and welfare. They found that, in order to maximise growth, patent breadth should be maximum, whereas patentability requirements should have an intermediate value. If the objective consists of maximising welfare instead, patent breadth and requirements should have intermediate values.

Chu, Cozzi, and Liao (2013) built a model with no patent races. Authority sets patent breadth (price markup) in order to maximise growth and welfare. They concluded that, to maximise growth, patent breadth should be maximum or minimum depending on a fertility parameter, whereas breadth has an ambiguous effect on welfare (simulation results suggest that extending patent breadth might have a small positive effect on social welfare).

Yang (2013) built a model with no patent races. Assuming that innovation is produced by cooperative research joint ventures, authorities set patent breadth (price markup) and a profit division rule (this might be interpreted as a dimension of patent breadth) in order to maximise growth and welfare. The author found that optimal patent breadth is finite. If research joint ventures are competitive, then optimal policy requires a 50-50 division rule. If they are cooperative instead, the division rule has no effect when patent breadth is optimal. The best patent breadth is narrower when joint ventures are cooperative.

Iwaisako and Futagami (2013) built a model with no patent races. Assuming that patent length is fixed and infinite, authorities set patent breadth (amount of compensation for damage with infringement, which is linearly related with price markup) in order to maximise growth. They found that patent breadth has a negative effect on capital growth and a positive effect on variety growth, which leads to an ambiguous effect of patent policy on overall growth. In a lab-equipment version of their original model, they found that a growth-maximising breadth exists and it is finite.

Chu and Pan (2013) built a model with no patent races. Assuming infinite lentgth patents and that each entrant infringes the patent of the incumbent, authorities set patent breadth (price markup) and a profit division rule in order to maximise the arrival rate of innovation and growth. Maximum patent breadth is innovation-maximising, and it is growth-maximising if division rule favours incumbents. If innovation step size is exogenous, division rule should favour entrants. However, if step size is endogenous, the rule should have an intermediate value.

Chu, Furukawa, and Ji (2016) built a model with no patent races. Probably assuming that patent length is infinite, authorities set patent breadth (price markup) and R&D subsidies in order to maximise growth. They found that patent breadth and R&D subsidies serve to increase economic growth. However, when market structure adjusts endogenously in the long run, R&D subsidies increase economic growth, whereas patent breadth reduces economic growth.

Huang, Yang, and Cheng (2017) built a model with no patent races. Assuming the existencie of cash-in-advance constraints, authorities set patent breadth (price markup) and interest rates in order to maximise welfare. They found that, given a low interest rate, an intermediate patent breadth is optimal. If both policies are set jointly, optimal patent length is slightly higher, but it is still finite, while optimal interest rate is 0.

Yang (2018) built a model with no patent races. Authorities set patent breadth (price markup), a profit-division rule, research subsidies and production subsidies in order to maximise growth and welfare. To maximise growth, patent breadth should be maximum, and the division rule should favour entrants. In order to maximise welfare, patent breadth and profit division rule should assume intermediate values.

Ikeshita (2018) built a model with no patent races. Assuming infinite patent length, authorities set patent breadth in order to maximise growth and welfare. The article also endogenises authorities decisions. The author found that optimal patent breadth has an intermediate value, which is located between the best value for type I households (those having access to finanacial assets) and the best for type II ones.

Chu and Cozzi (2018) built a model with no patent races. Authorities set patent breadth (price markup), a tax rate, R&D subsidies and general expenditures in order to minimise inequality in income and consumption. They found that minimum patent breadth is optimal.

Chu, Lai, and Liao (2019) built a model with no patent races. Authorities set patent breadth (price markup) and monetary policy in order to maximise economic growth and social welfare. They found that patent breadth has an ambiguous effect on growth and welfare, so optimal breadth should be minimum or maximum, but not intermediate.

Chu, Kou, and Wang (2020) built a model with no patent races. In an economy which transits through a sequence of stages, authorities set patent breadth in order to maximise growth. In the first stage (no innovation), breadth has a negative effect on output but leads to an earlier take-off. In the second (variety-expanding innovation) and third (quality-improving innovations) stages, maximum breadth is optimal. In the fourth stage (balanced growth path), minimum breadth is optimal.

Iwaisako (2020) built a model with patent races. Assuming infinite patent length, authorities set patent breadth in order to maximise growth and social welfare. They found that an exogenous shock in patent breadth, departing from a balanced growth path, induces an initial positive effect in growth, which gradually disappears. The effect of breadth on welfare is ambiguous. However, the author managed to found that an intermediate level of breadth is welfare-maximising (it also presents the condition for an interior solution).

Chu, Cozzi, Fan, Pan, and Zhang (2020) built a model with no patent races. Authorities set patent breadth (price markup) in order to maximise growth. They found that intermediate patent breadth is optimal. Below the optimal level, the absence of financial frictions makes the effect of breadth positive. Above that level, the presence of such frictions makes the effect negative.

Jerbashian (2021) built a model with no patent races. In a two-sector (or two-industry) scenario, authorities set sectoral patent breadth (productivity of research spillovers in a given industry) in order to maximise growth in a specific sector of the economy. The author found that the growth rate of a industry is maximised if its patent breadth is maximum and patent breadth in the other sector is minimum.

Tabata (2021) built a model with no patent races. Authorities set patent breadth (price markup), public investment in R&D and tax rate in order to maximise growth and welfare. Along the balanced growth equilibrium path, maximum patent breadth is the growth-maximising policy, whereas the effect on welfare is ambiguous. Some numerical results suggest that maximum patent breadth is also the welfare-maximising policy.

Chu, Furukawa, Mallick, Peretto, and Wang (2021) built a model with no patent races. Authorities set patent breadth (price markup) in order to maximise growth and minimise economic inequality. In the short run, maximum breadth is growth-maximising and it might be inequality-minimising (intermedium breadth might also be optimal from a distributive viewpoint). In the long run, however, minimum breadth is growth-maximising, but maximum breadth is inequality-minimising.

Lu (2022) built a model with no patent races. Assuming the existence of a cash-in-advance constraint, authorities set patent breadth (price markup), leading breadth (profit-division ratio or blocking patents), and monetary policy in order to maximise growth. For low leading breadth, minimum lagging breadth is optimal. For intermedium leading breadth, an intermedium value of lagging breadth is optimal. For high leading breadth, maximum lagging breadth is optimal. Minimum and maximum leading breadth are not optimal. Some simulation results suggest intermediate values for those variables.

Models with variable patent length and breadth:

Zeng, Zhang, and Fung (2014) built a model with no patent races. Authorities set patent length and breadth (price cap) in order to maximise growth and welfare. They found that growth-maximising patents have infinite length and maximum breadth. However, welfare-maximising patents have finite length and intermedium breadth. The authors also stated that, according to a few simulation results, patents with finite length and maximum breadth could be better or worse than patents with infinite length and intermediate breadth.

Diwakar, Sorek, and Stern (2021) built a model with no patent races. Assuming finitely lived agents, authorities set patent length and breadth (price markup) in order to maximise growth. Given infinite patent length, growth-maximising patents have intermediate breadth if depreciation rates are positive, but they have maximum breadth if those rates are zero. Furthermore, patents with optimal length and maximum breadth are better than patents with infinite length and intermediate breadth. Optimal length is finite if depreciation rates are positive, but it is infinite if they are zero. Finally, they showed that welfare-maximising patent breadth is lower than growth-maximising breadth.

6.3 Open economy extensions

Models with variable patent length and fixed patent breadth:

Bondarev (2018) built a two-country model with no patent races. Assuming that statutory and effective patent length coincide, southern authorities set patent length in order to maximise growth. The author found that infinite patent length is growth-maximising, but it may become an obstacle to structural change (modernization).

Models with fixed patent length and variable patent breadth:

Iwaisako and Futagami (2013) built a two-country model with no patent races. Assuming that patent length is fixed and infinite, authorities of an open small economy set patent breadth (amount of compensation for damage with infringement, which is linearly related with price markup) in order to maximise capital and output. They found that optimal patent breadth is finite when the main channel for technology transfer is licensing, whereas optimal breadth is minimum (no patents at all) when the main channel is imitation.

Tanaka and Iwaisako (2014) built a two-country model with no patent races. Authorities set patent breadth (risk of imitation in the South, which is interpreted by the authors as a measure of effective patent length), R&D subsidies, and foreign direct investment (FDI) subsidies in order to maximise aggregate rates of innovation and FDI. They found that optimal patents have maximum breadth. However, if the objective is to maximise Southern welfare, they have intermediate breadth. If it is to maximise Northern welfare instead, they may have minimum or maximum breadth.

Zheng, Huang, and Yang (2020) built a two-country model with no patent races. Authorities in both countries set patent breadth (price markup) in order to maximise relative wages (domestic wage against foreign wage), the rate of innovation in the North or the rate of technology transfer to the South. If the first variable is employed as the main criterion, Southern authorities prefer minimum patent breadth, whereas Northern authorities prefer maximum breadth. If the rate of innovation in the North or the rate of technology transfer from the North to the South were employed as the main criterion instead, southern authorities would prefer maximum patent breadth. Finally, if they had domestic innovation as their objective, northern authorities would prefer maximum patent breadth only if the North-South ratio of labor force is sufficiently large. Otherwise, they would choose minimum patent breadth.

6.4 Summary

	Objective Variable	Optimal Length	Optimal Breadth
Partial equilibrium models			
Adilov and Waldman (2013)	Welfare	Infinite	
Acemoglu and Akcigit (2012)	R&D Inv.		Max or Int
Grossmann (2013)	R&D Inv.		Intermediate
Y. Chen et al. (2014)	Innovation?		Max or Int
Bondarev (2016)	Innovations?		Int or Min
Çevikarslan (2017)	Welfare?	Finite	Intermediate
Parra (2019)	R&D Inv.?	Finite	Intermediate
Parra (2019)	Welfare?	Finite	Intermediate
Krasteva et al. (2020)	Innovation?	Infinite or finite	Intermediate
General equilibrium models			
Lin (2015)	Innovation?	Infinite	
Lin (2015)	Welfare	Finite	
Lin (2016)	Welfare	Infinite	
Acemoglu et al. (2012)	Growth		Intermediate
Acemoglu et al. (2012)	Welfare		Intermediate
Acemoglu and Akcigit (2012)	Welfare		Max or Int
Cysne and Turchick (2012)	Growth		Maximum
Cysne and Turchick (2012)	Welfare		Max or Int
Denicolò and Zanchettin (2012)	Growth		It depends
Chu et al. (2012)	Growth		Max or Int
Chu and Furukawa (2013)	Growth		Minimum
Chu and Furukawa (2013)	Welfare		Intermediate
Chu et al. (2013)	Growth		Max or Min
Chu et al. (2013)	Welfare		Ambiguous
Yang (2013)	Growth		Intermediate
Yang (2013)	Welfare		Intermediate
Iwaisako and Futagami (2013)	Growth		Int or Amb
Chu and Pan (2013)	Innovation?		Maximum
Chu and Pan (2013)	Growth		Max or Amb?
Chu et al. (2016)	Growth		Max or Min
Huang et al. (2017)	Welfare		Intermediate
Yang (2018)	Growth		Max and Min
Yang (2018)	Welfare		Intermediate
Ikeshita (2018)	Growth		Intermediate
Ikeshita (2018)	Welfare		Intermediate
Chu and Cozzi (2018)	Inequality (Y)		Minimum
Chu and Cozzi (2018)	Inequality (C)		Minimum
Chu et al. (2019)	Growth		Max or Min
Chu et al. (2019)	Welfare		Max or Min
Chu, Kou, and Wang (2020)	Growth		Ambiguous
Iwaisako (2020)	Growth		Maximum
Iwaisako (2020)	Welfare		Intermediate
Chu, Cozzi, et al. (2020)	Growth		Intermediate
Jerbashian (2021)	Growth		Max and Min
Tabata (2021)	Growth		Maximum
Tabata (2021)	Welfare		Ambiguous
Chu et al. (2021)	Short-run growth		Maximum
Chu et al. (2021)	Long-run growth		Minimum
Chu et al. (2021)	Short-run ineq.		Max or Int
Chu et al. (2021)	Long-run ineq.		Maximum
Lu (2022)	Growth		Intermediate
Zeng et al. (2014)	Growth	Infinite	Maximum

Zeng et al. (2014)	Welfare	Finite	Intermediate
Diwakar et al. (2021)	Growth	Finite	Maximum
Open economy extensions			
Bondarev (2018)	Growth	Infinite	
Iwaisako and Futagami (2013)	Capital		Int or Min
Iwaisako and Futagami (2013)	Output		Int or Min
Tanaka and Iwaisako (2014)	Innovation		Maximum
Tanaka and Iwaisako (2014)	Welfare (S)		Intermediate
Tanaka and Iwaisako (2014)	Welfare (N)		Max or Min
Zheng et al. (2020)	Wage (S)		Minimum
Zheng et al. (2020)	Wage (N)		Maximum
Zheng et al. (2020)	Innovation (S)		Maximum
Zheng et al. (2020)	Innovation (N)		Max or Min

7 Complementary innovations case

The case of complementary innovations arises when a product or process is covered by multiple patents, which may be held by two or more firms. This implies that developing a new idea requires accessing the technologies in two or more patents. In such settings, the last innovation may be inefficiently blocked by the patentees, a situation that is known as the "tragedy of the anticommons" (Heller & Eisenberg, 1998) in the economics of innovation literature.

7.1 Partial equilibrium models

Models with variable patent length and fixed patent breadth:

Llanes and Trento (2012) built a dynamic single-country model with no patent races, which resembles the "quality ladder" models of the previous section (see above). Authorities set patent length, given infinite (maximum) patent breadth, in order to maximise probability of innovation. They found that optimal patents have finite length, which is short or minimum (no patent system at all).

Models with fixed patent length and variable patent breadth:

Kwon (2012a) built a model with patent races. Assuming that firms may use secrecy instead of patents to protect their innovations, authorities set patent breadth (probability of imitation or inventing around) in order to maximise investment. An extension of the original model contemplates the case of complementary innovation. The author found that strengthening patents might discourage innovation, so minimun patent breadth might be optimal.

Kwon (2012b) built a model with patent races. Assuming that firms may use secrecy instead of patents, authorities set patent breadth (one minus probability of getting around the patent) to maximise investment in innovation. The author concluded that if patent propensity is between 0 and 1, minimum patent breadth is optimal. Assuming reasonable values for the parameters of the model, allowing for licensing contracts will not change this result.

Biagi and Denicolò (2014) built a model with no patent races. Assuming that patent length is infinite, authorities set patent breadth (the flow of profit that each innovator can obtain, or the royal rates that the patent holders are allowed to charge) in order to maximise expected consumer surplus (equal to social welfare under the assumptions of the model). They found that maximum patent breadth cannot be optimal. Their results suggest that intermediate levels of patent breadth must be optimal.

Marjit and Yang (2015) built a two-country model (firms, actually) and with no patent races. Authorities set patent breadth (decide if intellectual property rights are weak or strong) in order to maximise innovation and welfare. They found that patent breadth should be maximum if the "innovation effect" dominates the "competition effect". In the opposite case, it should be minimum.

Turner (2018) built a model with no patent races. Authorities set patent breadth (compulsory royalty rates) in order to maximise welfare. The author found that, under strict patentability requirements (patents for non-practising entities are not allowed), patent breadth has no effect on welfare. However, minimum patent breadth is optimal under lenient patentability requirements.

7.2 Summary

	Objective Variable	Optimal Length	Optimal Breadth
Partial equilibrium models			
Llanes and Trento (2012)	Innovation	Finite	
Kwon (2012a)	R&D Inv.		Min or Int?
Kwon (2012b)	R&D Inv.		Minimum
Biagi and Denicolò (2014)	Cons. Surplus		Intermediate
Marjit and Yang (2015)	Innovation		Max or Min
Marjit and Yang (2015)	Welfare		Max or Min
Turner (2018)	Welfare		Min or Any

8 Conclusion

A systematic review of the last ten years of theoretical contributions in optimal patent design literature has been carried out. Since the focus of this paper was on the trade-off between patent length and breadth, only the models featuring at least one of these variables were included in the review. The results were classified according to their conclusions in relation to the optimal levels of patent length and breadth. The main concern of the criteria employed was if the optimal values of the decision variables under analysis were their extreme values (i.e. the minimum or the maximum allowed by the restrictions of the problem) or intermediate ones.

According to the results, optimal patent length is finite in some models, whereas it is infinite in others. A similar diversity of results was found in models featuring patent breadth: some of them suggest that it should be minimum, others conclude that an intermediate value would be optimal, and others are more prone to favour maximum breadth. Few works seemed to conclude that two or more kinds of soutions were possible. The four kinds of models exhibited, up to a certain degree, this seeming inconsistency. As a consequence, works presenting sufficient conditions for different sorts of optimal solutions have not been found within this period.

It might be argued that these models are based on unrealistic assumptions, which may be true (Spulber, 2013). However, this is not enough to rule out the study of such models, because the analysis of these unrealistic theoretical constructions should be considered as a first approximation to the problem. The optimal solution of complex problems require the understanding of complex issues, which in turn require the study and the understanding of simpler cases (i.e., simpler versions of the situation at glance) as a previous step, because "one must learn to walk before learning to run" (Nordhaus, 1969, p. vii).

Having said all that, it is clear that there are some interesting lines of work which deserve to be developed in the future. First, it would be enlightening to propose sufficient conditions for different sorts of optimal solutions, as it has been said before. Secondly, more complex models might be necessary to perform more realistic analysis of patent design issues. The performing of research in these directions could lead to better recommendations for patent policy in the future. This would encourage better allocations in R&D markets and, after all, the whole economy, which could mean a significative improvement for the well-being of many people.

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