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DOCUMENTO DE TRABAJO 2020-29

Noviembre de 2020

Citar como:
Aryal, Gaurab, Eduardo Fajnzylber, Maria F. Gabrielli, y Manuel Willington (2020). Auctioning Annuities. *Documento de trabajo RedNIE, 2020-29*.

Auctioning Annuities*

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November 3, 2020

Abstract

We use data on annuities to study and evaluate an imperfectly competitive market where firms have private information about their (annuitization) costs. Our data is from Chile, where the market is structured as *first-price-auction-followed-by-bargaining*, and where each retiree chooses a firm and an annuity contract to maximize her *expected present discounted utility.* We find that retirees with low savings have the highest information processing cost, and they also care about firms' risk-ratings the most. Furthermore, while almost 50% of retirees reveal that they do not value leaving bequests, the rest have heterogeneous preference for bequest that, on average, increases with their savings. On the supply side, we find that firms' annuitization costs vary across retirees, and the average costs increase with retirees' savings. If these costs were commonly known then the pensions would increase for everyone, but the increment would be substantial only for the high savers. Likewise, if we simplify the current pricing mechanism by implementing English auctions *and* "shutting down" the risk-ratings, then the pensions would increase, but again, mostly for the high savers.

Keywords: Annuity, Auctions, Mortality, Annuitization Costs.

JEL: D14, D44, D91, C57, J26, L13.

*We are thankful to Lee M. Lockwood, Leora Friedberg, Gaston Illanes and Fernando Luco for their helpful suggestions. We also thank the seminar/conference participants at SEA 2019, APIOC 2019, 2020 ES NAWM, NYU Stern and ES World Congress 2020. Aryal acknowledges financial support from the Bankard Fund for Political Economy at U-Va., TIAA Institute and Wharton School's Pension Research Council/Boettner Center. Fajnzylber and Willington acknowledge financial support from CONICYT/ANID Chile, Fondecyt Project Number 1181960. The content is solely the responsibility of the authors and does not necessarily represent official views of the TIAA Institute or Wharton School's Pension Research Council.

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1 Introduction

Most countries have social security programs to help provide retirees with financial security. But, these programs are experiencing enormous enormous pressure to remain solvent and viable. For example, the OECD notes that "[P]ressure persists to maintain adequate and financially sustainable levels of pensions as population ageing is accelerating in most OECD countries," (OECD, 2019). At the same time, there is a fear that too many people do not have enough retirement-savings.[1] In light of these challenges, there have been several policy discussions and research on the level of benefits, taxes to finance these benefits, incentives to induce more savings, and even delay retirement, see, for example, Feldstein (2005) and Mitchell and Shea (2016). There is also an increasing awareness that these *fiscal measures* might be insufficient on their own, and that it might be fruitful to use a competitive market-based system to provide retirement products that can improve retirees' financial security.

Despite this, we know little about how the demand and the supply of a retirement product, e.g., an annuity, interact with each other to determine the equilibrium pensions and retirees' welfare. Our contribution in this paper, is to answer this question in the context of a privatized annuity market in Chile. Ultimately, we want to shed light on questions such as: Is there a need to reform this market for annuities? And if so, how? If there is a room for improvement, how can we refine the market? An annuity is an insurance against the longevity risk of outliving one's savings. So, it is considered to be an ideal retirement product (Yaari, 1965; Brown et al., 2001; Davidoff, Brown, and Diamond, 2005), and understanding how a market for annuities works is a hugely important knowledge from a policy perspective.

To this end, we propose an empirical framework to study an imperfectly competitive market for annuities, where insurance companies have private information about their annuitization costs, and retirees have heterogeneous mortality risks, savings, and preferences. We apply this framework to a rich administrative dataset from Chile and estimate preference parameters and annuitization cost distributions. Then we evaluate the current market, and quantify the effect of adopting simpler auctions on the pensions and retirees' welfare.

Chile provides an ideal setting to study and evaluate a market for annuity contracts. It is one of the first countries in the world to adopt a market-based system for annuities. In 1981, Chile replaced its public pay-as-you-go pension system with a new system of privately managed individual accounts. And since 2004, all retirees use a centralized exchange (known as SCOMP) to choose between an annuity, from among those offered by many insurance companies, or a programmed withdrawal option, which is a default "self-insurance" pension

[1] For example, in the U.S., Government Accountability Office (GAO) reported that 48% of households whose head of household is age 55 and over have no retirement savings (Vernon, Streeter, and Deevy, 2020).

product. Thus, Chile has a "mature" market, and because it represents the entire country it also provides an ideal setting for us to learn how demand (preferences, mortalities, and savings) and supply (costs and competition) affect equilibrium pensions and welfare.

Our empirical findings directly inform current policy debates in Chile. While the Chilean pension system has reached a large share of retirees (91% receive some pension), the level of retirement income is very low. For instance, the median replacement rate in Chile (ratio of initial pension to the last wage) is 44%, while ILO recommends 70%. According to the antitrust authority in Chile, lower pensions could be because of poor design of the selling mechanism, poor understanding (by the retirees) of the role of risk-ratings, and dubious role of intermediaries (Quiroz et al., 2018). There is an ongoing policy debate in Chile about the effectiveness of "shutting down" the risk-ratings. Whether that change improves pensions and retirees' welfare is unknown. And in this paper, we answer that question, among others.

These findings should also be relevant for other countries: who have either adopted the "Chilean model," and, or, are considering using annuities to improve their retirement system. For example, our findings would be useful for the U.S., where the *Setting Every Community Up for Retirement Enhancement Act of 2019* incentivizes businesses and communities to band together to offer annuities, but, the law, is silent about how to structure such market(s).

There are additional modeling and data advantages from considering the Chilean system. First, there are only *fixed annuities*, so this market is simpler to understand and model than if there were *variable annuities* (Brown et al., 2017). Second, because SCOMP is a centralized system, search frictions are less of a concern than elsewhere. Third, our data is of high quality and we observe everything about retirees that the firms observe. In particular, for each retiree, we observe her demographic information, savings, names of the participating firms and their offers for different types of annuities (e.g., immediate annuity, annuity with 10 years of guaranteed payments), her final choice and her date of death, whenever applicable.

Following the institution, we model the interaction between a retiree and a set of firms as a *first-price-auction-followed-by-bargaining* with selective entry. In particular, each retiree is a risk-averse auctioneer who chooses a firm and an annuity that gives her the highest expected present discounted utility. So, each auction-retiree is different in terms of the retiree's savings, demographic characteristics that affect her expected longevity, and her preferences for bequest and for the firms' risk-ratings. Thus, we have non-standard multi-attribute auction, i.e., *"beauty contest"* (Asker and Cantillon, 2008), where besides the pension, the retiree may also value bequests and firms' risk-ratings.[2] Moreover, these weights (and the expected utilities) vary across retirees and because they are unobserved they have to be estimated.[3]

[2]Bequest motive is an important determinant of annuity demand. See, for example, Kopczuk and Lupton (2007); Lockwood (2018); Illanes and Padi (2019) and Einav, Finkelstein, and Schrimpf (2010).

[3]Similar considerations arise when the U.S. states bid for firms (Slattery, 2019), and in Internet service

In Chile, however, there is uncertainty about the role of firms' risk-ratings in retirees' decision. First, bankruptcy is a rare event in Chile, and most firms have high risk-ratings, and second the government guarantees a minimal pension amount should a firm fail. Should the preference for risk-ratings be subjective or should it be objective and same across all retirees? To capture this uncertainty, we assume that retirees are *rationally inattentive* decision-makers (Sims, 1998) who do not know their preferences for risk-ratings, but can determine one by processing some information, which is costly. To keep the learning and updating process tractable, we use the discrete choice framework in Matějka and McKay (2015) to model the decision process in the first stage. In the second stage, however, we assume that the retirees know their preferences, and conditional on choosing an annuity product they choose the firm that maximizes their expected present discounted utility.

On the supply side, we assume that the life insurance companies observe everything about the retiree, and their annuitization cost before deciding to participate in the retiree-auction. The per-dollar annuitization cost of a firm, also known as the *Unitary Necessary Capital* (UNC), captures the cost of promising a survival-contingent stream of payments to retirees.[4] Participating firms bid simultaneously on all of the annuity products that the retiree has requested quotes for. If the retiree chooses from the first-round the game ends, or else it ensues a bargaining between her and the participating firms, where she has imperfect information about firms' annuitization costs and what they can offer.

We then establish the identification of our model parameters under the assumptions that preferences are homothetic with CRRA utility and mortality follows Gompertz distribution. To this end, we rely on exogenous variation in retirees' demographics, savings, and the market interest rates over our sample that spans more than a decade. These variations have differential effects across firms, which affect firms' entry decisions and the pensions.

For instance, to identify the distribution of bequest-preferences, we look at the winning firm, and consider the offers it made to the retiree in the first-round. All else equal, a retiree with stronger bequest preferences is more likely to choose annuities with larger present expected value of the bequest, such as annuities with longer guaranteed periods. So, if someone chooses an annuity with the lowest bequest then it provides an upper bound on her bequest preference, and vice versa. As these bounds vary across firms and retirees they identify the preference distributions. Some people might just not care about leaving bequest or they cannot even afford it. Others, especially the richer retirees, may have additional wealth that is not observed, and they may choose an annuity with lower bequest. If so, we

markets (Krasnokutskaya, Song, and Tang, 2020), where the "winner" is not necessarily the highest bidder.

[4]UNC is the expected amount of dollars required to finance a stream of payments of one dollar until retiree's death and any proportional obligations to her surviving relatives, if any. For example, if the UNC is 200 then it means that the expected cost for the firm to provide a pension of \$100 is \$20,000.

interpret this as retirees have low/zero preference for bequest. To capture this "mass" at zero, we use a mixture distribution to model heterogeneity in bequest preference. That is there might be two reasons for the "mass" point at zero, but the second one highlighted above is probably the one that correspondes to richer people. Thus, we contribute to the existing literature by providing a direct evidence of bequest motives than before.[5]

To identify the retirees' information processing costs, we use the fact that the elasticity of the choice probability with respect to the offered pensions is inversely proportional to the information processing cost. Our model ascribes those who are less responsive to pensions as someone with high cost of processing information.

For the identification of the retirees' preference for companies' risk-ratings, we focus only on those who choose in the second round. There, the chosen pension can be expressed as a linear combination of differences in risk-rating between the two most competitive firms and the annuitization costs. Relying on the within-retiree variation in pension offers across firms, and the variation in annuitization costs we can identify the risk-rating preferences. After that, we can identify the conditional distributions of annuitization costs by adapting the identification strategy used in English auctions (Athey and Haile, 2002).[6]

Our estimates suggest that those who have higher savings have lower information processing costs. This is consistent with the fact that those with larger savings tend to be more educated, and, so, possibly have better financial literacy. Interestingly, we find that those who use sales-agents or directly contact insurance companies behave as if they care a lot more about risk-rating than others. One interpretation of this result is that while everyone starts with a prior that puts a lot of weight on the risk-ratings, those with lower information processing cost revise their weights downwards.

We also find that close to 50% of retirees show no preference for a bequest, except for those in the highest savings quintile. There is, however, considerable heterogeneity among those who value bequests. For instance, those in the lowest and the highest savings quintiles, on average, respectively, care 1.92 times and 2.82 times more about their spouse than themselves.

Using our demographic information, we also estimate the survival probability for each retiree. Comparing the expected mortality with the model-implied annuitization costs, we find that retirees who are expected to live longer have larger annuitization costs. However,

[5]Typically, bequest motives are indirectly inferred from savings that marginally change the distribution of bequests across different mortality states rather than from direct choices of annuities that have smaller or larger built-in bequests. For more see Bernheim (1991); Kopczuk and Lupton (2007); Lockwood (2018).

[6]Our identification strategy does not rely on optimal bidding in the first stage, which involves submitting bids for several types of annuities; to identify the risk-rating preferences and cost distributions it is sufficient to focus on second-round bidding, which is considerably more tractable. In view of this, we do not characterize the equilibrium *multi-product* bidding strategy for the first stage. Without the modeling the first stage, we cannot estimate the ex-ante expected profit, which in turn means we cannot identify the entry costs.

there is significant heterogeneity in these costs across retirees' and across retirees' savings and the average annuitization costs increases with savings. Interestingly, our estimates also suggest that there is a non-negligible probability that firms have relatively more efficient in annuitizing retirees in the highest savings quintiles than self-annuitization.

To quantify the effect of asymmetric information on pensions and retirees' ex-post expected utilities, we simulate the equilibrium pension under the assumption that the firms observe each other's annuitization costs, while shutting down the risk-ratings. We find that the gap between the observed pensions and the complete-information pensions is the largest for retirees who belong to the top two savings quintiles. This is consistent with our estimates of the savings-quintile specific cost distributions: close to 20% of the time, firms are relatively more efficient to annuitize high savers than if the retirees had self-annuitizated.

Next, we evaluate the effect of replacing the current pricing mechanism with, a simpler, one-shot English auctions while also shutting down the role of risk-ratings. Similar to the complete information counterfactual, we find that using English auctions, either with or without reserve prices, increase pensions for everyone, but the gain is minimal for the retirees who have less than 60% of savings in our sample. In terms of the retiree's ex-post expected present discounted utilities, we find that these changes do not translate into large gains in utilities because either the pensions do not change (for those with lower savings) or they increase (for those with high savings) but the utility gains are minimal because of the diminishing marginal utilities. Taken together, our estimates highlight the roles of asymmetric information, mortality, differences in savings and costs on pensions and welfare.

In the remainder of the paper we proceed as follows. In Section 2 we summarize the literature, in Section 3 we introduce the institutional detail, and in Section 4 we describe our data. Section 5 presents our model and Section 6 discusses its identification. Section 7 and Section 8 present the estimation results and the counterfactual analysis, respectively. Section 9 concludes. The Appendix includes additional details not included in the main text.

2 Literature

Our paper contributes to several strands of literature in public finance, and in empirical industrial organization. First, and foremost, we contribute to a rich literature on annuities (Yaari, 1965; Brown, 2001; Mitchell and Smetters, 2003; Davidoff, Brown, and Diamond, 2005; Reichling and Smetters, 2015), and those that use Chilean data (Berstein, 2010; Alcalde and Vial, 2016, 2017; Morales and Larraín, 2017). The key innovation in our paper relative to these papers is that we study both demand and supply of annuities. So, we compliment Illanes and Padi (2019), who show that for the Chilean retirement market, to understand the impact of policy reforms in annuity market, we have to also consider the supply side.

Second, our paper is also related to Finkelstein and Poterba (2004) and Fajnzylber and Willington (2019) who test for adverse selection in U.K. and Chile annuity markets, respectively, and to Einav, Finkelstein, and Schrimpf (2010) who estimate the welfare cost of asymmetric information under monopoly seller in the U.K. In our paper, we do not focus on adverse selection, instead, our focus is to estimate retirees' welfare under an oligopolistic market with private information. Fajnzylber and Willington (2019) have identified that in Chile the adverse selection is of first-order importance in the dichotomous choice between an annuity or programmed withdrawal, but once we condition on choosing annuities, as in our case, the evidence of adverse selection is tenuous.

With this in mind, and to keep the supply-side model tractable, we assume that retirees and insurance companies have the same information about retirees' mortality. As a consequence, we only use information from retirees who choose annuities, even though many in our original sample do not choose an annuity. To model retirees' mortality as their private information, we would have to extend the model of informed principal, e.g., (Myerson, 1983) to allow risk-aversion and adverse selection, which is difficult.

Third, our paper is related to the literature that recognizes the role of information processing costs in annuity choices. It is widely understood that annuities in the U.S. are difficult to comprehend for most retirees (Brown et al., 2017). While such considerations are less of a problem in Chile because unlike in the U.S. the annuities sold through SCOMP are fixed annuities, which are simpler and easier to evaluate, there is significant uncertainty among retirees about the relevance of firms risk-ratings. We model this uncertainty using the rational inattention model with discrete choice (Matějka and McKay, 2015).

We allow the information processing cost associated with the rational inattention decision-maker to depend on middlemen, e.g., sales-agent. This way we can capture in a "reduced-form" the effect middlemen can have in this system. In that regard, we complement Alcalde and Vial (2017) and Hastings, Hortaçsu, and Syverson (2017) who study the role of middlemen in Chilean and Mexican retirement markets, respectively. The key innovation in our paper is that we uncover a new mechanism by which middlemen can influence decisions: by making it difficult for retirees to process information.

Finally, our paper also complements the papers that estimate bequest motives, see Kopczuk and Lupton (2007) and Lockwood (2018). A key difference is that we focus on the bequest motive among those who choose to annuities, use their choices across different types of annuities with different levels of bequests to provide a direct, revealed preference, measure of bequest preference. In that aspect, our approach is similar to Einav, Finkelstein, and Schrimpf (2010), except we assume that preferences are uncorrelated mortality risk.

3 Institutional Background

The Chilean pension system went through a major reform in the early 1980s, when it transitioned from a *pay-as-you-go* system to a system of fully funded capitalization in individual accounts run by private pension funds (henceforth, AFPs). Under this system, workers must contribute 10% of their monthly earnings, up to a pre-determined maximum (which in 2018 was U.S. $2,319), into accounts that are managed by the AFPs.[7]

Upon reaching the minimum retirement age –60 years for female and 65 years for men– individuals can request an old-age pension, transforming their savings into a stream of pension payments. In this paper, we focus only on those retirees who have savings in their retirement accounts, that are above a certain threshold, who can, and must, participate in the electronic annuity market.[8]

Regulation

The Chilean government regulates and supervises AFPs, who manage retirement savings during the accumulation phase, and life insurance companies, who provide annuities during the decumulation phase. In addition, at the time of retirement, the government provides subsidies to workers who fail to save enough during their work-lives (Fajnzylber, 2018).

Moreover, the life insurance industry is heavily regulated. The current regulatory framework for life insurance companies providing annuities recognizes that the main risks associated with annuities are the risk of longevity and reinvestment. Longevity risk is taken care of through the creation of technical reserves by insurers that sell annuities, which consider self-adjusting mortality tables. The government also regularly assesses the risk of reinvestment via the Asset Sufficiency Test established in 2007. Under this regulation, every insurance company is required to establish additional technical reserves, if and when there are "insufficient" asset flows, following the international norm of good regulatory practices in insurance industries.Bankruptcy among life insurance companies is rare in Chile, but the government guarantees every retiree pensions up to 100% of the basic solidarity pension, and 75% of the excess pension over this amount, up to a ceiling of 45 UFs (see footnote 7). Thus, there is enough safety nets for retirees to feel protected in case of a bankruptcy.

[7]This maximum, and annuities in general, are expressed in *Unidades de Fomento* (UF), which is a unit of account used in Chile. UF follows the evolution of the Consumer Price Index and is widely used in long-term contracts. In 2018 the UF was approximately equivalent to U.S. $39.6.

[8]The threshold is currently established as the amount required to finance a *Basic Solidarity Pension*, which is the minimum pension guaranteed by the State. Retirees with insufficient funds will receive them from the AFP based on a programmed withdrawal schedule.

3.1 Pension Products

Retirees participating in the electronic market have three main choices: Programmed Withdrawal (PW), immediate annuity (IA), and deferred annuity (DA).[9] Under PW, savings remain under AFP management and is paid back to the retiree following an actuarially fair benefit schedule. In the event of death, remaining funds are used to finance survivorship pensions or, in absence of eligible beneficiaries, become part of the retiree's inheritance. PW benefits are exposed to financial volatility and provide no longevity insurance so that, barring extraordinarily high returns, the pension steadily decreases over time.

Under both IA and DA, the retiree's savings are transferred to an insurance company of her choice that will provide an inflation-indexed monthly pension to her and her surviving beneficiaries. In the deferred annuities, pensions are contracted for a future date (usually between 1 and 3 years), and in the meantime the retiree is allowed to receive a temporary benefit that can be as high as twice the pension amount.

Thus, the main trade-off between an annuity and a PW is that an annuity provides insurance against longevity risk and financial risk whereas under a PW a retiree can bequeath all remaining funds in case of an early death. Moreover, while annuitization is an irreversible decision, a retiree who chooses a PW can switch and choose an annuity at a later date.

Annuities may also include a special coverage clause called a guaranteed-period (GP).[10] If an annuity includes, for instance, a 10 year guaranteed period, the full pension will be paid during this period to the retiree, eligible beneficiaries or other individuals. Once the guaranteed period is reached, the contracts reverts to the standard conditions (implying a certain percentage of the original pension and only for eligible beneficiaries).[11]

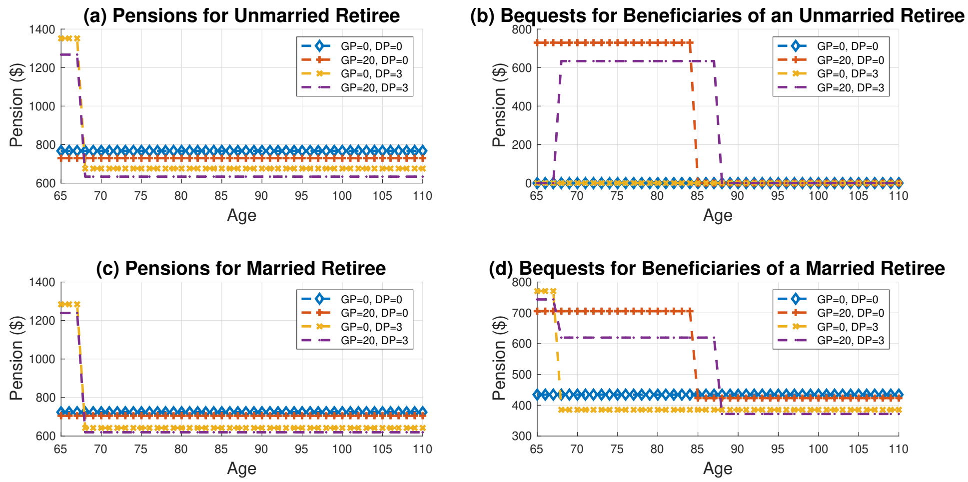
For illustration of how benefits change with the annuity products and marital status, consider a male retiree who is 65 years old, has a savings of U.S. $200,000 and is retiring in 2020. Suppose he is unmarried and chooses an annuity with GP=0 and DP=0, then he gets a constant pension until death (blue '$\diamond$' in Figure 1-(a)), but after that his beneficiaries gets nothing (blue '$\diamond$' in Figure 1-(b)). But if he chooses an annuity with GP=20, then while alive he gets lower pension (compare red '+' and blue '$\diamond$' in Figure 1-(a)), but if he dies within 20 years of retirement, his beneficiaries get a strictly positive amount (purple 'x' in Figure 1-(b)) for 20 years, and after that it they get nothing. If he was married, then even with GP=0 and DP=0 (blue '$\diamond$' in Figure 1-(c)), the beneficiaries will get a positive amount (blue '$\diamond$' in Figure 1-(d)) after the retiree dies.

[9] There is a fourth, rarely chosen, pension product which is a combination between a PW and an IA.

[10] Another rarely chosen clause is the spouse's percentage increase, which maintains the full payment to the surviving spouse, instead of the mandated 50% or 60% for regular contracts.

[11] In our sample, 99.9% of the chosen annuities correspond to contracts with 0, 10, 15, or 20 years of GP.

Figure 1: **Benefit Schedules, by Annuity Type**



Note. The figure shows the survival-contingent benefit schedules for retirees and their beneficiaries for a representative retiree in our data, who is a 65 years old male and with savings of U.S. $200,000. Subfigures (a) and (b) shows the pension and bequest schedules, respectively, for 4 types of annuities and if he is unmarried. Similarly, subfigures (c) and (d), respectively show the pension and benefit schedules when he is married. All calculations are performed by the authors using the official 2020 *mortality table*. GP stands for guaranteed period (in years) and DP stands for deferred period (in years).

3.2 Retirement Process

The process of buying an annuity begins when a worker communicates her decision of considering retirement to her designated AFP. We assume that she is then exogenously matched with one of four intermediaries or "channels" who can help her choose a product and firm. Out of these four channels, two (AFP and insurance company) are free and the other two (sales-agent and independent advisor) charge fees. Retirees must also disclose information on all eligible beneficiaries.[12] The AFP then generates a *Balance Certificate* that contains information about the total saving account balance (henceforth, just savings), and her demographic characteristics. Then the decision process can be described in the following steps:

1. The retiree requests offers for different types of pension products (described above).[13] Upon request, insurance companies in the system have 8 business days to make an offer (for every requested annuity products).

2. These offers (i.e., bids) are collected and collated by the SCOMP system and presented to the retiree as a *Certificate of Quotes.* The certificate is in the form of a table, one

[12]The main beneficiaries are the retiree's spouse and their children under age 24.

[13]Retirees can request quotes up to 13 different variations, including PW and annuities with different combinations of contractual arrangements.

for each type of annuity, sorted from the highest to the lowest pensions along with the company's name and risk-rating.[14]

3. The retiree can choose from the following 5 options: (i) postpone retirement; (ii) fill a new request for quotes (presumably for different types of annuities); (iii) choose PW; (iv) accept one of the first-round offers for a particular type of annuity; or (v) negotiate with companies by requesting second-round offers for one type of annuity. In the latter case, firms cannot offer lower than their initial round offers, and the individual can always fall back to any first-round offer.[15]

4 Data

Our data on the Chilean annuity market span between January 2007 and December 2017. We observe everyone who used SCOMP to buy an annuity or choose PW during this period. As mentioned before, we observe everything about a retiree that participating life insurance companies observe about them before they make their entry decisions and their first-round offers. For each retiree, we observe all the offers they received and their final choice, and whether they chose in the first round or the second round. Our working assumption is that most retirees use first-round offers to choose between different types of annuities–as all trade-offs between different guaranteed and deferred periods become apparent with these first-round offers–and conditional on choosing the annuity type, in second-round they bargain with companies for better pensions.

4.1 Retirees

We focus on individuals without eligible children, who are considering retirement within 10 years of normal retirement age (NRA), which is 60 years for a woman and 65 years for a man. The result is a data set with 238,891 retirees, with an almost even split between PW, immediate annuities and deferred annuities, see Table 1. Less than 1% of retirees choose annuity *with* PW and so we exclude them, leaving a total of 238,548 retirees.

In Table 2 we present the sample distribution, by retirees' marital status, gender, and age at the time of their retirement. Approximately 56% retire at their NRA, and close to 79% retiree at or at most within three years after NRA (rows 2 and 3), and married men are half of all retirees. Retirees also vary in terms of their savings; see Table 3. The mean

[14]In the case of guaranteed periods, the certificate also includes a discount rate that would be applicable in the event of death within the GP. In absence of legal beneficiaries, other relatives can receive the unpaid benefits in a lump sum, calculated with the offered discount rate. For an example see Figure 1.

[15]A firm that does not offer in the first-round cannot participate in the second-round.

Table 1: **Share of Pension Products**

Product	Obs.	%
PW	78,161	32.7
Immediate annuity	87,115	36.4
Deferred annuity	73,272	30.6
Annuity with PW	343	0.9
Full Sample	238,891	100

Note. The table shows the distribution of retirees across different annuity products. We restrict ourselves to annuities with either 0, 10, 15 or 20 years of guaranteed periods or at most 3 years of deferment.

Table 2: **Age Distribution, by Gender and Marital Status**

Retiring Age	S-F	M-F	S-M	M-M	Total
Before NRA	1,871	1,771	4,714	22,142	30,498
At NRA	20,789	22,475	17,114	72,572	132,950
Within 3 years after NRA	14,470	16,797	4,447	19,086	54,800
At least 4 years after NRA	6,900	6,715	1,251	5,434	20,300
Full Sample	44,030	47,758	27,526	119,234	238,548

Note. The table displays the distribution of retirees, by their marital status, gender and their retirement ages. Thus the first two columns 'S-F' and 'M-F' refer, respectively, to single female and married female, and so on. NRA is the 'normal retirement age,' which is 60 years for a female and 65 years for a male.

savings in our sample is $112,471, while the median savings is $74,515 with an inter-quartile range of $85,907. Savings are higher for men, and for those who retire before NRA.

Table 3: **Savings, by Retirement Age and Gender**

	Mean	Median	P25	P75	N
Retiring Age					
Before NRA	185,660	129,637	73,104	245,857	30,498
At NRA	89,907	60,023	41,521	103,680	132,950
Within 3 years after NRA	115,666	87,126	54,353	135,562	54,800
At least 4 years after NRA	141,673	101,594	58,815	168,202	20,300
Full Sample	112,471	74,515	46,449	132,356	238,548
Gender					
Female	97,308	81,180	51,817	121,633	91,788
Male	121,955	69,372	43,818	147,184	146,760
Full Sample	112,471	74,515	46,449	132,356	238,548

Note. Summary statistics of savings, in U.S. dollars, by retiree's age at retirement, and by retiree's gender.

4.1.1 First-Round Offers

A retiree receives approximately 10.6 offers, for several types of annuity, and the number of offers increases with savings. For instance, those with savings at the 75^{th} percentile of our sample get an average of 12.4 offers and those at the 25^{th} percentile get an average of 7.8 offers. It is reasonable to assume that retirees with larger savings are more lucrative for the firms, and therefore more companies are willing to annuitize their savings. If those with higher savings, however, also live longer than those with lower savings then it means that annuitizing higher savings are costlier for the firms. To determine which of these two opposing forces dominate, we estimate the annuitization costs and mortality, by savings.

Moreover, there is also substantial variation in the pensions offered, across both life insurance companies and retirees; see Table 4. On average, for an immediate annuity, retirees get an offer of $570 and for deferred annuities, the average offer is $446. Women, on average, get an offer of $479 for immediate annuities and $412 for deferred annuities, while for men they are $631 and $473, respectively. Both these features are consistent with men having higher savings *and* shorter life expectancy than women (see Table 7).

Table 4: **Monthly Pension Offers, by Annuity type and Gender**

Annuity Type	**Gender**	**Mean**	**Median**	**Savings Q1**	**Savings Q2**	**Savings Q3**	**Savings Q4**	**Savings Q5**
Immediate	Female	479	414	202	288	385	510	857
	Male	631	435	200	269	372	585	1329
	Full Sample	570	423	201	278	378	556	1152
Deferred	Female	412	374	190	258	349	463	714
	Male	473	356	187	241	331	529	1019
	Full Sample	446	365	189	248	339	500	882

Note. Summary of average monthly pensions (in U.S. dollars) offers received in the first-round.

In our empirical model, we rationalize this variation in pension offers by allowing firms to have heterogeneous costs (UNCs) of annuitization. We assume that only the firm knows its annuitization cost which can depend on the savings of the retirees. An important exogenous factor that can affect UNCs is the market interest rate, which affects the opportunity cost of offering a pension at retirement. Our sample spans a decade, so we observe substantial variation in interest rates, which causes exogenous variation in annuitization costs.

4.1.2 Chosen Annuities

Once the participating companies make first-round offers, one for each type of annuity the retiree requests quotes for, she can either choose from one of those offers or she can buy PW or initiate the second-round bargaining phase. Table 5 displays the distribution across these stages. Almost all retirees (98.1%) who choose in the first-round choose PW, and most of those who choose annuity (86.9%) opt for the second-round.

Table 5: **Number of Retirees who choose in First- or Second-Round**

Round/Choice	PW	1^{st} round	2^{nd} round	Total
1^{st} round	76,690	18,001	0	94,691
2^{nd} round	1,471	2,979	139,407	143,857
Total	78,161	20,980	139,407	238,548

Note. Round refers to whether retirees chose in the first- or in the second-round.

In Table 6 we present information about the chosen annuities: (i) the total number of accepted offers by the type of annuity; (ii) the average number of first-round and second-round offers received for the annuity that was eventually chosen; (iii) the number of accepted second-round offers; (iv) the average percentage increase in pension offers from first-round to second-round (only for the accepted choice); (v) the percentage of retirees who requested at least one second-round offer; (vi) the percentage of retirees who chose the highest paying alternative; and (vii) the percentage of retirees who chose a dominated option, in terms of either pension (with the same risk-rating) or risk-ratings (with the same pension) or both.

Table 6: **Summary of Accepted Annuities**

GP Months	# Accepted	Average # of 1^{st} Round Offers	# Accepted in 2^{nd} Round	Increase	Average % Requested 2^{nd} Round	Best	Dominated
			Immediate				
0	21,292	11.3	16,357	1.5	80	59	22
120	26,907	11.1	23,463	1.3	89	51	28
180	24,452	11.6	22,070	1.4	92	49	29
240	14,464	11.8	13,020	1.5	92	51	29
Total	87,115	11.4	74,910	1.4	88	53	27
			Deferred				
0	11,703	10.9	8,919	1.5	79	53	23
120	26,119	11.0	23,390	1.4	91	46	31
180	26,775	11.4	24,324	1.4	92	42	34
240	8,675	11.0	7,864	1.3	92	42	34
Total	73,272	11.1	64,497	1.4	90	45	31

Note. The table shows the number of chosen annuities by type of product, the average number of first-round offers received for the chosen annuity, the number of accepted offers that resulted from second-round offers, the average percentage increase between the first-round and second-round offers (for the accepted choice), the percentage of individuals who requested at least one second-round offer, the percentage of retirees who chose the highest paying alternative option and the percentage of individuals who chose an offer that was dominated by another alternative with same (or better) credit rating.

From Table 6, we see that some retirees *do not* choose the annuity with the highest pension. One way to rationalize this behavior is to recognize the fact that besides pensions, retirees also care about firms' risk-ratings. After all, risk-rating is a proxy of financial health, and it is also widely advertised as such. So a retiree can prefer lower pensions from healthier firms to a higher pension from a less healthy firm.

This rationalization, however, begs the ensuing follow-up questions: Is there an objective (i.e., correct) trade-off between pension and risk-rating and should it be homogeneous or vary across retirees? If it is heterogeneous should it increase or decrease with savings? On the one hand, because of the regulation, those with lower savings are less exposed to the risk of firms defaulting than those with higher savings, those with higher savings should care more about the risk-ratings than those with lower savings. On the other hand, because savings is positively correlated with education, those with higher savings will be able to determine the actual likelihood of default, which in the case of Chile suggests that retirees should not care much about the risk-rating. Finally, how does this trade-off vary with preferences for bequest? To determine which of these countervailing forces dominate, and how pensions and utilities would change under alternative market rules, later we estimate a structural model.

4.1.3 Mortality

An important determinant of annuity demand and supply is retiree's expected mortality. For every retiree, we observe when they entered our sample, i.e., their retirement age, and their age at death if they die by the end of our sample period. Using this information, we estimate a mixed proportional hazard model (defined shortly below) and use the estimated survival function to predict the expected life conditional on being alive at retirement.

Let the hazard rate for retiree i with socio-economic characteristics X_i at time $t \in \mathbb{R}_+$, that includes includes i's age, gender, marital status, savings and the year of birth, be $h_{it} = \lim_{dt\to 0} \frac{d\Pr(m_i \in [t, dt) | X_i, m_i \geq t)}{dt} = h(X_i) \times \psi(t)$, where m_i is i's realized mortality date, $\psi(t)$ is the baseline hazard rate. Furthermore, let the hazard function $\psi(t)$ be given by Gompertz distribution, such that the probability of i's death by time t is $F_m(t; \lambda_i, \mathfrak{g}) = 1 - \exp(-\frac{\lambda_i}{\mathfrak{g}}(\exp(\mathfrak{g}t) - 1))$, and let $\lambda_i = \exp(X_i^\top \tau)$.

The identification of such model is well established in the literature (Van Den Berg, 2001). The maximum likelihood estimated coefficients of the hazard functions suggest a smaller hazard-risk is associated with younger cohorts, individuals who retire at a later age, with females, those who are married and those with higher savings.[16] Using these estimates, the median expected lives, by gender and savings quintile, and their standard errors are reported in Table 7. Overall, 50% of males expect to live until 86 years and 50% of females

[16]For robustness, we also estimated the Gompertz model from a separate data set that includes retirees before the introduction of SCOMP, and thus has less censoring, and the estimates are qualitatively the same. For instance, the predicted median expected life at death is 85 and 96 for males and females, respectively. Both of these results are available upon request.

expect to live until they are 94.9 years old. As we can see, those who have larger savings also tend to live longer than those with lower savings.

Table 7: **Median Expected Life, by Savings Quintile**

Savings	**Male**	**Female**	**Overall**
Q1	85.15	93.80	86.89
	(5.79)	(6.03)	(5.82)
Q2	85.86	94.24	87.64
	(5.81)	(6.06)	(5.84)
Q3	86.45	94.83	88.23
	(5.83)	(6.09)	(5.88)
Q4	87.62	95.48	89.40
	(5.88)	(6.12)	(5.95)
Q5	90.87	97.25	93.52
	(6.01)	(6.21)	(6.11)
Total	86.75	94.91	89.57
	(5.82)	(6.09)	(5.94)

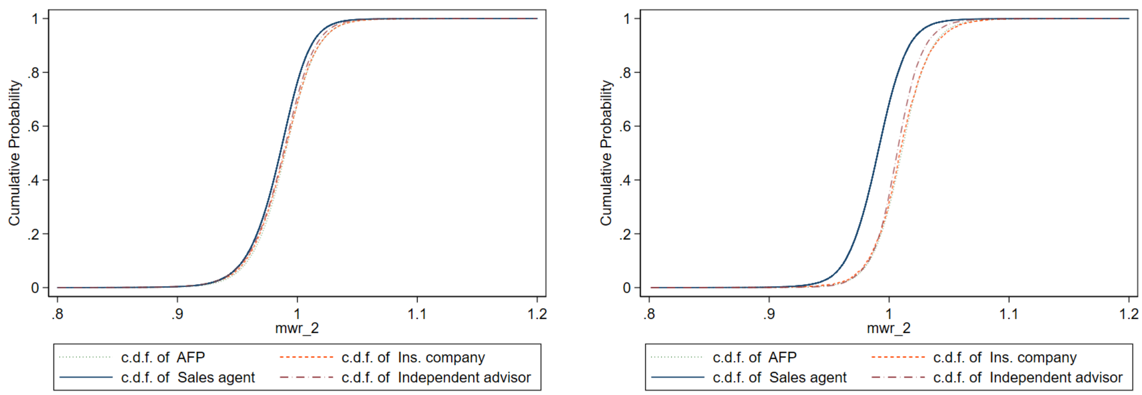
Note. The table shows the predicted median expected life at the time of retirement implied by our estimates of the Gompertz mortality distribution. Standard errors are reported in the parentheses.

4.2 Intermediary Channels

We observe retirees with one of the four intermediary channels (AFP, Insurance Company, Sales Agent or Independent Advisor) to assist them with their annuitization process. If and when the incentives of such an intermediary do not align with those of a retiree, then retirees do not always choose the "best" option for them. The misalignment of incentives may be particularly relevant for sales-agents, who receive their intermediation fee only if the retiree chooses the sales-agent's firm. In other words, it is possible and very likely that those with sales-agent would appear to value the non-pecuniary benefits of a company more than the pecuniary benefits. So, to capture this effect on the decision process, in our estimation we allow preferences for risk-ratings and information processing costs to depend on the channel.

To account for observed differences among retirees we consider the money's worth ratio (henceforth, `mwr`) which is the expected present value of pension per annuitized dollar. If `mwr` $= 1$ then it means the retiree expects to get \$1 pension (in present value) for every annuitized dollar. In Figure 2 we display the distributions of the `mwr` offered in the first-round (left panel) and `mwr` accepted by the retirees (right panel). The mean and the median `mwr` of the offers, by channels (AFP, Insurance Company, Sales Agent, Advisor), are $(0.989, 0.988, 0.984, 0.987)$ and $(0.990, 0.989, 0.986, 0.988)$, respectively, but the means and medians for accepted offers are $(1.010, 1.010, 0.990, 1.007)$ and $(1.010, 1.009, 0.991, 1.007)$, respectively. Thus, the final accepted offers are on average better than the first-round offers, and those with sales-agents have lower `mwr`.

Figure 2: **CDFs of Offered and Accepted MWR, by Channel**



Note. Distributions of the offered and chosen `mwr` (left panel vs. right panel), by channel.

We use a multinomial Logit model to consider if observed differences among retirees can explain the differences in their channels, see Table 8. In particular, we estimated the log-odds ratio of having one of the three intermediary channels relative to the AFP and find that some characteristics are correlated with the channel. For instance, those who have lower savings, retire early, are male or unmarried are more likely to use sales-agents, relative to AFP.

Table 8: **Intermediary Channel - Estimates from Multinomial Logit**

Regressors \ Channels	**Insurance Company**	**Sales-Agent**	**Advisor**
Savings ($million)	0.629***	-0.857***	-0.130***
	(0.128)	(0.0436)	(0.0447)
Age	0.0131	-0.0408***	-0.0816***
	(0.00857)	(0.00189)	(0.00218)
Female	0.437***	-0.0588***	-0.124***
	(0.0546)	(0.0120)	(0.0140)
Married	0.0245	0.0620***	0.0874***
	(0.0491)	(0.0107)	(0.0127)
Constant	-5.029***	2.333***	4.326***
	(0.560)	(0.123)	(0.142)
N	238,548	238,548	238,548

Note. Estimates of multinomial logit regression for channels, where the baseline choice is AFP. Standard errors are in parentheses, and ***,** ,* denote p-values less than 0.01, 0.05 and 0.1, respectively.

Although we cannot rule out the selection on unobservable retiree characteristics, for model tractability, we treat the channel as exogenous. There are two reasons why we believe this is not a strong assumption in our context as might appear. First, several anecdotal evidence from Chile suggests that most people rely on word-of-mouth when it comes to a channel. Second, and as mentioned previously, we observe everything the firm observes about a retiree at the time of making the first-round offers. When we estimate the preference parameters we estimate them separately for several groups that we define based on age, gender, savings, and channels. Estimating preference parameters separately for each group allows us to control for the effects of the potential selection on unobservable characteristics.

For instance from Table 9 we see that channels affect the outcomes. Out of 109,786 retirees who choose AFP, only 25.1% choose the second-round, whereas the shares are 85.2%, 92.0%, and 87.8% for Insurance Company, Sales Agents or Advisors, respectively. Most of those who choose PW have AFP, and those with sales-agents are least likely to choose PW.

Table 9: **Retiree choices, by Intermediary Channel**

	N	**Requests 2^{nd} Round**	**Chooses PW**	**Chooses in 2^{nd} Round**
AFP	109,786	0.251	0.661	0.235
Company	2,169	0.852	0.066	0.817
Sales-agent	79,120	0.920	0.030	0.907
Advisor	47,473	0.878	0.066	0.846
Full Sample	238,548	0.603	0.328	0.584

Note. Proportion of retirees separated by their choices and their channel.

Our empirical framework can capture the effect of channels on outcomes. In particular, we posit that channels affect the cost of acquiring information about the importance of risk-rating. For instance, we allow those retirees who use sales-agents to act "as if" they have a higher cost of acquiring information about the trade-off between risk-rating and pensions. We assume that in the first-stage, retirees are rationally inattentive with respect to their preference for risk-ratings, but in the second-stage they know their preferences.

4.3 Firms

In our sample, we observe 20 unique life insurance companies, and they differ in terms of their annuitization costs, which are unobserved, and in terms of their risk-ratings. The distribution of risk-ratings is displayed in Table 10. The ratings mostly remain the same over time, and most companies have high (at least AA) risk-ratings. For our empirical analysis, we treat these ratings as exogenous, and group them into three categories: 3 for the highest risk rating of AA+, 2 for all the risk-ratings from AA to A, and 1 for the rest.

Although there are 20 unique firms, not all of them are active at all times, and not all participate in every auction. On average, 11 companies participate in a retiree-auction, which suggests that the market is competitive.[17] We define potential entrants (for each retiree-

[17] The quarterly Hirschman-Herfindahl Index, measured at the level of annuity-type (e.g., immediate

Table 10: **Risk-Ratings**

Rating	Frequency	%	Cumulative %
AA+	155	24.64	24.64
AA	245	38.95	63.59
AA-	171	27.19	90.78
A+	2	0.32	91.1
A	15	2.38	93.48
BBB+	1	0.16	93.64
BBB	6	0.95	94.59
BBB-	15	2.38	96.98
BB+	19	3.02	100
Total	629	100	

Note. The table shows the distribution of quarterly credit-ratings from 2007-2018.

auction) as the set of active firms that participated in at least one other retiree-auction in the same month. In our sample, retirees have either 13, 14 or 15 potential entrants.

The participation rate, which is the ratio of the number of actual bidders to the number of potential bidders, varies across our sample from as low as 0.08 to as high as 1, with mean and median rates of 0.73 and 0.78, respectively, and a standard deviation of 0.18.[18] Thus, it is likely that a firm's decision to participate depends on its financial position at the time a retiree requests quotes, and this opportunity cost of participating can vary across retirees.[19] To capture this selection, in our empirical application, we follow Samuelson (1985) to model firms' entry decisions, which posits that firms observe their retiree-specific annuitization cost prior to entry. This is a reasonable assumption in our setting because firms have sophisticated models to predict retiree's mortality and the expected returns they can get from the savings.

For model tractability, we treat firms as symmetric bidders, whose annuitization costs are independently and identically distributed with some (unknown) distribution function. We do not observe firms annuitization costs, and so, we cannot directly test this assumption. But we can perform a diagnostic test and check if the firm-specific pension (bid) distributions are different from one another. If they are not different from one another then our symmetry assumption is a reasonable first step.

To perform this test, however, we have to "control" for all relevant factors that can affect the pension. For instance, retirees with high savings can be lucrative because the total gain

annuity) and the channel, is almost always below 1900.

[18]Using a Poisson regression of the number of participating firms on the retiree characteristics we find that one standard deviation increase in savings, which is approximately $87,000, is associated with roughly 1 more entrant. And women have 0.61 additional participating companies than men, while sales-agents and advisors are associated with approximately 0.19 fewer participants than the other 2 channels.

[19]We tested this selection by estimating a Heckman selection model with the number of potential bidders as the excluded variable and found strong evidence of negative selection among firms.

from annuitizing their savings will be large. But, as we have seen above, these retirees are expected to live longer. So to compare the bids across firms, we have to estimate the *expected discounted life* for each retiree, which we refer to as UNC_i where the subscript i refers to retiree i. This UNC_i is different from UNC_j, where the latter refers to a firm j's cost. We formally define UNC_i when we present the supply side of our model, and in Appendix A.1 we detail how we use the estimates from the mortality distribution to calculate UNC_i. But for now, it sufficient to know that UNC_i depends only on i's estimated mortality parameter and in the discount factor, such that a retiree who expects to live longer will have a larger UNC_i and will be costlier for firms to annuitize, but these costs are unobserved.

For each of the 20 firms, in Figure 3 we present the histograms and scatter plots of monthly pension per annuitized dollar (which is known as the monthly pension rate) and the UNC_is of all the retirees that the firms make offers to in the first stage. Using pension rates, instead of pensions, allows us to compare across different retirees. As we see, indeed UNC_i and pension rates are negatively correlated, and the are no big differences across firms. Now, using these UNC_i's we can compare pensions across firms. To this end, we homogenize the offered pension rates (ratio of monthly pension to annuitized savings) across firms and compare the distributions across firms and we say that firms are asymmetric if the distributions are different, and symmetric otherwise. For each firm we estimate

$$\begin{aligned}\texttt{Pension-Rate}_{i,j} &= \texttt{constant} + \beta_1 \times UNC_i + \beta_2 \times \texttt{Age}_i + \beta_3 \times \texttt{Gender}_i \\ &\quad + \beta_4 \times \texttt{Marital Status}_i + \beta_5 \times \texttt{Spouse's Age}_i \\ &\quad + \beta_6 \times \texttt{Guaranteed Months}_i + \beta_7 \times \texttt{Potential Bidders}_i + \varepsilon_{i,j}, \quad (1)\end{aligned}$$

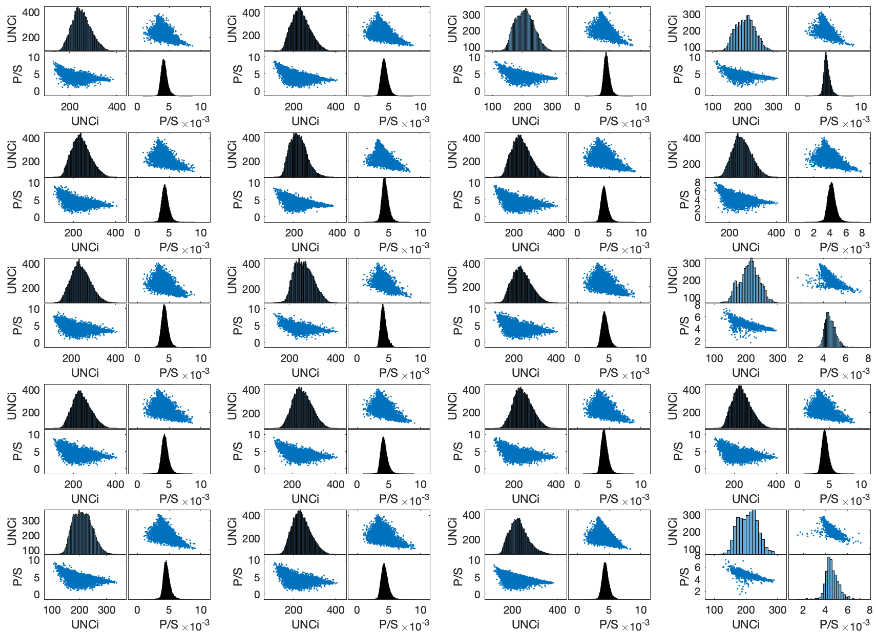
using ordinary least squares method, and predict the residual $\hat{\varepsilon}_{i,j}$ for retiree i and firm j. In Figure 4 we show the Kernel density estimate of the firm-specific distribution of $\hat{\varepsilon}_{i,j}$. We can see that these 20 distributions are very similar to each other, and so it is reasonable to say that firms have a symmetric cost distribution.

5 Model

In this section, we introduce our model. For the demand, we consider the decision problem facing a retiree who uses SCOMP to choose a company to annuitize her savings with. To model the utility from an annuity, we closely follow the extant literature on annuities, in particular Einav, Finkelstein, and Schrimpf (2010), with a modification that accounts for heterogeneous preferences for firm characteristics.

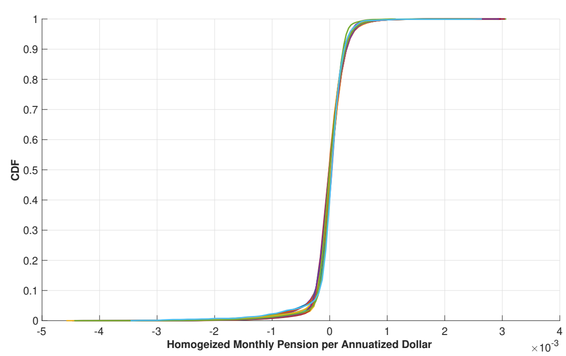
As we have shown before, retirees do not always choose the best offer. To rationalize

Figure 3: **Pension Rates and UNC_i for each Firm**



Note. These are histograms and scatterplots of monthly pension rate, i.e., the ratio of monthly pension to annuitized savings, and the UNC_i of the retirees the firms make an offer. There are twenty firms, so there are twenty sets of four subfigures each. Clockwise, the first sub-figure is the histogram of UNC_i, the second sub-figure is the scatterplot of the pension rates (x-axis) and UNC_i (y-axis), the third sub-figure is the histogram of the pension rates, and the last sub-figure is the scatterplot of UNC_i and the pension rates.

Figure 4: **Distributions of Homogenized Pension Rates, by Firms**



Note. Kernel estimates of the distribution residuals $\hat{\varepsilon}_{ij}$ from Equation (1), one for each firm.

this we posit that besides the pecuniary aspect of an annuity, retirees also care about the risk-ratings of a company, which is a proxy for the likelihood of default. That being said, we assume that all retirees have a prior that puts a lot of emphasis on risk-rating, and only those who spend some resources learning about the likelihood of default will update their prior and choose accordingly. To capture the trade-off between pension, risk-ratings, and information gathering, we follow Matějka and McKay (2015) and model the retiree as a rationally inattentive (Sims, 1998) decision-maker. If a retiree chooses to go to the second-round bargaining, then we assume that she knows her preferences for risk-ratings.

On the supply side, we model the imperfect competition using an extensive form game where the first stage is a first-price auction with independent private value and endogenous entry (Samuelson, 1985), and if there is a second stage then it is multilateral bargaining with one-sided asymmetric information. The winner of the game is not always the firm that offers the highest pension, because the probability of winning depends on the bids as well as on the preferences for risk-rating and bequest, which can vary across retirees.

5.1 Demand

Here, we consider the problem faced by an annuitant i who has already decided which annuity product to choose (e.g., an immediate annuity with 0 guaranteed period) and is considering between J_i firms who have decided to participate in the auction for i's savings S_i. The retiree will choose the firm that provides her the highest indirect utility.

We assume that the utility from an annuity consists of three parts: the *expected present discounted utility* from the monthly pension that the retiree enjoys while alive, utility she gets from leaving bequest (if any) to her kin, and her preference for firm's risk rating. Retirees may value the risk-ratings because they may dislike firms with lower risk-ratings. However, they may not know the "correct" weight to put on these risk-ratings. To capture this uncertainty we model retirees as rationally inattentive decision-makers. We explain this aspect shortly below, but for ease of exposition we begin without rational inattention.

Let (θ_i, β_i) denote i's preferences for bequest and risk-rating, respectively, and given savings S are distributed independently and identically across retirees as $\tilde{F}_\theta(\cdot|S) \times F_\beta(\cdot|S)$ on $[0, \overline{\theta}] \times [\underline{\beta}, \overline{\beta}]$. To capture the fact that retirees might not be able to afford bequest, and therefore will act as someone who does not care about bequest we allow $\tilde{F}_\theta$ to have a mass point at $\theta = 0$. Letting $\zeta \in (0, 1)$ be the probability that the retiree has $\theta_i = 0$, and let $F_\theta(\cdot) = \zeta \times H(0) + (1-\zeta) \times \tilde{F}_\theta(\cdot)$ where, $H(0)$ is a Heaviside function and $\tilde{F}_\theta$ is the continuous distribution on $(0, \overline{\theta}], \overline{\theta} < \infty$.

Let P_{ij} denote the pension offered by firm j to retiree i. Given the type of annuity and the pension P_{ij}, i's expected mortality and the mortality of her beneficiaries determine the bequest, which we denote by $B_{ij}(P_{ij})$. Whenever it is clear from the context, we suppress the dependence of B_{ij} on P_{ij}. Let i's indirect utility at retirement from choosing an annuity with pension and bequest (P_{ij}, B_{ij}) from firm j with risk rating $Z_{i,j} \in \{1,2,3\}$ be

$$U_{ij} = \underbrace{\mathfrak{U}(P_{ij}, B_{ij}; \theta_i)}_{i\text{'s discounted utility}} + \underbrace{\beta_i \times Z_j,}_{i\text{'s preference for } j\text{'s risk-rating}} - \underbrace{\mathfrak{U}_{0i}(S_i),}_{\text{outside utility}} \tag{2}$$

where the utility $\mathfrak{U}_{0i}(S_i)$ is the utility associated with the outside option.

Next, we explain the expected present discounted utility, $\mathfrak{U}(P_{ij}, B_{ij}; \theta_i)$. For simplicity, consider only the first month after retirement, and let q_i be the probability of being alive one month after retirement. Then, the expected present discounted utility will be

$$\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i) = u(P_{ij}) \times q_i + \theta_i \times v(B_{ij}(P_{ij})) \times (1 - q_i),$$

where $u(P_{ij})$ is the utility from P_{ij}, and $v(B_{ij})$ is the utility from leaving a bequest B_{ij}. Thus, the marginal utility from leaving a bequest B_{ij} upon death is $\theta_i \times (1 - q_{it}) \times v'(B_{ij})$. Now, if we consider two periods after retirement, the we would have to adjust the probability that the retiree survives two periods given that she is alive at retirement, and also take into account the fact that the bequest left upon death will also change, which in turn depends on whether the annuity product under consideration includes a guaranteed period.

In practice, we do not know for how long i expects to live. So, to determine expected longevity at retirement, we estimate a continuous-time Gompertz survival function for i and her spouse (if she is married) as a function of her demographic and socio-economic characteristics. Once we have the survival probabilities we can determine the expected discounted utilities as the product of $u(P_{ij})$ and the discounted number of months i expects to live, where the discounting is with respect to market interest rate.

Even with bequest, $\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i)$ has an intuitive structure: it is the sum total two terms, one is the product of $u(P_{ij})$ and the discounted number of months i expects to live and the other is the product of $v(B_{ij})$ times the discounted number of months i's beneficiaries expect to receive B_{ij}. Legally, i's spouse is legally entitled to 60% of the i's pension and given the possibility of having a guaranteed period during which 100% of the pension is paid, the amount B_{ij} may change over time.

Thus, we can write $\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i)$ as

$$\begin{aligned} \mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i) &:= u(P_{ij}) \times D_i^R + \theta_i \left(v(0.6 \times P_{ij}) \times D_i^S + v(P_{ij}) \times D_i^{S,GP} \right) \\ &\equiv \rho_i(P_{ij}) + \theta_i \times b_i(P_{ij}), \end{aligned} \tag{3}$$

where D_i^R is the discounted expected longevity of the retiree (in months, from the moment the annuity payments start), $D_i^{S,GP}$ is the discounted number of months that the spouse (or other beneficiaries) will receive the full pension because of the guaranteed period, and D_i^S is the discounted number of months that the spouse will receive 60% of the retiree's pension.[20] If the annuity has a deferred period, then until the annuity payment starts, the retiree gets twice her pension, so $\rho_i(P_{ij}) = u(P_{ij}) \times D_i^R + u(2P_{ij}) \times D_i^{R,DP}$ where $D_i^{R,DP}$ is the expected life during the deferred period.[21]

A retiree, however, can have additional wealth, besides S_i, that she can use for consumption or bequest, especially those who are wealthy. We, however, do not observe her consumption (after retirement) or her wealth, so following the literature (Mitchell et al., 1999; Scholz, Seshadri, and Khitatrakun, 2006; Einav, Finkelstein, and Schrimpf, 2010; Illanes and Padi, 2019) we assume that retirees have homothetic preferences. In particular, we assume that all retirees have CRRA utility $u(c) = v(c) = \frac{c^{(1-\gamma)}}{1-\gamma}$ with $\gamma = 3$. Homothetic preferences imply that the retiree's annuity choice does not depend on the unobserved wealth. In Appendix A.1 we detail the steps to estimate $\rho_i(P_{ij})$ and $b_i(P_{ij})$.

Substituting (3) in (2) we can express i's indirect utility from annuity P_{ij} from firm j as

$$U_{ij} \;\;=\;\; \rho_i(P_{ij}) + \theta_i \times b_i(P_{ij}) + \beta_i \times Z_{ij} - \mathfrak{U}_{0i}(S_i). \tag{4}$$

Thus (4) shows that there is a trade-off between higher pensions and lower risk-ratings, but as mentioned above, we assume that i does not know her β_i, but only its distribution.

We follow Matějka and McKay (2015) and assume that before the retirement process begins, i has a belief that $\beta_i \overset{i.i.d}{\sim} F_\beta(\cdot)$ with support $[\underline{\beta}, \overline{\beta}]$, and if i wants to learn her preference, she has to incur information processing cost, valued at $\alpha > 0$ per unit of information.

So, i has to first decide how much to spend learning about β_i, and after that make the decision. Matějka and McKay (2015) consider a similar discrete choice decision problem facing a rationally inattentive decision maker and determine the optimal decision rule. We use their solution. Let $\sigma : [\underline{\beta}, \overline{\beta}] \times \mathcal{P} \to \Gamma := \Delta([0,1]^{J+1})$ denote the strategy of a retiree with

[20] These "discounted life expectancies" can be reinterpreted as annuitization costs: assuming firms use the same mortality process as we have and they invest retirees' funds at an interest rate equal to the discount rate, then D_i^R is the necessary capital to provide a one-dollar pension to the retiree until she dies. Similarly, $D_i^{S,GP}$ is the necessary capital to finance a dollar of pension for the beneficiaries once the retiree is dead and until the guaranteed period expires, and D_i^S is the necessary capital to finance a dollar of pension for the beneficiaries between the retiree's death or the guaranteed period is over (whichever occurs later) and until the spouse dies. The gains from trade between a retiree and insurance companies come from the differences in risk-attitude between retiree and life insurance companies, and potential differences between the discount rate of the retiree and firms' investment opportunities.

[21] For simplicity, we are disregarding survival benefits during the deferment period. Deferred periods in our sample are at most 3 years thus the death probability is quite low.

preference parameter β, with offered pensions $\boldsymbol{P}_i := (P_{i1}, \dots, P_{iJ}) \in \mathcal{P}$. The strategy is a vector $\sigma(\beta, \boldsymbol{P}_i) \equiv (\sigma_1(\beta, \boldsymbol{P}_i), \dots, \sigma_J(\beta, \boldsymbol{P}_i), \sigma_{J+1}(\beta, \boldsymbol{P}_i))$ of probabilities, where $\sigma_j(\beta, \boldsymbol{P}_i) = \Pr(i \texttt{ chooses } j|\beta, \boldsymbol{P}_i) \in [0,1]$. For notational simplicity, we suppress the dependence of choice probabilities on the offers ($\boldsymbol{P}_i$).

Let i's expected utility from j be given by $\int U_{ij}\sigma_j(\beta)dF_\beta(\beta)$, and we further assume that the information processing cost has to be paid only in the first-round. By the time i decides to go to the second-round i knows her β_i. Let $\mathbb{E}U_i$ be the ex-ante expected utility from second-round. Then i's maximization problem can be stated as:

$$\max_{\{\sigma(\beta)\in\Gamma\}} \Big\{ \sum_{j=1}^{J} \int U_{ij}(\beta)\sigma_j(\beta)dF_\beta(\beta) - (\texttt{information cost}) + \sigma_{J+1}(\beta) \times \mathbb{E}U_i \Big\}, \tag{5}$$

where the information cost is equal to the reduction in uncertainty times α, where we use relative entropy to measure information and uncertainty.[22] In other words, the total information cost of updating the prior from $F_\beta(\cdot)$ to $F'_\beta(\cdot)$ is $\alpha \times \{\texttt{entropy of } F_\beta - \texttt{entropy of } F'_\beta\}$.

Let $\sigma_j^0 := \int_{\underline{\beta}}^{\overline{\beta}} \sigma_j(\beta)dF_\beta(\beta)$ be the unconditional probability of choosing option j. Then the expected reduction of entropy of i conditional on β is

$$I(\sigma, F_\beta) = -\sum_{j=1}^{J} \sigma_j^0 \log \sigma_j^0 + \int_{\underline{\beta}}^{\overline{\beta}} \left(\sum_{j=1}^{J} \sigma_j(\beta) \log \sigma_j(\beta) \right) dF_\beta(\beta),$$

and the information cost is $\alpha \times I(\sigma, F_\beta)$; see Matějka and McKay (2015). Substituting this cost in (5), we can re-write i's optimization problem as

$$\max_{\{\sigma_j(\beta)\}_{j=1}^{J+1}} \left\{ \sum_{j=1}^{J} \int_{\underline{\beta}}^{\overline{\beta}} U_{ij}\sigma_j(\beta)dF_\beta(\beta) - \alpha \times I(\sigma, F_\beta) + \sigma_{J+1}(\beta)\mathbb{E}U_i \right\}. \tag{6}$$

Then by adapting Matějka and McKay (2015)'s choice formula to two-periods, we can show that the probability that i chooses j is given by

$$\sigma_{ij}(\boldsymbol{P}_i) = \sigma_j(\beta_i, \boldsymbol{P}_i) = \begin{cases} \dfrac{\exp\left(\log \sigma_j^0 + \frac{U_{ij}}{\alpha}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}, & j = 1, \dots, J \\[2ex] \dfrac{\exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}, & j = J+1. \end{cases} \tag{7}$$

Thus if the information processing cost is large, say, $\alpha = \infty$, then the retiree's choice becomes

[22] Entropy of a continuous random vector β with density $f_\beta(\cdot)$ is $\mathbb{E}[-\ln(f_\beta(\beta)]$.

$\sigma_{ij} = \frac{\sigma_j^0}{1+\sum_{j'}^J \sigma_{j'}^0}$. Similarly, if two retirees have different information costs then their choice probabilities will reflect different degree of "elasticity" with respect to the pensions.

5.2 Supply

Next, we present the supply side, where J insurance companies participate in an auction run by "auctioneer" i with characteristics $X_i \equiv (S_i, \tilde{X}_i)$. For simplicity, we suppress the dependence on X_i and treat J as fixed, but account for selection in our empirical application.

Companies differ in terms of their UNCs. Thus, if j can annuitize i cheaper than j', then, j has an advantage over j' because all else equal j can offer a higher pension. Let UNC_j^R be j's unitary necessary capital to finance a dollar pension for the retiree. Similarly, we must consider the costs related to the bequest, which may come from two sources: a guaranteed period, during which after the death of the retiree the beneficiaries receive the full amount of the pension, and the compulsory survival benefit, according to which the spouse of the retiree receives, after the retiree died and after the guaranteed period is over, 60% of the pension until death, see Equation (A.1). We denote by $UNC_j^{S,GP}$ and UNC_j^S the present value of the cost of providing these two benefits. Then, j's expected cost of offering P_{ij} is

$$C(P_{ij}) := P_{ij} \times (UNC_j^R + 2 \times UNC_j^{R,DP} + 0.6 \times UNC_j^S + UNC_j^{S,GP}) \equiv P_{ij} \times UNC_j. \quad (8)$$

Here, the 2 in (8) follows from our assumption that the pension payments during the deferred period were made by the life insurance company. Let UNC_i be the unitary cost of a pension calculated with the retirees' discount rate and the mortality process we estimate. For the same retiree i, firms' UNCs may differ from UNC_i due to the differences in their (i) mortality estimates, (ii) investment opportunities, and (iii) expectations about future interest rates.[23] For these reasons, it is more likely that only firm j knows its UNC_j. Morevoer, the ratio of UNC_j to UNC_i captures j's margin from selling an annuity to i. Henceforth we call this ratio $r_{ij} \equiv \frac{UNC_j}{UNC_i}$, j's relative cost of annuitizing a dollar.

We assume the cost r_{ij} is private and is distributed independently and identically across companies as $W_r(\cdot|S)$, with density $w_r(\cdot|S)$ that is strictly positive everywhere in its support $[\underline{r}, \overline{r}]$. Thus, we assume that firms are symmetric, and this is consistent with what we observe in the data; Figure 4. Allowing the cost distribution to depend on S captures the fact that those who have higher savings tend to live longer and, therefore, costlier to annuitize.

Ignoring for now the second-round, and the multi-product nature of the first-round, j's

[23]Firms may also have different expectations about the interest rates in the future than the retirees.

net present expected profit from offering P_{ij}, to a retiree i with S_i is

$$\begin{aligned}\mathbb{E}\Pi^{I}_{ij}(P_{ij}) &= (S_i - P_{ij} \times UNC_j)) \times \Pr(\texttt{j is chosen by offering } P_{ij}|\mathbf{P}_{i-j}) \\ &= S_i \times (1 - r_{ij} \times \rho^*_i(P_{ij})) \times \sigma_{ij}(\mathbf{P}_i), \end{aligned} \tag{9}$$

where $\rho^*_i(P_{ij}) \equiv P_{ij} \times UNC_i/S_i$ is the money worth ratio (`mwr`) computed using the retirees' discount rate, and $\sigma_{ij}(\mathbf{P}_i)$ is the probability that i chooses j given the vector of offers $\mathbf{P}_i$. Considering the second round, and denoting by $\tilde{P}_{ij}$ the second-round offer of firm j

$$S_i \times (1 - r_{ij} \times \rho^*_i(P_{ij})) \times \sigma_{ij}(\mathbf{P}_i) + \sigma_{iJ+1}(\mathbf{P}_i) \times \mathbb{E}\Pi^{II}_j(\rho^*_i(\tilde{P}_{ij})|r_{ij}, \mathbf{P}_i), \tag{10}$$

is its ex-ante expected profit, where $\sigma_{iJ+1}(\mathbf{P}_i)$ from (7) is the probability that i takes the bargaining option in the second round with expected profit given by $\mathbb{E}\Pi^{II}_j$.

The two rounds are connected. First, more generous offers on the first round may lower the probability of the retiree choosing to go to the second round. Second, and more importantly, each firm's first-round offer is binding for the second round: a firm cannot make any second-round offer below its first-round one. Our focus in the empirical analysis will be on the second round. For the first period, it suffices for our purposes to argue that firms will never make first-round offers that, if accepted by the retiree, would render expected non-positive profits.

Now, when we include the fact that i might request offers from A_i types of annuities, insurance companies have to solve a multi-product bidding problem. As mentioned in the timing assumptions, once i receives all the offers $\{P^a_{ij} : a \in A_i, j \in J\}$, she chooses $a^* \in A_i$ and then chooses the firm. Thus, with a slight abuse of notations, we can express the expected profit of a firm $j \in J$ from an auction where i requests offers for A_i types of annuities as $\mathbb{E}\Pi_{ij} := \sum_{a\in A_i} \mathbb{E}\Pi_{ij}(a) \times \Pr(\texttt{i chooses } a|\{\mathbf{P}^b_i\}_{b\in A_i}; \theta_i)$.

Thus, in the first round, when choosing P^a_{ij}, firm j has to not only consider the competition from other firms for a and for all other types of annuities in $A_i\backslash\{a\}$, but also from its own offers $P^b_{ij}, b \in A_i, b \neq a$. This is the standard self-cannibalization consideration facing a multi-product seller. Determining the equilibrium bidding strategies for the first-round auction although conceptually straightforward, will require us to first determine the equilibrium in the bargaining phase. However, irrespective of the first-round offers, to estimate F_β and W_r it is sufficient to only consider the equilibrium outcome in the second-round. Under the assumption that by the second round the retiree would already know her β_i and has already decided which $a \in A_i$ to choose, the choice problem facing the retiree is relatively straightforward: choose the offer that maximizes the utility (3). Henceforth, we focus only on the second-round bargaining, which is relatively simpler to model and to use for estimation.

This multi-product feature means to fully characterize the equilibrium first-round offers we have to solve a multi-dimensional bidding problem, which is hard problem to solve and beyond the scope of this paper. The problem becomes more complex when we consider the fact that at the time of making the first-round offers, it is unlikely that firms know (β_i, θ_i).

In our empirical application we only use the chosen offers from the second-round to infer the distributions of the annuitization costs. And in the second-round, however, it is more reasonable to think that firms are able to learn retirees (β, θ) from the retirees. First, there is a lot of interactions between firms and retirees, so firms will be able to (at least) update their priors belief about β_i. We do not observe any communication in the bargaining process, so we cannot be definitive about the extent of this updating process. Second, given our assumption that retirees choose the type of annuity in the first-round, it is reasonable likely that in the second round firms will be able to know more about θ_i than they did in the first round.

We recognize that this is a strong assumption, but it allows us to keep the second-round bargaining game tractable. Otherwise, if the firms do not know the preference of the retirees, then it would lead to a bargaining game with two-sided asymmetric information. Even then we would have to make assumptions about firms' updated beliefs about (β_i, θ_i), and if and how the updating varies across retirees. So from here, we assume that firms know (β_i, θ_i) for those who opt for the second round.

The second-round is modeled as an alternating offer bargaining process. The timing of the game is as follows: In an arbitrary order, firms sequentially choose whether to improve their previous offer by a fixed amount ε or to "stay." The process ends after the round with all firms consecutively choosing to stay. Finally, the retiree then chooses any of the offers. In Lemma 1 we formalize the analysis, with the proof in the Appendix A.3.

Before we proceed we introduce some new notation. Let $P_{ij}^{\max}$ be the maximum firm j can offer to i without losing money, i.e., $P_{ij}^{\max}$ solves $C(P_{ij}^{\max}) = P_{ij}^{\max} \times UNC_j = S_i$, or equivalently $1 = r_{ij} \times \rho_i^*(P_{ij}^{\max})$ and let j_i^* denote the firm that can offer the highest utility without losing money, i.e.,

$$j_i^* := \arg\max_{j \in J} \rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij}.$$

Lemma 1. *In the bargaining game, firm j_i^* wins the annuity contract and, as ε goes to zero, ends up paying a pension $\tilde{P}_{ij_i^*}$ such that*

$$\beta_i \times Z_{ij_i^*} + \theta_i b_i(\tilde{P}_{ij_i^*}) + \rho_i(\tilde{P}_{ij_i^*}) = \max_{k \neq j_i^*} \left\{ \beta_i \times Z_{ik} + \theta_i b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \right\}. \tag{11}$$

The symmetric behavioral strategies that sustain this perfect Bayesian equilibrium are:

1. For the retiree, choose whichever firm made the best offer (including non-pecuniary attributes), i.e., retiree i chooses firm j_i^* if

$$j_i^* \;=\; \arg\max_{j \in J} \rho_i(\tilde{P}_{ij}) + \theta_i \times b_i(\tilde{P}_{ij}) + \beta_i \times Z_{ij},$$

   where $\tilde{P}_{ij}$ refers to the last offer of firm j (or to its first-stage offer if it did not raise it during the bargaining game).

2. For a firm j, play I iff $\tilde{\tilde{P}}_{ij} + \varepsilon < P_{ij}^{\max}$ and

$$\beta_i \times Z_{ij_i^*} + \theta_i b_i(\tilde{\tilde{P}}_{ij_i^*}) + \rho_i(\tilde{\tilde{P}}_{ij_i^*}) < \max_{k \neq j_i^*} \Big\{ \beta_i \times Z_{ik} + \theta_i b_i(\tilde{\tilde{P}}_{ik}) + \rho_i(\tilde{\tilde{P}}_{ik}) \Big\},$$

   where $\tilde{\tilde{P}}_{ik}$ refers to the standing offer of firm k (or to its first-stage offer when we are in the initial round of the bargaining game).

6 Identification and Estimation

In this section, we study the identification of the model parameters, which include the conditional distribution of bequest preferences $F_\theta(\cdot|S)$, the distribution of preferences for risk-ratings $F_\beta(\cdot)$, the distribution of costs $W_r(\cdot|S)$, and the channel- and savings-specific information processing cost α. We observe outcomes of the annuity process described above for N retirees who choose one of the several annuity products, where N is large.

For each retiree $i \in N$ we observe her socio-economic characteristics $X_i = (\tilde{X}_i, S_i)$, her consideration set $\mathbf{A}_i$, which is the list of annuity products that she solicits offers for, the set of firms $\tilde{J}_i$ who could participate, the set of participating firms $J_i \geq 2$, their risk-ratings $\{Z_j \in \mathbb{R} : j = 1, \ldots, J_i\}$ and their pension offers each product and the implied discounted expected utilities $\boldsymbol{\rho}_{ia} := (\rho_{1a}, \ldots, \rho_{J_i a})$ for all $a \in \mathbf{A}_i$. For each offer we can determine the corresponding bequest, if any. So, for each $a \in \mathbf{A}_i$ we also observe the implied discounted expected utilities from bequest $\boldsymbol{b}_{ia} := (b_{1a}, \ldots, b_{J_i a})$.

Let $D_i^1 \in \{1, \ldots, J+1\}$ denote i's choice in the first-stage, such that $D_i^1 = j$ means i chose firm j, and $D_i^1 = (J+1)$ means i chose to go to the second-round. Conditional on $D_i^1 = (J+1)$, we also observe j's final choice and the identity of the chosen company.

6.1 Distribution of Bequest Preference

Here we study the identification of the distribution of the preference for bequest $F_\theta(\cdot|S)$ with support $[0, \overline{\theta}]$. To this end, we rely on the fact that for each retiree we observe her final choice, which means we know her chosen bequest. Comparing the chosen bequest and the foregone bequests, relative to the difference in pensions, we can identify her bequest preference. In this exercise, we use only the offers made by the winning firms to "control" for the effect of risk-rating on choices.

For intuition, let's consider the case where the consideration sets have only two annuity products, where product 1 offers a smaller bequest–and larger pensions–than product 2. Let $a \in \{1,2\}$ denote the two products. Using (4) we can write the utility from product a as

$$U_{ij_i^*a} = \beta_i^\top Z_{ij_i^*} + \rho_{ij_i^*a} + \theta_i \times b_{ij_i^*a} - \mathfrak{U}_{0i}(S_i),$$

where $j_i^* \in J_i$ is the firm chosen by retiree i. Let $\chi_i \in \{1,2\}$ denote i's choice $a = 1$ or $a = 2$. Suppressing the index for retiree and winning firm, $\chi = 1$ if and only if $U_1 \geq U_2$, or equivalently $\theta \leq \frac{\rho_1-\rho_2}{b_2-b_1}$. Then the probability that a retiree with characteristics X chooses the annuity with the smallest bequest is

$$\Pr(\chi = 1|X) = F_{\theta|S}\left(\frac{\rho_1 - \rho_2}{b_2 - b_1}\middle| S\right) = F_{\theta|S}\left(-\frac{\Delta\rho_{12}}{\Delta b_{12}}\middle| S\right).$$

The left hand side probability $\Pr(\chi = 1|X)$ can be estimated, and we also observe the "indifference ratio" $\left\{\frac{\Delta\rho_{12}}{\Delta b_{21}}\right\}$. So if there is sufficient variation in $\tilde{X}$ across retires, and sufficient variation in the indifference ratios across retirees and firms, we can "trace" $F_\theta(\cdot|S)$ everywhere over $[0,\overline{\theta}]$. Formally, if for $t \in [0,\overline{\theta}]$ there is a pair $\{\Delta\rho_{12}, \Delta b_{21}\}$ in the data such that $t = -\Delta\rho_{12}/\Delta b_{12}$ then the distribution is nonparametrically identified.

If there are more than 2 products in the consideration set, i.e., $\mathbf{A} \geq 2$, then we can order them from that with the lowest bequest to the highest bequest, the probability that a retiree with X chooses the annuity with lowest bequest is given by

$$\Pr(\chi = 1|\tilde{X}, S) \;\; = \;\; \int_\Theta \mathbb{1}\left\{U_1 \geq U_a, a \in \mathbf{A}|\theta\right\} dF_{\theta|S}(\theta|S) = F_{\theta|S}\left(\min_{1<a\leq A}\left\{-\frac{\Delta\rho_{1a}}{\Delta b_{1a}}\right\}\middle|S\right).$$

Similarly, the probability of *not choosing* the annuity with the *largest* bequest is given by

$$\Pr(\chi \neq |\mathbf{A}|\Big|\tilde{X}, S) = F_{\theta|S}\left(\max_{1\leq a<A}\left\{-\frac{\Delta\rho_{Aa}}{\Delta b_{Aa}}\right\}\middle|S\right).$$

So, we can use these two equations to identify $F_\theta(\cdot|S)$, where, as mentioned above, the identifying source of variation are, $\tilde{X}$, annuitization costs across firms, number of participating firms, which in turn lead to variations will induce variation in pensions and bequests.

6.2 Information Processing Cost

Here, we verify that the channel- and savings-specific information processing cost can be identified from our data. Let $\mathcal{J}$ denote the unique values of J_i across all $i \in N$. Consider the subset of retirees with $|J_i| = J$. Then, we can identify the conditional choice probability for $j \in (J+1)$, including the option, being chosen, given $X = x$, $Z = z$ and $(\boldsymbol{\rho}, \mathbf{b})$, by

$$\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J) = \sum_i \frac{\mathbb{1}[D_i^1 = j, X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}{\sum_i \mathbb{1}[X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}; \quad \hat{\sigma}_{J+1}(\tilde{x}, z, \boldsymbol{\rho}, \mathbf{b}|J) = 1 - \sum_{j=1}^{J} \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}). \tag{12}$$

Applying (12), to the relevant subsample, we can identify $\{\sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J)\}_{j \in J}$ for all $J \in \mathcal{J}$. We can also identify the probability that there are J participating firms as $p(J) = \#\{\texttt{retirees with } J_i = J\}/N$, and together we identify $\sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}) = \sum_{J \in \mathcal{J}} \sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J) \times p(J)$. Integrating (7) with respect to F_β and using the definition of $\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b})$ gives

$$\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}) = \int \frac{\exp\left(\log \sigma_j^0 + \frac{U_{ij}}{\alpha}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ij}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)} dF_\beta(\beta). \tag{13}$$

Taking the derivative of (13) with respect to ρ_j identifies the cost $\alpha = \frac{\sigma_j(x,z,\boldsymbol{\rho},\mathbf{b})(1-\hat{\sigma}_j(x,z,\boldsymbol{\rho},\mathbf{b}))}{\frac{\partial \hat{\sigma}_j(x,z,\boldsymbol{\rho},\mathbf{b})}{\partial \rho_j}}$.[24]

Thus, the information processing cost depends on the elasticity of the choice probability with respect to ρ. Consider an extreme case when the choice for j is insensitive to changes in premium, i.e., $\frac{\partial \hat{\sigma}_j(x,z,\boldsymbol{\rho})}{\partial \rho_j} \approx 0$ then it implies that $\alpha \approx +\infty$ because the only way to rationalize the fact that retirees do not respond to changes in pension is that their information processing cost is extremely large. If the demand is elastic with respect to the pensions then the cost of processing information is low, and vice versa. To identify the cost as a function of the channel and savings, we can use the appropriate subsample, and follow the above steps.

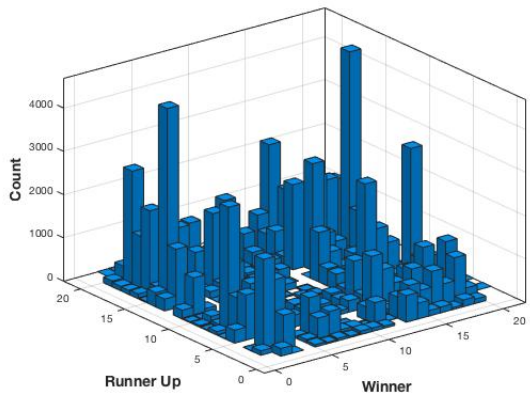
6.3 Risk-Rating Preferences and Annuitization Costs

To identify the preference distribution F_β and the cost distribution W_r it is sufficient to consider only those who buy annuities in the second round, where the chosen pension and bequests are given by (11). Let $\tilde{P}_{ij_i^*}$ be the chosen offer. Then from (11) $\tilde{P}_{ij_i^*}$ satisfies

$$\rho_i(\tilde{P}_{ij_i^*}) + \theta_i b_i(\tilde{P}_{ij_i^*}) = \max_{k \neq j_i^*} \left\{ \beta_i \times Z_{ik} + \theta_i b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \right\} - \beta_i \times Z_{ij_i^*}. \tag{14}$$

[24] To estimate α, we use a logit specification to model the LHS of (13) so the derivatives are well defined.

Figure 5: **Histogram of Winner and Runner Up**



Note. This is a histogram of the identity of winning firm (x-axis) and the identity of runner-up (y-axis) across retirees. The runner-up firm for a retiree is the firm that has the largest probability of being chosen by the retiree after excluding the firm that was chosen, where the probabilities are estimated using Logistic regression as explained in Appendix A.2.

Let k_i^* denote the runner-up company in i's auction. Then we can re-write (14) as

$$\begin{aligned}\rho_i(\tilde{P}_{ij_i^*}) + \theta_i b_i(\tilde{P}_{ij_i^*}) &= \mathfrak{U}(\tilde{P}_{ij_i^*}, B_{ij}(\tilde{P}_{ij_i^*}); \theta_i) = \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \theta_i b_i(P^{\max}_{ik_i^*}) + \rho_i(P^{\max}_{ik_i^*}) \\ &= \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \mathfrak{U}(P^{\max}_{ik_i^*}, B_{ik_i^*}(P^{\max}_{ik_i^*}); \theta_i) \\ &\equiv \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \varpi_i, \end{aligned} \tag{15}$$

where the first equality follows from (3), and $\varpi \sim F_\varpi$ is the highest gross utility that the runner up firm k_i^* can offer to retiree i. Notice that the left-hand side terms can be determined from the chosen annuity, and if we view ϖ as an error then, (15) is the random coefficient model. From the literature on random coefficient (Hoderlein, Klemelä, and Mammen, 2010) we know that the distributions F_β and F_ϖ are nonparametrically identified under our maintained assumption that $(Z_{ik_i^*} - Z_{ij_i^*})$ and β_i and ϖ_i are uncorrelated and there is sufficient variation in $(Z_{ik_i^*} - Z_{ij_i^*})$. The runner-up and the winner firm pairs vary across retirees which ensures the difference $(Z_{ik_i^*} - Z_{ij_i^*})$ also varies as can be seen in Figure 5.

Next step is to show that we can determine $W_r(\cdot)$ from $\{F_\beta, F_\theta\}$. The argument is based on the following steps. First, note that for each draw $\theta_i \sim F_\theta$, the distribution of the LHS in (15) is also the distribution of the second largest value of the RHS in (15). Second, from this distribution of the order statistics, we can identify the parent distribution of the RHS in (15). Third, this parent distribution is a convolution of the distribution of $\beta_i \times (Z_{ik_i^*} - Z_{ij_i^*})$ and the distribution of ϖ_i, which in turn identifies the distribution of ϖ_i. Fourth, we know that there is a one to one mapping from ϖ_i to $P^{\max}_{ik_i^*}$ –the maximum pension runner-up firm k_i^* can offer to retiree i, see Equation (A.7), which together with the definition that $C(P_{ik_i^*}^{\max}) = S_i$ identifies the distribution of $r = \frac{UNC_{k_i^*}}{UNC_i} = \frac{S_i}{P_{ik_i^*}^{\max} \times UNC_i}$. We formalize these steps in the following result, and provide the proof in Appendix A.3.

Lemma 2. $W_r(\cdot|S)$ can be nonparametrically identified from $\{F_\beta, F_\theta\}$.

Selective Entry. Let $\tilde{J}$ be the set of companies that are interested in selling annuities to i with characteristics X_i. When i requests offers for a product, company $j \in \tilde{J}$ observes its cost r_j, and all firms simultaneously decide whether or not to participate, and it costs (the same) $\kappa \geq 0$ for each company to participate. This cost captures the opportunity cost to participate, and it can vary across retirees. Let $J \subset \tilde{J}$ denote the set of participating companies. All the firms that participate simultaneously make their offers.

Under the symmetric Perfect Bayesian-Nash equilibrium the entry decision is characterized by a unique threshold $r^* \in [\underline{r}, \overline{r}]$ such that firms participate only if their costs are less than r^*. Then the cost distribution among the participating firms is $W_r^*(r; \tilde{J}) := W_r(r|r \leq r^*; \tilde{J}) = W_r(r)/W_r(r^*; \tilde{J})$. Let $r_{\tilde{J}}^*$ be the threshold with $\tilde{J}$ potential bidders, and suppose $\tilde{J} \in \mathcal{J} := \{\underline{J}, \ldots, \overline{J}\}$, where $\overline{J}$ is the maximum number of potential bidders and $\underline{J}$ is the smallest number of potential bidders. All else equal, $r_{\tilde{J}}^*$ decreases with $\tilde{J}$, so $W_r(r)$ is identified on the support $[\underline{r}, r_{\underline{J}}^*]$.

6.4 Estimation Steps for Risk-Rating Preference and Annuitization Cost Distribution

Here, we present the steps that we take to estimate the conditional distributions of β and r. Although we can nonparametrically identify $F_\beta(\cdot|X)$, we impose parametric assumption about the density for estimation. In particular, we divide retirees into separate groups based on gender, three age groups and savings quintiles and three channels, which gives us a total of $G = 90$ groups, and further assume that β_i in (15) is Normally distributed, $\beta_i \sim \mathcal{N}(\beta_{g(i)}, \sigma_{g(i)})$ where $g(i) \in G$ is i's group. Thus, we allow each group to have a group-specific mean and variance of β. Similarly, we assume that savings affect r through the savings quintiles S_q, i.e. $r \sim W_r(\cdot|S_q)$, where S_q is the $q \in \{1, \ldots, 5\}$ quintile of savings.

Let $N_{q,J}$ denote the subset of retirees in the $q^{th}-$quintile and have $J \in \{13, 14, 15\}$ potential bidders. Then, we can re-write our estimation Equation (15) with group-specific coefficients for each $(q, J) \in \{1, \ldots, 5\} \times \{13, 14, 15\}$ pair as

$$\rho_{j_i^*} + \theta_i \times b_{j_i^*} = \beta_{g(i)} \times (Z_{k_i^*} - Z_{j_i^*}) + \varpi_{k_i^*}; \quad i = 1, 2, \ldots, N_{q,J}, \tag{16}$$

and $\beta_g = \beta_0 + \upsilon_g$ where $\mathbb{E}(\upsilon_g) = 0$ and $\mathbb{E}(\upsilon_g^2) < \infty$. Applying GLS to equation (16) we estimate group specific β_g and $\varpi_{k_i^*}$ for all $i \in N_{q,J}$.

Next, using the estimated $\hat{F}_\theta(\cdot|S_q)$ we can "integrate-out" θ from the estimation equation. For each (q, J) and each $i \in N_g$, we generate i.i.d. samples $\{\theta_{i,\ell}\}_{i=1}^{|N_g(i)|} \sim \hat{F}_\theta(\cdot|S_q)$, and estimate $\{\hat{\beta}_g^\ell : g = 1, \ldots, G\}$ applying generalized least squared method to

$$\rho_{j_i^*} + \theta_i^\ell \times b_{j_i^*} = \beta_{g(i)} \times (Z_{k_i^*} - Z_{j_i^*}) + \varpi_{k_i^*}^\ell. \tag{17}$$

We repeat this exercise for $L = 10,000$ sample draws of θ, which, for each group $g \in G$ gives us 10,000 estimates $\{\hat{\beta}_g^\ell\}_{\ell=1}^L$, and averaging across those samples give $\hat{\beta}_g = L^{-1}\sum_{\ell=1}^L \hat{\beta}_g^\ell$.

To estimate $W_r(\cdot|S_q)$ we focus on the sub-sample of retirees that have the top two firms with the same risk-ratings. In our sample close to 60,000 retirees are in this group and have $(Z_{k_i^*} - Z_{j_i^*}) = 0$. Substituting this in (17), for $J \in \{13, 14, 15\}$ gives

$$\rho_{j_i^*} + \theta_i^\ell b_{j_i^*} = \varpi_{k_i^*}^\ell, \tag{18}$$

where the left hand is the known winning utility and the right-hand side is the unobserved maximum utility the runner-up firm can offer without incurring loss. Thus, the estimation problem in (18) becomes similar to the estimation problem in a standard English auction where only the winning bid is observed. The key difference here is that everything is expressed in terms of winning utility and not the bid. From the estimated distribution of $(\rho_{j_i^*} + \theta_i^\ell b_{j_i^*})$ we can estimate the parent distribution of $\hat{r}_{ij}$, i.e., $W_r^*(\cdot|S_q, J)$ using a Kernel Density Estimator.

7 Estimation Results

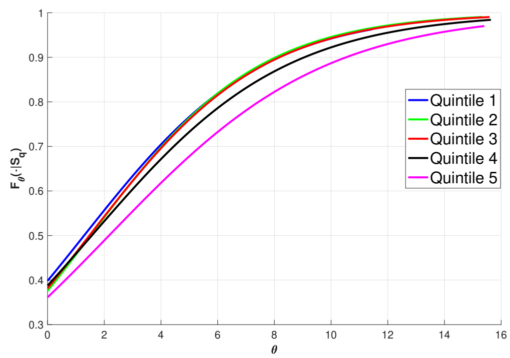
Preferences for Bequests.

In Figure 6 we display the estimated $\{\hat{F}_\theta(\cdot|S_q), q = 1, \ldots, 5\}$ conditional distributions of preferences for leaving bequests. Our estimates exploit the fact that there is a nonlinear relationship between the preference for bequests, the retiree's mortality risk, her savings, and the pension offers she received for different types of annuities, with different bequest.

Our estimates suggest three features of θ. First, irrespective of their savings, approximately 40% of retirees do not value leaving bequests. In fact, except for those in the highest savings quintile, the median θ is either 0 or very close to 0 (second column of Table 11). Among the rest, there is a lot of variation within and across different savings quintiles.[25]

[25] Kopczuk and Lupton (2007) provides a nice discussion about possible variation in bequest preference.

Figure 6: **Estimated Distributions of Bequest Preferences**



Note. This figure displays estimated conditional distribution of preference for bequests $F_\theta(\cdot|S_q)$ given savings quintile $S_q, q = 1, \ldots, 5$, as we move from the left to the right.

Second, the preference θ increases with savings and mean θ also increases with the savings, see Figure 6. This result is consistent with the hypothesis that with decreasing marginal utility from a pension, the marginal utility of bequest for an altruist person is the product of the preference θ and the marginal utility of the bequest recipients. Third, from Table 11, we also see that although the mean of θ suggests that, at the margin, retirees value bequest twice as much as they value self-consumption, the median is almost zero. This suggests that there is significant variation in preference for bequest. Indeed, both the standard deviations and the interquartile ranges of θ are larger than the mean and they increase with savings.

Table 11: **Summary Statistics of Preference for Bequests**

Savings	**Mean**	**Median**	**Std. Dev.**	**IQR**
Q1	1.92	0	2.82	3.34
Q2	2.22	0.1	3.22	3.77
Q3	2.25	0	3.27	3.85
Q4	2.41	0	3.5	4.13
Q5	2.82	0.35	3.82	5.01

Note. Mean, median, standard deviation and inter-quartile range of preference for bequests, by saving quintiles. These statistics are calculated using simulated θ from $\{\hat{F}_\theta(\cdot|S_q)\}_{q=1}^{5}$ as shown in Figure 6.

Preferences for Risk-Ratings and Information Processing Costs

Next, we present the estimates of the preference for risk-rating. Figure 7 displays the group-specific means of β, with their corresponding 95% confidence intervals. The mean of β is always non-negative, which suggests that retirees prefer firms with higher risk-rating, but the strength varies across groups. Interestingly, the estimates suggest that those in the lowest two savings quintiles care the most about firms' risk-ratings than the others. And even among these retirees, males exhibit a stronger preference for risk-ratings than females, and although the difference varies by retirement age and channels in some cases they are statistically equal.

In the face of it, our result that those mean β decreases with savings appear counter-intuitive, because first if the risk-rating is a proxy for financial health then everyone should have the same preference for risk-rating. Second, since those with higher savings are more exposed to the bankruptcy risk than retirees with lower savings because of the government's guarantee, it stands to reason that the former groups should exhibit a stronger preference for risk-rating. Yet, we see that those with higher savings on average do not value risk-ratings.

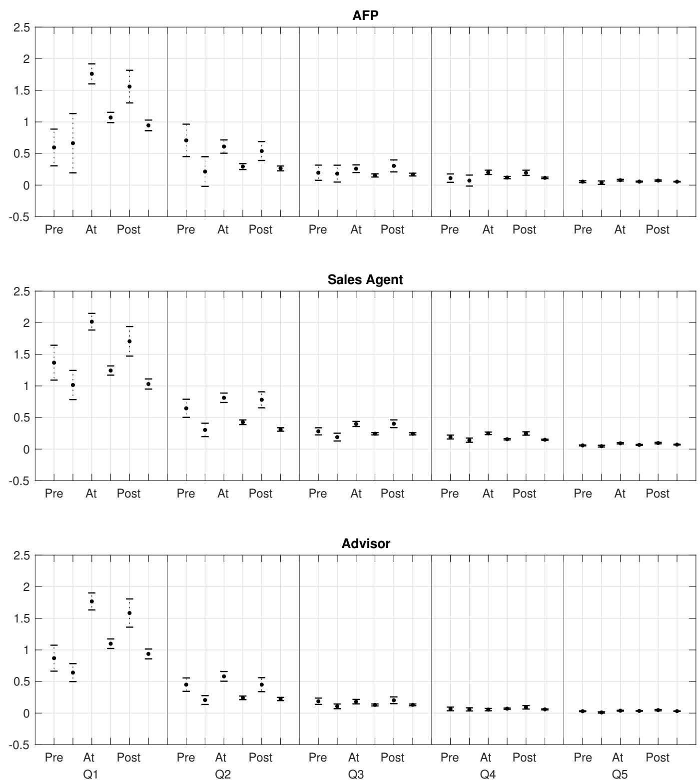
Our model suggests that one of the reasons for this discrepancy is the differences in information processing cost (α_g) across savings quintile as shown in Table 12. If, the prior mean of β is positive for every group, then depending on their respective information processing costs (α_g) retirees revise their beliefs through due diligence. The fact that bankruptcy is a very rare event in Chile and that many firms have the best risk-rating suggest that retirees should not care so those retirees "update" their beliefs and give less weight to risk-rating. Indeed, as shown in Table 12, we find that the information processing cost decreases with savings, and the absolute decrease is largest among the retirees with the lowest quintile and who have sales-agents. This could be because those with higher savings tend to be educated, who in turn can have lower information processing costs.

Table 12: **Information Processing Cost**

Savings	**AFP**	**Sales Agent**	**Advisor**	**Full Sample**
Q1	0.009	0.027	0.006	0.021
Q2	0.006	0.019	0.004	0.016
Q3	0.005	0.013	0.003	0.013
Q4	0.005	0.012	0.003	0.005
Q5	0.005	0.012	0.003	0.006
Overall	0.005	0.013	0.003	0.009

Note. Estimates of the median of information processing cost, by savings quintiles and intermediary channel.

Figure 7: **Group-Specific Mean of Preferences for Risk-Ratings**



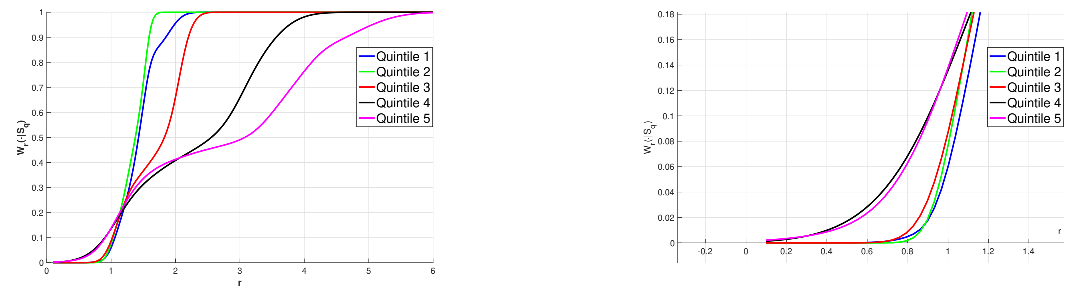
Note. These figures display the estimates for group-specific mean of $\mathbb{E}(\beta_g)$, from (15). Each panel (row) corresponds to a channel, and each channel is divided into five quintiles. And within each channel-quintile box, parameters are ordered by retirement age (before, after or at NRA), and for each age group, the two estimates correspond to male and a female respectively. The two bars represent 95% confidence intervals.

Annuitization Costs

In Figure 8i, we present the estimates of the conditional distributions of costs r, given the savings quintile. Recall that r_{ij} is the ratio of firm j's UNC to retiree i's UNC, so, larger r_{ij} means that firm j cost of annuitizing i's savings is large. The advantage of working with r_{ij} instead of UNC_j is that once we normalize UNC_j by UNC_i we get a unit-free measure of the cost that is comparable across retirees with different mortality force.

As we can see from Figure 8i, the relative annuitization cost increases with savings, although the average increase is not too big. The average cost varies between 2.4 to is 2.7, and the median is more or less constant at 3.12; see Table 13. This finding is consistent with the prior research that finds that even after conditioning on the initial health status, wealth

Figure 8: **Conditional Distributions of Annuitization Costs**



(i) Over the Full Support

(ii) Focusing on $r < 1$.

Note. The first sub-figure shows the estimated conditional distribution of relative annuitization costs, by savings quintiles. The second figure shows the same distributions but focuses only on the support $r < 1$.

rankings are important determinants of mortality, e.g., Attanasio and Emmerson (2003).

It is nonetheless interesting and important to consider the shapes of the distributions below $r = 1$ as shown in Figure 8ii. In equilibrium, pensions are determined by the lowest two order-statistics of the cost which in turn depends on the left tail of the distributions (Figure 8ii). The cost distributions cross around $r = 1$, which means if there are sufficiently many bidders, then in equilibrium winning firms have lower costs from retirees with higher savings.

To illustrate this better, for each quintile, we determine the maximum pension that can be offered to someone with median savings, within each quintile.

Table 13: **Summary Statistics of r**

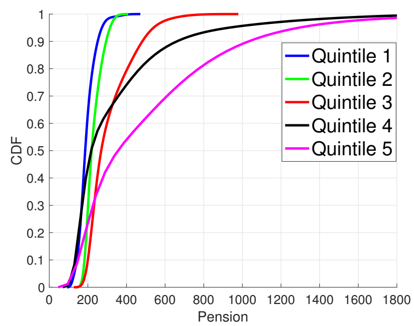
Savings	**Mean**	**Median**	**Std. Dev.**	**IQR**
Q1	2.74	3.1	1.47	2.7
Q2	2.75	3.11	1.47	2.7
Q3	2.73	3.07	1.46	2.69
Q4	2.77	3.12	1.47	2.69
Q5	2.76	3.12	1.48	2.72

Note. The table displays mean, median, standard deviation and inter-quartile range of the annuitization costs r. These statistics are calculated using simulated r from $\{\hat{W}_r(\cdot|S_q)\}_{q=1}^5$ as shown in Figure 8i.

In particular, we implement the following simulation exercise: (i) for each savings quintile we identify the retiree with the median income (among this subsample); (ii) simulate $\{r^{(\ell)} : \ell = 1, \ldots, 1000\}$'s from the relevant distribution $\hat{W}_r(\cdot|\cdot)$; (ii) using the savings and the estimated UNC_i of the retiree identified in step (i), for each draw $r^{(\ell)}$ determine UNC_j and from that the maximum pension a firm can offer without making a loss, i.e., we determine $P_{ij} = S_i/UNC_j$; and (iii) plot the distribution of this maximum pension.

The resulting distributions are shown in Figure 9. As can be seen from the figure, the fact that the cost distributions for higher savings are stochastically dominated by the cost for lower savings at $r \leq 1$ translates into larger pensions for those with higher savings. This has an important implication on the utility of the retirees as we discuss in the next section.

Figure 9: **Distributions of Maximum Pension $P^{\max}$**



Note. Conditional distributions of maximum pensions for retiree with median savings within each quintile. For each savings quintile $1 \leq q \leq 5$ we simulate several r's from $\hat{W}_r(\cdot|S_q)$ that is displayed in Figure 8i, and we determine the median savings among this group. Using these r and the median saving, we determine the maximum pensions $P^{\max}$ that firms can offer without making loss and estimate the distribution of $P^{\max}$.

8 Counterfactual Results

The results above suggest that the retirees in the lowest two savings quintiles exhibit lower elasticity with respect to the pension. According to our model estimates, the reason for this lower elasticity is that these retirees have higher information processing costs. Possibly because of the high information processing costs, their choices are consistent with those who care a lot about firms' risk-ratings. Based on these results, we consider ways to improve the market, some of which are also under debate in the Chilean parliament: (i) simplify the current system by replacing it with the standard English auction; (ii) remove risk-ratings from the supply side to increase competition by selecting the firm that pays the highest pension, and (iii) automate the system so retirees do not use risk-ratings to choose a firm.

The demand-side estimates, in particular the fact that lower savings retirees care more about risk-ratings, suggest that if we shut down risk-ratings then it will level the "playing field" and increase competition and pensions, especially for those with lower savings. Moreover, diminishing marginal utility means that higher pensions do not necessarily translate into higher discounted expected utility. We present gross utilities using both the estimated preference for risk-rating and setting the risk-rating to zero.

Our estimates are representative of the entire market, so we can use counterfactual simulations to evaluate the effects of these changes. For comparison of outcomes across different mechanisms, we present the first-best full-information outcomes, i.e., when firms' annuitization costs are publicly known and the winning firm offers the maximum possible pension (without making a loss). Next, we present pensions and retirees' gross utility under the current system, full information and English auction with and without reserve prices.

8.1 Complete Information

We begin by considering the effect of asymmetric information on the pension and the margin, and how that varies across different savings quintiles and competition. To determine the pensions under full information, we divide retirees into 15 groups based on their savings quintiles and their corresponding potential number of bidders. Then, for each retiree in a group, we simulate as many r from the appropriate $\hat{W}_r(\cdot|S_q)$ as the potential number of bidders present, and determine the lowest cost among those draws. The winner will be the bidder with the lowest cost. Then, we determine the zero-profit pension the winning firm can offer. For every retiree, we repeat this step 10,000 times and calculate the average pension.

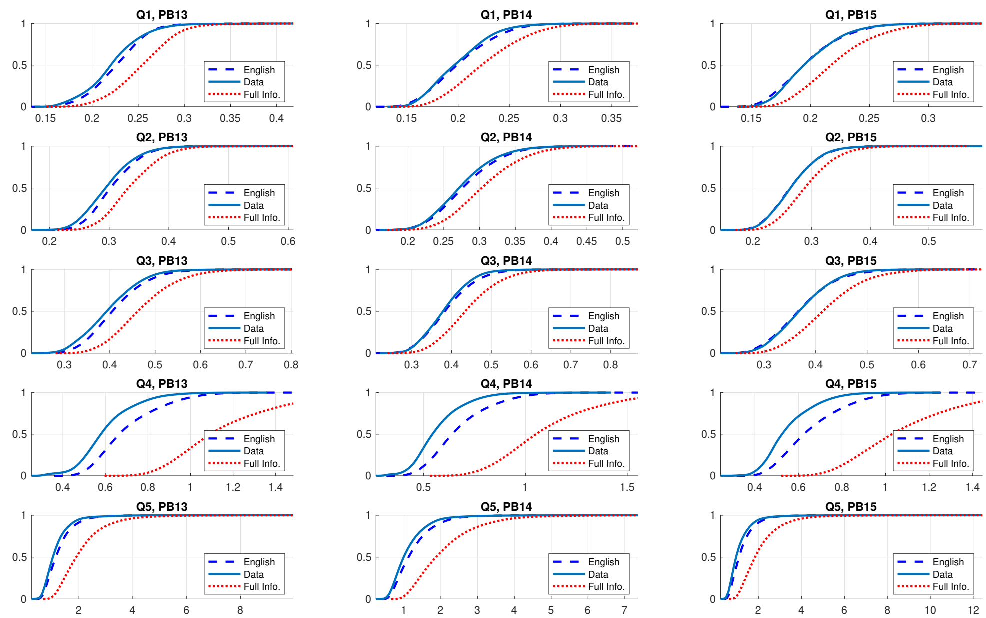
In Figure 10 we present the distributions of chosen pensions (in solid blue line) and the pension if there was no private information about r (in the dotted red line).[26] As expected, the pension distribution under full-information first-order stochastically dominates the distribution of the observed pensions. Interestingly, we find that the gap between the two distributions is largest for those with higher savings, suggesting that the firms have a larger margin from this group.

Another way to present this pattern is to consider current pension as a percentage of the pension under full information, and take an average across all retirees within each group. These numbers are displayed in Table 14, where we also display the median ratio to account for possible outliers. The table includes the median ratio for both the current scheme and the English auction. Consistent with Figure 10, we see that for the lowest three quintiles the pensions under the current system are close to 90% of the pension under full information, on average. Similarly, the median is also high for these three quintiles. However, for those with the top two quintiles, the offered pension is only 60%.

Next, we also consider the money's worth ratio for each group. As we have explained earlier in Section 4, the money's worth ratio is the return a retiree can expect to earn per

[26]The dotted blue line in the figures represents pension under English auction when we shut down risk-rating. We explain this later in the next section.

Figure 10: **Estimated Distributions of Pensions**



Note. Distributions of pensions (in thousands of U.S. dollars) under the current system (solid blue), under English auction (dashed blue) and under full-information (dotted red), by savings quintiles (rows) and the number of potential bidders (columns). The sample includes only those who choose in the second-round.

Table 14: **Pensions under Current and English auctions, relative to Full Info.**

Savings \Potential Bidders	13	14	15
Q1	(87%, 87%)	(91%, 91%)	(93%, 93%)
	(88%, 88%)	(89%, 89%)	(89%, 89%)
Q2	(89%, 90%)	(94%, 93%)	(97%, 97%)
	(90%, 90%)	(91%, 91%)	(91%, 91%)
Q3	(88%, 90%)	(92%, 93%)	(95%, 96%)
	(87%, 87%)	(88%, 88%)	(88%, 88%)
Q4	(54%, 55%)	(55%, 56%)	(56%, 56%)
	(60%, 60%)	(60%, 60%)	(60%, 60%)
Q5	(55%, 56%)	(56,% 57%)	(57%, 57%)
	(60%, 60%)	(60%, 59%)	(59%, 59%)

Note. Mean and median of pensions under the current system and under English Auction, expressed as a percentage of the pension under full information, separated by savings quintile (rows) and the number of potential bidders (columns). Each entry has two rows, the first row corresponds to the current system and the second row corresponds to the English auction.

annuitized dollar. If this ratio is greater (less) than one, then the retiree expects to earn more (less) than she annuitizes.[27]

Given our interest, instead of presenting individual money's worth ratios in Table 15, we present the group-specific money's worth ratios, which are equal to the ratio $(\sum_i P_i \times UNC_i)/\sum_i S_i$, where the sum is over all retirees in the respective group. As we can see from the first column, under the current system, those with AFP (the first row within each quintile) get better moneys' worth ratio than the other two channels. We also see that those with higher savings get a slightly better offer than those with lower savings. If we compare the first and the last columns in Table 15, we see that as before, the gap between the current system and that under the full-information is the largest for those with higher savings.

Table 15: **Money's Worth Ratio, by Savings Quintile and Channel**

Savings Quintile	Channel	Current	English	Optimal	Full Info.
Q1	AFP	0.99018	0.93229	0.93234	1.04419
	Sales-Agent	0.95663	0.93128	0.93128	1.04327
	Advisor	0.95969	0.93019	0.93019	1.04237
Q2	AFP	1.02480	0.95833	0.95837	1.04920
	Sales-Agent	0.99589	0.95728	0.95727	1.04841
	Advisor	0.99624	0.95608	0.95609	1.04748
Q3	AFP	1.04418	0.96340	0.96347	1.08998
	Sales-Agent	1.02315	0.96216	0.96218	1.08906
	Advisor	1.01623	0.96067	0.96073	1.08796
Q4	AFP	1.06109	1.13492	1.13504	1.86677
	Sales-Agent	1.04144	1.13166	1.13161	1.86129
	Advisor	1.03278	1.12759	1.12760	1.85429
Q5	AFP	1.09793	1.12368	1.12392	1.87748
	Sales-Agent	1.07350	1.12027	1.12038	1.87109
	Advisor	1.06609	1.11688	1.11711	1.86514

Note. Each row denotes a different group, and each entry is money's worth ratio for that group, which is equal to $(\sum_i P_i \times UNC_i)/\sum_i S_i$, where the sum is taken over all retirees in the group. There are 15 groups based on 5 savings quintiles and 3 channels. Each column corresponds to a different pricing mechanism, where English is the standard English auction and optimal is the English auction with optimal reserve price.

8.2 English Auction

One way to increase pensions is to make the system more competitive. And for that, we can replace the current system with the standard English auction, and also "shut down" the risk-ratings in the supply side by picking the winner to be the firm that offers the highest pension. Simplifying the process should improve outcomes for those who choose in the first round,

[27]Formally, the money's worth ratio for i, under the current system, is equal to i's chosen pension times her UNC_i divided by her savings S_i. For more on the use of money's worth ratio to study the generosity of an annuity contract see Mitchell et al. (1999).

where the pensions tend to be lower than in the second round. Similarly, shutting down risk-rating should force firms to bid more aggressively, which should benefit retirees with lower savings more than those with higher savings because the former have stronger preferences for risk-ratings, which means without risk-ratings the firms should be more aggressive if the retiree is of lower savings. On the other hand, as we saw before, the gap between the chosen pension and the full information pension is the largest for those with higher savings, so they may benefit the most from the new mechanism.

Next, we implement the standard English auction by treating the potential bidders as the actual bidders. Our results are an upper bound on the effect of English auction on pensions and retirees' ex-post expected present discounted gross utilities. We follow the same steps as in the full-information counterfactual, except under the English auction, the winning pension is the maximum pension a firm with the second-lowest cost (r) can offer, at zero profit.

We present the Kernel density estimates of the distributions of winning pensions under English auction in Figure 10. Although English auction leads to higher pensions, most of the benefits accrue to those in the top two savings quintiles. We can also see this in Table 14 second row for each quintile, where we present the mean and the median pension under English auction expressed as a percentage of the pension under full-information. Similar results hold with the money's worth ratio, see the first two columns in Table 15.

We are also interested in determining what is the effect of using English auction on retiree's utilities. Although without estimating the utility from the outside option, which in the case of annuities can be very involved, we cannot calculate the net ex-post expected present discounted utility, we can determine the ex-post gross utility which is equal to $\beta_i \times Z_j + \rho_{ij} + \theta_i b_{ij}$. Here, ex-post refers to the utility after the mechanism has been implemented. It is important to distinguish ex-post from ex-ante utilities because we know that despite the revenue-equivalence, a risk-averse retiree prefers First-Price auction to English-Auction. After all, the latter has more variance than the former.

For each retiree and each mechanism using the "winning" pensions, we first determine the bequest (if any) and then calculate the ex-post expected present discounted utilities. To shed light on the effect of shutting down risk-ratings on the retirees' utilities, for each mechanism we calculate two utilities: one with $\beta_i \times Z_j$ and one without Z_j by setting $\beta_i = 0$. To calculate the utility from the risk-rating, we use simulated data under the assumption that β_i is Normal with estimated group-specific mean and variance.

We present the average utilities across different groups in Tables 16 and 17. In Table 16 we group retirees, by their savings quintiles and the potential number of bidders, and in Table 17 we group retirees by their savings quintiles and their channels. In each table, and for each mechanism, we have two columns, one with and one without (asterisk) β, respectively.

Note that for each quintile in Table 16, by comparing the rows we can see that the utilities increase with the number of bidders, because the pensions increase when there are more firms. However, despite the large gap between the pensions under the current system or the pensions under English auction and the pension under the full information (Figure 10) our estimates show that the gap in utilities are almost negligible.

Similarly, from Table 17 we can see that similar results hold even if we group retirees by their savings quintile and their channel. Nonetheless, what is new and interesting is that those who have sale agents (second row in each savings quintile) have higher utilities than other channels, and this difference decreases with savings.

Table 16: **Average Gross Utility, by Savings Quintile and Potential Bidders**

Current	**English**	**Optimal**	**Full Info.**	**Current***	**English***	**Optimal***	**Full Info.***
8.8176	8.8180	8.8179	8.8191	-0.0054	-0.0049	-0.0049	-0.0039
6.9852	6.9851	6.9850	6.9866	-0.0073	-0.0073	-0.0073	-0.0058
11.9204	11.9200	11.9198	11.9215	-0.0073	-0.0077	-0.0077	-0.0061
3.5616	3.5618	3.5618	3.5622	-0.0027	-0.0026	-0.0026	-0.0021
3.5055	3.5054	3.5054	3.5061	-0.0038	-0.0040	-0.0040	-0.0033
4.2757	4.2753	4.2753	4.2760	-0.0042	-0.0046	-0.0046	-0.0039
2.5903	2.5903	2.5903	2.5907	-0.0015	-0.0014	-0.0014	-0.0011
2.6788	2.6787	2.6787	2.6791	-0.0018	-0.0020	-0.0020	-0.0015
2.8087	2.8084	2.8084	2.8089	-0.0021	-0.0024	-0.0024	-0.0018
2.4089	2.4091	2.4091	2.4095	-0.0007	-0.0005	-0.0005	-0.0002
2.4462	2.4464	2.4464	2.4468	-0.0009	-0.0007	-0.0007	-0.0003
2.4724	2.4726	2.4726	2.4731	-0.0010	-0.0008	-0.0008	-0.0003
2.3357	2.3358	2.3359	2.3359	-0.0003	-0.0002	-0.0002	-0.0001
2.2684	2.2684	2.2683	2.2686	-0.0004	-0.0003	-0.0003	-0.0001
2.3018	2.3019	2.3019	2.3021	-0.0004	-0.0004	-0.0004	-0.0001

Note. The table displays the gross utility, see Equation (4), under 4 (current, English auction, English auction with optimal reserve price and full information) pricing mechanisms, averaged over subgroups defined by savings quintile and potential bidders. Each quintile is separated by a horizontal line, and within each line, the rows reflect the number of potential bidders $\{13, 14, 15\}$. The first four columns use the estimated β (c.f. Figure 7) in calculating the utility and the last four columns (with asterisk) set $\beta = 0$ in (4).

8.3 English Auctions with Reserve Price

Next, we explore if we improve the outcomes for retirees when implementing optimally chosen reserve price. On the one hand, having a reserve price should increase the pension, especially for retirees with higher savings where the gap between the current pension and the first-best pension is the largest. On the other hand, we know that the optimal reserve price does not depend on the number of bidders, and with 13 to 15 bidders, the effect of reserve price can be small.

Table 17: **Average Gross Utility, by Savings Quintile and Channel**

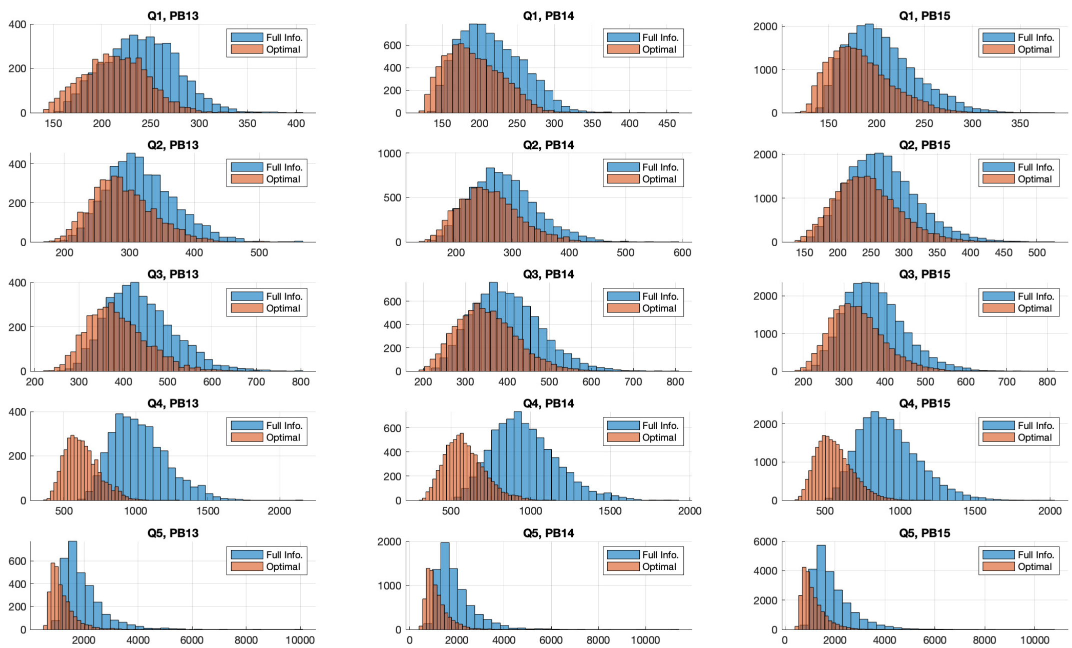
Current	English	Optimal	Full Info.	Current*	English*	Optimal*	Full Info.*
9.2078	9.2073	9.2071	9.2087	-0.0066	-0.0071	-0.0071	-0.0057
11.7779	11.7778	11.7777	11.7794	-0.0075	-0.0075	-0.0075	-0.006
9.239	9.2388	9.2388	9.2402	-0.0068	-0.0069	-0.0069	-0.0055
3.7995	3.799	3.7991	3.7998	-0.0038	-0.0043	-0.0043	-0.0036
4.4095	4.4092	4.4093	4.4099	-0.0041	-0.0043	-0.0043	-0.0036
3.585	3.5848	3.5847	3.5854	-0.0038	-0.004	-0.004	-0.0033
2.6741	2.6738	2.6738	2.6743	-0.0019	-0.0022	-0.0022	-0.0017
2.9609	2.9607	2.9607	2.9611	-0.002	-0.0022	-0.0022	-0.0017
2.5351	2.535	2.5349	2.5354	-0.0019	-0.0021	-0.0021	-0.0016
2.4637	2.4639	2.4639	2.4644	-0.0009	-0.0008	-0.0008	-0.0003
2.5845	2.5847	2.5847	2.5852	-0.001	-0.0008	-0.0008	-0.0003
2.2824	2.2826	2.2826	2.283	-0.0009	-0.0007	-0.0007	-0.0003
2.3075	2.3076	2.3076	2.3078	-0.0004	-0.0003	-0.0003	-0.0001
2.3537	2.3537	2.3537	2.354	-0.0004	-0.0004	-0.0004	-0.0001
2.2215	2.2216	2.2216	2.2218	-0.0004	-0.0003	-0.0003	-0.0001

Note. The table displays the gross utility, see Equation (4), under 4 (current, English auction, English auction with optimal reserve price and full information) pricing mechanisms, averaged over subgroups defined by savings quintile and the three channels: AFP, Sales-Agents and Advisors. The first four columns use the estimated β (Figure 7) to calculate the utility and the last four columns (with asterisk) set $\beta = 0$ in (4).

Recall that in an auction where the seller's outside option is v_0 and the bidder's valuation is distributed as F_v with density f_v then the optimal reserve price t solves $t = v_0 - \frac{1-F_v(t)}{f_v(t)}$. Holding the retiree fixed, we can transform our problem in terms of the cost $r \sim \hat{W}_r$ into an equivalent problem where the valuation of a firm is the maximum pension a firm with cost r can offer the retiree without making a loss. In that setting we can determine the distribution of the maximum pension (i.e., the valuation) from $\hat{W}_r(\cdot|\cdot)$. With this transformation, to determine an optimal reserve price, we need v_0. One way to determine v_0 is to assume that a retiree i with savings S_i and UNC_i consumes $\frac{S_i}{UNC_i}$ every month, and so, we set $v_{0i} = \frac{S_i}{UNC_i}$.

For each retiree, we first determine the optimal reserve price and then solve for the winning pension under optimal auction. Like before, we still shut down the effect of risk-ratings on the supply side. In Table 15, column 3 we present the money's worth ratio under optimal auctions, and in Figure 11 we present the histograms of pensions under optimal auction, along with the histogram of pensions under full-information for comparison. We can see that the pensions increase slightly under optimal auction, but the gap in pensions between optimal auction and full-information is still the largest for the top-two quintiles.

Figure 11: **Histogram of Pensions Under Full Information and Optimal Auction**



Note. This figure displays the distributions of pensions (in thousands of dollars) under three regimes: data, English Auction, and full-information, separated by savings quintiles (rows) and number of potential bidders (columns). The sample includes everyone, those who choose in the first round or in the second round.

9 Conclusion

In this paper, we develop an empirical framework to study an imperfectly competitive market for annuities. We used a rich administrative data set from the Chilean annuity market to estimate our model. In the market, risk-averse retirees use *first-price-auction-followed-by-bargaining* to select from different types of annuity contract and a firm. Life insurance companies have private information about their annuitization costs and for each retiree-auction they decide whether to participate, and upon participating compete by making pension offers. The Chilean data gives us a unique opportunity to examine the role of private information about cost, retiree's preferences, and market structure on the outcomes of a very important market like that of annuities.

Our main contribution is to study the current market system by estimating both the demand and supply of annuities, and evaluating simpler mechanism that may improve the system. We find that while there is a gap between the observed pensions under the current system and pensions under the full-information regime, the gap is significantly larger for those with higher savings. We also determine the effect of replacing the current system with a simpler one-shot English auction, where the winning firm offers the highest pension, on pensions and on ex-post expected present discounted utilities. We find that while the new mechanism increases pensions for almost every retiree, pensions increase the most for those in the top two savings quintiles, albeit the increase in utility is minimal. Using an English auction with an optimal reserve price does not lead to a large increase in pensions, which is consistent with the fact that the benefit of reserve price decreases with the number of bidders, and in our sample, there are at least 13 to 15 potential bidders.

There are several possible avenues for future research on related topics. First, we can also include the choice between PW and annuities, and consider an imperfectly competitive market with two-sided asymmetric information. On the demand side, retirees will have private information about their mortality forces and their preferences for bequest and on the supply side, as in our case, firms have private information about their annuitization costs. Another interesting extension of our model is to allow for the possibility that bargaining in the second-round is costly for some retirees. This would require us to embed search friction into the second stage, but it might provide us with a complete picture of the market.

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Appendix

A.1 Expected Discounted Present Utilities

Here we explain how we determine the discounted expected utility given by Equation (3). To provide intuition, while keeping the notations manageable we only explain a simple case where the mortalities are known and common across all individuals. Once we understand this simpler case, it is straightforward to allow for individual specific longevity prospects but notationally messy, and for brevity, we do not describe that case here.

The major difficulty in determining Equation 3 is the fact that unlike pension which is fixed, bequest (the wealth left for her estate) varies over time and across retirees. In particular, it depends on having legal beneficiaries, the type of annuity (in particular, whether it has a guaranteed period), and the time of death (before or after the guaranteed period). Chilean law states that certain individuals are eligible to receive survivorship benefits upon the death of a retiree. As mentioned in Section 4, we focus on retirees without eligible children (but with or without spouses), which is the most common case in our sample. The spouse is eligible for a survivorship annuity equivalent to 60% of the retiree's original pension.

When the annuity includes a guaranteed period (of G months), and the annuitant dies before G, say in $G' < G$ months, her spouse will continue to get the same pension for the next $(G - G')$ months and after that he gets 60% of the original pension. If at the time of death there is no surviving spouse (either because the retiree was single when contracting the annuity or because the spouse died before the retiree), the 100% is paid to the designated beneficiaries in the annuity contract. We assume that the retiree values her spouse or other beneficiaries in the same way, with utility $v(B_{it})$. Using these rules we can write Equation (3) as

$$\begin{aligned} w(P, B; \theta_i) &= u(P) \times D_i^R + \theta_i \times \left(\sum_{t=0}^{G} \frac{(1 - q_{it})}{(1 + \delta_t)^t} \times v(P) + \sum_{t=G+1}^{T} \frac{(1 - q_{it}) q_{it}^*}{(1 + \delta_t)^t} \times v(0.6 \times P) \right) \\ &= u(P) \times D_i^R + \theta_i \times \left(v(P) \sum_{t=0}^{G} \frac{(1 - q_{it})}{(1 + \delta_t)^t} + v(0.6 \times P) \sum_{t=G+1}^{T} \frac{(1 - q_{it}) \times q_{it}^*}{(1 + \delta_t)^t} \right) \\ &= u(P) \times D_i^R + \theta_i \times \left(v(P) \times D_i^S + v(0.6 \times P) \times D_i^{S,GP} \right), \qquad \text{(A.1)} \end{aligned}$$

where q_{it}^* is the probability that the spouse will be alive in t.

Next, we explain how to calculate the Net Present Expected Value (NPEV) of an annuity (ρ_{ij}, b_{ij}) from pension offer P_{ij} in Equation (3). For this, we model the force of mortality as a continuous random variable distributed as Gompertz distribution. Let t_0 denote the age at retirement, expressed in months and let $\delta \in (0,1)$ denote the discount factor. An annuity pays a constant benefit P from t_0 until retiree's death so NPEV is calculated at t_0. We start by considering immediate annuity with no spouse. Such annuity does not pay anything to the beneficiaries upon death therefore, $b_{ij} = 0$.

Let $F_m(t|X)$ be the conditional distribution function for the time of death of retiree with characteristics X, and let $f_m(t|X)$ be the corresponding conditional density. For notational simplicity, we suppress the dependence on X. The probability of being alive at time t, i.e., that death occurs after t, is given by the survivor function $\overline{F}_m(t) := 1 - F_m(t)$. Since the analysis is from the perspective of a retiree who is alive at t_0, henceforth, all relevant functions are conditional on being alive at t_0. Then the NPEV is

$$\rho_{ij} = \int_{t_0}^{\infty} u(P)\overline{F}_m(t|t > t_0)e^{-\delta(t-t_0)}dt. \tag{A.2}$$

As introduced in Section 4.1.3, we assume that F_m is a Gompertz distribution, so the conditional survival functions as $\overline{F}_m(t \mid t > t_0; \lambda, \mathfrak{g}) = e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t} - e^{\mathfrak{g}t_0})}$. Substituting this in (A.2) gives

$$\rho = u(P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t_0}} \int_{t_0}^{\infty} e^{-\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t}} e^{-\delta t} dt \right\} = u(p) \times D^R. \tag{A.3}$$

To allow demographic characteristics X to affect the mortality, we let $\lambda = \exp\left(X^\top \tau\right)$, and estimate the parameters $(\mathfrak{g}, \tau)$ using maximum likelihood method. Finally, we set the discount factor $\delta = \ln(1 + \tilde{r}_{t_0})$, where $\tilde{r}_{t_0}$ is the annual market rate of return at t_0.

Deferred Annuity. If the annuity contracts include a deferred period clause for d months, then the pensions start from $t_0 + d$. In the meantime, the retiree receives a "temporal payment," which is almost always twice the pension. The annuity component of the NPEV expression in (A.3) remains the same, except the lower limit is $t_0 + d$ and an additional term reflecting the temporal payment to be received during the transitory period:

$$\rho = u(2P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t_0}} \int_{t_0}^{t_0+d} e^{-\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t}} e^{-\delta t} dt \right\} + u(P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t_0}} \int_{t_0+d}^{\infty} e^{-\frac{\lambda}{\mathfrak{g}} e^{\mathfrak{g}t}} e^{-\delta t} dt \right\}. \tag{A.4}$$

Annuity with Guaranteed Periods. In addition to deferment, annuity contracts can also have a guaranteed period clause, which implies that if the retiree dies within a certain period (denoted as g months) from the start of the payment (either t_0 or $t_1 = t_0 + d$), the total pension amount (P) will be paid to the retiree's spouse or other beneficiaries specified in the contract until the end of the guaranteed period. The NPEV of benefits to be received by the retiree is the same as (A.3) if $d = 0$ and (A.4) if $d > 0$. As the retiree's beneficiaries are now eligible for benefits in the event of death within the guaranteed period, we let b as the NPEV of benefits to be received by these beneficiaries, i.e., bequests. Recall that the instantaneous utility associated with beneficiaries receiving a pension P is given by $\theta \times v(\cdot)$.

The bequest b, assuming a deferment period until $t_0 + d$ and a guaranteed period of g is similar to (A.2), except that the upper integration limit is given by the guaranteed period and the instantaneous probability function corresponds to $F_m(t \mid t > t_0; \lambda, \mathfrak{g})$:

$$\begin{aligned} b &= \theta \times v(P) \times \left\{ \int_{t_1}^{t_1+g} F_m(t|t > t_0; \lambda, \mathfrak{g}) e^{-\delta(t-t_0)} dt \right\} \\ &= \theta \times v(P) \times \left\{ \int_{t_0+d}^{t_0+d+g} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t} - e^{\mathfrak{g}t_0})}) e^{-\delta(t-t_0)} dt \right\}. \end{aligned} \tag{A.5}$$

Allowing for Eligible Spouse When a participant is married at the time of retirement, the spouse is eligible for a survivorship benefit in the case he or she outlives the retiree. This benefit is until death and, in the absence of eligible children, equivalent to 60% of the original pension benefit. Once again, the formula for the NPEV associated with benefits to be received by the retiree (ρ) is not affected by the presence of spouse (except for the fact that the offered pension will be lower, to account for the additional contractual entitlements).

The formula for the NPEV of bequest must then include an additional term, to account for the additional benefits to be paid in the case the spouse outlives the retiree, after the guaranteed period has elapsed. We assume that the two mortality processes are independent and follow the same Gompertz distribution (same $\mathfrak{g}$ parameter, but different λ_{sp} parameter for the spouse). In this case, the expression for NPEV of bequest is given by:

$$\begin{aligned} b &= \int_{t_0+d}^{t_0+d+g} \theta \times v(P) \times F_m(t|t > t_0; \lambda, \mathfrak{g}) \times e^{-\delta(t-t_0)} dt \\ &\quad + \int_{t_0+d+g}^{\infty} \theta \times v(0.6 \times P) \times F_m(t|t > t_0; \lambda, \mathfrak{g}) \times \overline{F}_m(t - \Delta|t - \Delta > t_0 - \Delta; \lambda_{sp}, \mathfrak{g}) \times e^{-\delta(t-t_0)} dt \\ &= \theta v(P) \int_{t_0+d}^{t_0+d+g} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t} - e^{\mathfrak{g}t_0})}) \times e^{-\delta(t-t_0)} dt \\ &\quad + \theta \times v(0.6 \times P) \times \int_{t_0+d+g}^{\infty} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t} - e^{\mathfrak{g}t_0})}) \times (e^{-\frac{\lambda_S}{\mathfrak{g}}(e^{\mathfrak{g}(t-\Delta)} - e^{\mathfrak{g}(t_0-\Delta)})}) \times e^{-\delta(t-t_0)} dt, \end{aligned} \tag{A.6}$$

where Δ is the age difference between the retiree and the retiree's spouse.

A.1.1 Recovering Pension from Expected Present Value

In this section, we consider the reverse problem of determining pension P from ρ and b for a retiree with bequest preference θ. This exercise is important because, if we can uniquely determine pension from the expected present value then it will allow us to go back-and-forth between the monetary value of an annuity (for the supply side) to utility for the retiree (for the demand side). From (3) we know that $w(P, B; \theta) = \rho(P) + \theta b(P)$, and letting $\varpi = w(P, B; \theta)$ we get

$$\begin{aligned} \varpi &= u(P) \times D^R + u(2 \times P) \times D^{R,DP} + \theta \left(v(P) \times D^S + v(0.6 \times P) \times D^{S,GP} \right) \\ &= \frac{P^{-2}}{-2} \left(D^R + \frac{D^{R,DP}}{4} + \theta \left(D^S + \frac{D^{S,GP}}{0.36} \right) \right), \end{aligned}$$

where the second equality follows from $u(c) = v(c) = \frac{c^{-2}}{-2}$. Then we can solve for the pension as

$$P = \sqrt{\frac{\left(D^R + \frac{D^{R,DP}}{4}\right) + \theta \left(D^S + \frac{D^{S,GP}}{0.36}\right)}{-2 \times \varpi}}. \tag{A.7}$$

A.2 Determining the Runner Up Firm

We define the runner-up firm in round-one as the firm with the highest prob of being chosen in the first-round once we exclude the chosen firm. And under the assumption that the runner-up in round-one is one of the two most competitive firms in the second-round then we can identify the runner-up firm for the second round as well.

To construct a measure of the probability of being selected in the first round, we estimate a series of *alternative-specific conditional Logit model* of McFadden (1974). To allow for the most general estimation, we divided the sample into 90 different groups, based on the age at retirement (below, at, and above the NRA), gender, channel (recall that we combine insurance companies and sales-agents into one so there are three channels) and balance quintiles. For each group, we estimate the model where the choice of an individual depends on firms' characteristics such as the ratio of reserves to assets, the fraction of sellers employed by each firm, the ratio between the fraction of complaints and premium of each firm, and the risk rating and also the `mwr`. The random utility associated with j's offer to i is given by the following expression

$$\eta_{ijt} = \gamma_j^0 + \gamma^1 \times Z_{jt} + \gamma^2 \times \texttt{mwr}_{ij} + \varepsilon_{ijt}, \tag{A.8}$$

where γ_j^0 is a company-specific constant, and γ^1 is a coefficient vector for firm-specific variables. Then the probability of observing a particular choice is then given by $\Pr(D_i^1 =$

$j) = \frac{\exp(\hat{\eta}_{ijt})}{\sum_{j=1}^{J}\exp(\hat{\eta}_{ij})}$. Using these estimated probabilities for a retiree i, we say that a company j is the runner-up if j provides the highest utility to individual i among the set of companies ultimately not chosen by i.

A.3 Proofs

Proof of Lemma 1.

Proof. Note first that, given the proposed strategies, as ε goes to zero, the winner is the firm with the maximum $\rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij}$. We introduce some notation and then check that the proposed strategies are optimal for any $\varepsilon > 0$:

- Given a history $\mathfrak{H}$, let $\tilde{\tilde{\mathbf{P}}}_{\mathbf{i}}$ be the vector of standing offers.
- Given a history $\mathfrak{H}$ at which j plays, let $\mathcal{E}_1$ be the event that $j = \arg\max_{j\in J}\Big\{\rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij}\Big\}$; and let $\mu_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_1)$.
- Given a history $\mathfrak{H}$ at which j plays and player k is winning (it could be the case that $j = k$), let $\mathcal{E}_2$ be the event that $\tilde{\tilde{P}}_{il} + \varepsilon > P_{il}^{\max}$ for all $l \neq j$ and $l \neq k$. Let $\tilde{\mu}_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_2)$ and $\tilde{\tilde{\mu}}_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_1 \wedge \mathcal{E}_2)$.
- Given $\mathfrak{H}$ and conditional on $\mathcal{E}_1$, define P_{ji}^* as the expected value of P such that

  $$\beta_i \times Z_{ij_i^*} + \theta_i \times b_i(P) + \rho_i(P) = \max_{k\neq j}\Big\{\beta_i \times Z_{ik} + \theta_i \times b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max})\Big\}.$$

  Note that $P_{ji}^* \leq P_{ij}^{\max}$.
- Given $\mathfrak{H}$ and conditional on $\mathcal{E}_1 \wedge \neg\mathcal{E}_2$, define $\tilde{P}_{ji}^*$ as the expected value of P such that

  $$\beta_i \times Z_{ij_i^*} + \theta_i \times b_i(P) + \rho_i(P) = \max_{k\neq j}\Big\{\beta_i \times Z_{ik} + \theta_i \times b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max})\Big\}.$$

Assume first $\mathfrak{H}$ is such that j is not the current winner, then j's expected payment from choosing I is greater than the one from choosing S:

$$\mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*) \geq (1 - \tilde{\mu}_j(\mathfrak{H})) \times \mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*).$$

Assume $(\mathfrak{H})$ is such that j is the current winner. Then j's expected payoff of choosing S_i is

$$\mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*) = (\mu_j(\mathfrak{H}) - \tilde{\tilde{\mu}}_j(\mathfrak{H})) \times (S_i - UNC_j \times \tilde{P}_{ji}^*) + \tilde{\tilde{\mu}}_j(\mathfrak{H}) \times (S_i - UNC_j \times \tilde{\tilde{P}}_{ji}^*),$$

which is greater than or equal the expected payment of choosing I, so that

$$(\mu_j(\mathfrak{H}) - \tilde{\tilde{\mu}}_j(\mathfrak{H})) \times (S_i - UNC_j \times \tilde{P}^*_{ji}) + \tilde{\tilde{\mu}}_j(\mathfrak{H}) \times (S_i - UNC_j \times (\tilde{\tilde{P}}^*_{ji} + \varepsilon)).$$

□

Proof of Lemma 2.

Proof. For notational simplicity, we denote the LHS of Equation (15) as $\mathfrak{U}$ and the RHS as a sum $\tilde{\beta} + \varpi$. Consider auctions with J firms. From the observed chosen pensions and F_θ we can identify the distribution of $\mathfrak{U}$, which is, by definition, also the distribution of the second-highest value of the sum $\tilde{\beta} + \varpi$. We denote the latter distribution by $F^{(J-1:J)}_{\tilde{\beta}+\varpi}(\cdot)$. However, there is a one-to-one mapping between the distribution of order-statistics and the "parent" distribution. In particular, the parent distribution of the sum $F_{\tilde{\beta}+\varpi}(\cdot)$ is pinned down by $F_{\mathfrak{U}}(t) = F^{(J-1:J)}_{\tilde{\beta}+\varpi}(t) = J(J-1)\int_0^{F_{\tilde{\beta}+\varpi}(t)} (\xi^{J-2} \times \xi) d\xi$.

And since $F_{\tilde{\beta}+\varpi} = F_{\tilde{\beta}} * F_\varpi$, is a convolution, where $*$ is the convolution operator, we can identify the distribution of ϖ via deconvolution. Lastly, we observe that there is a one to one mapping from ϖ to $P^{\max}$ –the maximum pension runner-up firm can offer to retiree (see Equation A.7), which we denote by a function $P^{\max} = m(\varpi) = S/UNC_k$. Then we get

$$\begin{aligned}
W_r(\xi) &= \Pr(r \leq \xi) = \Pr\left(\frac{UNC_k}{UNC_i} \leq \xi\right) = \Pr\left(\frac{S}{P^{\max}} \leq \xi \times UNC_i\right) \\
&= \Pr\left(P^{\max} \geq \frac{S}{\xi \times UNC_i}\right) = 1 - \Pr\left(P^{\max} \leq \frac{S}{\xi \times UNC_i}\right) \\
&= 1 - \Pr\left(m^{-1}(P^{\max}) \leq m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right) = 1 - \Pr\left(\varpi \leq m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right) \\
&= 1 - F_\varpi\left(m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right), \qquad (\because P^{\max} = S/UNC_k).
\end{aligned}$$

□