

Interest Rate Uncertainty and Sovereign Default Risk

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Interest Rate Uncertainty and Sovereign Default Risk *

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Abstract

International data suggests that fluctuations in the level and volatility of the world interest rate (as measured by the US treasury bill rate) are positively correlated with both the level and volatility of sovereign spreads in emerging economies. We incorporate an estimated time-varying process for the world interest rate into a model of sovereign default calibrated to a panel of emerging economies. Time variation in the world interest rate interacts with default incentives in the model and leads to state contingent effects on borrowing and sovereign spreads which resemble those found in the data. The model delivers up to one-half of the positive comovement between the level and volatility of world interest rate and the level of sovereign spreads seen in emerging economies. Moreover, the model also delivers significant positive co-movements between the volatility of the spread and the process for the world interest rate which is also consistent with the data. Our model provides one potential source for the observed bunching in default probabilities observed across nations, namely the world interest rate process. Our model generates a positive and significant correlation (0.51) between the spreads of two nations with uncorrelated income processes. This is close to the observed mean correlation in the data (0.61).

JEL classification: F34, F41, E43, E32.

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1 Introduction

The emerging economy business cycle literature has shown that shocks to country spreads play an important role in accounting for domestic business cycles. In addition, a large body of empirical work has traced variation in emerging economy spreads and default risk to both domestic and global factors.¹ Motivated by this work, the sovereign default literature provides a framework in which time varying default probabilities generate endogenous variation in sovereign yields.² However, this literature has emphasized the role of domestic factors with little attention to the global interlinkages highlighted by the empirical work.³ Our paper addresses this gap by focusing on the relationship between uncertainty in the world interest rate and sovereign default risk, while also retaining a role for domestic factors. Our focus on variation in the world interest rate as a global factor is consistent with several studies. For example, [González-Rozada and Levy Yeyati \(2008\)](#) finds that movements in US treasuries as well as in proxies for global risk explain about half of the long run volatility in emerging economy interest rates.⁴

Some recent episodes highlight the importance of the behavior of US interest rates for world debt markets. A notorious example, usually referred to as the “taper tantrum,” occurred in May of 2013 when former US Fed chairman Ben Bernanke suggested the possibility of a reduction in future bond purchases by the Fed. This triggered a sharp market adjustment in emerging market economies featuring a reversal in capital flows and a spike in government bond yields. On average, sovereign yields across emerging economies rose by 1% ([Rai and Suchanek, 2014](#)). An example of policy makers’ dislike of uncertainty about the world interest rate occurred in 2015, as summarized by the following quotes reported in the Financial

¹See early work in [Edwards, 1984](#), [Cantor and Packer, 1996](#) and [Eichengreen and Mody, 2000](#). For some recent examples of studies that highlight global factors, see [Hilscher and Nosbusch \(2010\)](#), [González-Rozada and Levy Yeyati \(2008\)](#) [Akinci \(2013\)](#), and [Maltritz \(2012\)](#).

²We use the terms “sovereign” and “government” interchangeably throughout the paper.

³Below we discuss a few papers that do highlight the role of global risk aversion.

⁴These findings are re-iterated using a variety of empirical methods and proxies for global risk, different time periods, and different countries in other work. For example, [Akinci \(2013\)](#) uses a structural VAR on a panel of emerging economies while [Maltritz \(2012\)](#) uses Bayesian model averaging on a panel of European nations and both replace the high yield spreads used by [González-Rozada and Levy Yeyati \(2008\)](#) (as a measure of global risk) with corporate bond spreads captured by BAA bonds. [Hilscher and Nosbusch \(2010\)](#) add VIX as a measure of global uncertainty and find that it is statistically significant in explaining credit default swap (CDS) spreads of Mexico, Turkey, and Korea.

Times (September 9, 2015):

“We think US monetary policymakers have got confused about what to do. The uncertainty has created the turmoil.”

Mirza Adityaswara, Sr. Deputy Governor, Indonesia Central Bank.

“The uncertainty about when the Fed hike will happen is causing more damage than the Fed hike will itself.”

Julio Velarde, Governor, Peru Central Bank.

Motivated both by the empirical evidence on the importance of global factors in the movement of emerging economies' sovereign spreads as well as recent events and policy makers' concerns, we develop an equilibrium model of sovereign default (in the tradition of [Eaton and Gersovitz, 1981](#)) to study the relationship between endogenous country spreads and movements in both the level and the volatility of the world interest rate. To do so, we introduce stochastic volatility into the process of the world interest rate (as modeled by [Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe, 2011](#)) in an otherwise standard quantitative model of long term sovereign debt (following [Hatchondo and Martinez, 2009](#)). We use the model to separate out the role that shocks to the level of the interest rate play from the role that time-varying volatility in the world interest rate plays in explaining both the level and the volatility of the sovereign spread, the borrowing levels chosen by the sovereign, as well as the cross-country correlation in spreads induced by the world interest rate process.

Our model implies that the impact of shocks to the world interest rate are highly state contingent and depend on income and existing debt levels, as well as on the state of world interest itself (its level and volatility). In order to disentangle the implied co-movements between the objects of interest in the model-generated data and in our emerging economy data set, we regress the country spread on the level and volatility of the world interest rate adding the debt-to-GDP ratio, income growth and a measure of risk-aversion (as well as country fixed effects) as conditioning variables. We find that spreads are increasing in both

the level and the volatility of the world interest rate in the model. As expected, they increase with debt and fall with income growth and rise during periods of heightened risk aversion. Our panel of emerging economies generates similar conditional co-movement patterns. Since our model can be viewed as a mechanism that transmits shocks in the process for the world interest rate into fluctuations in default probabilities and hence in sovereign spreads, we can use the model to infer how much cross-country co-movement in spreads is generated by this “global factor”. We find that even when two economies face income processes that are uncorrelated, the common world interest rate process generates a correlation of 0.51 between their spreads. Interestingly, we find that the mean correlation between country spreads in our data is 0.61.

A number of observers have highlighted the importance of volatility in sovereign spreads to the business cycle of emerging economies. Looking beyond the impact of the world interest rate process on emerging economy spreads, we explore the implications for the volatility of these spreads and compare the co-movement patterns with those in our panel data set. Once more, the conditional co-movement patterns found in the data are close to those in the model. Both the level and volatility of the world interest rate are positively correlated with the volatility of the spread. Finally, in both model and data, borrowing is lower when the world interest rate increases and also when its volatility increases. If all fluctuations in the interest rate process could be eliminated, welfare in the average emerging country would increase by 0.21% of permanent consumption. These findings emerge from a model calibrated to the average dynamics observed in a panel of 66 emerging economies.

The rest of the paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 describes the model and defines the equilibrium. Section 4 discusses the numerical solution and the calibration. Section 5 presents the results and section 6 concludes. An appendix presents details about our data and computation, as well as robustness exercises.

2 Related Literature

There is ample evidence that movements in the international risk-free rate (usually proxied by the US T-bill rate) have macroeconomic consequences for emerging economies. [Neumeyer and Perri \(2005\)](#) report that real country interest rates in emerging economies are strongly countercyclical and tend to lead the cycle. They also find that exogenous interest rate shocks can account for up to 50 percent of the volatility of output in Argentina. [Uribe and Yue \(2006\)](#) find a strong relationship between the world interest rate, the country spread and emerging market fundamentals. In particular, they show that US interest rate shocks and country spread shocks can explain the large movements seen in the aggregate activity of emerging economies. [García-Cicco, Pancrazi and Uribe \(2010\)](#) also find that the country spread shock is one of the most important drivers of emerging economies business cycles. All these papers take the country spread as an exogenous variable with a time-invariant volatility, while our work endogenizes both the level and the time-varying volatility of the spread (as a result of default incentives on the part of the sovereign).⁵ There are also a number of empirical papers that include U.S. monetary policy variables (including interest rates) as determinants of sovereign spreads. We discuss these later in conjunction with our own regression results.

[Fernández-Villaverde et al. \(2011\)](#) study the impact of exogenous time-varying volatility on the macroeconomic dynamics of a small open economy. They examine the effects on the business cycles of Argentina, Ecuador, Venezuela, and Brazil. We follow [Fernández-Villaverde et al. \(2011\)](#)’s approach to modeling the stochastic behavior of the world interest rate, while departing from their approach to modeling the country spread: as already noted, our model is one of endogenous spreads. Then, we explore the mechanism by which world interest rate uncertainty affects the country spreads and default risk in emerging economies. We see our work as complementary to theirs. [Guimaraes \(2011\)](#) highlights the importance of shocks to the level of world interest rates in a theoretical sovereign default model. His work differs from ours in that it does not consider time variation in the volatility of the world

⁵In a recent study, [Reyes-Heroles and Tenorio \(2017\)](#) document the existence of two regimes in the volatility of interest rates at which emerging economies borrow and show that these regimes are closely related to the occurrence of sudden stops in these economies.

interest rate and does not carry out a quantitative evaluation of the model.

Our paper builds on the quantitative literature on sovereign defaults (following [Eaton and Gersovitz, 1981](#), [Aguiar and Gopinath, 2006](#), and [Arellano, 2008](#)). [Lizarazo \(2013\)](#) explores how risk aversion on the part of international lenders influences debt and default dynamics of the borrowing country and therefore how the risk premium affects sovereign spreads. [Verdelhan and Borri \(2010\)](#) also explore the role of time-varying risk aversion of lenders in a model with many small open economies that have endowments which are partially correlated with the lender's endowment process. They also find that risk aversion plays an important role in determining spreads and borrowing levels. Building on these two papers, we incorporate risk aversion in our modeling of foreign lenders.

Within the sovereign debt literature, our paper is particularly related to two recent studies. [Seoane \(2019\)](#) studies how changes in aggregate income volatility affect sovereign spreads of Greece, Italy, Portugal, and Spain. He presents a model in the spirit of [Arellano \(2008\)](#) and incorporates time-varying volatility of the income process which generates substantial variability in spreads. Our work complements his: we keep the income process with a time-invariant volatility and introduce time-varying volatility in the world interest rate process. The second paper is the one by [Pouzo and Presno \(2016\)](#). They study the problem of a small open economy that can default on its obligations in the presence of model uncertainty. In their model, lenders fear that the probability model of the underlying state of the borrowing economy is misspecified and hence may demand higher returns on their investments. Even though our paper tackles a different type of uncertainty (i.e. time-varying volatility of the world interest rate) the results are consistent: more uncertainty leads to higher and more volatile spreads.⁶

Finally, our paper is also related to the literature on uncertainty shocks in macroeconomic models.⁷ For instance, [Justiniano and Primiceri \(2008\)](#) and [Bloom \(2009\)](#) study the effect of changes in the volatility of technology shocks in general equilibrium models for closed economies. [Justiniano and Primiceri \(2008\)](#) study the changes in volatility in postwar US

⁶Another study in the sovereign debt literature that deals with time-varying volatility is [Gu and Stangebye \(2017\)](#). They study costly information acquisition in a model of defaultable debt and show how this can create time-varying volatility in the spread.

⁷[Fernández-Villaverde and Rubio-Ramírez \(2013\)](#) and [Bloom \(2014\)](#) provide thorough accounts of the growing literature dealing with uncertainty shocks and time-varying volatility in macroeconomics.

data by estimating a large-scale dynamic stochastic general equilibrium model allowing for time variation in the structural innovations. They find that shocks specific to investment are mostly responsible for the observed “great moderation.” [Bloom \(2009\)](#), on the other hand, shows that uncertainty shocks can generate short sharp recessions and recoveries.

3 Model

We consider a small open economy populated by a continuum of households. The economy trades long-duration non-state-contingent bonds with a mass of competitive foreign lenders and has no commitment to repaying its debts. The world interest rate (which matters for bond prices) is time-varying. Time is discrete and goes on forever: $t = 0, 1, 2, \dots$

3.1 Domestic Economy

There is a single tradable good. As is standard in the sovereign default literature, the economy receives a stochastic endowment stream of this good y_t , where

$$\log(y_t) = \rho_y \log(y_{t-1}) + \varepsilon_t^y \quad (1)$$

with $|\rho_y| < 1$, and $\varepsilon_t^y \sim N(0, \sigma_\varepsilon^2)$. The government’s objective is to maximize the expected life-time utility of the representative agent in the economy, namely

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (2)$$

where \mathbb{E} denotes the expectation operator, c_t is consumption, $\beta \in (0, 1)$ denotes the subjective discount factor, and the $u(\cdot)$ is a period utility function which satisfies $u' > 0$, $u'' < 0$.

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.

The government has access to an international financial market where it trades long-duration non-contingent bonds with competitive foreign investors at a price q_t . As in [Hatchondo and Martinez \(2009\)](#), we assume that a bond issued in period t promises an infinite stream of coupons, which decrease at a constant rate δ . In particular, a bond issued

in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$. Let b_t (b_{t+1}) denote the number of outstanding coupon claims at the beginning of the current (next) period. A positive value of b_t implies that the government was a net issuer of bonds in the past. The number of bonds issued by the government is given by $[b_{t+1} - (1-\delta)b_t]$. The resource constraint for the repayment case is then given by:

$$c_t + b_t = y_t + q_t [b_{t+1} - (1-\delta)b_t]. \quad (3)$$

If the government declares a default, it is excluded from financial markets and remains in financial autarky for a stochastic number of periods. While the government is in default, it cannot issue debt and domestic aggregate income is reduced by $\phi(y)$. As in [Arellano \(2008\)](#) and [Chatterjee and Eyigungor \(2012\)](#), we assume that it is proportionally more costly to default in good times ($\phi(y)/y$ is increasing in y).⁸ Following most studies of sovereign default, the income-cost of defaulting is not a function of the size of the default.⁹ Thus, when the government defaults, it does so on all current and future debt obligations. As argued in [Hatchondo, Martinez and Sosa-Padilla \(2016\)](#), this is consistent with the behavior of defaulting countries.¹⁰ Following previous studies, we also assume that the recovery rate for debt in default is zero. The resource constraint for the default case is given by:

$$c_t = y_t - \phi(y_t). \quad (4)$$

⁸[Arellano \(2008\)](#) and [Chatterjee and Eyigungor \(2012\)](#) show that this property is important in accounting for the dynamics of the sovereign debt interest rate spread. [Mendoza and Yue \(2012\)](#) show that this property of the cost of defaulting arises endogenously in a setup in which defaults affect the ability of local firms to acquire a foreign intermediate input good.

⁹See [Sosa-Padilla \(2018\)](#) for a model of endogenous default costs, where the output-cost-of-default is a function of the amount of debt that is defaulted upon.

¹⁰Sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current. The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued (see [IMF, 2002](#)). For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed.

3.2 Foreign Lenders

Foreign creditors are risk averse and their stochastic discount factor is given by:

$$m_{t,t+1} = e^{-r_{t+1} - \kappa_y (\varepsilon_{t+1}^y + 0.5 \kappa_y \sigma_\varepsilon^2)}, \quad \text{with } \kappa_y \geq 0. \quad (5)$$

This formulation introduces a positive risk premium because bond payoffs are more valuable to lenders in states in which the government is more likely to default (i.e., in states in which income shocks in the domestic economy, ε^y , are low). Here, r is the time-varying world interest rate, and κ_y is the parameter governing the magnitude of the risk premium. A higher value of κ_y can be seen as capturing how correlated the small open economy is with respect to the lenders' income process, or alternatively, the degree of diversification in foreign lenders' portfolios.¹¹

Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition. The price per bond is then given by:

$$q_t = \mathbb{E}_t \{m_{t,t+1} (1 - d_{t+1}) [1 + (1 - \delta)q_{t+1}]\} \quad (6)$$

where d_{t+1} and q_{t+1} represent the government's default decision and equilibrium bond price in period $t + 1$, respectively.

3.3 Law of Motion for the World Interest Rate

Following Fernández-Villaverde et al. (2011) we specify the international risk-free rate faced by investors as:

$$r_t = \bar{r} + \varepsilon_{r,t} \quad (7)$$

¹¹This modeling of risk-averse foreign lenders follows Vasicek (1977) and has been used recently in the sovereign debt literature (Arellano and Ramanarayanan, 2012, Bianchi and Sosa-Padilla, 2020, etc.). The functional form for the lenders' stochastic discount factor (SDF) makes it explicit that the variance of the domestic income matters, but it is less clear that the variance of r_t also affects the SDF through the level of r_t (see equation 8). Since r_t is time-varying its volatility will induce additional time variation in the SDF. The appendix shows that our results are robust to using a richer specification for (5) (one that explicitly includes the variance of r_t).

where \bar{r} is the mean of world risk-free real rate, and $\varepsilon_{r,t}$ represents deviations from this mean. In particular, we assume the following AR(1) behavior for $\varepsilon_{r,t}$:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \quad (8)$$

where $u_{r,t}$ is a normally distributed shock with mean zero and unit variance. The crucial ingredient in this stochastic process is that the standard deviation ($\sigma_{r,t}$) is not constant but time-varying, and itself follows another (independent) AR(1) process:

$$\sigma_{r,t} = (1 - \rho_{\sigma_r}) \bar{\sigma}_r + \rho_{\sigma_r} \sigma_{r,t-1} + \eta_r u_{\sigma_r,t} \quad (9)$$

where $u_{\sigma_r,t}$ is a normally distributed shock with mean zero and unit variance. We further assume that $u_{r,t}$ and $u_{\sigma_r,t}$ are independent of each other. The parameters $\bar{\sigma}_r$ and η_r measure the degree of mean volatility and stochastic volatility in the international risk free rate. A high $\bar{\sigma}_r$ corresponds to a high mean volatility and a high η_r corresponds to a high degree of stochastic volatility in the international risk free rate.

3.4 Timing

The timing of events, for a government that is not excluded from financial markets, is as follows. The government starts with an initial bond position b_t and observes the realizations of the income level (y_t), the world interest rate level (r_t) and the interest rate volatility ($\sigma_{r,t}$), and then decides whether to repay its outstanding debt. If it decides to repay, it chooses b_{t+1} subject to the resource constraint, taking the bond price schedule $q_t(b_{t+1}; y_t, r_t, \sigma_{r,t})$ as given. Finally, consumption takes place.

On the other hand, if the government decides to default it gets excluded from financial markets and suffers a direct income loss. In case of default, there is no other decision to be made as the level of consumption equals the (reduced) income level. The government will re-access financial markets in the following period with probability μ (and it will remain excluded from financial markets with probability $1 - \mu$).

3.5 Recursive Equilibrium

We now turn to recursive notation, where *primes* denote next-period value of the variables. Let $\mathbf{s} = \{y, r, \sigma_r\}$ denote the aggregate exogenous state. Given a number of outstanding coupon claims at the beginning of the next period b' and a realization of \mathbf{s} , the price of a bond satisfies:

$$q(b', \mathbf{s}) = \mathbb{E}_{\mathbf{s}' \mid \mathbf{s}} \left\{ m(\mathbf{s}', \mathbf{s})(1 - d') \left[1 + (1 - \delta)q(b'', \mathbf{s}') \right] \right\} \quad (10)$$

where d' is the next-period default decision, and b'' is the next-period debt choice. The optimal default decision is taken as:

$$v^0(b; \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1 - d)v^c(b; \mathbf{s}) + dv^d(\mathbf{s}) \right\} \quad (11)$$

where d equals 1 (0) if the government chooses to (not to) default. Under no-default, the government solves the following problem:

$$v^c(b; \mathbf{s}) = \max_{b'} \left\{ u(y + q(b'; \mathbf{s})(b' - (1 - \delta)b) - b) + \beta \mathbb{E}_{\mathbf{s}' \mid \mathbf{s}} [v^0(b'; \mathbf{s}')] \right\} \quad (12)$$

Under default, the value function is given by:

$$v^d(\mathbf{s}) = u(y - \phi(y)) + \beta \mathbb{E}_{\mathbf{s}' \mid \mathbf{s}} [\mu v^0(0; \mathbf{s}') + (1 - \mu)v^d(\mathbf{s}')] \quad (13)$$

where, in order to keep the environment as simple as possible, we assume that when the government gains re-access to financial markets it does so with no debt obligations (i.e. it gets a “fresh start”).¹² Next, we define the recursive equilibrium of this economy.

Definition 1. *The recursive equilibrium for this economy is characterized by*

1. a set of value functions v^0 , v^c , and v^d ,
2. a default policy rule d and a borrowing policy rule b' ,

¹²For studies with positive recovery rates and renegotiation between sovereigns and lenders, see for example [Yue \(2010\)](#), [D’Erasco \(2011\)](#), and [Hatchondo et al. \(2016\)](#).

3. a bond price function q ,

such that:

- (a) given the default and borrowing policy functions, v^0 , v^c , and v^d satisfy equations (11) – (13) when the government can trade bonds at q ;
- (b) given the default and borrowing policy functions, the bond price function q is given by equation (10);
- (c) the default and borrowing policy functions d and b' solve the dynamic programming problem defined by equations (11) – (13) when the government can trade bonds at q .

4 Numerical Solution

We solve the model numerically using value function iteration with a discrete state space. We focus on Markov-perfect equilibria. We solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium objects as the infinite-horizon economy equilibrium objects.

The functional form for the period utility is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (14)$$

where γ is the coefficient of relative risk aversion. As in [Chatterjee and Eyigunor \(2012\)](#), we assume a quadratic loss function for income during a default episode:

$$\phi(y) = \max\{0, d_0y + d_1y^2\} \quad (15)$$

As explained by [Chatterjee and Eyigunor \(2012\)](#), this functional form for the income loss $\phi(y)$ is flexible enough to accommodate many cases. If $d_0 > 0$ and $d_1 = 0$, then the cost is proportional to income; if $d_0 = 0$ and $d_1 > 0$, then the cost increases more than proportionately with income; if $d_0 < 0$ and $d_1 > 0$, then the cost is zero in a region

Table 1: Parameters of Full Model Economy

Household risk aversion	γ	2	Standard value
Household's discount factor	β	0.96	Standard value
Mean int'l risk-free rate	\bar{r}	0.01	Standard value
Income autocorrelation coefficient	ρ_y	0.933	Estimated
Std. dev. of income innovations	σ_ε	0.027	Estimated
Probability of re-entry	μ	0.0385	Chatterjee and Eyigungor (2012)
Coupon decay rate	δ	0.0341	Average debt duration
<hr/>			
Lenders' risk aversion	κ_y	2.5	Calibrated to fit targets
Default cost parameter	d_0	-0.14	Calibrated to fit targets
Default cost parameter	d_1	0.24	Calibrated to fit targets

$(0 < y < -d_0/d_1)$ and then increases faster than income (for $y > -d_0/d_1$). This last case is similar to [Arellano \(2008\)](#)'s cost-of-default function.

4.1 Calibration

We define the “full model” as one in which all the shocks are present. This full model is calibrated to a quarterly frequency using data from a panel of 66 emerging economies for the period 1990 — 2017.¹³ Table 1 summarizes the parameter values.

We estimate equation (1) using quarterly real GDP for our panel of countries. The re-entry probability μ is set to 0.0385 according to [Chatterjee and Eyigungor \(2012\)](#).¹⁴

We assume the representative agent in economy has a coefficient of relative risk aversion γ of 2, the typical value in the literature. The average risk-free rate and the domestic discount factor ($\bar{r} = 0.01$ and $\beta = 0.96$) are standard in quantitative business cycle and sovereign default studies. We set $\delta = 3.41\%$. With this value and the targeted level of sovereign spread, sovereign debt has an average duration of 5.6 years in the simulations, which is close to the average duration found in previous literature.¹⁵

¹³As is common in the sovereign default literature, we focus on time series that exclude default crises. The appendix has details of the country coverage.

¹⁴This value for μ implies an average financial exclusion of 6.5 years. [Gelos, Sahay and Sandleris \(2011\)](#) report an average exclusion of 4.7 years for emerging economies. [Uribe and Schmitt-Grohé \(2017\)](#) find an average exclusion of 8 years.

¹⁵We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = (1 + i^*)/(\delta + i^*)$, where i^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, [Cruces, Buscaglia and Alonso \(2002\)](#) find an average duration of 4.77 years, with a standard deviation of 1.52 years. [Bai, Kim and Mihalache \(2017\)](#) report an average debt duration of 6.7

Table 2: Estimates of the World Interest Rate Process

Autocorrelation risk-free rate	ρ_r	0.908
Mean volatility of int'l risk-free rate	$\bar{\sigma}_r$	-6.2869
Autocorrelation interest vol. shock	ρ_{σ_r}	0.8742
Stochastic vol. of int'l risk-free rate	η_r	0.2632

We are left with three parameters to assign values to: the parameter controlling the risk premium, κ_y , and the coefficients of the default cost function, d_0 and d_1 . We calibrate these three parameters to the median values of the debt-to-income ratio (48%), the sovereign spread (3.4%), and the standard deviation of the spread (1.5%). All the data counterparts are the medians observed in our panel of emerging economies.

Table 2 presents the parameterization of the stochastic processes that govern the behavior of the world interest rate. We estimate equations (8) and (9) using data on the real international risk free rate for the period 1990 — 2017.¹⁶ We obtain this rate by subtracting expected inflation from the quarterly US T-bill rate. Following [Neumeyer and Perri \(2005\)](#) and [Fernández-Villaverde et al. \(2011\)](#), we compute expected inflation as the average of the US CPI inflation in the current quarter and in the 3 preceding quarters. Parameter values in Table 2 correspond to the median of the posterior estimates. These posterior estimates imply annualized average standard deviations for the risk-free interest rate of 74 basis points (with only mean volatility) and 97 basis points with both mean and stochastic volatility.

4.2 Model fit

Having calibrated the model, we first verify its ability to reproduce basic features of emerging economy business cycles and that the targets used in calibration are closely approximated.

Table 3 reports several key moments in the data and in our simulations of the full model.¹⁷

years in a panel of 11 emerging economies.

¹⁶We use the `stochvol` R package, which implements an efficient algorithm for Bayesian estimation of stochastic volatility models via MCMC methods. See [Kastner \(2016\)](#) for more details on the estimation procedure.

¹⁷As in previous studies, we report results for pre-default simulation samples. We simulate the model for 500 samples of 1,500 periods each. We then discard the initial 1,000 periods of each sample as a burn-in and from the remaining data we extract 500 samples of 26 consecutive years before a default. 26 years (or 104 quarters), is the length of the time series for the US T-bill rate used to estimate the process of the world interest rate.

Table 3: Model fit – targeted and non-targeted moments

	Data	Full Model
Debt/ y (in %)	48	48
Spread (in %)	3.4	3.4
SD (Spread) (in %)	1.5	1.5
$sd(c)/sd(y)$	1.2	1.3
$corr(c, y)$	0.7	1.0
$corr(tb/y, y)$	-0.4	-0.6
$corr(Spread, y)$	-0.4	-0.7

Note: the standard deviation of a variable x is denoted by $sd(x)$ and the correlation between two variables x and z is denoted by $corr(x, z)$. We detrend (the log of) income (y), (the log of) consumption (c) and the trade-balance (tb/y) using the Hodrick-Prescott filter, with a smoothing parameter of 1,600. We report deviations from the trend.

The moments reported in Table 3 are chosen to illustrate the ability of the full model to replicate distinctive business cycle properties of economies with sovereign risk. This table shows that the full model approximates well the moments used as targets (the debt-to-income ratio, and the level and volatility of the sovereign spread) and it is broadly consistent with non-targeted moments in the data: consumption is procyclical and more volatile than income; the trade balance is countercyclical; and the sovereign spread is also countercyclical.

5 Results

First, we study the effects of introducing time variation in both the level and the volatility of the world interest rate on default risk, sovereign spreads and debt levels. Having presented the key mechanisms through which the model operates, we then turn to our main results and compare the co-movement patterns seen in the model to those found in the data by running identical panel regressions on simulated data from the model as well as on our data set of emerging economies. Finally, we compare our model’s predictions with those of an otherwise identical model but with a constant world interest rate: we highlight differences in long-run moments and measure the welfare cost of interest rate uncertainty.

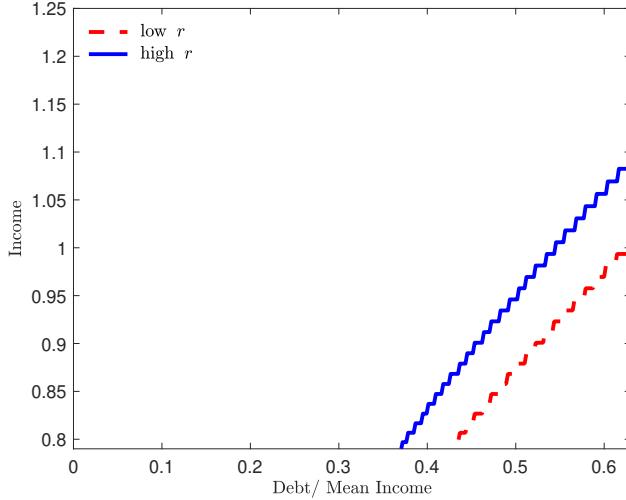


Figure 1: Default sets. The solid blue line corresponds to a high level of r and the red dashed line is for a low level of r . Each line is the respective default set contour: the government defaults *south* of the line. The figure assumes that the volatility of the world interest is at its average value.

5.1 Effects of Uncertainty about the World Interest Rate

In this first subsection we present the mechanisms and intuition underlying our main results (which are discussed next, in section 5.2). Specifically, we show how shocks to the level and volatility of the world interest rate interact with the dynamics typically studied in the sovereign default literature. We examine these effects in the following order : default incentives, sovereign spreads, and borrowing decisions. Throughout this section we refer to ‘low’ and ‘high’ values of r and σ_r to illustrate the state-contingent nature of the effects.¹⁸

Default incentives. Figure 1 shows the effect of different levels of r on default incentives, holding the volatility state at its mean. The graph shows combinations of debt and income states, and divides the space into two regions. To the north of the default contour, the sovereign chooses to repay outstanding claims while it chooses to default at or south of the contour. We plot contours for two levels of the world interest rate, low and high. The message from the figure is clear: other things equal, the default set is increasing in the level of the world interest rate.

In order to understand this result first note from equations (5) and (6) that if lenders face

¹⁸Low and high values of r and σ_r refer to values that are 4 standard deviations below and above mean.

a higher r , then the lenders' stochastic discount factor ($m_{t,t+1}$) and the bond price (q) will mechanically fall (implying higher sovereign yields). In addition to this purely mechanical pass-through of higher borrowing costs (from the lenders to the borrower), there will be additional effects due to the equilibrium response of the sovereign.

The higher yields change the default incentives of the sovereign through two main forces. First, higher borrowing costs imply that more consumption must be sacrificed in order to roll-over existing debt, *ceteris paribus*, which lowers the desire to repay. Second, since r follows a persistent process, higher borrowing costs today are likely to remain in place in the near future. This makes the threat of financial exclusion less severe, as the periods in which the sovereign would be unable to borrow would likely be periods of high borrowing costs. Both forces go in the same direction and make the default set larger when r is high.

Figure 2 presents the effect of shifts in the volatility state (σ_r) on default incentives. Panels (a) and (b) of this figure show the default set for two different values of the volatility of the world interest rate (σ_r), conditional on facing either a low level of r (panel a) or a high level of r (panel b). Comparing panels, we see that the effect of volatility is state contingent.

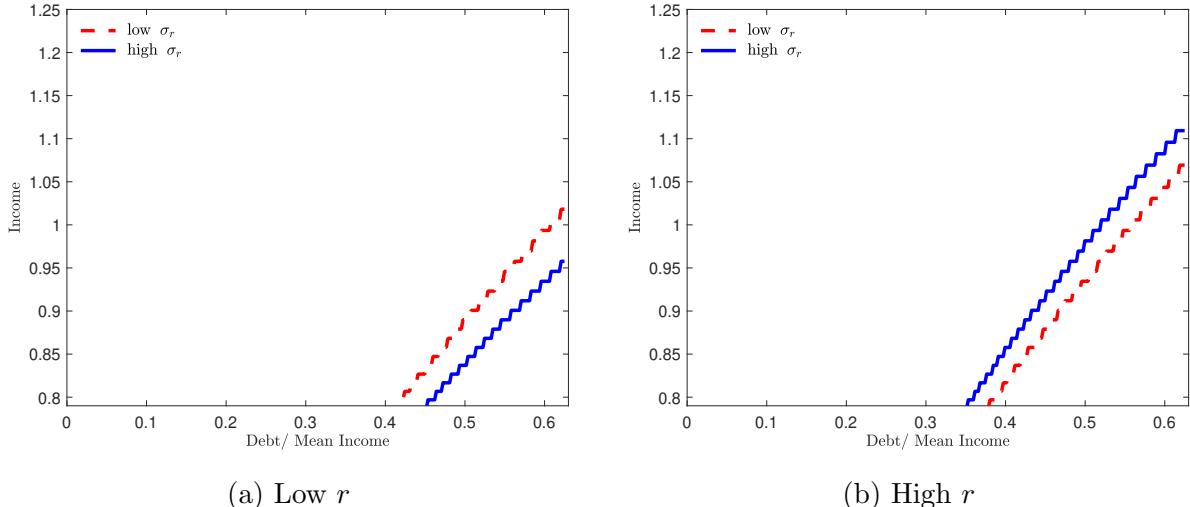


Figure 2: Effect of volatility on default incentives. The left panel is for a low level of r and the right panel is for a high level of r . The solid blue line corresponds to the high σ_r and the red dashed line to the case of low σ_r . Each line is the respective default set contour: the government defaults *south* of the line.

As discussed above, we know that when r is low q will be high (panel a). Since it is cheap to borrow, the government wants access to the markets which requires repayment.

This incentive to repay is stronger if the volatility of r is high because a higher volatility state means it is more likely (compared to the low volatility state) that rates are going to increase, making borrowing in the near future more expensive. As a result, the sovereign will want to borrow now therefore it is less willing to default. When rates are high today (panel b), the opposite is true, and volatility now enlarges the default set since it is a good time to default, *ceteris paribus*. We showed in Figure 1 that a high level of r increases the incentive of the government to default. Panel b of Figure 2 shows that if the country faces *both* a high level *and* a high volatility of r , then default incentives are even higher.

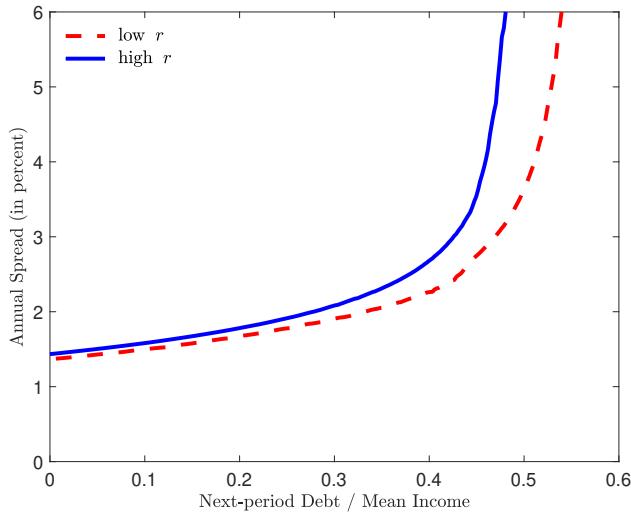


Figure 3: Spread-debt menus. The solid blue line corresponds to a high level of r and the red dashed line is for a low level of r . The figure assumes that the volatility of the world interest and the domestic income level are at their mean values.

Spreads-debt menus. The effects we just described affect as well the equilibrium spread-debt menus facing the small open economy. Figure 3 shows the menu of spreads and next-period debt choices offered by lenders to the sovereign, for two levels of the world interest rate (while keeping the volatility at its mean). As expected, for both high (solid blue line) and low r (dashed red line) choosing more debt comes with higher spreads. At high enough debt levels, spreads rise so much as to act as an effective endogenous debt limit. Comparing lines, we see that a higher level of r leads to lower debt capacity: the steeply rising part of the spread-debt menu moves to the left in the solid blue line when compared with the dashed red line (low r). For all debt levels we obtain a quite natural result: when r is higher, the

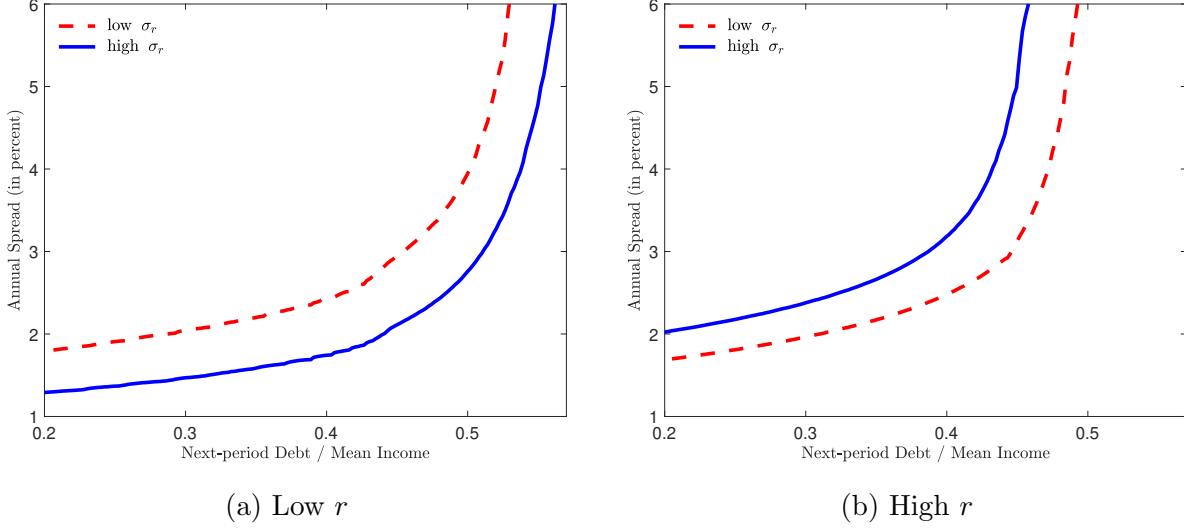


Figure 4: Effect of volatility on spreads. The left panel is for a low level of r and the right panel is for a high level of r . The solid blue line corresponds to the high σ_r and the red dashed line to the case of low σ_r . Both panels are for the mean income level.

economy faces worse borrowing opportunities. At the median debt level in the simulations (roughly 48% of income), the spread is more than 200 bps higher under a high r .

Figure 4 shows that the effect of σ_r on spreads is highly state contingent. Panel (a) is for a low value of the level of r : in this case, we see that higher volatility translates into lower spreads. Similarly to our intuition for the results in Figure 2, when facing a low interest rate the government finds it cheaper to repay and is more incentivized to do so if volatility is high: there is a higher chance that rates are increasing in the future and therefore the economy chooses to ‘lock-in’ favorable rates today. Panel (b) shows the effect in the opposite case (when the rate is high today). It naturally goes the other way: now higher volatility worsens the borrowing terms.

Borrowing decisions. Figure 5 shows the effect of different levels of the world interest rate on borrowing decisions, holding volatility at its average value. It is clear from this figure that the small open economy borrows more when lenders face a low world interest rate compared to when lenders face a high interest rate. The b' function for the low r level lies consistently above the one for high r level. Recalling that the spread facing the economy rises with r , it is not surprising that when facing better terms the government’s reaction is to lever up and take advantage of the lower borrowing costs.

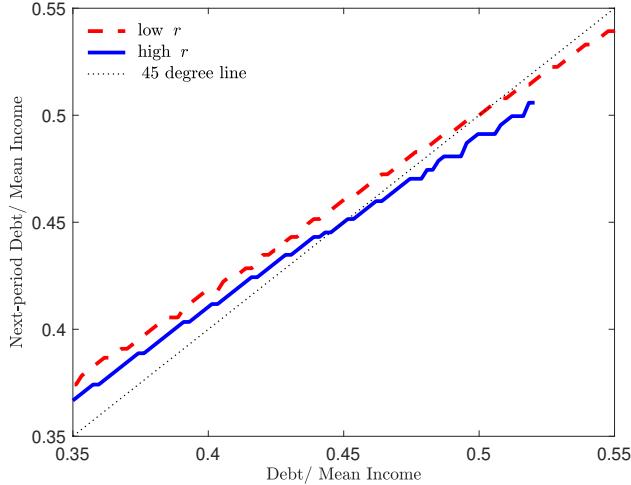


Figure 5: Borrowing policy functions for different levels of r . The solid blue line corresponds to the case of a low level of r while the red dashed line is for the case of a high level of r . Both lines are for the mean volatility (σ_r) and the mean income level.

The figure also highlights a well known aspect of borrowing decisions in this class of models. At low levels of debt, the policy rules lie above the 45 degree line and at high levels they lie below the 45 degree line. This occurs because spread-debt menus offered by lenders imply spreads increase in the amount of debt chosen. The sovereign realizes this and optimally chooses to lower debt when it is relatively high. Note also that the debt level at which borrowing is curtailed is lower in the solid blue line when interest rates are high. Relatedly, we see that the solid blue line “disappears” before the end of the graph: this indicates that for those levels of initial debt (over 52% of mean income) the government is defaulting (and hence not borrowing that period, due to financial exclusion). This is a manifestation of the bigger default sets associated with higher r discussed above.

Figure 6 presents the effect of varying the volatility of the world interest rate on borrowing choices. Panels (a) and (b) of this figure each show the borrowing policy functions for two different values of the volatility of the world interest rate (σ_r), conditional on facing a low level of r (panel a) or a high level of r (panel b). The main takeaway from the two panels is that the effect of σ_r on borrowing choices is highly state contingent and quantitatively small. With low r , a higher volatility state involves marginally more borrowing than the low volatility state: the solid blue line sits above the dashed red line at most points in debt to mean income space, but their difference is rather small. We also see that the sovereign

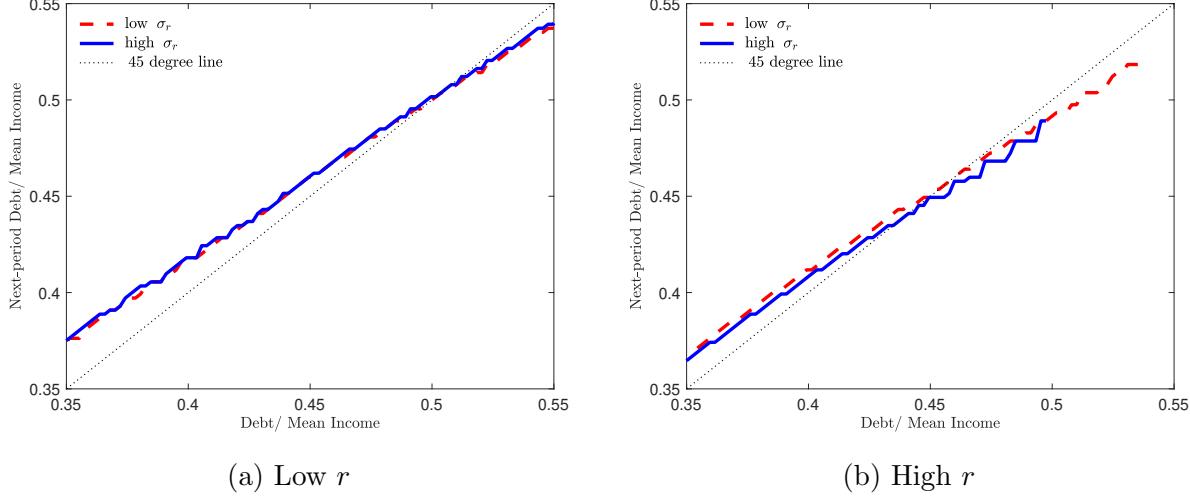


Figure 6: Effect of volatility on the borrowing policy functions. The left panel is for a low level of r and the right panel is for a high level of r . The solid blue line is for high σ_r while the red line is for low σ_r . Both panels are for the mean income level.

prefers to increase its debt level for most values of initial debt: when interest rates are low, there is a strong incentive to “lock in” the good times. This incentive is marginally stronger when the volatility is higher (as it is more likely that in the future the level of r will increase).

In the right panel of Figure 6, the state contingent nature of the sovereign’s decisions become further clarified. Before discussing the impact of volatility, note that the presence of a high level of r shifts the borrowing rules ‘down and to the right,’ making the sovereign more likely to lower debt when compared to the left panel and this is true for both volatility regimes. Turning to the impact of volatility, we see that the location of the two lines is reversed: the red dashed line (low volatility) sits uniformly above the solid blue line (high volatility). At higher debt levels, both volatility states lead to lower borrowing but the effect is more pronounced in high volatility states. The explanation for this is similar to the previous paragraph: the sovereign sees that rates are unfavorable today so a high volatility regime implies a higher chance (given the high σ_r) that rates will decrease in the near future compared to the chance in a low volatility state. As a result, it is optimal for the sovereign to wait for better times and consequently to inter-temporally substitute current borrowing for future borrowing, when interest rates are lower. Coupled with the desire to avail of lower spreads, the sovereign lowers borrowing more in the high volatility state.

5.2 Main results: comparing model implications to the data

From our discussion so far it is clear that the sovereign's responses to variation in the level and volatility of the world interest rate are highly state contingent. As a result, looking at raw first and second moments such as correlations can be misleading. Our approach to this issue is to tease out *conditional* co-movement patterns in the data and model using panel regressions that include the level and volatility of the world interest rate along with the debt-to-GDP ratio and output growth. We begin with spread levels, followed by spread volatility and end with the relationship between debt accumulation and the level and volatility of r .

Effect on sovereign spreads. We begin this subsection by exploring the co-movement patterns between the international risk-free rate and sovereign debt spreads for our panel of 66 emerging economies and compare them with the patterns that emerge from our model. In particular, we are interested in uncovering the conditional correlations between the level of emerging economies' sovereign spreads and the level and volatility of world interest rates as proxied by the real U.S. 3 month treasury yield. Beyond the level of sovereign spreads, we are also interested in the relationship between the volatility of spreads and the dynamics of the world interest rate which we also tease out using a panel regression.

Table 4 presents the results of our regressions on the level and volatility of sovereign spreads for a panel of 66 nations in columns 1 and 3, and for a panel of simulated data from our model in columns 2 and 4.¹⁹ In column 1 we see that a higher world interest rate is positively associated with higher emerging economy spreads. A similar positive association is seen between our estimated time series on the volatility of the world interest rate and the level of emerging nation spreads. Our regression includes country fixed effects and uses the debt-to-GDP ratio, output growth and a risk aversion dummy (based on the VIX) as conditioning factors since these elements are also present in our model. Spreads increase with debt and risk aversion and fall with output growth. All five estimated coefficients

¹⁹The spread, the world interest rate and its volatility are measured in annualized percentage points. The volatility of the spread is measured as 3-year rolling standard deviations, also expressed in annualized percentage points. Similar results hold if we measure the volatility as the absolute deviation from mean. The risk aversion dummy is an indicator variable that takes the value of 1 if the year is labeled as a high risk aversion year: this is based on the VIX (for the data) and the foreign lenders' stochastic discount factor (for the model). Details of the construction of all variables and country coverage are in the appendix.

Table 4: Spread regressions: data and model

Dep. variable:	Spread		Spread volatility	
	Data	Model	Data	Model
r^w	0.26*** (0.05)	0.09*** (0.02)	0.10*** (0.02)	0.06*** (0.01)
r^w volatility	0.79** (0.39)	0.36*** (0.07)	0.35*** (0.12)	0.17*** (0.03)
Debt/GDP	0.02*** (0.01)	0.43*** (0.02)	0.01** (0.003)	0.12*** (0.01)
GDP growth	-0.20*** (0.04)	-0.10*** (0.01)	-0.03 (0.02)	-0.06*** (0.01)
Risk aversion dummy	1.07*** (0.19)	0.13* (0.07)	0.39*** (0.08)	0.06* (0.04)
No. of countries/samples	66	66	66	66
Adjusted R ²	0.78	0.96	0.59	0.68

Note: All specifications include country/samples fixed effects. r^w stands for the world interest rate. Robust standard errors are reported in parentheses. * $p<0.1$; ** $p<0.05$; *** $p<0.01$.

are significant at conventional levels. In column 2, we present the results of an identical regression using a panel of model-simulated data. As in column 1, our model displays a positive association of the spread with the level and the volatility of world interest rates, debt and the risk aversion dummy and a negative association with output growth. Once again, all coefficients are statistically significant.

The model implies that a one percentage point increase in the world interest rate will increase the spread by 9 basis points while a one percentage point increase in volatility will increase the spread by 36 basis points: these effects are between 1/3 and 1/2 of the mean effects seen in the data. When comparing the quantitative response of the model to the empirical patterns, we see that both feature spreads that are more positively correlated with interest rate volatility than with interest rate levels. Overall, these results are broadly consistent with the empirical literature on spreads (see discussion below), and as such justify the worries of emerging economy policy makers regarding a rise in uncertainty around the world interest rate (driven, for example, by uncertainty regarding US monetary policy).²⁰

²⁰The results in columns 2 and 4 of table 4 are not very sensitive to using risk neutral (instead of risk

Effect on the volatility of sovereign spreads. Columns 3 and 4 of Table 4 turn attention to the volatility of sovereign spreads. This measure is regressed on the same five variables as before: the level and volatility of U.S. interest rates, the debt to GDP ratio, output growth, and the risk aversion dummy. We find that both higher and more volatile world interest rates are associated with more volatile sovereign spreads. The estimated relationship between spread volatility and the volatility of the U.S. interest rate is again larger than that with the volatility of the interest rate. Higher debt levels and periods of higher risk aversion are also associated with a rise in volatility of sovereign spreads, while GDP growth lowers the volatility. All coefficients except that of GDP growth are significantly estimated at conventional levels. Once again we explore the ability of our model to deliver these empirical patterns with identical regressions using model-simulated data in column 4. Our model predicts that a rise in the level and volatility of the world interest rate lead to a rise in the volatility of the spread. A rise in debt as well as periods of heightened risk aversion are associated with increases in the volatility of the sovereign spread. Output growth has a negative (and significant) coefficient in the model regression. All model based coefficients have the same sign as in the emerging economy data. Comparing columns 3 and 4, we note that the model generates roughly half of the positive co-movement between spread volatility and the (level and volatility of the) world interest rate estimated from our panel of emerging economies.

Our empirical results are consistent with existing studies that explore these relationships. [Arora and Cerisola \(2001\)](#) explore the empirical determinants of sovereign spreads using data from 11 emerging economies with special emphasis on US monetary policy with controls typical in the literature. Like us, they find that the level of spreads is increasing in the level of the interest rate as well as a proxy of volatility. Unlike us, they use the federal funds rate as their measure of interest rate and their volatility proxy is constructed from the estimated values of the conditional standard error from an ARCH model for the difference between the three-month U.S. treasury bill yield and the federal funds rate. Despite these differences the results are strikingly similar with one difference being that we find a larger relationship between volatility and spread than between level of the US interest rate and spread. [Foley-](#)

averse) foreign lenders. Results for this case are available upon request from the authors.

[Fisher and Guimaraes \(2013\)](#) also find that an unexpected increase in US inflation-indexed bond yields increases sovereign spreads. [Longstaff, Pan, Pedersen and Singleton \(2011\)](#) and [Fender, Hayo and Neuenkirch \(2012\)](#) also include a measure of US interest rates in their study of the determinants of sovereign CDS data. Like us, they also control for the impact of global risk appetite (as measured by VIX).

International comovement in sovereign spreads. Time variation in the world interest rate not only increases the level and volatility of sovereign spreads, but can potentially create comovement between the spreads of various countries. To illustrate and quantify this natural implication we do the following exercise: (i) we draw multiple random (and independent) samples for the income process, (ii) we select the two samples that have the lowest correlation (in absolute value), (iii) we feed these sequences of shocks into our full model along with the a common (and independent) sequence of shocks to the world interest rate, (iv) finally, we compare the time series profile of spreads between these two samples, which may be thought of as symbolizing two countries.

Since the correlation between the income processes hitting our simulated economies is essentially zero (1.17×10^{-6}), if these economies were to face a constant interest rate, then the correlation between their spreads should not be significantly different from zero.²¹ In contrast, when these economies face a common and time-varying world interest rate, the correlation between their spreads should increase (driven by the common global factor). Indeed we find that the correlation between spreads is significant and equal to 0.51.²²

Using data from a quarterly panel of emerging economies, we find that the mean pairwise correlation in sovereign spreads is 0.61.²³ This simple exercise highlights the importance of global shocks (in this case the world interest rate) in explaining the observed comovement between sovereign spreads.

²¹We confirm this insight by running this same exercise in our ‘basic model’ with a constant r (explained below in section 5.3) and indeed find a correlation between the spreads of 0.03 (and insignificant).

²²This behavior is reminiscent of the waves of insolvency problems in ‘periphery countries,’ as documented by [Kaminsky and Vega-García \(2016\)](#).

²³For this exercise we use quarterly spread data from [Longstaff et al. \(2011\)](#). The appendix has the details of the data as well as a complete spread correlation matrix.

Effect on debt accumulation. We are also interested in understanding how the behavior of the world interest rate correlates with the amount that emerging economies are able to borrow. We regress the growth rate of the debt-to-GDP ratio on the level and the volatility of the U.S interest rate. Table 5 presents results for both the emerging economy panel and the model-simulated data. We find that both the level and volatility of the world interest rate have negative coefficients which are significant at conventional levels. Simulated data from our quantitative model also displays this negative association.

Table 5: Debt regressions: data and model

	Dep. variable: Debt growth	
	Data	Model
r^w	−0.020*** (0.006)	−0.004*** (0.0005)
r^w volatility	−0.178*** (0.032)	−0.01** (0.002)
N. of countries/samples	66	66
Adjusted R ²	0.12	0.07

Note: we include country/samples fixed effects. r^w stands for the world interest rate. Robust standard errors are reported in parentheses. * $p<0.1$; ** $p<0.05$; *** $p<0.01$.

5.3 Keeping r constant: the basic model

Having shown that both the level and the volatility of the world interest rate have important influences on the debt and default dynamics of our “full model”, we now compare the average properties of this model to a “basic model” where the world interest rate is constant (i.e. we set $u_r = u_{\sigma_r} = 0$ while leaving all other parameters unchanged at their values in the full model). We simulate and compute statistical moments from the basic model in the same way as we did for the full model.²⁴

Table 6 reports the main moments of interest for both models. It highlights three clear results: on average, shocks to the world interest rate lead to (i) higher spreads, (ii) more volatile spreads, and (iii) lower debt carrying capacity.

²⁴Additional decomposition exercises where the full model and basic model are compared to an intermediate case with shocks to the level of world interest rates but no shocks to the volatility are available in the appendix.

The average level of the sovereign spread is 10% higher (roughly 30bps) and 15% more volatile (roughly 20bps) in the full model compared to the basic model. As we showed in section 5.1, uncertainty about the world interest rate can in some cases decrease borrowing and increase it in others. Table 6 shows that the precautionary forces dominate, on average.²⁵

Table 6: Main moments of interest: Full and basic models

	Full Model	Basic Model
Debt/ y (in %)	48	50
Spread (in %)	3.4	3.1
SD(Spread) (in %)	1.5	1.3

Note: the moments' computation and de-trending method are the same as those described in the footnote to Table 3. There is no recalibration in the basic model.

Comparing borrowing opportunities. Panel (a) in Figure 7 shows the spread-debt menus in both economies.²⁶ This figure shows that uncertainty regarding the world interest rate shrinks the opportunity set for the small open economy: the spread-debt menu shifts ‘up and to the left.’

The figure also provides an illustration of the main differences between the two models highlighted in Table 6. For the mean debt level (roughly 48% of income), the equilibrium spread in the full model would be about 100 basis points higher than in the basic model (even with r and σ_r at their means). The figure also shows the equilibrium choices of the government in the two models: the precautionary forces dominate and the government chooses a lower debt level (and yet pays a higher spread).

Welfare effects. Having established that our quantitative model is able to replicate the broad patterns of interest rates and debt levels, we turn to a natural question to ask: what is the welfare cost of being exposed to shocks to the world interest rate? Or equivalently, what are the welfare gains for the average emerging nation that borrows on world markets of

²⁵These results are, as expected, consistent with the findings in regression tables 4 and 5.

²⁶This relates to the findings in section 5.1, except that it compares the spread-debt menus between the full model and the basic models.

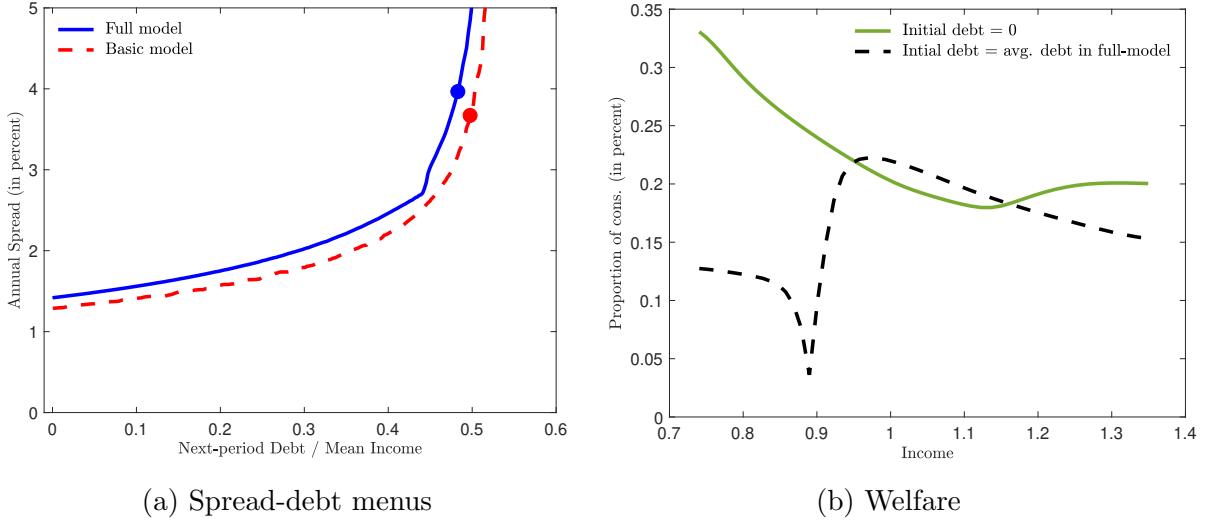


Figure 7: Comparison between “full” and “basic” models. Panel (a) shows the spread-debt menus (computed for mean values of $\{r, \sigma_r\}$ in the full model, and the mean income level for both models). Panel (b) shows welfare gains of moving from the full to the basic economy. The solid green line is for zero initial debt and the dashed black line assumes the initial debt level equals the average in the simulations.

getting rid of the world interest rate uncertainty? Panel (b) in Figure 7 plots these gains as a function of the income level. The gains are expressed as the constant proportional change in consumption that would leave a consumer indifferent between living in the full model or the basic model where r is constant. We present results for two scenarios. In scenario 1, initial debt is zero while in scenario 2, the economy starts at the mean level of debt.

When initial debt is zero (solid green line), the average (across income levels) welfare gain is 0.21% of permanent consumption. Note that in this case the welfare gains are decreasing in the income level. For the case with positive initial debt (dashed black line), there are some interesting non-monotonicities at work. At low income levels, the welfare gains are particularly low since default is more likely and the value of defaulting ($v^d(s)$) under no interest rate uncertainty is not dramatically higher than with uncertainty. However, for intermediate levels of income, the welfare gains are higher because it is precisely in these states where the basic model implies that the government is able to repay existing debt and also borrow at cheaper rates than in the full model. The average welfare gain of eliminating all uncertainty about the world interest rate in this case (with initial debt equal to the mean level observed in the simulations) is equal to a 0.20% constant increase in consumption.

6 Conclusions

We have introduced time-varying volatility in the world interest rate in a standard sovereign default model with long term debt. The process for the world interest rate follows the work of [Fernández-Villaverde et al. \(2011\)](#) and includes both mean volatility (i.e. shocks to the level of the interest rate) and stochastic volatility (i.e. shocks to the volatility of the interest rate). Time variation in the world interest rate interacts with default incentives and its effect on borrowing and sovereign spreads is state contingent.

We disentangle these state contingent effects by running panel regressions using both actual and model-simulated data (recall our model is calibrated to a panel of emerging economies). We include the level and the volatility of the world interest rate as regressors along with model relevant covariates (i.e., debt levels, output growth and a measure of risk-aversion). We find, both in the data and in the model, significant positive relationships between the level and the volatility of the world interest rate and the level of sovereign spreads. We also uncover a positive association between the volatility of spreads and the level and volatility of the world interest rate. Consistent with the data, the model implies that debt growth is decreasing in both variables.

In our model, the common process for the world interest rate acts as a global factor with the potential to generate international comovement in sovereign spreads. We quantify this by comparing simulations from the basic and full models: independent economies featuring uncorrelated spreads under the basic model would produce times series for their spreads that have a positive and significant correlation under the full model (i.e., with a common stochastic process for the world interest rate). The estimated correlation (0.51) is close to the cross-country spread correlation found in international data (0.61).

The welfare gains from eliminating uncertainty about the world interest rate amount to a 0.21% percent permanent increase in consumption. Put differently, our work helps to understand the concerns expressed by policy makers in the face of an increase in uncertainty about the path of the world interest rate.

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ONLINE APPENDIX

A Data

A.1 Time and country coverage

For our empirical analysis (calibration targets and regressions) we use annual data from a panel of 66 emerging economies for the period 1990 — 2017. The list of countries is the following:

1. Algeria	23. Ethiopia	45. P. N. Guinea
2. Angola	24. Fiji	46. Pakistan
3. Argentina	25. Gabon	47. Panama
4. Azerbaijan	26. Georgia	48. Peru
5. Barbados	27. Ghana	49. Philippines
6. Belarus	28. Guatemala	50. Poland
7. Belize	29. Honduras	51. Romania
8. Brazil	30. Hungary	52. Russian Federation
9. Bulgaria	31. India	53. Senegal
10. Chile	32. Indonesia	54. Slovakia
11. China	33. Jamaica	55. South Africa
12. Colombia	34. Jordan	56. Sri Lanka
13. Congo	35. Kazakhstan	57. Tanzania
14. Costa Rica	36. Latvia	58. Thailand
15. Côte d'Ivoire	37. Lebanon	59. Trinidad and Tobago
16. Croatia	38. Lithuania	60. Tunisia
17. Czech Rep.	39. Malaysia	61. Turkey
18. Dominican Republic	40. Mexico	62. Ukraine
19. Ecuador	41. Mongolia	63. Uruguay
20. Egypt	42. Morocco	64. Venezuela
21. El Salvador	43. Mozambique	65. Vietnam
22. Estonia	44. Nigeria	66. Zambia

A.2 World interest rate (r)

We obtain this rate by subtracting expected inflation from the quarterly US T-bill rate. Following [Neumeyer and Perri \(2005\)](#) and [Fernández-Villaverde et al. \(2011\)](#), we compute expected inflation as the average of the US CPI inflation in the current quarter and in the 3 preceding quarters. Both of these time series are obtained from FRED for the period January 1990 - March 2017.

As explained in the main text, we estimate equations (8) and (9) using the `stochvol` R package, which implements an efficient algorithm for Bayesian estimation of stochastic volatility models via MCMC methods. Our measure for the ‘volatility of the world interest rate’ in the regressions is $\exp(\sigma_{r,t})$.

The process for r is estimated at a quarterly frequency. We obtain annualized time series by taking the year's mean of both r_t and $\sigma_{r,t}$ (but all results hold if we use the median, or the last quarter's value).

A.3 Other variables' definitions

As is common in studies of emerging economies, we exclude crisis years. Whenever possible, we take the data from the online appendix of [Catão and Mano \(2017\)](#). We also follow them in terms of variable definitions for debt, spreads, and crisis years. Here, we provide a brief description of these variables:²⁷

1. Gross Domestic Product (GDP): as reported in IMF's International Financial Statistics. 'GDP growth' is defined as the three-year moving average of the growth rate of GDP.
2. Debt: we focus on external debt. The source is the World Bank's Global Development Finance database.
3. Spreads: the main source for emerging market spreads is JP Morgan's EMBI spreads. The volatility of the country spread is measured as its standard deviation in three-year rolling windows.
4. Risk aversion dummy: is an indicator variable taking a value of 1 if the VIX is above its mean in a given year, and taking a value of 0 otherwise. The VIX is the commonly used volatility index.
5. Crisis years: these are defined as years in which a given country experienced a "credit event." These events are defined as all the years in between the initial default and full (or near full) settlement of arrears as per the Standard and Poor's definition.

A.4 Model-generated data used in the regressions

As mentioned in the main body of the text we use pre-default samples of 104 quarters (26 years). Here are some details about the time series used in the model regressions:

1. GDP growth: similarly to the panel data, it is defined as the three-year moving average of the growth rate of annual GDP.
2. Annualized debt-to-GDP in quarter t : $b_t^{\text{annualized}} = b_t / \sum_{j=t-3}^t y_j$.
3. Annualized spread in quarter t : $s_t^{\text{annualized}} = \left(\frac{1+i^*}{1+i^{\text{df}}} \right)^4 - 1$, where i^* is the constant yield-to-maturity implicit in the sovereign bond price and i^{df} is the constant yield-to-maturity of a default free bond with identical coupon structure.
4. Risk aversion dummy: an indicator variable that takes a value of 1 if the expected stochastic discount factor (SDF) of the international lenders ($\mathbb{E}_t m_{t,t+1}$) is below its mean, and zero otherwise. A low expected SDF correlates with a higher degree of risk aversion.

²⁷See [Catão and Mano \(2017\)](#)'s data appendix for further details.

B A different specification for the lender's SDF

As anticipated in section 3.2 of the main body of the text, we also solve the model allowing for a richer specification of the lenders' stochastic discount factor (SDF). In particular, we allow the lenders' SDF to be affected by r_t , the innovations to GDP (ε_t^y), the variance of innovations to GDP (σ_ε^2), *as well as* the variance of the innovations to the world interest rate.

The conditional variance of the innovations to the world interest rate is given by:

$$Var(e^{\sigma_{t+1}} u_{t+1} | \sigma_t) = (\tilde{\mu}_\sigma)^2 \times \exp(2\eta_r) \equiv \Omega_{t+1}$$

where $\tilde{\mu}_\sigma \equiv E(e^{\sigma_{t+1}} | \sigma_t)$ and η_r measures the degree of stochastic volatility in the international risk free rate (see section 3.3 in the paper). Therefore, our new SDF is:

$$m_{t,t+1} = \exp \left(- (r_{t+1} + \kappa_y (\varepsilon_{t+1}^y + 0.5 \kappa_y \sigma_\varepsilon^2) + \kappa_r \Omega_{t+1}) \right),$$

where now $\{\kappa_y, \kappa_r\}$ control the degree of risk aversion. Since we now have an additional parameter (κ_r) we recalibrate our model targeting one additional moment (the unconditional default frequency). The table below shows that the calibration of the model with the 'new SDF' produces almost identical simulated moments.²⁸

Table A1: Model comparison – targeted and non-targeted moments

	Data	Benchmark	New SDF
Debt/ y (in %)	48	48	48
Spread (in %)	3.4	3.4	3.4
SD (Spread) (in %)	1.5	1.5	1.5
Defaults per 100 years	1.7	1.9*	1.7
$sd(c)/sd(y)$	1.2	1.3	1.3
$corr(c, y)$	0.7	1.0	1.0
$corr(tb/y, y)$	-0.4	-0.6	-0.6
$corr(Spread, y)$	-0.4	-0.7	-0.7

Note: table 3's footnote applies here. The default frequency is computed using all simulation periods. The benchmark model's calibration does not target the default frequency (*).

In order to illustrate that all our results are robust to using this richer SDF, we expand regression table 4 to include columns estimated with simulated data from this new model. The table below shows that almost all the coefficients of interest are still significant and are of similar magnitude across the models.

²⁸The re-calibration is such that: $\kappa_y = 3.25$ and $\kappa_r = 3.0$. All other parameters are unchanged. The default frequency is computed using data for default events from [Catão and Mano \(2017\)](#).

Table A2: Spread regressions: comparison between model specifications

Dep. variable:	Spread			Spread volatility		
	Data	Benchmark	New SDF	Data	Benchmark	New SDF
r^w	0.26*** (0.05)	0.09*** (0.02)	0.11*** (0.02)	0.10*** (0.02)	0.06*** (0.01)	0.07*** (0.01)
r^w volatility	0.79** (0.39)	0.36*** (0.07)	0.31*** (0.05)	0.35*** (0.12)	0.17*** (0.03)	0.08*** (0.03)
Debt/GDP	0.02*** (0.01)	0.43*** (0.02)	0.45*** (0.02)	0.01** (0.003)	0.12*** (0.01)	0.13*** (0.01)
GDP growth	-0.20*** (0.04)	-0.10*** (0.01)	-0.08*** (0.01)	-0.03 (0.02)	-0.06*** (0.01)	-0.05*** (0.01)
Risk aversion dummy	1.07*** (0.19)	0.13* (0.07)	0.05 (0.07)	0.39*** (0.08)	0.06* (0.04)	0.04 (0.04)
No. of countries/samples	66	66	66	66	66	66
Adjusted R ²	0.78	0.96	0.96	0.59	0.68	0.66

Note: All specifications include country/samples fixed effects. r^w stands for the world interest rate. Robust standard errors are reported in parentheses. *p<0.1; **p<0.05; ***p<0.01.

C Comovement of sovereign spreads in the data

As explained in section 5.2 of the main body of the text, we compute the cross country pairwise correlations in sovereign spread using quarterly data from [Longstaff et al. \(2011\)](#).²⁹ Using quarterly data for this particular exercise is more appropriate since low frequency data (like annual data) could mask the true comovement between spreads.

As can be seen in the figure below, every time the pairwise correlation is significant it is also positive, as our model predicts. The mean (median) is 0.61 (0.68).

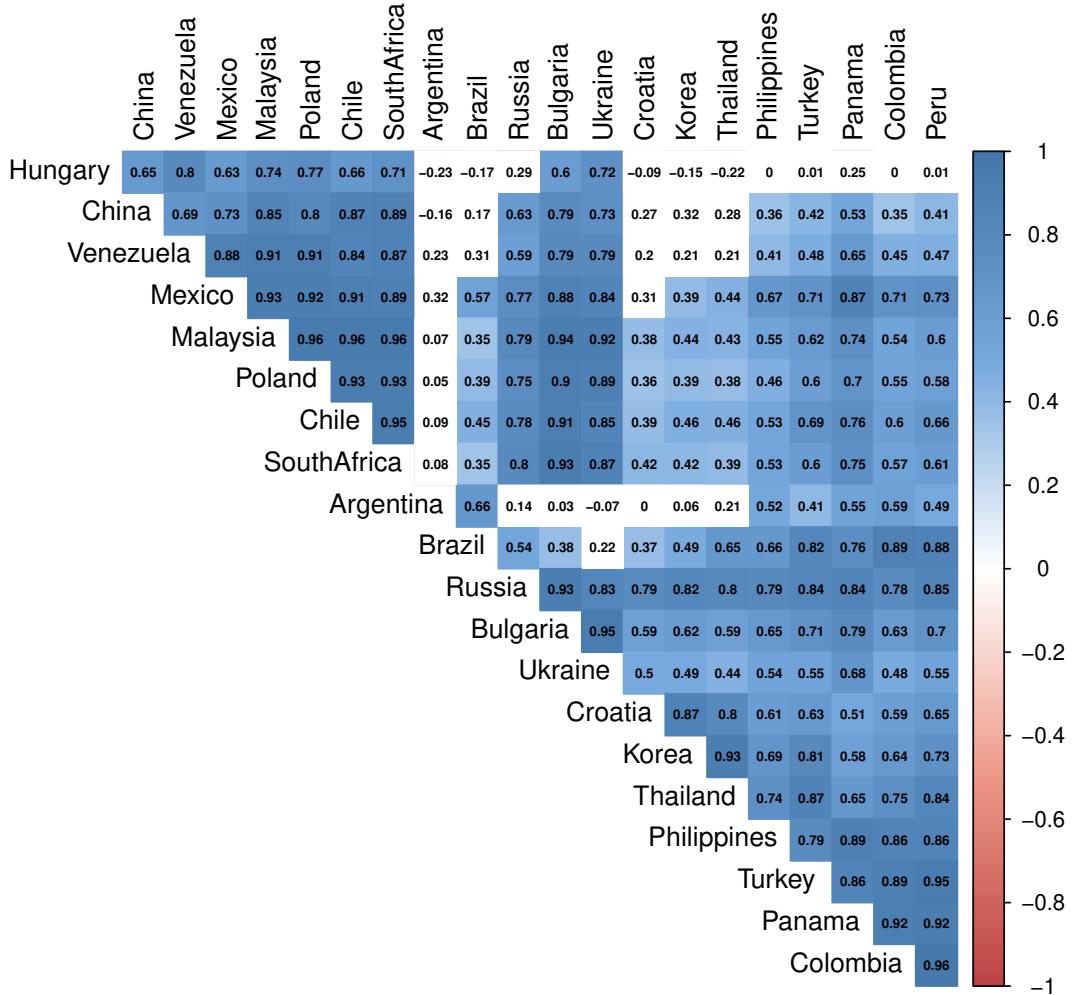


Figure A1: Correlation matrix for spreads. Non-white cells denote pairwise correlations that are statistically significant at the 5% level.

²⁹We take [Longstaff et al. \(2011\)](#)'s monthly data and use quarter-end observations to obtain quarterly time-series. Results are similar if we use quarter averages.

D The ‘intermediate’ model

As pointed out in section 5.3, one could do a further de-composition of the effect of a time-varying world interest rate. To illustrate this we define the ‘intermediate model’ as the one that features a time-varying level of the interest rate but with a constant volatility. Essentially, what we do is to set $u_{\sigma_r,t} = 0$ and rewrite the process for the world interest rate as

$$\begin{aligned} r_t &= \bar{r} + \varepsilon_{r,t} \\ \varepsilon_{r,t} &= \rho_r \varepsilon_{r,t-1} + \Gamma u_{r,t} \end{aligned}$$

where Γ is given by

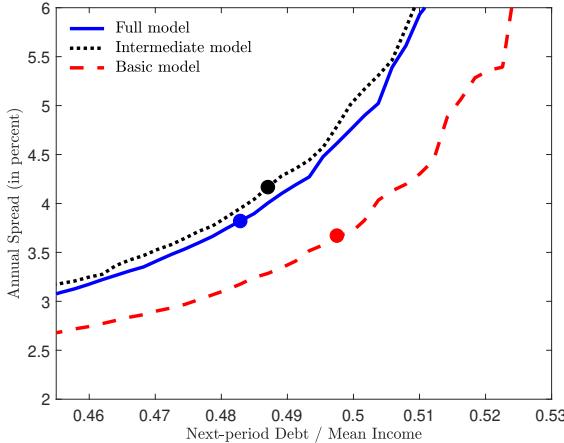
$$\Gamma \equiv E[e^{\sigma_t}] = e^{\bar{\sigma} + \frac{1}{2}\eta^2/(1-\rho_\sigma^2)},$$

and represents the unconditional mean volatility of the world interest rate in the full model. Defining the intermediate model in this way guarantees that both models, full and intermediate, face the same mean volatility of r (even when the intermediate model does not have stochastic volatility).

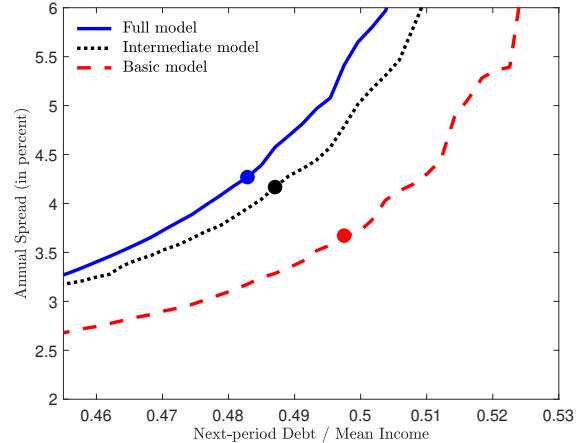
Table A3: Model comparison: basic, intermediate and full

	Basic	Intermediate	Full
Debt/ y (in %)	50	49	48
Spread (in %)	3.1	3.4	3.4
SD (Spread) (in %)	1.3	1.5	1.5
$sd(c)/sd(y)$	1.35	1.33	1.33
$corr(c, y)$	0.98	0.96	0.96
$corr(tb/y, y)$	-0.77	-0.57	-0.56
$corr(Spread, y)$	-0.76	-0.71	-0.70

The table above presents simulation moments for all three versions of the model: full, intermediate and basic. The main take away from this cross-model comparison of simulation moments is that mean volatility in the world interest rate (present in both full and intermediate models) is quantitatively more important than stochastic volatility (present only in the full model), *on average*. This is not to say that the time-variation in the volatility of the world interest rate is not important. On the contrary, our paper shows that the effect of volatility is highly state contingent (see discussions in sections 5.1 and 5.2). Moreover, this is also evident when comparing the full model against the intermediate model, as the figure below shows.



(a) Low σ_r



(b) High σ_r

Figure A2: Spread-debt menus across the three different models. The left panel is for a low level of σ_r and the right panel is for a high level of σ_r . The solid blue line is for the full model, the dotted black line is for the intermediate model and the red dashed line is for the basic model. Both panels are for the mean income level and mean level of r . The solid dots are the median debt levels observed in the simulation of each model.

Figure A2 shows the spread-debt menus for the full and intermediate models for two cases: low and high σ_r (the basic model is also shown, for completeness).³⁰ When σ_r is low the full model actually features less volatility in r than the intermediate model.³¹ The left panel shows that in this case, the spread-debt menu is more favorable in the full model. The right panel shows the opposite case: then σ_r is high, then the full model faces uniformly worse borrowing terms.

³⁰We zoom into the relevant debt range in order to highlight better the differences across models.

³¹Recall the intermediate model was constructed so that it features the same unconditional volatility in r seen in the full model. So, if the full model is in a low σ_r state it effectively becomes a mean-preserving *contraction* of the intermediate model, and naturally features less uncertainty (in that period).

E Robustness to using finer grids

We solve our model numerically using value function iteration on a discrete grid. We use [Tauchen \(1986\)](#)'s method to discretize the income shock and Rouwenhorst's method to discretize the interest rate level shock and the interest rate volatility shock (as suggested by [Kopecky and Suen, 2010](#)).³² Our benchmark calibration is done on a state space of the following dimension: $Nb = 300$, $Ny = 50$, and $Nr = Nr_{vol} = 7$.

In this section of the appendix we show that the average behavior of our model is robust to using finer grids. To do so we increase each dimension of the state space by 50%, one at a time. Lastly, we increase all grid sizes by 50% at the same time. Throughout this exercise we keep parameter values unchanged. As table [A4](#) shows, our results are very robust to increasing grid sizes.

Table A4: Robustness to finer grids

Case	Benchmark	1	2	3	4
Nb	300	450	300	300	450
Ny	50	50	75	50	75
$Nr = Nr_{vol}$	7	7	7	11	11
Debt/ y (in %)	48	48	48	48	48
Spread (in %)	3.4	3.5	3.5	3.5	3.5
SD (Spread) (in %)	1.5	1.5	1.5	1.5	1.4
$sd(c)/sd(y)$	1.3	1.3	1.3	1.3	1.3
$corr(c, y)$	0.96	0.96	0.96	0.96	0.96
$corr(tb/y, y)$	-0.56	-0.58	-0.58	-0.57	-0.58
$corr(Spread, y)$	-0.70	-0.71	-0.72	-0.71	-0.72

Finally, we show that the main regressions results are also robust: these results are presented in table [A5](#). As can be see from this table, our main regression results are very similar across cases. The only noticeable changes are that the coefficient on r is slightly larger as we increase grid sizes and the coefficient of r volatility is slightly smaller, but both remain significant at the 1% level throughout all cases.

³²In their numerical examples, [Kopecky and Suen \(2010\)](#) show that a 5-point grid (their benchmark value) provides a good approximation to persistent AR (1) process using Rouwenhorst's method. Our simulations never use less than 7 points for the interest rate process, and never less than 50 points for the income process.

Table A5: Spread regressions: robustness to finer grids

Panel A – Dependent variable: Spread					
	Benchmark	Case 1	Case 2	Case 3	Case 4
r^w	0.09*** (0.02)	0.13*** (0.02)	0.12*** (0.02)	0.13*** (0.02)	0.12*** (0.02)
r^w volatility	0.36*** (0.07)	0.33*** (0.06)	0.30*** (0.05)	0.28*** (0.05)	0.29*** (0.07)
Debt/GDP	0.43*** (0.02)	0.49*** (0.03)	0.47*** (0.03)	0.47*** (0.03)	0.47*** (0.03)
GDP growth	-0.10*** (0.01)	-0.09*** (0.01)	-0.10*** (0.02)	-0.08*** (0.02)	-0.11*** (0.02)
Risk aversion dummy	0.13* (0.07)	0.14** (0.07)	0.10 (0.06)	0.12* (0.07)	0.07 (0.07)

Panel B – Dependent variable: Spread volatility					
	Benchmark	Case 1	Case 2	Case 3	Case 4
r^w	0.06*** (0.01)	0.08*** (0.01)	0.07*** (0.01)	0.09*** (0.01)	0.08*** (0.01)
r^w volatility	0.17*** (0.03)	0.11*** (0.03)	0.10*** (0.03)	0.09*** (0.03)	0.11*** (0.04)
Debt/GDP	0.12*** (0.01)	0.15*** (0.01)	0.14*** (0.01)	0.14*** (0.01)	0.14*** (0.01)
GDP growth	-0.06*** (0.01)	-0.05*** (0.01)	-0.05*** (0.01)	-0.05*** (0.01)	-0.06*** (0.01)
Risk aversion dummy	0.06* (0.04)	0.09** (0.04)	0.09** (0.04)	0.03 (0.04)	0.05 (0.04)

Note: All specifications include country/samples fixed effects. r^w stands for the world interest rate. Robust standard errors are reported in parentheses. * $p<0.1$; ** $p<0.05$; *** $p<0.01$.