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ASSESSMENT OF COLLUSION DAMAGES IN FIRST PRICE AUCTIONS*

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ABSTRACT. We propose a structural method for estimating the revenue losses associated with bidding rings in symmetric and asymmetric first-price auctions. It is based on the structural analysis of auction data and is consistent with antitrust damage assessment methodologies: we build a but-for (competitive) scenario and estimate the differences between the two scenarios. We show in a Monte Carlo exercise that our methodology performs very well in moderate size samples. We apply it to Ohio Milk Data Set analyzed by Porter and Zona [1999] and find that damages are around 7%. Damages can be assessed without any information about unaffected markets.

Keywords: Collusion; First price auctions; Damages.
JEL: C1, C4, C7, D44, L4.

1. INTRODUCTION

Auctions and procurements are widely used mechanisms in the private and public sectors to buy and sell goods and services. Collusion among bidders, however, is a permanent concern for auctioneers and antitrust authorities, as the benefits of handling an auction critically depend on the bidders behaving competitively. In the

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US, most of the Section 1 violations of the Sherman Act cases are related to collusion in auctions or procurements; the evidence of collusion is overwhelming.[1]

From the perspective of antitrust authorities, it is of paramount importance to use publicly available bidding data to (1) "flag" bidders whose bids are inconsistent with competitive bidding, and (2) quickly estimate potential damages if anticompetitive behavior is suspected. These analysis should help authorities "police" the market, especially before they launch a costly full investigation. For instance, it might not even be worthwhile to start an investigation if the damages are smaller than the cost of an investigation.

Related to the second issue, in this paper we propose a tractable yet general method to estimate the damages caused by collusive behavior using only bids data. We use structural analysis of auctions methodologies (Laffont and Vuong [1996]; Guerre, Perrigne, and Vuong [2000]) to recover players' underlying valuations, and then we estimate counterfactual (competitive) bids. The comparison of the data with the estimated counterfactual behavior gives us the damages.

The main advantages of our method are that it is simple to implement, it does not require additional information on markets not affected by the anticompetitive behavior, and it can easily be adjusted depending on the exact nature of the ring. We envision that it can be used by antitrust agencies both for an initial and quick assessment of damages and for a more in-deep analysis once a full investigation is underway. To provide a proof-of-concept, we apply our method to the data from Ohio milk cartel from Porter and Zona [1999] and we find the estimated damage to be around 7%, which is similar to what Porter and Zona [1999] find.

[1] See Comanor and Schankerman [1976]; Feinstein, Block, and Nold [1985]; Lang and Rosenthal [1991]; Porter and Zona [1993]; Froeb, Koyak, and Werden [1993]; Baldwin, Marshall, and Richard [1997]; Bajari [2001]; Porter and Zona [1999]; Pesendorfer [2000]; Marshall and Meurer [2004]; Asker [2010]; Harrington [2008]; Marshall and Marx [2012]; and [Conley and Decarolis, 2016]. According to the D.O.J.'s website, since 2000, 399 cases involving a bid rigging accusation were filed.

Markets as timber, highway constructions, off-shore wildcat, and school-milk programs among others have been extensively studied in the empirical auction literature; see Athey and Haile [2006] and the papers they cite. The main focus of the empirical literature on collusion in auctions has been on the possibility of distinguishing competitive vs. collusive behavior (either explicit or tacit) from economic data; see Porter and Zona [1993]; Baldwin, Marshall, and Richard [1997]; Bajari [1997]; Porter and Zona [1999]; Bajari and Ye [2003]; Aryal and Gabrielli [2013]; Conley and Decarolis [2016]; Kawai and Nakabayashi [2014]; Chassang and Ortner [2019], and Harrington [2008] for a review of several methodologies.[2]

Motivated by particular bid-rigging cases, there are also several papers that attempt to quantify the auctioneer damages associated to collusion. Most of this literature (we briefly review it below) consists of reduced-form empirical analysis, where the actual performance of the rigged market is compared to a non-rigged market. The comparator could be the same market in a different period, a similar market that is not suspected of collusion or unsuspected firms acting in the same market where collusion is suspected. This approach is widely used to assess damages for different anti-competitive conducts (see Rubinfeld [2012]).

Two advantages of the comparator based techniques are that they are relatively easy to implement and simple to communicate, which might be relevant in litigation contexts. Its main disadvantage, on the other hand, is that the reliability of the results critically hinges on the validity of the comparator market *and* the possibility of controlling for those additional variables that may explain differences between the two markets. The availability of information on relevant control variables and/or non-affected markets conditions the possibility of using this reduced-form approach.

[2] There is also myriad of theoretical literature, analyzing how different auction formats may be more vulnerable to collusion than others, when and how efficient agreements can be sustained, when firms have an incentive to join a ring, etc. See McAfee, Williams, and Hendricks [2014] for a brief review of this literature and Klemperer [2002]; Kovacic, Marshall, Marx, and Raiff [2006]; Marshall and Marx [2009, 2012] for different perspectives on how the details of the auction design may hinder collusion.

On the other hand, the structural approach to analyze auction data is a framework that has had a tremendous development over that last 30 years. The main reason is the richness of the analysis that can be done, mainly due to the possibility of using game theoretical tools, which economists are now very familiar and confortable with. It is fair to say that using structural models has become a cornerstone of the analysis made by both applied and theoretical industrial organization researchers to study different problems of interest for IO economists.

It is not an exaggeration to say that the structural approach to analyze auction data has transformed the way of studding these markets. As emphasized by Laffont and Vuong [1996] in their influential paper, auction models appear especially well suited for structural estimation "because of the availability of many data sets and the well-defined game forms associated with auctions". The main objective in using the structural approach with auction data is to recover or estimate bidders' private valuations. Given these estimates, one can then study different policy counterfactuals.

Structural empirical work on auctions has examined, for example, the division of rents in auctions of public resources, whether reserve prices in government auctions are adequate, the effects of mergers on procurement costs, whether changes in auction rules would produce greater revenues, whether bundling of procurement contracts is efficient, the value of seller reputations, the effect of information acquisition costs on bidder participation and profits, whether bidders' private information introduces adverse selection, and whether firms act as if they are risk averse. Yet we see that the use of structural methods in antitrust cases in the context of auctions is scant. In this paper we give a step towards filling this gap.

To apply our methodology we require that the identity of the cartel members is known and the functioning of the cartel has been established.[3] The identity of the cartel members is necessary to properly recover the valuation of the serious

[3]Alternatively, in an initial phase of an investigation, methodologies as the one proposed by Bajari and Ye [2003] can be used to preliminary identify the members of the ring and different assumptions can be made about the cartel functioning.

bidder of the cartel, as the first-order equation that characterizes his/her behavior is different from the one of non-colluded players. The functioning of the cartel –e.g., whether the cartel members follow a *phases-of-the-moon* scheme to coordinate or the cartel is efficient and the serious bidder is the one who values the object the most– is key to construct the counterfactual scenario.

In terms of the required information, we need to observe both winning and non-winning bids, but only of the affected market (unlike with comparator based techniques). If the (repeated) auctions are heterogeneous, it is also critical to observe those variables that allow to control for the heterogeneity (e.g., engineers' cost estimates in road maintenance procurements).

To validate our methodology, we perform Monte Carlo exercises generating valuations based on the family of the Power Law distribution function. The advantage of using this function is that, following Cheng [2006], we can calculate theoretical equilibrium bids for all serious players in the collusive and competitive scenarios, and compare the estimated results to the true ones. The results show that our method is robust and provides accurate results for the cases of symmetric and asymmetric players.

We then apply our methodology to the school districts' milk procurement markets in Ohio in the 80s, where collusion was established. A cartel of three firms with their main plants in the Cincinnati area operated for eleven years (between 1980 and 1991). Porter and Zona [1999] thoroughly analyze these markets by comparing bidding participation decisions and bid levels of defendants and non-defendants.

Consistent with the confession of two of the cartel members, they find that cartel members were more likely to submit bids even if their plants were far away from the school district and that this (phony) bids were insensitive to distance (unlike serious bids). They also estimate a reduced form price equation including as controls market structure variables, auction specific variables, and a variable that captures how affected by the ring is the school district. They find that collusion raised prices by 6.5% when considering all school districts where cartel firms had plants within

75 miles, and about 21% in those districts-years where a cartel member was an incumbent.

When applying our methodology to the milk-procurement market we adapt it to consider endogenous entry. We find that the damages in terms of revenue are around 7%. Moreover, when we restrict the sample to those school districts where at least two members of the cartel were potential bidders, we find that damages are approximately 1 percentage point higher on average. This is an interesting robustness check for our methodology, as with this subsample there is no unaffected market being used as a comparator.

As mentioned above, auction markets are an ideal setting for the structural approach, since game theoretic models of auction are widely accepted and can be easily adjusted to consider collusive and competitive scenarios. In our view, measuring the damages from anticompetitive practices with structural analysis makes the analysis more reliable and transparent, since all assumptions behind the theoretical model are explicit and can eventually be challenged and damages re-assessed under different assumptions.[4] Moreover, in many collusion cases pertaining auctions an appropriate competitive comparator may not be available, so a structural approach becomes the only option. The progress done in terms of estimation methods and computational issues, makes this approach even more attractive and easy to implement.

Related literature on damage assessment in auctions. Several papers have assessed damages in bid-rigging cases, most of them performing reduced-form estimations following the before-after approach. Hendricks and Porter [1989] first propose a methodology to assess damages estimating a relationship between costs estimates and prices in non-rigged contracts, and then applying this relationship to

[4] For example, in our estimation of damages we assume non-cartel firms are unaware of the cartel's existence. If that is not the case, then a differential equation (or a system of differential equations in the case of asymmetric firms) should be solved to obtain the counterfactual scenario. In terms of how the cartel works, we consider two polar cases: that the cartel is efficient (i.e., that the cartel member with the highest valuation makes the serious offer and that the internal functioning of the cartel does not distort its offer) or that the serious bidder is randomly chosen. Alternative cases could be considered with minor adjustments.

calculate but-for prices in rigged contracts. With a richer dataset, Froeb, Koyak, and Werden [1993] follow the same logic to estimate price overcharges between 23% and 30% in a frozen-perch fillets case in the eighties. The available information includes a post-conspiracy period, so they follow a before-after methodology using the price of fresh perch fillets as an additional control. In the context of an English auction, Nelson [1993] follows Hendricks and Porter [1989] reduced-form approach and estimates damages between 17% and 28% in the sales of used police-cars in the city of New York.

Though structural methods are widely used in antitrust analysis there is scant literature on structural methodologies for damage assessment in the context of bidding rings.[5] Kwoka [1997] estimates damages between 22% and 32% on the rigging of real estate properties in the DC area. His calculations make use of the observed price at the formal English auction and the price resulting from a knock-out auction the cartel members held right after the formal one. Interestingly, his calculation of damages explicitly considers the fact that the distribution rule of ring profits generated incentives to bid below one's own valuation. Therefore, damages can not be simply assimilated to the observed difference between the two auctions.

Closer to our paper, Asker [2010] estimates efficiency costs and damages in the context of a stamp dealers cartel. The cartel used an internal knockout auction to allocate the good among the cartel members before participating in an English auction. Interestingly, the sharing rule among ring members induced overbidding in the knockout auction that was carried to the target auction. As a consequence, for some auctions, the presence of the ring generated damages to non-ring members, efficiency losses, and a gain for the auctioneer. In 19% of the auctions the ring won the price was 7% higher than without the ring; and in 27% of the auctions won by the ring the price was, on average, 17% lower than without the ring.

[5] Strucutural methods have been extensively used in the context of merger simulations. Early works include Hausman, Leonard, and Zona [1994], Shapiro [1996], and Werden [1997]. See Werden and Froeb [2008] for a survey and Peters [2006] and Weinberg [2011] for empirical assessments of the accuracy of merger simulations. See Zona [2011] for a thorough discussion of the use of structural methods based on imperfect competition models to assess damages in price-fixing cases.

Our methodological proposal is similar in spirit to Asker [2010], in the sense that in a first stage the underlying distribution of valuations is recovered from the observed bids using the econometric model derived from the corresponding economic model, and in a second stage counterfactual data –assuming competition among all players– is generated from the recovered valuations. There are several obvious differences though, as our focus is on first-price auctions and therefore the counterfactual differs. Also, in our setup point valuations can be recovered and therefore damages and efficiency losses can be computed for each actual auction, being unnecessary to simulate auctions from the recovered valuation distributions. We are not aware of any structural damage estimation performed in the context of first-price auctions.

The rest of the paper is organized as follows. In Section 2 we first present a fairly standard asymmetric first-price auction setup and then describe our methodological proposal to assess collusion damages in terms of revenue and efficiency losses. In Section 3 we describe our Monte Carlo experiments and present the results we obtain, comparing theoretical and estimated damages. In Section 4, we present the results of our methodology as applied to the Ohio Milk data and in Section 5 we conclude. In Appendix A-1 we present results of our Monte Carlo experiments for the symmetric efficient cartel case and in Appendix A-2 those corresponding to the non-efficient cartel case.

2. Model setup and damage assessment methodology

2.1. **Model and notation.** We consider L homogeneous first-price sealed bid auctions with n_0 low-valuation bidders –type 0– and n_1 high-valuation bidders –type 1. In every auction a type k bidder draws his/her valuation, i.i.d. across all other bidders, from $F_k(\cdot)$ with support $[\underline{v}, \overline{v}]$. We assume a proper subset of type 1 bidders form a collusive ring of size W and that participation in each auction is exogenous and known to all bidders.[6]

[6] The assumptions of two types of firms is to keep the model (and the notations) relatively simple, but the extension to more types is straightforward. In the empirical application we consider a more general case where the number of cartel firms may be different in different auctions and where entry

We denote the symmetric-type bidding strategy of a k-type firm that does not belong to the cartel by $s_k(v_k|n_0,n_1)$ and the strategy of the type-1 firm that makes the serious offer of the cartel by $s_c(v_1|n_0,n_1,W)$. The maximization problems for the firms that do not belong to the cartel and for the serious bidding firm of the cartel are, respectively:

$$\max_{b_i^k}(v_i - b_i^k)(F_k(s_k^{-1}(b_i^k)))^{n_k-1}(F_{k'}(s_{k'}^{-1}(b_i^k)))^{n_{k'}}; \quad k \neq k' \in \{0,1\} \tag{Pk}$$

and

$$\max_{b_i^c}(v_i - b_i^c)(F_1(s_1^{-1}(b_i^c)))^{n_1-|W|}(F_0(s_0^{-1}(b_i^c)))^{n_0}. \tag{Pc}$$

We assume that non-members of the cartel are unaware of the ring's existence. This is reflected in the fact that $s_c(v_1)$ plays no role in Problems (Pk), and also in the power of $F_k(\cdot)$ and $F_{k'}(\cdot)$ when $k=1$ and $k=0$ respectively.[7]

The first-order conditions of these two problems can be written as:

$$v_i = b_i^k + \frac{1}{\frac{(n_k-1)f_k(s_k^{-1}(b_i^k))}{s_k'(b_i^k)F_k(s_k^{-1}(b_i^k))} + \frac{n_{k'}f_{k'}(s_{k'}^{-1}(b_i^k))}{s_{k'}'(b_i^k)F_{k'}(s_{k'}^{-1}(b_i^k))}}; \quad k \neq k' \in \{0,1\} \tag{1}$$

and

$$v_i = b_i^c + \frac{1}{\frac{(n_1-|W|)f_1(s_1^{-1}(b_i^c))}{s_1'(b_i^c)F_1(s_1^{-1}(b_i^c))} + \frac{n_0 f_0(s_0^{-1}(b_i^c))}{s_0'(b_i^c)F_0(s_0^{-1}(b_i^c))}}. \tag{2}$$

We denote the distribution function of non-colluded type-k bids by $G_k(\cdot|n_0,n_1)$ and the c.d.f. of the serious offers from the cartel by $G_c(\cdot|n_0,n_1,W)$. The assumption that the equilibrium strategies are strictly increasing implies $G_k(b) = F_k(s_k^{-1}(b))$ and $g_k(b) = f_k(s_k^{-1}(b))/s_k'(s_k^{-1}(b))$ for $k=0,1$ (for simplicity we omit the dependence on n_0, n_1). This observation is key to identifying the distribution of private values

is endogenous. We discuss below the role of the assumption that the ring does not include all firms of one particular type and how it can be relaxed.

[7] We discuss at length the implications of relaxing this assumption or estimating a misspecified model in section 3.4.

from observed bids. Note that, given our assumption that colluded firms are only of type-1, both $G_1(b)$ and $G_c(b)$ allow us to recover $F_1(c)$.

2.2. **Methodology to assess damages and efficiency costs.** In a nutshell, the methodology that we follow consists in estimating the underlying valuations of all serious bidders (i.e., non-colluded firms and the serious bidder of the cartel) as described above, and the bidding function of type-1 non-colluded firms. We then use this information to calculate the bid of the serious member of the cartel in the counterfactual scenario. For non-colluded players the factual and counterfactual bids are identical, since we assume they are unaware of the cartel's existence.

Once we have the counterfactual bids of the serious member of the cartel, we are able to compute the counterfactual winning bid and the identity of the winner if there had been no collusion. We then simply calculate the difference between winning bids in the two scenarios –i.e., the revenue cost of collusion– and the difference between the estimated valuations of the winners in the two scenarios –i.e., the efficiency cost of collusion.

The methodology we propose is flexible and can be adjusted depending on how the cartel works. In particular, we consider two main cases: that the cartel members take turns to make the serious offer vs. that the cartel is efficient and, therefore, selects the most efficient member to bid seriously.[8]

The methodology consists of four steps:

(1) We estimate G_0 and g_0 non-parametrically from the observed bids of all type 0 firms, G_1 and g_1 from the observed bids of non-colluding type 1 firms, and G_c and g_c from the observed bids of the serious member of the cartel.
(2) With the observed bids and the distribution functions estimated in the previous step, we follow Guerre, Perrigne, and Vuong [2000] to recover point estimates of all serious players' valuations. We consider two cases here: First, if

[8] Note that equation 2 is derived assuming that the internal agreement of the cartel does not distort the bidding incentives of the serious bidder. If this were the case, the first-order condition should be derived explicitly considering the sharing rule.

the cartel is efficient, the recovered valuation of the cartel serious bidder corresponds to the valuation of the most efficient member and we use it in the next step. Second, if the cartel randomly selects the serious bidder, the recovered valuation may not correspond to the most efficient member of the cartel. In this case, as an intermediate step, we estimate a distribution function F_1 from competitive type 1 players and use it to randomly generate valuations for the non-serious bidders. The highest valuation of all cartel members will be the relevant one for the next step.

(3) We use a polynomial approximation to deduce the relationship between valuations and bids for the competitive type-1 bidders. We then apply this polynomial to the cartel most efficient member's valuation –identified in the previous step– to obtain his counterfactual bid.

(4) Finally, we identify the winner for each auction in the but-for scenario and simply calculate the difference in valuations and winning bids between the collusive and competitive scenarios.

Within the three first steps there is considerable leeway on how to implement them. We now provide a detailed description of what we do in our Monte Carlo experiments and briefly discuss other alternatives.

2.2.1. *Step 1: Estimating the distributions of bids.* In the first step we estimate the distribution and density of bids by means of nonparametric estimators. In order to minimize boundary problems, a typical problem for kernel density estimation, we use the boundary corrected kernel method proposed by Hickman and Hubbard [2015].

2.2.2. *Step 2: Recovering valuation distributions.* The first order conditions (1) and (2) together with the boundary conditions $s_k(\underline{v}) = \underline{v}$, $k \in \{0, 1\}$ fully characterize the symmetric-type strategies that solve problems (Pk) and (Pc). Following Guerre, Perrigne, and Vuong [2000], the first order conditions (1) and (2) can be conveniently re-written in terms of bids' distributions rather than valuations' distributions:

$$v_i = b_i^k + \frac{1}{(n_k - 1)\frac{g_k(s_k^{-1}(b_i^k))}{G_k(s_k^{-1}(b_i^k))} + n_{k'}\frac{g_{k'}(s_{k'}^{-1}(b_i^k))}{G_{k'}(s_{k'}^{-1}(b_i^k))}}; \quad k \neq k' \in \{0,1\} \tag{1'}$$

and

$$v_i = b_i^c + \frac{1}{(n_1 - |W|)\frac{g_1(s_1^{-1}(b_i^c))}{G_1(s_1^{-1}(b_i^c))} + n_0\frac{g_0(s_0^{-1}(b_i^c))}{G_0(s_0^{-1}(b_i^c))}}. \tag{2'}$$

We know from Guerre, Perrigne, and Vuong [2000] and Aryal and Gabrielli [2013] that this model is non-parametrically identified. We see that for each auction and each participant i of type k, each bid uniquely determines the valuation through the first order condition, thereby identifying $\{F_0(\cdot), F_1(\cdot)\}$ that are consistent with the data. Moreover, for each serious bidder in each auction we can obtain a pseudo valuation simply using the above equations.

Note that for the non-serious bidders of the cartel, it is impossible to infer their valuations as they do not bid according to the first order condition (2′). If the cartel is efficient this is irrelevant, since the most efficient cartel member would outbid all other ring members in the counterfactual scenario.

However, under the assumption that the cartel members randomly select who will be the serious bidder (or that they would take turns), we need to allow for the possibility that non-serious bidders could win the auction in the counterfactual scenario. Since we cannot recover pseudo valuations from their observed bids, we first estimate non-parametrically the distribution of valuations $F_1(\cdot)$, and then randomly generate valuations for each non-serious bidder and pick the highest one.

2.2.3. *Step 3: Calculating counterfactual bids for the cartel members.* This step is key and there are several alternatives that could be followed. In particular we fit a polynomial of degree two to obtain the bid strategy as a function of the pseudo valuations of type-1 competitive bidders (recall that the observed bids of this group are the same in the data and counterfactual scenario). This is a relative simple option to calculate counterfactual bids and, as we will show in our Monte Carlo experiments, it is quite accurate.

Once we learn this relationship between valuations and bids, we use it together with the pseudo valuations of the cartel members from the previous step to calculate the counterfactual bids. Under the assumption that the cartel is efficient we will have one counterfactual bid per auction that will change. Under the assumption that the cartel members take turns, we will have a counterfactual bid for each cartel member (although the only relevant bid is the one of the most efficient member).

There are a few alternatives that could also be applied: One (cumbersome) option would be to solve the system of two differential equations that characterize the (counterfactual) equilibrium. This can be done numerically following the method proposed in Fibich and Gavish [2011], that proposes a novel boundary-value method for computing the equilibrium strategies of asymmetric first-price auctions. This approach would be fully consistent with the structural methodology followed to recover valuations, at the expense of considerable computational and programming costs.

A second alternative is to fit parametric functions to the valuation distributions recovered in steps one and two. Then, under these functional forms, it might be possible to analytically solve the two differential equation system or, alternatively, it would facilitate the numerical solution of the system.

Clearly the main advantage of performing a polynomial approximation is ease, robustness and speed with which it can be implemented, which might be relevant in the context of litigation. As long as this alternative provides a good fit of the data we consider it a reasonable compromise.[9]

Note, however, that our approach of fitting a polynomial is restrictive: we need to observe bids of type 1 players behaving competitively to fit the polynomial. There are two scenarios in which this would not be possible: if the cartel is all-inclusive

[9] In our Monte Carlo experiments we present two scenarios: a symmetric one and an asymmetric one with two types of players. In the first case it is easy to numerically solve the first-order differential equation and we do so. In the asymmetric case and in the empirical application we opt for the polynomial approximation.

of type-1 players, or if the non-ring members are aware of the cartel's existence. In the first case simply there are no bids of type-1 non-ring players. In the second case the problem is that non-colluded type-1 firms will bid differently in the factual and counterfactual scenarios. In these cases it would be necessary to rely on one of the alternatives discussed.[10]

2.2.4. *Step 4: Calculating revenue and efficiency losses.* Once the relevant counterfactual bid has been obtained, it is trivial to identify the winner in the counterfactual scenario and to simply calculate the auctioneer's cost and the efficiency cost. Let b_ℓ^{*d} and b_ℓ^{*c} be the winning bids in auction ℓ in the data and counterfactual scenarios, respectively; and similarly let v_ℓ^{*d} and v_ℓ^{*c} be the estimated valuation of the winner in each scenario. Then:

$$\text{Auctioneer's Damages of Collusion} = \sum_{\ell=1}^{L} (\, b_\ell^{*c} - b_\ell^{*d} \,) \tag{3}$$

and

$$\text{Efficiency Cost of Collusion} = \sum_{\ell=1}^{L} (\, v_\ell^{*c} - v_\ell^{*d} \,). \tag{4}$$

3. Monte Carlo Experiments

3.1. **Design of the experiment.** In order to assess the performance of our proposed methodology we perform a number of Monte Carlo simulations based on Power Law distribution functions. The key advantage of using the Power Law distribution is that, following Cheng [2006], we have closed-form solutions for the bidding strategies in the collusive scenario and in the counterfactual. This allows us to compare at the end of the experiment the estimated damages and efficiency losses with the true ones.

For each experiment, our procedure is as follows:

(1) Generate valuations for all auction participants assuming a sample of size of a thousand auctions.

[10] Naturally, if non-ring members were aware of the cartel's existence, we would also need to adjust their optimization problem and their respective first-order conditions to reflect that.

(2) Based on closed-form solutions, compute competitive bids under the assumption of no collusion; this defines the "counterfactual-theoretical" scenario.
(3) Define a cartel formed by $|W|$ firms and, based on closed-form solutions, compute the serious collusive bids. Together with the competitive bids of non-colluded firms, these bids define the "actual scenario" or "data".
(4) Compute revenue and efficiency in the two previous scenarios and the differences between them to obtain the theoretical efficiency loss and auctioneer's damages in terms of revenue.
(5) Start over from real-scenario bids (the data) and, following steps 1 and 2 of the methodology, estimate the distribution of bids and the valuations for non-cartel players and for the serious cartel bidder.
(6) Only if the serious bidder of the cartel is picked randomly (i.e., when the cartel is non-efficient): Estimate F_1, generate random valuations for the non-serious bidders, and identify the most efficient member of the cartel.
(7) From the estimated valuations, estimate the counterfactual bids of the most efficient cartel member following step 3 in our methodological section; these bids together with those of non-colluded bidders define the "counterfactual-empirical" scenario.
(8) Compute revenue and efficiency in the counterfactual-empirical scenario and calculate the differences with the collusive scenario or data to obtain the estimated efficiency loss and auctioneer's damages.
(9) Repeat 1,000 times.
(10) Compare theoretical and estimated efficiency losses and damages.

We perform two sets of experiments. In the first one we consider two types of players as described in the methodological section. In the second one, whose results are relegated to Appendix A-1, all valuations are drawn from the same distribution function (symmetric case). In the first case we consider only different cartel sizes while in the symmetric case we vary both the cartel size and the underlying distribution of valuations.

3.2. **Asymmetric Case - Efficient Cartel.** In this section we assume that bidders are ex-ante asymmetric and a subset of the strong players form a cartel. Asymmetric models are harder to analyze, as there is no analytical solution in general for the system of differential equations that characterizes the equilibrium. In our case, however, we use the Power Law Distribution precisely to have a theoretical solution for the equilibrium strategies, both under competition and under collusion. These solutions allow us to compute "true" damages and efficiency losses, and compare them with the estimated ones.

Our setup considers a total of $n = 6$ bidders. There are $n_0 = 2$ weak or type 0 bidders and $n_1 = 4$, strong or type 1 bidders. Type 1 bidders draw their valuation iid from the distribution $F_1(v) = v^2$ with support $[0, 1]$, while type 0 bidders draw from the uniform distribution $F_0(v) = v/m$ with support $[0, m]$, where $m = 80/81$.[11] In the collusive environment a subset of type 1 bidders form a cartel. We consider two cartel sizes, $|W| = \{2, 3\}$.

Table 1 presents the results obtained for these cases in terms of damages and efficiency losses under the assumption that the cartel player with the highest valuation makes the serious offer; i.e., it is an efficient cartel. For the two cartel sizes considered we present two sets of results: Those corresponding to the true data generating process (DGP), i.e., using the true valuations and the theoretical formulas to compute observed and counterfactual bids (indicated with a T), and those corresponding to the proposed methodology (indicated with an E). In the latter we first recover valuations (pseudo values) from observed bids, we then fit a second degree polynomial to estimate the relationship between valuations and bids of type-1 players, and finally we use it to compute counterfactual bids of the cartel.

Median estimated damages are 10.3 and 39.7 for the cartels of 2 and 3 players respectively. These losses can be compared to the revenue in all 1,000 auctions (Total Revenue) or against the revenue in the set of "Affected auctions"; i.e., those in which the winning bid differs in the collusive (data) and counterfactual scenario.

[11] The adjustment in the support of type 0 bidders is necessary to obtain a closed form solution for the bidding strategies; see Cheng [2006] for details.

Measured against the latter, damages are 3.2% and 8.2% for the two cartel sizes. Efficiency losses are relatively small even when compared to the valuations in the affected auctions, which is expected given the assumption that the most efficient member of the cartel is the serious bidder.

Beyond the absolute values of revenue and efficiency losses, for our purposes it is particularly interesting to highlight how remarkably close to each other are the estimated and theoretical distributions of damages and number of affected auctions.[12]

TABLE 1. Asymmetric players case - Efficient Cartel

POWER LAW ($r_0 = 1, r_1 = 2$)

W=2	5p	25p	50p	75p	95p
Revenue Loss (T)	9.69	10.08	10.34	10.66	11.06
Efficiency Loss (T)	1.21	1.40	1.52	1.66	1.84
Affected Auctions (T)	375	390	400	410	424
Revenue Loss (E)	9.29	9.95	10.33	10.78	11.41
Efficiency Loss (E)	0.87	1.05	1.18	1.32	1.50
Affected Auctions (E)	372	388	400	410	426
Total Revenue	804.22	806.53	808.17	809.62	811.91
Total Efficiency	902.56	905.13	906.93	908.58	911.13
Rev. Affected Auctions	302.17	315.01	322.87	331.24	343.07
Eff. Affected Auctions	339.94	354.39	363.23	372.65	385.95
W=3					
Revenue Loss (T)	37.91	38.97	39.68	40.45	41.74
Efficiency Loss (T)	8.82	9.51	10.01	10.50	11.20
Affected Auctions (T)	576	590	600	610	625
Revenue Loss (E)	35.61	38.13	39.79	41.59	44.44
Efficiency Loss (E)	7.68	8.55	9.14	9.70	10.52
Affected Auctions (E)	571	589	601	614	632
Total Revenue	804.22	806.53	808.17	809.62	811.91
Total Efficiency	902.56	905.13	906.93	908.58	911.13
Rev. Affected Auctions	464.40	477.17	484.71	492.99	504.99
Eff. Affected Auctions	522.45	536.81	545.30	554.62	568.11

[12] We have additional results available upon requests with the mean revenue, efficiency, and profits per firm type, distinguishing the data and counterfactual scenario. As expected, both colluded and non-colluded firms benefit from the ring's existence. Moreover, results show that a non-colluded type-1 firm would benefit joining a cartel that grows from two to three members.

3.3. **Asymmetric Case - Non-efficient Cartel.** Here we present the results when the cartel is assumed to be inefficient in the sense that the serious bidder is randomly chosen. Table 2 has an identical structure to Table 1. As in the previous case, the methodology we propose performs remarkably well in the sense that true and estimated values are really close.

The median revenue losses are 24.3 and 66.9 for the two sizes of the cartel considered. As a percentage of the revenue of affected auctions, damages are 7.5% and 13.8%. Efficiency losses are, respectively, 6% and 10.7% of the affected auctions. These figures, as expected, are much larger than for the efficient cartel case.[13]

TABLE 2. Asymmetric players case - Non Efficient Cartel

POWER LAW ($r_0 = 1, r_1 = 2$)

W=2	5p	25p	50p	75p	95p
Revenue Loss (T)	21.70	23.19	24.25	25.35	26.90
Efficiency Loss (T)	18.88	20.40	21.66	22.84	24.59
Affected Auctions (T)	375	390	400	410	424
Revenue Loss (E)	20.86	22.81	24.11	25.46	27.39
Efficiency Loss (E)	17.62	19.66	21.00	22.49	24.67
Affected Auctions (E)	365	384	399	412	430
Total Revenue	804.22	806.53	808.17	809.62	811.91
Total Efficiency	902.56	905.13	906.93	908.58	911.13
Rev. Affected Auctions	302.17	315.01	322.87	331.24	343.07
Eff. Affected Auctions	339.94	354.39	363.23	372.65	385.95
W=3					
Revenue Loss (T)	62.20	65.02	66.87	69.02	72.06
Efficiency Loss (T)	53.31	56.09	58.47	60.75	63.87
Affected Auctions (T)	577	590	600	611	626
Revenue Loss (E)	60.42	64.23	66.97	69.96	74.13
Efficiency Loss (E)	51.01	54.97	57.39	60.03	64.04
Affected Auctions (E)	565	584	600	615	636
Total Revenue	804.08	806.34	808.04	809.53	811.79
Total Efficiency	902.29	904.90	906.79	908.53	910.89
Rev. Affected Auctions	466.00	476.49	484.98	494.13	506.34
Eff. Affected Auctions	524.25	536.05	545.61	555.89	569.63

[13] Unlike the case of an efficient cartel, when we compare the average profits of colluded and non-colluded firms we conclude that there are no individual incentives to join an inefficient cartel (increasing its size from two to three firms) and there are no joint incentives to form an inefficient cartel of size two or three.

3.4. **Umbrella Pricing and Damages.** An often discussed issue in antitrust is who is entitled to collect damages and whether damages associated to "umbrella pricing" can be claimed. In the context of auctions, umbrella pricing emerges naturally if non-ring firms are aware of the cartel's existence. On the contrary, if non-ring firms are unaware of the existence of the cartel (as we have assumed so far), they would behave as if there were no cartel and there would be no umbrella pricing effect.

A related issue is whether those who did not buy from a cartel firm can claim damages. They could be hurt because non-ring firms are pricing above the competitive level or, even without umbrella pricing, because in the counterfactual scenario they would have bought from a more aggressive cartel firm. The two arguments why a non-direct purchaser of the cartel may suffer damages make perfect economic sense, but in litigation it may be hard for them to successfully claim damages.[14]

Our assumption that non-cartel firms are unaware of the cartel's existence directly implies that there is no pricing under the umbrella of the cartel. The assumption is certainly debatable and it may be valid or not depending in the particular case under scrutiny.[15]

What changes would be required to our methodology if umbrella pricing is relevant? First of all, problem (Pk) should be modified to reflect that the relevant level of competition faced by competitive type 1 bidders is given by $n_1 - |W|$ and, if the cartel is efficient, its valuation is drawn from $F_1^{|W|}$. The main difficulty arises once the pseudo valuations have been recovered (using the new first order conditions) and

[14] See Maier-Rigaud [2014] for a discussion (and a critique) on why purchasers outside a vertical chain of purchasers are not considered in damage claims. In the recent judgement of *Ohio vs. Amex* merchants not buying from *Amex* were denied standing to sue for damages. They alleged that in a counterfactual scenario with *Amex* behaving competitively *Visa* would have had lower prices too, since in the factual scenario they were pricing under the umbrella of *Amex* (anticompetitive) prices. The situation in Europe, however, could be different after the ECJ judgement in *Kone AG and Others v. ÖBB Infrastruktur* (Case C-557/12); see Franck [2015] for a discussion of the ECJ ruling and on the desirability of allowing umbrella claims and the risk of over-deterrence.

[15] In our damage assessment methodology the assumption allows us to easily calculate counterfactual bids for cartel firms in Step 3: By assuming that observed bids of non-cartel bidders are competitive, we are able to estimate a relationship between their estimated private costs and their observed bids and use the results to calculate the counterfactual bids of cartel members once we recover their private costs.

the counterfactual bids need to be calculated. In the context of asymmetric players, this would require numerically solving a system of first order differential equations (see Fibich and Gavish [2011] for a method developed precisely in the context of auctions) or, alternatively, estimating functional forms from the recovered pseudo valuations and solve the system of differential equations based on this functional forms.

What is the bias of ignoring the umbrella pricing effect? If non-cartel firms are aware of the cartel's existence and we ignore it in our calculations we would underestimate damages. Both for the cartel and non-cartel firms we would be assuming that firms behave as if they were facing more aggressive competitors, and this would lead us to underestimate all pseudo-valuations.

The bias will be larger for non-ring members as we are ignoring a first-order effect related to the number of competitors. For ring members this error is of second order of magnitude, as we are missing the adjustment the cartel should made in response to the non-cartel firms being less aggressive when they know about the cartel's existence.

Our estimates would be a lower bound of the true damages when the non-cartel firms are aware of the cartel's existence. However, the bias should be less severe when we focus on the damages of those who bought from the cartel, since we can safely ignore the damages due to the umbrella effect.

To confirm our intuitions, we perform an additional Monte Carlo experiment for the particular case of asymmetric bidders and a non-efficient cartel. We first draw valuations, compute observed bids assuming all firms are aware of the cartel's existence and counterfactual bids assuming competition. We compare revenue in the two scenarios to compute true damages in all auctions and in the subset of auctions in which the winner in the data was a member of the cartel. Second, starting from the computed collusive bids (the data), we apply our methodology "as is", ignoring that all firms are aware of the cartel's existence and then we compute estimated damages with the misspecified model.

In Table 3 we report the median damages for the true model and the estimated ones with the misspecified model. Confirming our intuition total economic damages are severely underestimated when we estimate the wrong economic model (compare damages of 44.9 vs. 23.9 when the cartel has two members), but the bias is much smaller when we restrict the calculation to the auctions won by the cartel members in the collusive scenario (compare 11.56 vs. 10.26 when $|W| = 2$ and 40.81 vs. 32.68 when $|W| = 3$).

TABLE 3. Median Damages in the True Model and the Misspecified One

POWER LAW (r0=1, r1=2)

W=2	True Model	Misspecified Model
Total Revenue Loss	44.90	23.90
CF Total Revenue Affected Auctions	806.78	351.58
# Affected Auctions	1000	443.00
Revenue Loss - Ring Winner	11.56	10.26
CF Revenue - Ring Winner	202.67	200.90
# Auctions - Ring Winner	251	250
W=3		
Total Revenue Loss	118.25	63.78
CF Total Revenue	804.17	513.00
# Affected Auctions	1000	675
Revenue Loss - Ring Winner	40.81	32.68
CF Revenue - Ring Winner	269.14	261.30
# Auctions - Ring Winner	333	333

4. Empirical Illustration

In this section we apply the proposed methodology to real data from the well known bid rigging case in school milk procurement contracts in Ohio during the 1980's. Porter and Zona [1999] analyze this market and find strong economic evidence of collusion among three firms in the Cincinnati area. Moreover, two of the firms confessed the agreement and describe the functioning of the cartel as one of respecting incumbency, with the other firms submitting phony complementary bids.[16]

Each school district awards an annual contract for the supply of school milk and other products. The process of soliciting bids is done independently by each school district every year between May and August. In response to these solicitations, interested dairies in a position to supply school milk submit bids. Typically, the low bidder is selected to supply milk in half-pints to the schools during the following school year. As described in Porter and Zona [1999], there are several features of the market that may facilitate collusion: the policy of making public all bids and the identity of bidders and the fact that competition is only on prices both simplify the monitoring of any agreement. Also, the fact that there are multi contact markets and the auctions do not occur at the same time help the firms adjust their behavior to comply with the agreement. And, more importantly, the set of potential competitors in a given auction is quite stable and known to all firms, since transportation costs limit competition to local daries.[17]

We obtained our data from Robert Porter who shared a database cleaned by Wachs and Kertész [2019]. It consists of 7003 observations from 496 different school districts for the period 1980-1991 that the cartel lasted (with the exception of years 1983 and 1989 that the agreement broke down). It includes information of all submitted

[16] The presence of a cartel including these three firms is also identified by Wachs and Kertész [2019], who develop a network-based method to detect cartels in auction markets.

[17] For a detailed discussion about the institution, and the collusion scheme and description of the data we referred the interested reader to Porter and Zona [1999].

bids (per-pint prices), identifying the submitting firm and the winning one for every auction. We enlarged the dataset by including for each observation the distance between the submitting firm's closest plant and the school district.[18]

We closely follow Porter and Zona [1999]'s description of the cartel functioning and assume that the cartel operates *only* in those auctions where three copulative conditions are verified: that at least two members of the cartel are among the potential bidders, that one of the members of the cartel is the incumbent of that school district, and that the year of the auction is not 1983 or 1989.

In order to model this environment we use the Independent Private Value (IPV) paradigm with ex-ante asymmetric bidders. For each auction, we classify as strong or type 1 bidders (i.e. with cost advantages) those who have a plant within 75 miles from the school district and as weak or type 0 bidders those that are further away.[19] On the other hand, the product is relatively homogeneous so in our empirical model we do not need to control for observed heterogeneity.

For the empirical exercise we need to depart from the simplified model presented in Section 2. In particular, the assumption that entry is exogenous and the number of bidders is known to all players might be problematic. In our data we observe fluctuations in the number of bidders in a particular school district through the years and, as discussed in Porter and Zona [1999], bid preparation costs are relevant. Both elements point to a model with endogenous entry.

We consider a model where firms first learn their costs and then decide whether to participate in the auction as in Samuelson [1985]. We define the potential set of bidders for a particular contract as those firms that participated in the corresponding school district at least once in the eleven years of our sample.

[18] Our data set unfortunately does not include all relevant information analyzed in Porter and Zona [1999]. It lacks, for example, information on whether the bidder is a processor or a distributor and also potentially relevant information on wether a district is already on the route of a particular firm.

[19] Porter and Zona [1999] estimate an effect of distance on bidding prices of roughly one cent per pint per hundred miles, which is approximately a 10% of total incremental cost.

The original dataset of 7003 observations from 3754 contracts is reduced once we eliminate 174 observations from 127 contracts with missing information, 239 contracts (and bids) for which there is only one potential bidder, and additional 25 contracts (64 observations) in which all potential bidders are members of the cartel.[20] Once we exclude 176 phony bids by cartel firms we end up considering 6350 serious offers corresponding to 3363 contracts; in 148 of them the cartel acted as such.

Table 4 provides some descriptive statistics for all bids and for the winning bids. We distinguish between ring firms (5% of the serious bids) and type 0 and type 1 competitive firms (25% and 70% of the serious bids respectively). Average bids are very similar for the three groups (around 0.1 cents per pint) and type 0 and type 1 competitive firms both have similar success rates (53% and 55% respectively). This figure raises to 84% for the cartelized firms.

When we consider all contracts in the sample there are 4.3 potential bidders, while this number rises to 5.6 when considering only those contracts where the cartel participated as a ring (as defined above).

Given the previous considerations and taking into account that we deal with a procurement problem, we need to restate the firms' problems and adjust our notation. $|W|$ is the number of potential players of the cartel, n_k^* is the number of potential bidders of type-k, $c_i \in [\underline{c}, \overline{c}]$ denotes the private cost of firm i that is drawn i.i.d. from $F_k(\cdot)$, $s_k(c_i)$ the bidding equilibrium strategy of non-cartel type k firms, $s^c(c_i)$ the bidding equilibrium strategy of the serious bidder of the cartel, and $G_k(\cdot)$ and $G^c(\cdot)$ their respective distribution functions.[21,22]

[20] For these contracts we cannot model the bidders' behavior without incorporating additional features to the model (e.g., a reservation price). Our results must therefore be considered as a lower bound to the true damages, since we are excluding those auctions where presumably the cartel was more damaging.

[21] For brevity, we are omitting that all strategies $s(\cdot)$ and their distributions $G(\cdot)$ are conditional on the number of potential bidders of each auction.

[22] For convenience, we are considering that all potential members of the cartel are type 1. We observe 124 serious offers of the cartel from firms that would be classified as type 1 and 24 bids for type 0, which is insufficient to perform the econometric analysis in each separate group.

TABLE 4. Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
All bids					
bid (c/pint)	0.102	0.012	0.073	0.165	6350
distance (miles)	51.83	47.59	0	268.59	6350
Type 1 competitive bids					
bid (c/pint)	0.102	0.012	0.073	0.162	4540
distance (miles)	28.24	20.67	0	74.65	4540
Type 0 competitive bids					
bid (c/pint)	0.101	0.012	0.073	0.165	1662
distance (miles)	117.68	38.36	75.17	268.59	1662
Cartel bids					
bid (c/pint)	0.1	0.009	0.081	0.13	148
distance (miles)	35.92	39.35	0	147.54	148
Winning bids					
All bids (c/pint)	0.1	0.011	0.073	0.159	3363
Type 1 bids (c/pint)	0.1	0.012	0.073	0.159	2475
Type 0 bids (c/pint)	0.099	0.011	0.073	0.155	888
Cartel bids (c/pint)	0.101	0.009	0.081	0.13	125
Potential Number of bidders					
all bidders	4.285	1.632	2	9	3363
type-0 bidders	1.248	1.284	0	6	3363
type-1 bidders	3.037	1.533	0	7	3363
cartel bidders	0.363	0.807	0	3	3363
cartel contracts	5.615	1.286	3	8	148

The maximization problems for the firms that do not belong to the cartel and decide to enter the auction and for the serious bidding firm of the cartel are, respectively:

$$\max_{b_i^k \leq \bar{b}} (b_i^k - c_i)(1 - F_k(s_k^{-1}(b_i^k)))^{n_k^*-1}(1 - F_{k'}(s_{k'}^{-1}(b_i^k)))^{n_{k'}^*} - K; \quad k \neq k' \in \{0,1\} \tag{Pk'}$$

$$\max_{b_i^c \leq \bar{b}} (b_i^c - c_i)[1 - F_1(s_1^{-1}(b_i^c))]^{n_1^*-|W|}[1 - F_0(s_0^{-1}(b_i^c))]^{n_0^*-|W|} - K \tag{Pc'}$$

where $\bar{b}$ is the maximum price firms can offer and K is the bid preparation cost that we assume identical for all firms. We denote by c_k^* the critical cost level that, in equilibrium, leaves a type-k player indifferent betwen participating or not, which are determined by the equation:

$$(\bar{b} - c_k^*)[1 - F_k(c_k^*)]^{n_k^*-1}[1 - F_{k'}(c_{k'}^*)]^{n_{k'}^*} - K = 0; \quad k \neq k' \in \{0, 1\} \tag{5}$$

Following the identification argument of Guerre, Perrigne, and Vuong [2000], the first-order conditions that characterize the bidding function for the non-cartelized firms who enter and for the serious member of the cartel can be rewritten as:

$$c_i = b_i^k - \frac{1}{(n_k^* - 1)\frac{g_k(s_k^{-1}(b_i^k))}{1-G_k(s_k^{-1}(b_i^k))} + n_{k'}^* \frac{g_{k'}(s_{k'}^{-1}(b_i^k))}{1-G_{k'}(s_{k'}^{-1}(b_i^k))}}; \quad k \neq k' \in \{0, 1\} \tag{1'}$$

$$c_i = b_i^c - \frac{1}{(n_1^* - |W|)\frac{g_1(s_1^{-1}(b_i^c))}{1-G_1(s_1^{-1}(b_i^c))} + n_0^* \frac{g_0(s_0^{-1}(b_i^c))}{1-G_0(s_0^{-1}(b_i^c))}}. \tag{2'}$$

Unlike the theoretical model presented in Section 2 and the Monte Carlo application in Section 3, here we need to consider explicitly that the number of potential players differs between auctions. This is taken into consideration in equations (1′) and (2′), but it also must be considered when estimating the counterfactual bids of the cartel (step 3 of the proposed methodology). For that purpose, we adjust a second order polynomial of all type-1 non-cartel bids on the recovered pseudo-costs and the number of potential players.

Finally, an additional element to be considered is that the cartel worked through the years respecting the incumbency of other cartel members. This implies that, to be consistent with our theoretical model where firms draw i.i.d. costs for each auction, the serious cartel firm needs not be the most efficient one, so we estimate the model assuming that the cartel is inefficient.

4.1. **Results.** As described in our methodological section, to calculate damages we simply compare observed and counterfactual bids in all auctions. In Table 5 we report the estimated damages for alternative definitions of what can be understood as the set of "affected auctions". All damages are reported as a % of counterfactual revenue in the same of set of auctions where damages are computed. We consider four different measures.

In the first line damages are computed only for those contracts whose winner changes in the two scenarios (there are only 69 such auctions). In these auctions average damages are around 12.5%. This is a restrictive measure of damages, as it excludes all cases where a cartel member wins both in the data and in the counterfactual scenario.

A more sensible definition considers all contracts in which the winning bid between the two scenarios changes (there are 131 such auctions). This is reported in the second line and average damages in this case are 7.33%. A third alternative is to consider all contracts where the cartel's existence could have had an effect (even if it did not had an effect), defined as the set of contracts where two or more cartel members were potential players (there are 148 such contracts). Naturally, the total damages of our second and third measure damages are identical if measured in dollar terms, they differ only in the total revenue we consider in the denominator (all affected auctions vs. all potentially affected auctions).

In the fourth line we consider the damages in those contracts in which the observed winner was a member of the ring. This measure underestimates total economic damages. However, it is interesting to report it, because it might correspond to the damages that can be recouped in court. This would be the case if those auctioneers who do not buy from the cartel in the collusive scenario are not granted antitrust standing in court. For these 125 contracts average damages are 6.65% of the total counterfactual revenue.

In the third column of Table 5 we report bootstrap confidence intervals for each measure. To build them, we constructed 1000 bootstrap samples at the auction level with replacement and calculated the 5 and 95 percentile of each bootstrap distribution.

TABLE 5. Damages calculation

	# Contracts	**(%)**	**95 % Bootstrap CI**
Criterion	Revenue Loss		
Different winner	69	12.45	[11.16, 14.92]
All affected contracts	131	7.33	[6.36, 8.51]
All potentially affected contracts	148	6.14	[5.18, 6.93]
Observed winner from the cartel	125	6.65	[5.76, 7.67]

4.2. **Discussion.** We find that the collusive agreement in Ohio's milk school market generated damages around 6 or 7%, depending on the set of auctions used. This figure is consistent with some of the estimates by Porter and Zona [1999], that report average damages of 6.5%.

From an economic perspective, we consider that our second and third indicators in Table 5 properly reflect economic damages. However, it is the fourth indicator (i.e., those contracts where the winner in the data is a ring member) the one that would probably be accepted in a court. From a legal perspective, those school districts/year that did not buy from a cartel member could be prevented from standing to sue for damages.

The damages we find can be considered a lower bound on the true damages for two reasons. First, as mentioned above, we are excluding from our sample 25 contracts where the only potential bidders were members of the cartel (that is 20% of the total number of contracts the cartel won). Presumably, in these contracts the cartel profited the most. Second, as we discussed in section 3.4, our assumption that non-cartel members are unaware of the ring's existence, if not true, leads to an underestimation of damages, particularly of the second and third indicators and, to a much lesser extent, of the fourth indicator.

As a robustness check for our methodology, we present in Table 6 the same set of results reported in Table 5, but performing all estimations with a subsample that includes only those school districts where at least two members of the cartel are potential players. I.e., we are only using the information of the affected markets. This is relevant because in many antitrust cases it may not be possible to have information on unaffected markets. From the comparison of the two tables it is apparent that the results are quite robust although, as expected, the confidence intervals are wider when estimations are performed with the smaller sample.[23]

TABLE 6. Damages calculation: Subsample of affected markets

	# Contracts	**(%)**	**95 % Bootstrap CI**
Criterion	Revenue Loss		
Different winner	62	14.28	[12.49, 16.74]
All affected contracts	113	9.93	[8.35, 11.02]
All potentially affected contracts	146	7.26	[5.98, 8.28]
Observed winner from the cartel	123	7.75	[6.51, 9.20]

5. CONCLUSION

We propose a conceptually simple structural method to empirically assess the damages and efficiency costs associated with bidding rings in (repeated) first-price auctions. We make use of well-established empirical methods (Guerre, Perrigne, and Vuong [2000], Hickman and Hubbard [2015]) that allow us to recover, from observed bids, the underlying unobserved valuations of all participating firms. The estimated valuations of colluded firms, together with the estimation of an empirical relation between valuations and bids for competitive firms, allow us to build the but-for scenario and to calculate the bid of the serious bidder of the cartel if there had been no collusion. Once this counterfactual bid is obtained, then it is a matter of simple calculations to find the procurement and efficiency costs that can be associated with the collusive behavior.

The proposed structural method has the advantages of being relatively easy to implement, being quite intuitive for courts to follow, and not being as data intensive as other damage assessment methodologies (e.g., before-after and diff-in-diff estimations).

[23] For this exercise and given the (sub)sample size, we considered a single type model.

We illustrate our methodology with a series of Monte Carlo experiments, considering ex-ante symmetric and asymmetric cases, and the cases of an efficient cartel and one that randomly picks the serious bidder. We show how damages change depending on the parameters and the functioning of the cartel and, more importantly, we are able to compare the estimated damages and efficiency losses with the true ones. The methodology works remarkably well in all cases.

We apply our methodology to the well-known case of collusion in the provision of milk for schools in Ohio. The case was analyzed in Porter and Zona [1999], who fully describe the market and show that the behavior of three firms –in terms of participation decisions and the level of bidding– in the Cincinnati area is consistent with the hypothesis of collusion. Their reduced form estimation of damages is consistent with our findings.

The empirical exercise performed is just one example, and therefore the positive estimated damages and efficiency costs found need not be relevant in other contexts. Several of the empirical decisions made are related to the empirical exercise we perform (e.g., the existence of two-types of firms) and may need to be adapted in different contexts. Other empirical decisions –e.g., to assume the cartel is an inefficient one– are directly related to the assumptions of the model, and may also need to be revised if a different model is assumed.[24] The logic of the exercise, however, is quite general and widely accepted in antitrust analysis.

[24]As explained in Zona [2011] though, it is not enough to have consistency between the modeling assumptions and the empirical decisions, the modeling assumptions also need to be consistent with the economic facts of the case to conform with the Daubert criteria.

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APPENDIX

A-1. SYMMETRIC CASE - EFFICIENT CARTEL

We consider here a number of cases in which the players are ex-ante symmetric and the cartel works efficiently. We assume there are $n = 6$ risk-neutral bidders that draw their valuations iid from $F(v) = v^r$ with support $[0, 1]$. In the collusive scenario, a subset W colludes. We consider 9 cases by varying the distribution of valuations, namely $r = 0.5, 1, 2$ and also the size of the cartel, $|W| = 2, 3, 4$.

In Table A-1.1 we present damages and efficiency losses results for the efficient cartel. Following the structure of Table 1, for each of the nine cases considered there are two sets of results: Those calculated using the true valuations and the true distributions (T), and those estimated following our methodology (starting from the same collusive bids; indicated by (E)). Beyond calculating damages and efficiency losses, we also indicate the number of auctions in which collusion mattered, in the sense that the winner in the counterfactual scenario is a member of the cartel and, therefore, the winning bid is different in the two scenarios.

Some basic intuitions can be confirmed inspecting Table A-1.1: First, both damages and efficiency losses are more severe as the cartel size increases; i.e., as we move down in any column. Second, damages and efficiency losses also increase as the coefficient of the power distribution function diminishes; i.e., as large valuations by the competitors are less likely. In the worst case, i.e. when the cartel size is 4 and $r = 0.5$, median damages are about 91 and the efficiency loss about 22. These figures can be compared to the total revenue in the 1,000 auctions (535.7) or in the subset of affected auctions (356.7).

Some further analysis available upon request shows that, as expected, the larger is the cartel the larger is the difference between the estimated collusive and counterfactual densities of bids. A larger cartel faces less competition (as the total number of firms is fixed), and therefore it will be more aggressive in its shading. Similarly, the

TABLE A-1.1. Symmetric Players - Efficient Cartel

	POWER LAW (r=0.5)					POWER LAW (r=1)					POWER LAW (r=2)				
W=2	5p	25p	50p	75p	95p	5p	25p	50p	75p	95p	5p	25p	50p	75p	95p
Revenue Loss (T)	10.22	10.74	11.09	11.49	12.01	8.12	8.53	8.80	9.08	9.46	5.25	5.53	5.70	5.88	6.14
Efficiency Loss (T)	0.76	0.91	1.05	1.18	1.37	0.63	0.75	0.84	0.93	1.09	0.41	0.49	0.55	0.61	0.70
Affected Auctions (T)	309	322	332	343	358	311	324	334	344	358	309	325	335	344	359
Revenue Loss (E)	9.42	10.19	10.80	11.48	12.83	7.95	8.42	8.82	9.23	9.78	5.15	5.47	5.70	5.91	6.20
Efficiency Loss (E)	0.74	0.92	1.06	1.20	1.45	0.60	0.74	0.84	0.93	1.10	0.41	0.49	0.55	0.61	0.70
Affected Auctions (E)	307	321	331	340	355	311	324	334	344	357	309	323	333	343	358
Total Revenue	528.91	532.77	535.68	538.59	542.83	708.78	711.98	714.10	716.54	719.40	835.86	837.80	839.19	840.66	842.69
Total Efficiency	740.47	745.87	749.95	754.03	759.97	850.53	854.37	856.92	859.85	863.28	919.45	921.58	923.11	924.73	926.95
Rev. Affected Auctions	164.08	172.05	177.85	184.08	192.18	221.16	231.21	238.56	245.62	255.90	259.04	272.61	280.47	288.64	301.29
Eff. Affected Auctions	229.71	240.86	248.99	257.71	269.06	265.39	277.45	286.27	294.74	307.09	284.94	299.87	308.51	317.50	331.42
W=3															
Revenue Loss (T)	35.59	36.89	37.86	38.80	40.17	28.91	29.98	30.71	31.38	32.46	19.06	19.75	20.21	20.69	21.40
Efficiency Loss (T)	5.66	6.34	6.80	7.33	8.11	4.71	5.23	5.58	5.96	6.49	3.21	3.54	3.75	4.00	4.32
Affected Auctions (T)	475	489	500	511	527	476	489	500	510	527	474	489	500	511	527
Revenue Loss (E)	33.34	35.66	37.45	39.34	42.21	27.64	29.47	30.91	32.42	34.72	18.44	19.49	20.23	20.98	22.22
Efficiency Loss (E)	5.54	6.26	6.82	7.42	8.17	4.72	5.22	5.62	6.05	6.67	3.12	3.47	3.70	3.97	4.41
Affected Auctions (E)	470	486	498	509	524	473	490	501	513	529	473	489	500	511	526
Total Revenue	528.48	532.50	535.59	538.72	542.81	709.01	712.00	714.46	716.47	719.22	835.57	837.68	839.08	840.45	842.51
Total Efficiency	739.87	745.51	749.82	754.21	759.94	850.81	854.40	857.35	859.76	863.06	919.12	921.44	922.99	924.49	926.76
Rev. Affected Auctions	253.07	261.03	268.20	274.34	283.12	339.66	349.20	357.19	364.41	375.76	397.92	410.52	419.89	428.36	441.61
Eff. Affected Auctions	354.30	365.44	375.48	384.08	396.36	407.59	419.04	428.63	437.29	450.92	437.71	451.57	461.88	471.20	485.77
W=4															
Revenue Loss (T)	87.02	89.20	90.86	92.70	94.90	74.07	75.99	77.49	78.78	80.80	50.52	51.94	52.81	53.71	55.01
Efficiency Loss (T)	19.68	21.28	22.29	23.44	25.14	17.53	18.94	19.81	20.72	22.23	12.49	13.22	13.82	14.45	15.32
Affected Auctions (T)	643	656	666	677	689	641	657	667	677	690	642	656	667	676	690
Revenue Loss (E)	79.15	84.84	89.12	94.36	101.06	69.46	74.10	77.65	81.24	86.62	47.42	50.86	53.15	55.52	58.89
Efficiency Loss (E)	18.80	20.73	22.24	23.92	26.55	17.03	18.64	19.85	21.09	22.95	11.98	13.05	13.89	14.68	15.85
Affected Auctions (E)	634	651	662	675	694	638	656	668	681	699	634	655	669	681	702
Total Revenue	528.35	532.75	535.69	538.74	542.58	708.75	712.06	714.24	716.52	719.69	835.90	837.92	839.15	840.55	842.46
Total Efficiency	739.70	745.84	749.97	754.24	759.61	850.50	854.47	857.09	859.83	863.62	919.49	921.71	923.07	924.60	926.70
Rev. Affected Auctions	342.77	350.76	356.71	363.08	371.03	457.17	468.89	476.16	483.43	493.86	539.09	550.56	559.28	567.60	579.46
Eff. Affected Auctions	479.88	491.06	499.39	508.31	519.44	548.61	562.67	571.39	580.11	592.63	593.00	605.62	615.21	624.36	637.40

collusive and counterfactual densities of bids differ more the smaller r is, as it implies that high valuations from competitors are less likely, and therefore the shading of the cartel is larger for high valuations.

A-2. SYMMETRIC CASE - NON-EFFICIENT CARTEL

We present in Table A-2.2 the main results for the symmetric case when the cartel is non-efficient, in the sense that they simply take turns or randomly decide which member wil make the serious offer.[25] As in the efficient case, basic intuitions about how the cartel size and the parameter of the valuations distribution r can be easily checked. For our purpose, however, it is interesting to highlight that the methodology performs very well in the sense that the distribution of estimated revenue losses and efficiency losses are very close to the theoretical ones.

[25] As before, we are implicitly assuming that the allocation rule within the cartel is not affecting the bidding incentives of the cartel bidder, and therefore equation (2) characterizes its behavior.

Table A-2.2 also illustrates that both damages and efficiency losses are much larger when the cartel is inefficient than when it is efficient (compare to Table A-1.1), and these differences are larger the lager the cartel is. This is expected, since a direct effect of the inefficient cartel is to eliminate bids from potentially efficient players.

TABLE A-2.2. Symmetric Players - Non-Efficient Cartel

	POWER LAW (r=0.5)					**POWER LAW (r=1)**					**POWER LAW (r=2)**				
W=2	5p	25p	50p	75p	95p	5p	25p	50p	75p	95p	5p	25p	50p	75p	95p
Revenue Loss (T)	27.86	30.17	31.81	33.53	36.10	22.04	23.67	25.10	26.29	28.31	14.01	15.18	16.11	16.95	18.30
Efficiency Loss (T)	30.76	33.92	36.13	38.57	42.26	20.76	22.81	24.42	25.83	28.29	12.08	13.37	14.31	15.23	16.67
Affected Auctions (T)	311	325	334	343	356	310	324	334	344	357	309	323	334	344	358
Revenue Loss (E)	23.83	28.32	31.20	34.91	39.74	19.45	22.50	25.02	27.50	31.69	12.31	14.48	16.11	17.74	20.05
Efficiency Loss (E)	29.77	33.47	35.80	38.42	42.66	20.22	22.31	24.12	25.82	28.47	11.68	13.14	14.21	15.20	16.83
Affected Auctions (E)	291	313	331	348	375	292	315	334	351	380	287	315	335	353	377
Total Revenue	528.43	532.90	535.89	538.77	542.76	708.92	712.01	714.33	716.50	720.03	835.79	837.84	839.30	840.69	842.66
Total Efficiency	739.80	746.06	750.25	754.28	759.87	850.70	854.42	857.20	859.80	864.04	919.37	921.62	923.22	924.76	926.92
Rev. Affected Auctions	165.26	173.42	178.83	184.23	191.61	221.28	231.49	239.00	245.68	255.21	258.85	271.12	280.38	288.85	300.46
Eff. Affected Auctions	231.36	242.78	250.36	257.92	268.25	265.54	277.79	286.81	294.81	306.25	284.74	298.23	308.41	317.73	330.50
W=3															
Revenue Loss (T)	70.44	73.78	76.21	78.93	83.06	56.90	59.74	61.94	64.11	67.43	37.39	39.35	40.79	42.34	44.71
Efficiency Loss (T)	78.08	82.64	86.31	89.92	95.27	53.86	57.24	59.66	62.29	66.43	32.29	34.41	35.96	37.54	40.06
Affected Auctions (T)	473	490	501	512	527	473	490	501	511	525	475	490	500	512	526
Revenue Loss (E)	62.43	69.41	75.08	80.69	89.84	51.22	57.52	61.89	66.32	72.91	34.56	38.28	40.90	43.50	48.21
Efficiency Loss (E)	75.05	81.48	86.04	90.41	98.28	52.07	56.45	59.54	62.68	67.19	31.47	34.05	35.87	37.62	40.42
Different Winner (E)	448	473	495	515	545	446	478	499	520	553	448	478	500	521	553
Total Revenue	528.26	532.93	535.74	538.40	542.41	708.66	712.11	714.24	716.28	719.36	835.84	837.90	839.23	840.56	842.56
Total Efficiency	739.57	746.11	750.04	753.76	759.37	850.39	854.53	857.08	859.53	863.23	919.43	921.69	923.16	924.61	926.82
Rev. Affected Auctions	252.55	262.01	268.44	274.23	283.05	337.47	349.71	357.44	365.23	375.92	397.96	410.80	419.76	428.92	441.62
Eff. Affected Auctions	353.57	366.81	375.81	383.92	396.27	404.96	419.65	428.93	438.28	451.11	437.76	451.88	461.74	471.82	485.78
W=4															
Revenue Loss (T)	135.83	140.20	143.60	146.90	151.46	115.61	119.91	123.18	126.16	131.16	79.16	82.22	84.47	86.48	89.51
Efficiency Loss (T)	147.87	154.23	159.14	163.70	170.29	106.84	112.09	115.77	119.58	124.62	66.80	70.13	72.36	74.51	77.75
Affected Auctions (T)	641	656	667	677	691	642	657	667	676	692	641	656	667	677	691
Revenue Loss (E)	119.47	131.97	140.02	148.11	161.50	106.75	115.90	123.22	131.05	142.92	73.12	79.76	84.31	88.75	95.83
Efficiency Loss (E)	140.75	150.68	158.15	165.38	176.21	103.30	110.11	115.40	120.95	129.40	64.12	69.01	71.98	75.50	79.65
Affected Auctions (E)	609	639	657	675	702	615	645	666	688	721	610	643	666	687	718
Total Revenue	528.77	532.67	535.81	538.63	542.85	708.65	712.07	714.21	716.14	719.20	835.93	837.73	839.21	840.52	842.32
Total Efficiency	740.28	745.74	750.13	754.08	759.99	850.37	854.48	857.06	859.37	863.04	919.52	921.50	923.13	924.58	926.55
Rev. Affected Auctions	342.66	351.46	357.25	363.39	371.37	458.11	469.28	476.20	483.03	494.50	538.14	551.44	559.68	568.23	579.20
Eff. Affected Auctions	479.72	492.05	500.15	508.74	519.91	549.74	563.13	571.44	579.64	593.40	591.95	606.58	615.65	625.05	637.12

Some additional results we are not reporting show that participating in a non-efficient cartel might be a "bad idea". In the examples we considered firms never have an incentive to *individually* join an existing cartel, while in the case of the efficient cartel the incentive exists. Moreover, profits for cartel firms are larger in the counterfactual scenario than in the factual one, and therefore firms do not even have a *joint incentive* to form a cartel. These results, logically, would revert if we consider an all-inclusive cartel and/or other parameters in our setup.