Red Nacional de Investigadores en Economía

Graphical Representation of Multidimensional Poverty: Insights for Index Construction and Policy Making

Rodrigo García Arancibia (Universidad Nacional del Litoral/CONICET)

Ignacio Girela (Universidad Nacional de Córdoba/CONICET)

DOCUMENTO DE TRABAJO N° 233

Abril de 2023

Citar como:
Arancibia, Rodrigo y Ignacio Girela (2023). Graphical Representation of Multidimensional Poverty: Insights for Index Construction and Policy Making. *Documento de trabajo RedNIE N°233.*

Graphical Representation of Multidimensional Poverty: Insights for Index Construction and Policy Making

Rodrigo García Arancibia[1,2] and Ignacio Girela[2,3]

[1]Instituto de Economía Aplicada Litoral (FCE), Universidad Nacional del Litoral

[2]Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina

[3]Facultad de Ciencias Económicas, Universidad Nacional de Córdoba

Abstract

By means of probabilistic graphical models, in this paper, we present a new framework for exploring relationships among indicators commonly included in the Multidimensional Poverty Index (MPI). In particular, we propose an Ising model with covariates for modeling the MPI as an undirected graph. First, we prove why Ising models are consistent with the theoretical distribution of MPI indicators. Then, a comparison between our estimates and the association measures typically used in the literature is provided. Finally, we show how undirected graphs can complement the MPI policy relevant properties, apart from discovering further insightful patterns that can be useful for policy purposes. This novel approach is illustrated with an empirical application for the global MPI indicators of Guinea and Ecuador, taking living areas and monetary poverty as covariates, respectively.

Keywords: MPI, Markov Random Fields, Ising Model, Conditional Dependency, Deprivations.

JEL Classification: I3, C18, C35.

1 Introduction

The idea of poverty as an intrinsically multidimensional phenomenon has gained consensus in academic and policy circles over the last decades. As a result, diverse methodologies of multidimensional poverty measurement have emerged with the aim of providing a more comprehensive assessment of poverty. In particular, the framework proposed by Alkire and Foster (2011) has received the most prominent scientific and political weighing due to its simplicity and functionality.

A Multidimensional Poverty Index (MPI) based on Alkire-Foster (AF) method satisfies a set of properties that are insightful for policy purposes. Namely, it can be disaggregated into very informative and consistent partial indices such as the incidence and the intensity of poverty. In addition, the MPI can be decomposed by different population subgroups (age, gender, region, or ethnic group) as well as broken down by the indicators that composite the MPI. In particular, these two properties provide a deeper understanding of multidimensional poverty and contribute to determining the priorities and the needs of specific groups for policy action (Alkire, 2020).

These informative properties are derived from the fact that the MPI contemplates the joint distribution of deprivations. In line with recognizing that the poor may face multiple deprivations simultaneously, each indicator included in the MPI is computed using the same benchmark population. This aspect is important from the first steps of building the MPI to the later coordination of policies as well. Therefore, capturing the joint distribution of deprivations is one of the cornerstones of the institutionalization of a MPI.

Firstly, before launching a MPI, there are significant normative decisions to make, such as determining the dimensions of poverty and which indicators are representative of those dimensions (Alkire, 2013). Multiple indicators may mean comprehensiveness. However, the more indicators, the greater the risk of including redundant measures. This is impractical for the further policy uses of the MPI. Hence, it is a matter of interest to study the composition of the indicators also by exploring their overlaps in deprivations.

Even when an already published MPI is used for establishing policy goals, it must be recognized that building a MPI intrinsically engages an eclectic list of stakeholders (different government levels - from ministries to local communities-, statistical offices, NGOs, private sector, etc.). This means that MPI-oriented poverty reduction agenda requires a schematic coordination to efficiently achieve policy objectives.

The Colombian MPI (C-MPI) is considered a good example of how a MPI is designed with the view of facilitating multisector public policy coordination for poverty reduction (Angulo, Díaz, & Pardo, 2016). The C-MPI was included as part of the National Development Plan. Since diverse stakeholders are involved, the overall goal on the C-MPI levels coordinates sector-specific strategies and actions. The coordination process is set by the *National Roundtable to Reduce Poverty and Inequality* which performs the monitoring role of the C-MPI and involves all the ministries directly related to the national poverty reduction strategy as well as several institutions that operate social programs in Colombia (Angulo, 2016). How coordination works depends on the context, but it is always facilitated by the MPI for addressing interconnected deprivations and managing changes over time (Alkire, 2020).

Evidently, an analysis of the joint distribution of indicators is central throughout the whole process of an MPI (from design to implementation). Notwithstanding, in the academic literature, joint deprivations analysis is based mostly on bivariate statistical techniques (two-way contingency tables, Venn diagrams, crosstab-based measures of associations, as well as factor analysis). These techniques study the relationship between each pair of indicators ignoring the rest and, as a consequence, not contemplating the whole picture.

Unveiling this missing information is not trivial and can be explained as follows. Since each MPI indicator is represented by a binary variable, it is possible to assume that they jointly have a Multivariate Bernoulli distribution (MBD). Then, the issue is that the joint distribution of q binary variables requires $2^q - 1$ parameters to be specified since the MBD probability density function involves terms of third and higher order moments of the binary random variables (Dai, 2013; Koller & Friedman, 2009). In this vein, the so-called Ising models (1925) emerged as a more parsimonious way of modeling interactions among binary variables.

In this paper, we will model the joint distribution of the MPI indicators using an Ising Model, i.e., where the interaction between indicators is modeled and estimated conditioned on the rest. Then we will show how this carries advantages for designing the MPI as well as for policy making. In particular, we depart from the model proposed by Cheng, Levina, Wang, and Zhu (2014) which also permits studying how exogenous variables, such as living area, have influence on the association between a pair of indicators complementing the relevant-for-policy property of subgroup decomposability.

An Ising model is one type of a wider set of methods for modeling conditional dependencies between random variables called Probabilistic Graphical Models. Among these, directed graphs, also called Bayesian networks, can be distinguished from undirected ones in which Ising Models are included. All interactions in a Bayesian Network assume a causal relationship, whilst in undirected graphs, estimated relationships between variables are symmetrical. Both are characterized by their flexibility in encoding probability distributions over complex domains (from bioinformatics to statistical physics).

In fact, there also has been an interest in modeling interactions among different socioeconomic well-being indicators (Stiglitz, Sen, & Fitoussi, 2009). More recently, Ceriani and Gigliarano (2020) adopted a Bayesian Network approach to study relationships between multiple dimensions of well-being based on the Life in Transition Survey (LITS II). They showed how the estimated dependence structure helps to simulate the impact of policies and suggested that the strength of the interactions may be used to build a composite multidimensional index. Although it seems promising, in terms of policy guidance, the idea of assuming a causal relationship between indicators, specially in the MPI approach, is difficult to sustain. Furthermore, they also included control variables (age and household size) as if they were additional indicators which is methodologically questionable. Duarte, Forzani, García Arancibia, Llop, and Tomassi (2021), on the other hand, used the Ising model to represent socioeconomic binary variables and combine them with other types of variables, not to investigate conditional dependencies between those variables, but rather to develop a framework for supervised dimension reduction for predictive indices.

By estimating MPI indicators with an Ising model it is possible to preserve the symmetrical nature of the interconnections between all deprivations. In addition, the model proposed by Cheng et al. (2014) permits exploring how associations between indicators change conditioned on an exogenous variable without taking it as another indicator.

We apply this model in two countries: Guinea and Ecuador. In the former, we explore relationships between MPI indicators in a context of high marginal probabilities, high levels of poverty, and a predominantly rural country. We will show that the association measures used in the literature are not robust to this issue, as it is the Ising model, and later we will explore how this association changes by conditioning on rural-urban areas.

In the latter, we estimate conditional associations between indicators in a context of low multidimensional poverty levels. Furthermore, Ecuador's available data allows computing both multidimensional and monetary poverty. In particular, studying the relationships between these measures has been a matter of interest in the poverty studies literature. Since policy programs are determined by how people is identified as poor it is important to learn if monetary measures are still enough to capture the poor shortfalls and how the MPI identifies deprivations to a larger extent. Recent empirical studies found that the relationship between monetary and multidimensional poverty measures are far from being perfect (e.g. Iceland & Bauman, 2007; Roelen, 2017; Suppa, 2016; Tran, Alkire, & Klasen, 2015; Wang, Feng, Xia, & Alkire, 2016). In general, the association analysis in these kinds of papers are mainly performed using contingency tables, correlation measures such as

Cramer's V or Redundancy (R) indices between pairs of indicators (e.g. Evans, Nogales, & Robson, 2020; Santos & Villatoro, 2018; Tran et al., 2015, among others). Others analyze the relationship between each MPI indicator and monetary poverty using logistic regression models where each non-monetary indicator is modeled as a function of monetary poverty (e.g. Salecker, Ahmadov, & Karimli, 2020; Wang et al., 2016). Unlike all these studies, in this paper, what is being modeled is the level of conditional association between pairs of non-monetary indicators as a function of monetary poverty as a novel way of studying how these two measurement methodologies are related and how they can be complemented in social programs.

This paper is structured as follows. In section 2, we first enumerate the AF method steps for building an MPI and briefly describe the properties that are relevant for policy uses. Subsequently, we mention the most generally used measures in the literature to analyze the relationship between MPI indicators and the associated deficiencies of each one. Section 2.3 justifies why we assume the MPI indicators to have a Multivariable Bernoulli distribution and its properties. Section 2.4 presents the Ising Model used in this study and how it is estimated. Next, we present the data for both countries and carry out an analysis based on MPI disaggregation properties and how they can be informative for policy purposes. In section 4, it is possible to find the results for the estimated graphs and a comparison with other measures of association used in the literature. Before ending with concluding remarks, in section 5, we briefly discuss how this framework may be suitable for building the MPI and implementation of policies.

2 Methodology

2.1 Multidimensional Poverty Index: measurement and policy relevant properties

Alkire and Foster (2011) proposed a multidimensional poverty measurement method (AF method) based on a "dual cutoff counting approach" for identification and aggregation of the poor. In a nutshell, constructing a MPI based on AF method consists of the following steps (see, for example, Alkire & Santos, 2014):

1. Defining a set dimensions considered relevant for human development. For measuring purposes, each dimension is represented by a subset of indicators.
2. Defining a set of q indicators to be included in the MPI. Data source should be the same for all indicators.

3. Establishing a deprivation cutoff z_j for each indicator j, where z_j represents a level of achievement considered sufficient in order not to be considered deprived in the j-th indicator for $j = 1, \dots, q$.

4. Applying each cutoff z_j to determine whether the i-th observation is deprived or not in each indicator for $i = 1, \dots, n$ and building a deprivation matrix $\mathbb{D}^0_{n \times q} = [d_{ij}]$ where $d_{ij} = 1$ if $x_{ij} < z_j$ and $d_{ij} = 0$ otherwise, and x_{ij} is the achievement of the i-th individual or household in the j-th indicator. We define $\mathbf{d}_{i\,1\times q}$ as the deprivation vector for i-th observation.

   It is important to note that, in the population, each cutoff define a vector of binary random variables $\mathbf{D} = (D_1, \dots, D_q)$ which in its sample version is represented by the deprivation matrix $\mathbb{D}^0_{n \times q}$.

5. Setting a vector of indicator relative weights $w = w_1, \dots, w_q$ such that $\sum_{j=1}^{q} w_j = 1$.

6. Determining a poverty cutoff k as the proportion of weighted deprivations an individual or household needs to experience to be considered multidimensionally poor.

7. Calculating the deprivation score for each observation, $c_i = \sum_{j=1}^{q} w_j\, d_{ij}$ and comparing it with k to identify the poor. If $c_i \geq k$, the i-th unit of analysis is multidimensionally poor.

   From the identification step, it is possible to obtain the censored measures such as the censored deprivation matrix $\mathbb{D}^0_{n \times q}(k)$, where each element is defined as follows:

$$d_{ij}(k) = \begin{cases} d_{ij} & \text{if} \quad c_i \geq k \\ 0 & \text{otherwise.} \end{cases}$$

8. Computing the Adjusted Headcount Ratio (MPI or M_0):

$$M_0 = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{q} w_j\, d_{ij}(k) = \frac{n_d}{n} \times \frac{1}{n_d} \sum_{i=1}^{n} \sum_{j=1}^{q} w_j\, d_{ij}(k) = H \times A \qquad (1)$$

   where n_d is the number of individuals or households identified as poor; H and A are the *incidence* (proportion of individuals or households who are multidimensionally poor) and the *intensity* (average deprivation share across the poor) of multidimensional poverty, respectively.

Policymakers may use either H or A to reduce overall poverty. If they aim to mitigate levels of deprivation of the poorest, A is a useful measure for monitoring poverty reduction. Whereas, if they focus on the number of poor people, H is a more appropriate guideline.

Furthermore, the MPI can be disaggregated by population subgroups, i.e., overall poverty can be expressed as the population-share weighted sum of the subgroup Adjusted Headcount Ratios.

$$M_0 = \sum_{l=1}^{L} \frac{n_l}{n} M_0^l \tag{2}$$

where n_l is the population and M_0^l is the poverty measure in the l-th population subgroup for $l = 1, \ldots, L$.

On the other hand, it is possible to break down the MPI by indicator and examine the contribution of each indicator to overall poverty and the composition of multidimensional poverty. We can rewrite equation (1) as follows:

$$M_0 = \sum_{j=1}^{q} w_j \frac{1}{n} \sum_{i=1}^{n} d_{ij}(k) = \sum_{j=1}^{q} w_j \, h_j(k) \tag{3}$$

where $h_j(k)$ is the censored headcount ratio of the j-th indicator (the proportion of people who are both deprived in that indicator *and* multidimensionally poor). The contribution of each indicator to the MPI can be expressed as:

$$\phi_j = w_j \frac{h_j(k)}{M_0} \tag{4}$$

Decomposition by population subgroup or indicators are two policy relevant properties because they help to establish intervention priorities. Both properties have been used for coordinating policy programs, improving budget allocation and access to public services by sector or geographical area, as well as doing group-based or geographically-based household targeting of social programs (Alkire, 2020).

2.2 Joint analysis of deprivations and relationships among indicators

One of the major advantages of MPI is that it accounts for the joint distribution of the deprivations due to the use of a single dataset. Consequently, it is possible to examine all the interlinkages between indicators. This is relevant from the first moment of building a consistent MPI (i.e., are two indicators capturing different aspects of poverty or are they redundant?) to the further policy uses (social programs design and coordination).

Since the focus is mainly on the joint deprivations, the most widely used measure in MPI construction literature is the *redundancy R* coefficient (Alkire et al., 2015) which exclusively concentrates on deprivations overlappings. For any j and j' indicators (with $j, j' = 1 \ldots, q, j \neq j'$), this redundancy measure is given by

$$R = p_{11}^{jj'} / \min \{p_{+1}^{j'}, p_{1+}^{j}\} \in [0, 1] \tag{5}$$

where $p_{11}^{jj'}$ is the proportion of people simultaneously deprived in j and j' and $p_{+1}^{j'}$ and p_{1+}^{j} are the proportion of people deprived only in indicator j' and j, respectively, also named as the uncensored headcount ratios of each indicator and represented by h_j and $h_{j'}$ (alternatively: marginal distributions or prior probabilities).

Apart from focusing on the joint distribution of two indicators, the R redundancy measure provides easy-to-interpret results. For example, if $R = 0.7$, it means that 70% of the people suffering deprivations in the indicator with the lowest headcount ratio are also deprived in the other indicator.

High levels of R may indicate that two indicators are redundant. If they capture the same poverty phenomenon, it may be considered to drop one of them. Nevertheless, this measure is affected by indicators with high headcount ratio values. Hence, two indicators may reflect absolutely different aspects of poverty (namely, School Attendance and Cooking Fuel) and show high values of R if 90% of people are deprived in one of them. In these cases, there are normative reasons to preserve both indicators since they capture different deprivations despite the high redundancy.

As a consequence, in spite of being useful for achieving a consistent MPI, the R measure may state that there are a number of redundant indicators that are based on different normative decisions which undermines its utility for informing policy afterwards. The well-known Cramer's V as a correlation measure for nominal variables seems to be an alternative. However, it misses the focus on joint deprivations $p_{11}^{jj'}$ when overlappings between indicators are high in the simultaneously non-deprived, $p_{00}^{jj'}$.

On the other hand, factor analysis has been used for an empirical analysis on how comprehensive the included indicators are, assuming poverty as a common cause latent phenomenon (Santos & Villatoro, 2018). Estimated factors yield an interpretation on how indicators are associated, but these results are highly sensitive to the selected factors rotation technique (Alkire et al., 2015).

Even more importantly, note that all these methods take the joint distribution only between pairs of indicators, but not all of them. R and V measures are built from a two-way contingency table and factor analysis estimates correlations between indicators assuming an underlying bivariate normal distribution.

In this context, we will show that there is a better way to represent the joint distribution of the MPI indicators in a tractable manner in order to analyze the conditional associations between pairs of indicators, i.e., if being deprived in the j-th indicator is likely to being deprived in another indicator j' *given the rest of the indicators.*

2.3 The Joint Distribution of MPI indicators

In the theoretical construction of a MPI (according to the steps 2 to 4 in subsection 2.1) we have q indicators that jointly conform a vector $\mathbf{D}$ of q binary variables that describe the different dimensions of poverty; that is, $\mathbf{D} = (D_1, \ldots, D_q)$ where $D_j \in \{0, 1\}$ indicating whether an individual or household is deprived in the j-th indicator. Given this definition of $\mathbf{D}$, it is natural to assume a Multivariate Bernoulli distribution (MBD) to model it. Let $p_{d_1,d_2,\ldots,d_q} = P(D_1 = d_1, D_2 = d_2, \ldots, D_q = d_q)$ with $d_1, d_2, \ldots, d_q = 0, 1$, then the joint probability function of MPI indicators is given by

$$P(\mathbf{D} = \mathbf{d}) = p_{0,0,\ldots,0}^{\prod_{j=1}^{q}(1-d_j)} \times p_{1,0,\ldots,0}^{\prod_{j=2}^{q} d_1(1-d_j)} \times p_{0,1,\ldots,0}^{\prod_{j=3}^{q}(1-d_1)d_2(1-d_j)} \times \ldots \times p_{1,1,\ldots,1}^{\prod_{j=1}^{q} d_j}, \tag{6}$$

with $\mathbf{d} = (d_1, d_2, \ldots, d_q)$. Following Dai, Ding, and Wahba (2013), equation (6) can be written as member of exponential family distribution as follows

$$P(\mathbf{D} = \mathbf{d}) = \exp\left(\sum_{k=1}^{q}\left(\sum_{1\leq j_1\leq j_2\leq\ldots\leq j_k\leq q} f^{j_1 j_2\cdots j_k} B^{j_1 j_2\cdots j_k}(\mathbf{d})\right) - \log\frac{1}{p_{0,0,\ldots,0}}\right) \tag{7}$$

where $f^1, f^2, \ldots, f^q, f^{12}, \ldots, f^{1,\ldots,q}$ are the natural parameters and B is called interaction function, where $B^{j_1 j_2\cdots j_k}(\mathbf{d}) = d_{j_1} d_{j_2} \ldots d_{j_k}$. To understand what natural parameters are, suppose we have $q = 3$ indicators, then $f^1 = \log\frac{p_{100}}{p_{000}}, f^2 = \log\frac{p_{010}}{p_{000}}, f^3 = \log\frac{p_{010}}{p_{000}}$, $f^{12} = \log\frac{p_{110}p_{000}}{p_{100}p_{010}}$, $f^{13} = \log\frac{p_{101}p_{000}}{p_{100}p_{001}}, f^{23} = \log\frac{p_{011}p_{000}}{p_{010}p_{010}}$ and $f^{123} = \log\frac{p_{111}p_{100}p_{010}p_{001}}{p_{110}p_{101}p_{011}p_{000}}$. Then, as Dai et al. (2013) proved, the independence of indicators can be viewed from these natural parameters. Specifically, the components of the random vector $\mathbf{D}$ are independent if and only if $f^{j_1 j_2\cdots j_k} = 0$ for all $1 \leq j_1 \leq j_2 \leq \ldots j_k \leq q$ with $k \geq 2$.

With this formulation we can see that the Multivariate Bernoulli distribution involves not only pairwise interactions but also those of higher order (among more than two indicators), which, in addition to being complex to represent, it could be computationally unfeasible to estimate (Duarte, 2016). For this reason, it is sought to obtain a simplified version of such distribution. For this purpose, and mainly in the field of computational statistics and machine learning, in recent years the so-called Ising graphical model has been widely used (Wainwright, Jordan, et al., 2008). This model can be viewed as a special case of the multivariate Bernoulli with $f^{j_1 j_2\cdots j_k} = 0$ for all

$1 \leq j_1 \leq j_2 \leq \ldots j_k \leq q$ and $k \geq 3$. Therefore, with such a graph, it is possible to model the pairwise correlations between the variables/indicators, contemplating their joint distribution. In fact, as highlighted by Ravikumar, Wainwright, and Lafferty (2010), this assumption of pairwise interactions does not imply a loss of generality since higher order interactions can be taken into account by introducing additional variables in the same framework of the Ising model. Details of such a procedure to include higher order interactions can be found in Wainwright et al. (2008). Taking this into account, we consider that a good strategy to model the joint distribution of the MPI indicators is through undirected graphical models, where the associations between pairs are modeled in conditional terms (i.e., conditioning the rest of the indicators), and such conditional distributions are distributed as Bernoulli, which is derived from the multivariate distribution of all the indicators (Dai et al., 2013). In the following subsections, we will present this approach in more detail.

2.4 Modeling the joint distribution of deprivations: the MPI as a graph

2.4.1 Probabilistic Graphical Models

Probabilistic graphical models aim to find a graph structure that compactly encodes a joint probability distribution over a high-dimensional space. They are composed by a set of random variables $\mathbf{D}$ and a graph $\mathcal{G} = (\mathbf{V}, E)$ where $\mathbf{V}$ is the set of vertexes or nodes, each one associated with a random variable $D_j \in \mathbf{D}$ and E is the set of edges or links that expresses dependence relationships between pair of variables in $\mathbf{D}$. Estimated graphs provide a simple way to visualize the structure of a probabilistic model and obtain insights about relationships among a set of random variables (Bishop, 2006). Thus, graphs preserve the simple communicative essence of the MPI.

Graphs can be distinguished between directed (also known as Bayesian Networks) or undirected (also known as Markov Random Fields). The former takes into account the directionality of the links to explain causality, whereas in the latter, associations between variables have no direction. Since there are no a priori reasons to set causality among the MPI indicators, it is assumed their interactions are described in a symmetric manner. Therefore, undirected graphs are preferable for exploring the joint distribution of deprivations. Besides, forcing directionality may give rise to models that are unintuitive and incapable of capturing independencies in the domain (Koller & Friedman, 2009).

2.4.2 Undirected graphs for binary variables: the Ising Model

As it was emphasized earlier, the MBD involves all q-order intersections among variables, so to obtain a more parsimonious representation of the joint distribution of $\mathbf{D}$, in the context of graphical models, it is usual to take the special specification given by the so-called Ising model (Cheng et al., 2014; Dai, 2013; Dai et al., 2013). The Ising model includes only the second-order interactions between pairs of binary indicators. Additionally, the dependency between variables contained in $\mathbf{D}$ could depend on other variables, resulting in different empirical graphs according to the value of these conditioning co-variables. In this way, the graph can be enriched by including possible effects of other socioeconomic, demographic, and/or regional variables on the association between pairs of MPI indicators. Therefore, let $Y \in \mathbb{R}^r$ with $r \geq 1$ being a vector of covariates. Following Cheng et al. (2014), conditioning on Y, the vector of q binary variables $\mathbf{D}^T = (D_1, D_2, \ldots, D_q)$ has the following joint density function

$$\begin{aligned} P(\mathbf{D} \mid Y = y) &= P(D_1, \ldots, D_q \mid Y = y) \\ &= \frac{1}{G(\mathbf{\Theta}^y)} \exp\left(\sum_{j=1}^{q} \theta_{jj}^y D_j + \sum_{1 \leq j < j' \leq q} \theta_{jj'}^y D_j D_{j'} \right), \end{aligned} \quad (8)$$

where the θ's are the model parameters that depend on the values of the covariates $Y = y$ and $G(\mathbf{\Theta}^y)$ ensures that all 2^q probabilities add up to 1. For $j, j' = 1, \ldots, q$ we have

$$\theta_{jj}^y = \log\left(\frac{P(D_j = 1 \mid \mathbf{D}_{-j} = 0, y)}{1 - P(D_j = 1 \mid \mathbf{D}_{-j} = 0, y)} \right),$$

$$\theta_{jj'}^y = \log\left(\frac{P(D_j = 1, D_{j'} = 1 \mid \mathbf{D}_{-j,-j'} = 0, y) P(D_j = 0, D_{j'} = 0 \mid \mathbf{D}_{-j,-j'} = 0, y)}{P(D_j = 1, D_{j'} = 0 \mid \mathbf{D}_{-j,-j'} = 0, y) P(D_j = 0, D_{j'} = 1 \mid \mathbf{D}_{-j,-j'} = 0, y)} \right),$$

$$G(\mathbf{\Theta}^y) = \sum_{j=1}^{q} \sum_{\{D_j = 0,1\}} \exp\left(\sum_{j=1}^{q} \theta_{jj}^y D_j + \sum_{1 \leq j < j' \leq q} \theta_{jj'}^y D_j D_{j'} \right),$$

with $\mathbf{D}_{-j} = (D_1, \ldots, D_{j-1}, D_{j+1}, \ldots, D_q)$, $\mathbf{D}_{-j,-j'} = (D_1, \ldots, D_{j-1}, D_{j+1}, \ldots, D_{j'-1}, D_{j'+1}, \ldots, D_q)$ and $\mathbf{\Theta}^y$ is a symmetric matrix with elements $[\mathbf{\Theta}^y]_{jj'} = \theta_{jj'}^y$. Following Cheng, Li, Levina, and Zhu (2017), each $\theta_{jj'}^y$ is modeled as a linear function of Y. Specifically, we assume that

$$\theta_{jj'}^y = \theta_{jj',0}^* + \boldsymbol{\theta}_{jj'}^T Y, \qquad j, j' = 1, \ldots, q \quad (9)$$

where $\boldsymbol{\theta}_{jj'}^T = (\theta_{jj',1}, \ldots, \theta_{jj',r})$ is a parameters vector (independents of Y) and $\theta^*_{jj',0}$ is the intercept for all (j, j').

Using this parametrization (9), the joint distribution for the Ising model (8) can be written as

$$P(\mathbf{D} \mid y) = \frac{1}{G(\boldsymbol{\Theta}_y)} \exp\Bigg(\sum_{j=1}^{q} \theta^*_{jj0} D_j + \sum_{j=1}^{q} \boldsymbol{\theta}_{jj}^T y D_j + \sum_{1 \le j < j' \le q} \theta^*_{jj'0} D_j D_{j'} + \sum_{1 \le j < j' \le q} \boldsymbol{\theta}_{jj'}^T y D_j D_{j'} \Bigg). \tag{10}$$

Considering a particular dichotomous indicator j and conditioning on the rest $\mathbf{D}_{-j}$ and Y, we obtain

$$\log \frac{P(D_j = 1 \mid \mathbf{D}_{-j}, Y)}{P(D_j = 0 \mid \mathbf{D}_{-j}, Y)} = \theta^*_{jj0} + \boldsymbol{\theta}_{jj}^T Y + \sum_{j \ne j'} \theta^*_{jj'0} D_{j'} + \sum_{j < j'} \boldsymbol{\theta}_{jj'}^T Y D_{j'} \tag{11}$$

In this way, conditional log-odds of D_j are linear in the parameters, so that maximum likelihood estimators can be obtained from logistic regression of D_j on $(Y, \mathbf{D}_{-j}, Y\mathbf{D}_{-j})$. Therefore, by fitting q uni-variate logit models, we can obtain estimators for $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}$. These parameters inform us about which edges exist and if they depend on covariates (e.g. income poverty or population subgroups). In particular, if the vector $\left(\theta^*_{jj'0}, \boldsymbol{\theta}_{jj'}\right)$ is zero, then the MPI indicators D_j and $D_{j'}$ will be conditionally independent given Y and the rest of MPI indicators. Additionally, $\boldsymbol{\theta}_{jj'}$ describes the size of the conditional contribution of the predictors Y on the edge between D_j and $D_{j'}$, and $\theta_{jj'k} = 0$ being zero (for some $k = 1, \ldots, r$) implies that the conditional association between those MPI indicators does not depend on Y_k.

In particular, for a sample of households with n i.i.d. observations $(\mathbf{d}_i, y_i) \equiv (d_{i1}, \ldots, d_{iq}, y_i)$ with $i = 1, \ldots, n$, for each poverty dimension j, with $j = 1, \ldots, q$, the log-likelihood is given by

$$\ell_j(\boldsymbol{\theta}_0, \boldsymbol{\theta}; \mathbf{d}_i, y_i) = \frac{1}{n} \sum_{i=1}^{n} \log P(d_{ij} \mid \mathbf{d}_{i,-j}, y_i) = \frac{1}{n} \sum_{i=1}^{n} \left(d_{ij}\epsilon_{ij} - \log(1 + \exp(\epsilon_{ij})) \right) \tag{12}$$

with

$$\epsilon_{ij} = \log \frac{P(d_{ij} = 1 \mid \mathbf{d}_{i,-j}, y_i)}{P(d_{ij} = 0 \mid \mathbf{d}_{i,-j}, y_i)} = \theta^*_{jj0} + \boldsymbol{\theta}_{jj}^T y_i + \sum_{j \ne j'} \theta^*_{jj'0} d_{ij'} + \sum_{j \ne j'} \boldsymbol{\theta}_{jj'}^T y_i d_{ij'}.$$

Therefore, we can obtain an estimator for $(\boldsymbol{\theta}_0, \boldsymbol{\theta})$ in order to maximize (12) for all j. By symmetry, $\theta^y_{jj'} = \theta^y_{j'j}$, so we have two estimators for the same parameter, i.e., one from the logistic regression of D_j on $(Y, \mathbf{D}_{-j}, Y\mathbf{D}_{-j})$ and another one from the regression of $D_{j'}$ on $(Y, \mathbf{D}_{-j'}, Y\mathbf{D}_{-j'})$ given that $D_{j'} \in \mathbf{D}_{-j}$ and $D_j \in \mathbf{D}_{-j'}$. As there are no guarantees that the estimates we obtain satisfies $\theta^*_{jj'0} = \theta^*_{j'j0}$ and $\boldsymbol{\theta}_{jj'} = \boldsymbol{\theta}_{j'j}$, we need to select some criteria so that the symmetry is fulfilled (Cheng et al., 2014; Meinshausen & Bühlmann, 2006).

2.4.3 Estimation

Cheng et al. (2014) proposed two algorithms that induce sparsity in the fitted graphs and in the number of selected covariates. This is a good strategy given that the dimension of $\boldsymbol{\theta}$ can be large in some applications and the gains in terms of interpretability can be very important as it is emphasized by empirical studies of networks. Specifically, using regularization with the L_1-norm, they propose two methods to maximize (12). In both, only $\boldsymbol{\theta}$ is penalized, but one does it separately for each j, while the other does it jointly (for more details see Cheng et al., 2014). In this paper we apply the joint regularization, estimating $(\boldsymbol{\theta}_0, \boldsymbol{\theta})$ from

$$\min_{(\boldsymbol{\theta}_0, \boldsymbol{\theta})} \sum_{j=1}^{q} -\ell_j(\boldsymbol{\theta}_0, \boldsymbol{\theta}; \mathbf{d}_i, y_i) + \lambda \|\boldsymbol{\theta}\|_{L_1}. \tag{13}$$

An advantage of joint regularization is that the symmetry is automatically imposed solving (13) but with a higher computational cost. In the present paper, we only have $q = 10$ binary indicators, so we use the joint method using a Matlab code to estimate the Ising model which was provided by the authors[1]. Then we use the R package `qgraph` to plot graphs of conditional dependencies between pairs of indicators of the MPI.

We exemplify the application of the Ising model for the MPI indicators in two countries: Guinea and Ecuador. We take $q = 10$ MPI indicators presented in Table 4 in Appendix 6.1, and including a different covariate (i.e. Y) in each case. Specifically, for Guinea we consider the living area (i.e., urban or rural) and for Ecuador the monetary poverty. Therefore, in both cases we have $r = 1$ with Y as a dummy or binary covariate.

3 Data and Multidimensional Poverty Analysis

This section describes data for two selected countries: Guinea and Ecuador. For the former we use the 2017/2018 Demographic and Health Survey (DHS). This country provides a good example about how the presence of very high uncensored headcount ratios in some indicators bias redundancy measures as it occurs with Cooking Fuel indicator in most Subsaharian countries. The latter is based on an official household survey (Encuesta de Condiciones de Vida - ECV 2013/2014) which

[1] Matlab code available in `http://onlinelibrary.wiley.com/doi/10.1111/biom.12202/`

permits to calculate both multidimensional and monetary poverty. Hence, it is possible to examine the linkages between these poverty measurement approaches[2].

Selected dimensions, indicators, weights and cutoffs are based on the global MPI which is an international measure of acute multidimensional poverty for the developing world proposed by Alkire and Santos (2014). All these parameters were originally established following international comparability purposes subjected to the available datasets for most developing countries. They are described in Table 4 in Appendix 6.1[3].

Table 1 shows the sample size, the share of the population and the multidimensional poverty estimates for Guinea and Ecuador disaggregated by their population subgroups: living areas and monetary poverty, respectively.

At a glance, we can appreciate that the multidimensional poverty in Guinea is higher than in Ecuador. In the former, the multidimensional poverty level is 0.369 whilst in the latter, it is 0.018. This is mainly because 65.60 % of the Guinean population are multidimensionally poor (H) while only 4.56 % of Ecuador's people are poor according to the global MPI indicators. However, both countries exhibit high levels of intensity (A). The poor are deprived in more than half of the indicators on average (56.28 %) in Guinea and poor Ecuadorians are deprived in more than one dimension on average (39.88 %).

Table 1: Disaggreated multidimensional poverty measures

Country	Subgroup	n	% Population	MPI	H %	A %
Guinea	National level	24215	100	0.369	65.60	56.28
	Rural	15821	65.30	0.501	86.18	58.14
	Urban	8394	34.70	0.121	26.87	45.05
Ecuador	National level	109694	100	0.018	4.56	39.88
	Non-poor	91385	90.11	0.011	2.82	37.96
	Poor	18309	9.89	0.086	20.40	42.30

Source: Own elaboration based on 2017/2018 DHS data for Guinea and ECV 2013/2014 for Ecuador.

Deeper insights can be obtained with the subpopulation decomposition property. Briefly, Guineans who live in rural areas are poorer than people from urban areas mostly because of

[2]The data for Guinea and Ecuador that support the findings of this study are available in the DHS website (`https://dhsprogram.com/methodology/survey/survey-display-539.cfm`) and in the official ECV 13-14 website (`https://www.ecuadorencifras.gob.ec//documentos/web-inec/ECV/ECV_2015/`)

[3]The code to compute the global MPI is available in `https://cloud-ophi.qeh.ox.ac.uk/index.php/s/7WCbyFaeHPaq78f`

considerably high levels of incidence of poverty. With regard to the intensity of poverty, urban areas also face less deprivations on average than their rural counterparts.

In the Ecuadorian case, large mismatches between multidimensional and monetary poverty measures are observed. Only 20.40 % of the monetary poor are also poor from a multidimensional perspective (according to \$1.90 a day and $k = 33\%$ poverty thresholds). These results are in line with other empiral studies. Evans et al. (2020) found that average identification overlap (either poor or non-poor by both measures) reaches 84% in Ecuador largely due to a vast majority of the population being classed as non-poor by both approaches to poverty.

On the other hand, Tables 2 and 3 show the disaggregated uncensored and censored headcount ratios (h_j and $h_j(k)$) for each indicator in Guinea and Ecuador, respectively. In Guinea, the Cooking Fuel and Sanitation indicators yield the higher levels of deprivation, which is also reflected in the high R measure values (see Appendix 6.3), i.e., the rest of the indicators exhibit high redundancy values against these two indicators due to the high uncensored headcount ratios. This is not informative on how these pair of indicators are related.

The censored headcount ratios are always lower (or equal) than the uncensored headcount ratios, as should be noted. The magnitudes of these differences can be very informative for targeted policy. For instance, in rural areas, if a person is deprived in one indicator, she is very likely to be multidimensionally poor since $h_j(k)$ is close to h_j except in Cooking Fuel and Sanitation. This is not the case in urban areas where differences are greater (e.g., only half of the households deprived of nutrition are multidimensionally poor). In Ecuador, it is remarkable that those who are both multidimensional and monetary poor show higher levels of deprivation in terms of Nutrition, Cooking Fuel, Sanitation, Drinking Water and Housing than those who are multidimensional but not monetary poor.

Additional insights can be drawn from these tables. However, most of this information is condensed in the relative contribution of indicators (Figures 1 and 2) which also takes into account the weight of each indicator. In terms of policy guidance, this simple graphical tool provides information about which indicators policymakers should focus on in order to achieve a greater impact on poverty reduction by means of household targeting or community-based programs, readjusting budget allocation in specific areas, and improving access to public services.

The contribution of each indicator in Guinean rural areas is similar at a national level (Figure 1). Years of Education, Sanitation, Nutrition are the indicators that contribute the most to multidimensional poverty in rural Guinea. Even larger than Cooking Fuel despite showing lower censored headcount ratios. This occurs due to the fact that these indicators receive higher rel-

Table 2: Uncensored and censored headcount ratios (%) for Guinea

Indicators	National		Rural		Urban	
	h_j	$h_j(k)$	h_j	$h_j(k)$	h_j	$h_j(k)$
Nutrition	41.31	35.31	46.55	45.48	31.24	16.20
Child Mortality	12.56	11.95	16.19	15.85	5.71	4.62
Years of Education	48.46	45.67	65.58	63.98	16.23	11.24
School Attendance	43.40	39.46	53.16	51.91	25.03	16.04
Cooking Fuel	97.48	65.14	99.80	86.08	93.10	25.73
Sanitation	73.57	54.93	83.09	74.14	55.63	18.76
Drinking water	42.94	36.57	56.10	51.28	18.17	8.88
Electricity	54.98	48.57	76.79	69.64	13.92	8.90
Housing	42.55	38.79	59.96	56.01	9.79	6.38
Assets	26.36	23.92	36.43	33.96	7.42	5.03

Source: Own elaboration based on 2017/2018 DHS data for Guinea and ECV 2013/2014 for Ecuador.

Table 3: Uncensored and censored headcount ratios (%) for Ecuador

Indicators	National		Non-Poor		Poor	
	h_j	$h_j(k)$	h_j	$h_j(k)$	h_j	$h_j(k)$
Nutrition	13.69	2.93	11.56	1.58	33.07	15.22
Child Mortality	5.19	1.48	4.6	1.02	10.51	5.60
Years of Education	4.68	1.61	4.33	1.24	7.92	5.06
School Attendance	2.05	0.96	1.36	0.45	8.33	5.59
Cooking Fuel	4.45	1.70	2.38	0.71	23.31	10.78
Sanitation	14.22	2.84	11.69	1.64	37.25	13.84
Drinking water	14.31	2.46	12.14	1.45	34.10	11.70
Electricity	1.25	0.70	0.60	0.31	7.14	4.27
Housing	11.09	2.37	8.79	1.45	31.99	10.78
Assets	4.11	1.67	2.54	0.81	18.39	9.44

Source: Own elaboration based on 2017/2018 DHS data for Guinea and ECV 2013/2014 for Ecuador.

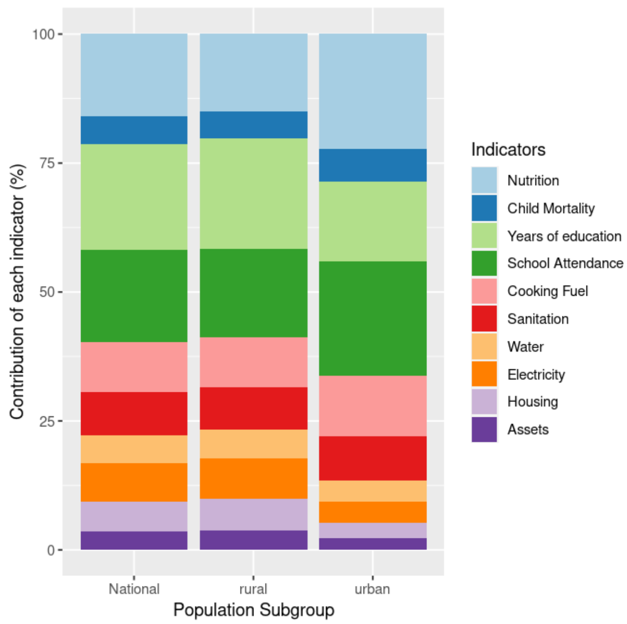


Figure 1: Indicator Contribution to Multidimensional Poverty by Area in Guinea

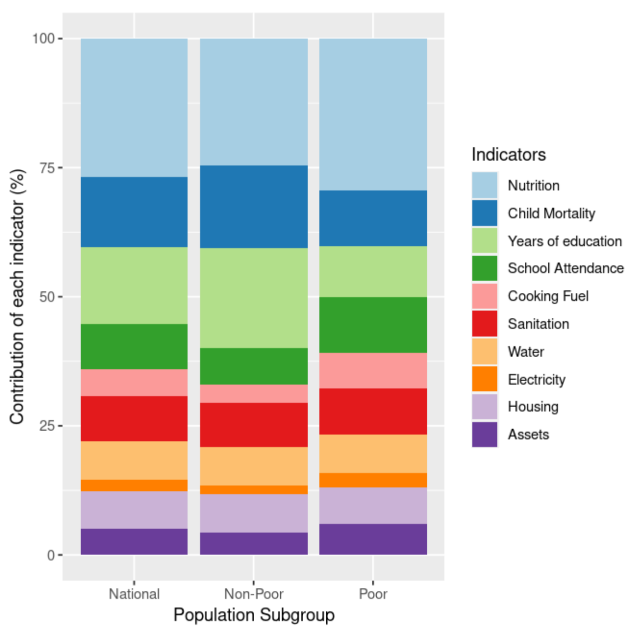


Figure 2: Dimension Contribution to Multidimensional Poverty by Monetary Porverty in Ecuador

ative weights, as it can be appreciated from equation (4) and Table 4. Years of Education and Nutrition also contribute proportionally more to poverty in urban than in rural Guinea. In urban areas, contributions show different patterns. Here Nutrition and School Attendance contribute the most to the MPI. This invites to coordinate existing programs in urban areas oriented to reduce malnutrition and to improve access to education.

As shown in Figure 2, Nutrition is clearly the indicator that has more influence in Ecuador's poverty and even greater for monetary poor people. For those who are multidimensionally poor, but monetary non-poor, deprivations are mostly explained by Years of Education. Hence, in Ecuador, policies aim to reduce indicators from Health and Education dimension will have greater impact on multidimensional poverty.

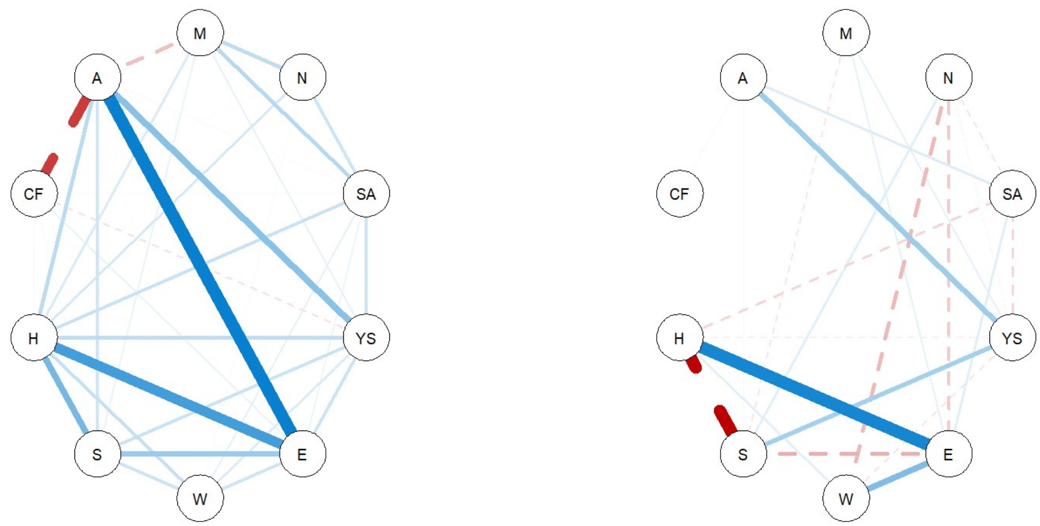
4 Results

In this section we present the estimated graphs $\hat{\mathcal{G}}$ of the deprivation matrix $\mathbb{D}$ conditioned on rural-urban areas in the case of Guinea and specified with monetary poverty for Ecuador. The estimated parameters of the Ising model used in the graphs' construction are presented in Tables 5 and 6 in Appendix 6.2.

4.1 Results for Guinea

Figure 3 shows the estimated graphs for Guinea, where each node represents a MPI indicator. The links or edges between nodes, indicate the presence of a conditional association between the two variables. The thicker the edge, the greater the association. Conversely, the absence of edges between nodes means that both indicators are independent given the rest of the MPI indicators. In addition, the estimated conditional associations can be either positive or negative, which is shown in the graph by the color and style of the edges, which are either blue-solid or red-dashed, respectively.

Left-hand panel of Figure 3 shows the *main* graph represented by the *intercept* $\hat{\boldsymbol{\theta}}_0$. In order to explore how relationships among indicators change if we conditioned by living areas, we include a binary covariate with value 0 if the household lives in a rural area and 1 if it is located in an urban area. Hence, the right-hand panel of Figure 3 presents the effects of living in a urban context on the conditional dependencies of the MPI indicators, quantified by the $\hat{\boldsymbol{\theta}}_{ij}$ for all $i, j = 1, \ldots, q$. Here blue edges mean that the conditional association between MPI indicators is positively stronger than those revealed in the *main* graph of Figure 3(a) whereas red dashed edges mean more negative or weaker association between those nodes.



(a) Main Graph (b) Urban areas effects

M: Mortality; N: Nutrition; SA: School Attendance; YS: Years of Schooling; E: Electricity; W: Water; S: Sanitation; H: Housing; CF: Cooking Fuel; A: Assets.

Figure 3: Conditional dependency between MPI deprivations in Guinea

In the main graph we find a strong positive conditional dependency between Electricity (E) and the Assets (A) indicators (E—A). This means that, given the rest of the MPI indicators, if a household has no access to electricity, it is very likely that it is also poor in terms of assets. Considering that several household assets are electrical, this high revealed association is reasonable. A similar high conditional association occurs between Electricity (E) with Housing Materials (H) and Sanitation (S). In fact, Housing (H) has positive associations with every indicator except for Cooking Fuel (CF). Therefore, if a household is deprived in the housing indicator, it is likely that it is also deprived in another indicator, with the exception of Cooking Fuel (CF).

Striking results are obtained with respect to Cooking Fuel (CF) indicator. First, it is conditionally independent or has a very weak relationship with almost all the indicators. Secondly, Cooking Fuel (CF) has a strong negative conditional association with the Assets (A) indicator, which can be interpreted as if a household is poorly endowed with assets, it is likely to compensate with a better supply of cooking fuel. Considering that in this country the vast majority of households (more than 95%) use solid fuels for cooking, the conditional association result seems to respond more to deprivation in Assets (A). This result reveals how robust sparse graphical models are to high-valued uncensored headcount ratio indicators and how new insights may emerge in comparison with R measure.

We identify a positive but weak conditional association between Child Mortality (M) and Nutrition (N), two indicators of the Health dimension. In fact, the conditional odd ratio of the edge connecting the two nodes is less than 1.5 which is consistent with a small association in terms of effect size of Cohen's d (Chen, Cohen, & Chen, 2010; Cohen, 2013). This low conditional association could reveal that these indicators are not redundant, but instead each of them seems to provide additional information to characterize that dimension of poverty. Furthermore, with respect to the other dimensions, low or null associations are also observed between indicators. This means that if a household is deprived in any aspect of Health dimension, little can be inferred from its level of poverty in terms of Education or Living Standards dimensions. When non-null associations are revealed, these are generally positive (M—SA, M—YS, N—SA), with some exceptions such as between Child Mortality (M) and Assets (A).

On the other hand, as in Health dimension, indicators of Education dimension also show low conditional association between them and the rest of the indicators. In particular, for School Attendance (SA) we find lower or null conditional associations with the rest of the MPI indicators. However, a relevant positive relationship between Years of Schooling (YS) and Assets (A) is estimated. Given the other MPI indicators, this indicates that if a household lacks assets, it is likely to have members with low level of education.

Finally, if we consider urban areas' effects, some changes in conditional associations are detected. First, it can be appreciated that in urban areas, the positive conditional association between the Housing (H) and Electricity (E) indicators is strongly accentuated. Therefore, if a household has no electricity, it is very likely that it also lives in a materially precarious dwelling, and this is revealed to a greater extent in urban areas. Other positive relationships increase in an urban area context, albeit to a lesser extent, such as E—W, YS—S and YS—A. Nonetheless, we see that in urban areas, the positive relationship between housing material deprivation (H) and improved sanitation (S) is weakened, as it is indicated by the thick red dashed line. This could be explained by the fact that in urban areas there are generally better sanitary conditions, covering households with significant deprivations in terms of housing quality or access to electricity. This is further supported by the fact that all of the indicators of the living standards dimension greatly decline in urban households, therefore it is expected that fewer matches will be found between pairs when conditioned on the rest (see Table 2). However, this is not trivially so. As it occurs between Electricity (E) and

Housing (H) or Water (W), although poor households according to such indicators are smaller in urban areas, the conditional dependencies between them increase. This shows that Ising models can reveal new interesting patterns that are not possible to deduce from the MPI decomposition properties.

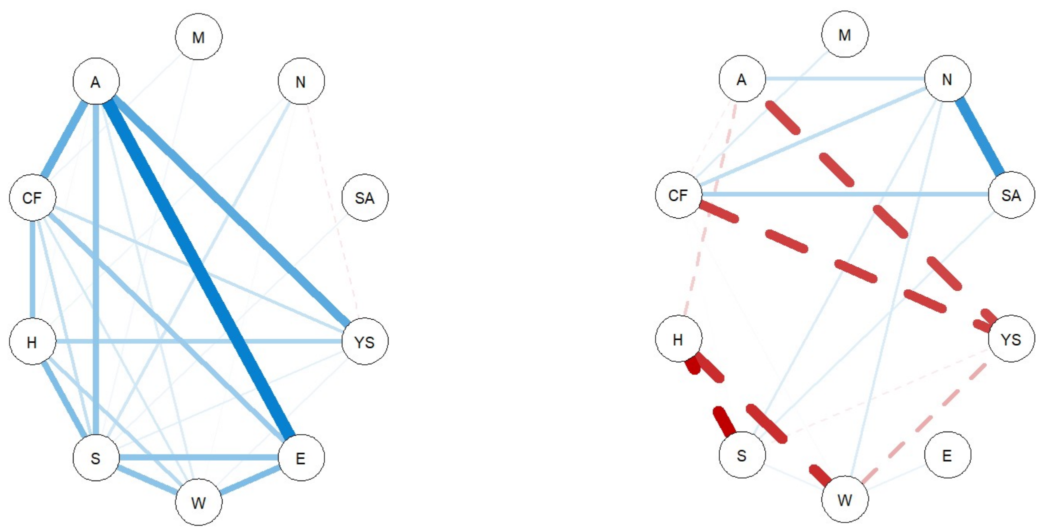
Finally, for the Nutrition indicator (N) we find that in urban areas the conditional dependencies with respect to Water (W) and Electricity (E) are now negative, while the main graph exhibits conditional independence. This indicates that even if an urban household does not suffer deprivations in terms of Nutrition, it is likely to have no access to drinking water or electricity, which is another intriguing aspect discovered by the MPI graph.

4.2 Results for Ecuador: The Role of Monetary Poverty on MPI associations

We estimate an Ising model for Ecuador for non-monetary indicators of the MPI considering monetary poverty as a covariate. Specifically, we include the binary indicator Y of monetary poverty based on the \$1.90 a day poverty line for income as a covariate in the graph, in order to investigate its effect on the conditional association of non-monetary poor indicators.

In Figure 4(a) we show the main graph of conditional associations between indicators. Here we can observe a strong conditional dependence between Assets (A) and Electricity (E). In other words, given the rest of the MPI indicators, if a household is deprived in terms of assets, it is very likely that it is also deprived in terms of electricity. Furthermore, highly positive conditional associations are observed between the Assets and Years of Schooling (A—YS) and Assets with Cooking Fuel (A—CF) indicators. Also, among the indicators of the Living Standards dimension, positive conditional relationships are observed, although of lesser magnitude than those mentioned above. In addition, we highlight the conditional independence that Mortality (M), Nutrition (N) and School Attendance (SA) indicators have with respect to the rest of the MPI indicators.

On the other hand, when we conditioned by monetary poverty (see Figure 4(b)), interesting patterns emerge. Firstly, for monetary poor people, we find a strong positive dependency between Nutrition (N) and School Attendance (SA). This means that in monetary poor households where there is a child not attending school, they are very likely to have at least one child with a nutritional deficit. Remarkably, positive relationships in the main graph between A—YS, CF—YS, H—W, H—S become negative when we condition by monetary poverty. These estimated relationships by the graph enable characterizing monetary poverty with respect to multidimensional poverty indicators. For example, monetary poor households with adequate housing materials are likely to have no



(a) Main Graph (b) Monetary Poor Effects

M: Mortality; N: Nutrition; SA: School Attendance; YS: Years of Schooling; E: Electricity; W: Water; S: Sanitation; H: Housing; CF: Cooking Fuel; A: Assets.

Figure 4: Conditional dependency between MPI deprivations in Ecuador taking monetary poverty as covariate

access to drinking water or improved sanitation. This result may be useful for household targeting in the improvement of access to public services since housing materials are easy-to-observe features (Kidd & Wylde, 2011; Klasen & Lange, 2015). Analogously, monetary poor households, although with sufficient educational attainments (i.e., no deprivations in terms of Years of Schooling) are very likely to face deprivations in terms of Assets (A) and Cooking Fuel (CF). Other new associations are revealed by conditioning on monetary poverty, but to a lesser extent than those described above.

4.3 Comparison with Unconditional Measures of Association

We compare the results of the main estimated graphs and the Cramer's V and Redundancy R measures for Guinea and Ecuador (see Appendix 6.3). First of all, since the latter are measures of unconditional association between indicators, the results can differ from those found from conditional graphs. In fact, this will always happen when at least one indicator is connected to the pair of indicators whose association is being analyzed.

In the Guinean case, we observe that estimated Cramer's V values are generally very low. Most of the values are close to zero or indicate weak association, and the highest three values are between 0.30 and 0.44. However, we can notice some similarities with the graphs. For example, despite being low, the highest values for the V measure are E and H; E and A; E and YS in that order. In the estimated graph, we found very strong and positive conditional associations in E—A and E—H but weaker in E—YS.

A strong association A—YS is estimated in the graph but not in the V. Besides, although Cramer's V pairwise associations are very low, we can identify that relationships between Housing (H) and the rest of the indicators follow similar behaviors in $\hat{\mathcal{G}}$ and V. Some low associations computed by Cramer's V (M with N and SA both with YS and the two health dimension indicators) show positive non-negligible associations in the graph. Other very low or close to zero V values (W with M or N, YS with N or H with CF to mention a few) are ruled out from the graph due to the induced sparsity in the graphical model. This indicates conditional independence between the indicators and it is better in terms of interpretability.

With respect to the R measure, we first observe the extremely high redundancy values between Cooking Fuel (CF) and the rest of the indicators. Similarly, it occurs with Sanitation (S). This happens due to the high uncensored headcount ratios in these two indicators for the case of Guinea and the fact that the R measure focuses on overlapping deprivations. In this case, it is possible to see that graphs are robust to this problem. Observing $\hat{\mathcal{G}}$, the conditional independence between CF and almost all of the indicators shows one of the advantages of estimating the association of a pair of indicators conditioned on the rest. On the other hand, considering that 0.70 is taken as a high redundancy value, the graph can also capture these positive associations (see for example the edges and R values of YS—E, YS—A, E—W, E—H, E—A). Low redundancy values are seen in the low positive conditional associations of N—SA and N—H, which are very close to zero in V values. Interestingly, cases such as W with N or M and A with N or SA or W show almost null V values and low R values. This indicates that the two indicators are not highly associated considering overlapping deprivations. The graph estimates are also enough robust to detect conditional independencies among these pairs of indicators.

Finally, we highlight that the strong negative conditional association A—CF in the graph is captured neither by the Cramer's V nor by the R measure as well as the negative association A—M. Moreover, the conditional independence of CF and almost all of the rest of the indicators is a clear difference between the graph and the unconditional pairwise measure. We tend to think that one or more indicators are connected to those pairs of indicators where remarkable differences with the graph are found.

More interesting results can be drawn from Ecuador. Again, Cramer's V heatmap states very low associations between variables. The highest values for the V measure are sortly E with A, A with CF, CF with E, and A with S, having a rank between 0.34 and 0.45. In spite of these low values, estimated associations from the graphs and V measures follow a similar pattern. In the estimated graph $\hat{\mathcal{G}}$, strong positive conditional associations A—E, A—CF are contemplated whilst CF—E and A—S in a lesser extent. Moreover, bearing in mind their differences, Cramer's V measures follow similar behavior to Cooking Fuel (CF) estimated associations in the graph, with the remarkable difference that a Sparse Ising Model can shrink to zero those negligible associations. Other very low Cramer's V associations are reflected by conditional independences in the graph such as Electricity with indicators from Health and Education dimensions and between each pair of indicators in these mentioned dimensions. Interestingly, the very weak negative relationship between N and A computed by V is also present in the graph.

On the other hand, the R values greater than 0.7 are E with S, E with A ,and E with W. These high redundancies are reflected mainly in the strong association E—A and the weaker positive relationships E—S and E—W estimated by the graph. Other similarities can be found between relatively high redundancies and the positive associations A—S and E—CF. On the contrary, the low redundancy values among all pairs of Health and Education indicators are shrunk to zero in the graph, i.e., given the rest of indicators, these ones are independent of each other. Other positive associations estimated by the graph are represented with low R values: H—S, H—CF, and H—YS.

Last but not least, neither Cramer's V nor redundancy can capture the relatively strong association YS—A revealed by the graph. Crearly, other indicators are having an influence in the association between YS and A. This is not not possible to capture with the unconditional measures used in the literature.

Finally, we do not carry out a factor analysis and compare the results mainly because, as we mentioned earlier, the interpretability of the MPI indicators' relationships depends on the selected factor rotation technique and the list rotation methods is large. Not to mention that factor analysis comes with a theoretical assumption, that the MPI indicators are manifest (observed) variables of a latent (unobserved) phenomenon: poverty. Some researchers may not agree with this perspective. As Kruis and Maris (2016) proved, the observed associations between variables estimated with Ising models can be explained by three distinct theoretical frameworks: 1) *common cause* (latent variables cause co-variation in observed variables), 2) *reciprocal affect* (observed associations are a consequence of mutualistic relationships between these variables) and 3) *common effect* (variables are marginally independent with respect to each other, yet observed relationships lead to the occurrence -or absence- of some common effect). This is a remarkable property and a solid reason for using Ising models to examine how MPI indicators are related to each other.

5 Discussion

From the previous section, it is possible to identify some advantages of using an undirected graph for modeling the joint distribution of the MPI indicators. In this section, we continue with further comments.

Despite its limitations, the Redundancy coefficient has been a useful measure for studying how two indicators are associated to build a consistent MPI. The Ising model can accomplish this task as well. By construction, it can detect two redundant indicators. Furthermore, it proved to be robust in cases where one indicator shows high headcount ratio levels, which severely affect the R measure.

On the other hand, the Ising model used in this study (Cheng et al., 2014) permits estimating the association between indicators conditioned not only on the rest but also on exogenous variables, which can be very informative for policies.

In the field of policy design, for instance, we found in both countries that, even if we condition by living area or monetary poverty, there is a strong positive conditional relationship between Electricity and Assets. Departing from this information, it is possible to deduce that the lack of asset endowments in some households is due to lack of access to electricity. Hence, policies oriented to improving access to electricity could increase households assets acquisition (such as refrigerator, TV or mobile phones). As another example, Ecuador's results show that monetary poor households where there is at least one child with a nutritional deficit are more likely to have a child who is not attending school. This information could suggest the creation of a conditional cash transfers (CCT) scheme targeted to the poorest households with children oriented to improve food security with the condition of children to attend school. In both examples, the reduction of multidimensional poverty may be larger and they illustrate how the Ising model used in this study can broaden the scope of the MPI as a policy relevant tool.

In terms of policy coordination, we emphasized that capturing joint deprivations enables us to design the best coordinated policy action for poverty reduction and that effective coordination requires each stakeholder's responsibility on the MPI to be clearly defined. Notwithstanding, the information platform for these schemes is limited. In this context, conditional associations between two indicators estimated by the graph definitely call for a synergetic complementation of the programs designed to tackle that pair of indicators. In addition, conditional independences between indicators help to consolidate each stakeholder's responsibility within the overall poverty reduction goal.

In sum, the MPI as a graph provides a more insightful understanding of what poverty means and results in a useful tool for developing cost-efficient and high-impact policies while moving towards an integrated poverty eradication agenda.

6 Conclusion

In this paper, we first highlighted how the MPI works as a high-resolution lens on poverty. All the MPI disaggregation properties (equations (1), (2), (3), (4)) provide a prominent information platform for policy purposes and we illustrated this by taking two structurally different countries as examples: Guinea and Ecuador.

We mentioned how the disaggregation property by incidence and intensity can guide overall poverty reduction goals and how new insights are revealed by means of the subgroup decomposability property (differences in the number of poor people and the intensity of the suffered deprivations between subgroups). In addition, we presented how this property conveys informative aspects for policy when it is combined with indicator breakdown property. For example, to what extent differences between h_j and $h_j(k)$ could be useful for targeting, and how each indicator's relative contribution could be used to coordinate different programs.

These useful properties are derived from the contemplation of the joint distribution of the indicators, for which we provided grounded reasons why we can assume that it follows a Multivariate Bernoulli distribution. We showed that the Ising Model is a more parsimonious way for studying interactions among indicators and that modeling the MPI as a graph carries a number of advantages: 1) All associations are condensed into a graphical structure that is simple to understand. This preserves the structural and communicational simplicity of the AF measurement method. 2) Estimating the associations between indicators considering the rest may come up with new powerful insights hidden in data. 3) The graph can be used throughout the entire operative process of an MPI, from the index construction to policy recommendations. 4) In particular, in this paper we used the Ising model proposed by Cheng et al. (2014) which can be specified with covariates. We proved how this framework can complement, for example, the subgroup decomposability property. Revealing outcomes may emerge if we can estimate how the conditional associations between indicators change while also considering the influence of, for example, belonging to a population subgroup.

Further research can be done by modeling the joint distribution of the MPI indicators with undirected graphs. 1) In this study, we only used one dummy covariate to examine how conditional associations change. More interesting analysis may emerge by incorporating other demographic features (such as, age or ethnic group, household head's sex, and regions or provincies). 2) For two main reasons, we estimated the graph on the uncensored deprivation matrix. Firstly, it helped to compare the graph estimate with the unconditional association measures used in the literature. Secondly, the censored deprivation matrix depends on the poverty cutoff k which is a decision based on analysis or policy goals. Therefore, we focused on the uncensored deprivation matrix to present this novel approach. Nevertheless, it would be interesting to study how interactions also change by censoring observations for different poverty cutoffs k. 3) In the graph, we estimate the conditional log-odds ratio between pairs of indicators, given the rest. This information could be used for predictive purposes in targeted poverty alleviation programs. 4) Finally, although the Ising model is the more parsimonious way to represent a MBD, it estimates only up to second-order interactions. Future analysis should explore the possibility of developing a hierarchically penalized graphical model for estimating higher order interactions.

Appendices

6.1 The global MPI

The global MPI has been published annually by OPHI and UNPD since 2010 and covers more than 100 developing countries with the aim of complementing globally comparable monetary poverty measures such as the \$1.90 day line. Table 4 describes all the selected dimensions, indicators, deprivation cutoffs and weights.

Poverty cutoff k is equal to 0.33. This means that a person is identified as poor if she is deprived in a third or more of ten (weighted) indicators or, in other words, if she is deprived in one dimension.

Table 4: MPI Dimensions, Indicators, Deprivation cutoffs, and Weights

Dimensions of poverty	**Indicator**	**Deprived if living in a household where...**	**Weight**
Health (1/3)	Nutrition	Any person under 70 years of age for whom there is nutritional information is undernourished.	1/6
	Child mortality	A child under 18 has died in the household in the five-year period preceding the survey.	1/6
Education (1/3)	Years of schooling	No eligible household member has completed six years of schooling.	1/6
	School attendance	Any school-aged child is not attending school up to the age at which he/she would complete class 8.	1/6
Living Standards (1/3)	Cooking fuel	A household cooks using solid fuel, such as dung, agricultural crop, shrubs, wood, charcoal, or coal.	1/18
	Sanitation	The household has unimproved or no sanitation facility or it is improved but shared with other households.	1/18
	Drinking water	The household's source of drinking water is not safe or safe drinking water is a 30-minute or longer walk from home, roundtrip.	1/18
	Electricity	The household has no electricity.	1/18
	Housing	The household has inadequate housing materials in any of the three components: floor, roof, or walls.	1/18
	Assets	The household does not own more than one of these assets: radio, TV, telephone, computer, animal cart, bicycle, motorbike, or refrigerator, and does not own a car or truck.	1/18

6.2 Parameters Estimates of Ising Models

Tables 5 and 6 show the parameters estimates of the graphs for Guinea and Ecuador, respectively.

Table 5: Parameter estimates of Ising Model for Guinea

$\hat{\boldsymbol{\theta}}_0$ (main)	M	N	SA	YS	E	W	S	H	CF	A
M	-2.422	0.338	0.380	0.136	0.000	0.000	0.112	0.195	0.000	-0.318
N		-0.625	0.262	0.000	0.000	0.030	0.000	0.226	-0.025	0.000
SA			-0.740	0.269	0.098	0.143	0.000	0.292	-0.034	-0.036
YS				-0.752	0.270	0.192	0.283	0.336	-0.146	0.618
E					-1.163	0.262	0.524	0.993	0.116	1.301
W						-0.880	0.266	0.307	0.002	-0.021
S							0.085	0.718	0.052	0.256
H								-2.197	0.065	0.358
CF									4.812	-0.998
A										-1.657
$\hat{\boldsymbol{\theta}}_{jj'}$ (urban areas)										
M	-0.294	0.000	0.000	0.059	0.056	0.000	-0.052	0.000	0.000	0.000
N		-0.116	-0.057	-0.028	-0.109	-0.147	0.064	0.000	-0.006	0.000
SA			-0.263	-0.089	0.066	0.000	0.000	-0.086	-0.003	0.073
YS				-0.826	0.000	-0.054	0.200	-0.033	-0.001	0.213
E					-1.070	0.279	-0.166	0.517	0.000	0.000
W						-0.586	-0.009	0.066	0.000	0.000
S							-0.081	-0.544	0.000	0.023
H								-0.463	0.000	0.000
CF									-1.334	0.021
A										-0.453

Note: M: Mortality; N: Nutrition; SA: School Attendance; YS: Years of Schooling; E: Electricity; W: Water; S: Sanitation; H: Housing; CF: Cooking Fuel; A: Assets.

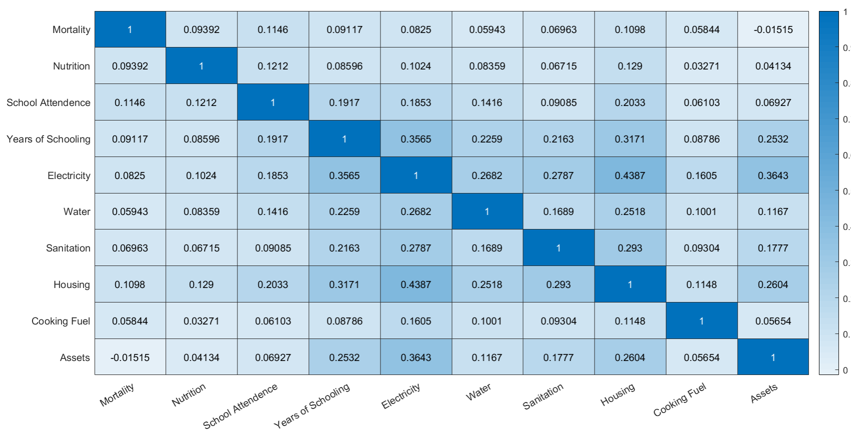
Table 6: Parameter estimates of Ising Model for Ecuador

$\hat{\boldsymbol{\theta}}_0$ (main)	M	N	SA	YS	E	W	S	H	CF	A
M	-2.732	0.017	0.000	0.000	0.000	0.000	0.117	0.000	0.148	0.000
N		-1.828	0.000	-0.199	0.000	0.085	0.400	0.127	0.005	0.000
SA			-3.841	0.000	0.000	0.023	0.150	0.000	0.000	0.000
YS				-3.198	0.000	0.147	0.230	0.739	0.514	1.443
E					-5.405	1.152	1.002	0.000	0.871	2.167
W						-2.006	1.013	0.642	0.368	0.290
S							-2.160	1.126	0.515	0.975
H								-2.646	0.940	0.102
CF									-3.351	1.363
A										-3.653
$\hat{\boldsymbol{\theta}}_{jj'}$ (monetary poverty)										
M	0.237	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.031	0.000
N		0.370	0.202	0.000	0.000	0.037	0.032	0.000	0.061	0.059
SA			0.475	0.000	0.000	0.000	0.030	0.000	0.084	0.002
YS				0.000	0.000	-0.079	-0.017	0.000	-0.182	-0.176
E					0.113	0.021	0.000	0.000	0.000	0.000
W						0.246	0.019	-0.199	0.007	0.000
S							0.300	-0.242	0.003	0.000
H								0.390	0.000	-0.047
CF									0.564	-0.015
A										0.471

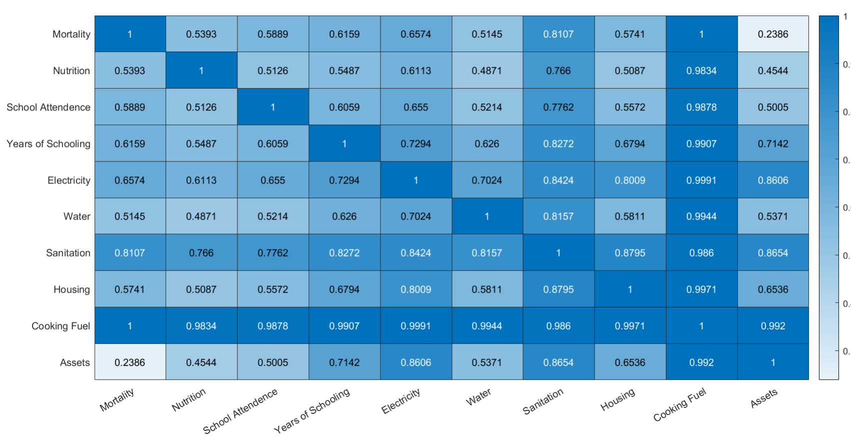
Note: M: Mortality; N: Nutrition; SA: School Attendance; YS: Years of Schooling; E: Electricity; W: Water; S: Sanitation; H: Housing; CF: Cooking Fuel; A: Assets.

6.3 Redundancy Measures

Figures 5 and 6 show the estimated unconditional association measures (Cramer's V and Redundancy R) for Guinea and Ecuador, respectively.

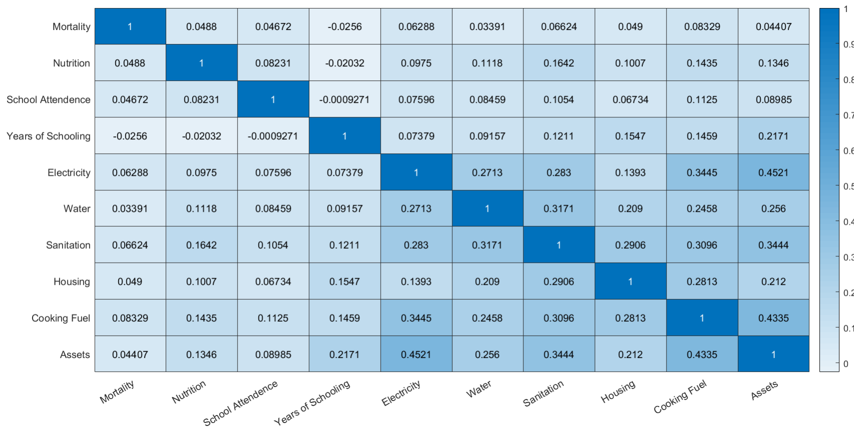


(a) Cramer's V

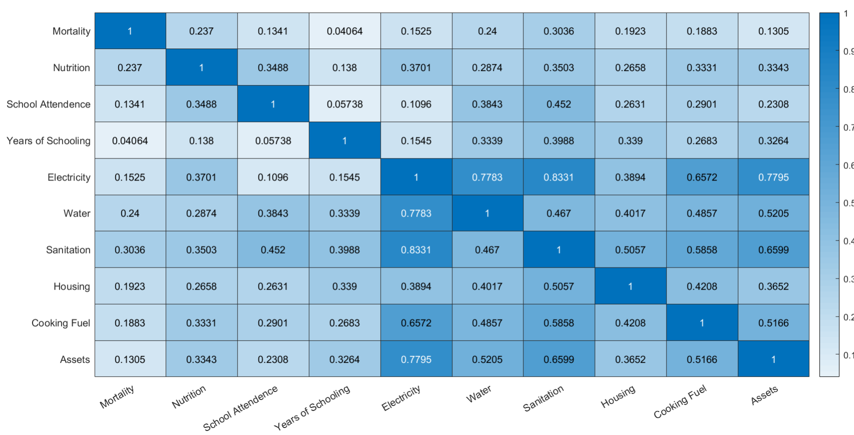


(b) Redundancy R

Figure 5: Heatmaps of unconditional association measures for Guinea



(a) Cramer's V



(b) Redundancy R

Figure 6: Heatmaps of unconditional association measures for Ecuador.

Authors' contributions

These authors contributed equally to this work.

Funding

This work was supported by ANPCYT Grant PICT 2018-04377, CONICET Grant PIP 11220200101595CO and UNL Grant CAI+D 506-201901-00041LI.

Availability of data and materials

The data for Guinea and Ecuador that support the findings of this study are available in the DHS website (`https://dhsprogram.com/methodology/survey/survey-display-539.cfm`) and in the official ECV 13-14 website (`https://www.ecuadorencifras.gob.ec//documentos/web-inec/ECV/ECV_2015/`).

References

Alkire, S. (2013). Choosing dimensions: The capability approach and multidimensional poverty. In N. Kakwani & J. Silber (Eds.), *The many dimensions of poverty* (p. 89–119). London: Palgrave Macmillan.

Alkire, S. (2020). Multidimensional poverty measures as policy tools. In V. Beck, H. Hahn, & R. Lepenies (Eds.), *Dimensions of poverty: Measurement, epistemic injustices, activism* (p. 197–214). Springer.

Alkire, S., & Foster, J. (2011). Counting and multidimensional poverty measurement. *Journal of Public Economics*, *95*(7), 476–487.

Alkire, S., Roche, J. M., Ballon, P., Foster, J., Santos, M. E., & Seth, S. (2015). *Multidimensional poverty measurement and analysis*. Oxford University Press.

Alkire, S., & Santos, M. (2014). Measuring acute poverty in developing world: Robustness and scope of the multidimensional poverty index. *World Development*, *59*, 251–274.

Angulo, R. (2016). *From multidimensional poverty measurement to multisector public policy for poverty reduction: lessons from the colombian case* (OPHI Working Papers No. 133). Queen Elizabeth House, University of Oxford.

Angulo, R., Díaz, Y., & Pardo, R. (2016). The colombian multidimensional poverty index: Measuring poverty in a public policy context. *Social Indicators Research*, *127*(1), 1–38.

Bishop, C. (2006). *Pattern recognition and machine learning*. Springer.

Ceriani, L., & Gigliarano, C. (2020). Multidimensional well-being: A bayesian networks approach. *Social Indicators Research*, *152*, 237–263.

Chen, H., Cohen, P., & Chen, S. (2010). How big is a big odds ratio? interpreting the magnitudes of odds ratios in epidemiological studies. *Communications in Statistics - Simulation and Computation*, *39*(4), 860–864.

Cheng, J., Levina, E., Wang, P., & Zhu, J. (2014). A sparse ising model with covariates. *Biometrics*, *70*(4), 943–953.

Cheng, J., Li, T., Levina, E., & Zhu, J. (2017). High-dimensional mixed graphical models. *Journal of Computational and Graphical Statistics*, *26*(2), 367–378.

Cohen, J. (2013). *Statistical power analysis for the behavioral sciences*. Routledge.

Dai, B. (2013). MVB: Multivariate bernoulli log-linear model [Computer software manual]. Retrieved from `https://CRAN.R-project.org/package=MVB` (R package version 1.1)

Dai, B., Ding, S., & Wahba, G. (2013). Multivariate Bernoulli distribution. *Bernoulli*, *19*, 1465–1483.

Duarte, S. (2016). Modelos lineales generalizados: regresión de rango reducido y reducción suficiente de dimensiones. *Ph.D Dissertation, FIQ-Universidad Nacional del Litoral*.

Duarte, S., Forzani, L., García Arancibia, R., Llop, P., & Tomassi, D. (2021). Socioeconomic index for income and poverty prediction: A sufficient dimension reduction approach. *Review of Income and Wealth*. doi: https://doi.org/10.1111/roiw.12529

Evans, M., Nogales, R., & Robson, M. (2020). *Monetary and multidimensional poverty: Correlations, mismatches and joint distributions* (OPHI Working Papers No. 133). Queen Elizabeth House, University of Oxford.

Iceland, J., & Bauman, K. J. (2007). Income poverty and material hardship: How strong is the association? *The Journal of Socio-Economics*, *36*(3), 376–396.

Ising, E. (1925). Beitrag zur theorie des ferromagnetismus. *Zeitschrift für Physik*, *31*, 253–258.

Kidd, S., & Wylde, E. (2011). *Targeting the poorest: An assessment of the proxy means test methodology* (AusAID Report). AusAID.

Klasen, S., & Lange, S. (2015). *Targeting performance and poverty effects of proxy means-tested transfers: Trade-offs and challenges* (IAI Discussion Papers). Ibero-America

Institute for Economic Research.

Koller, D., & Friedman, N. (2009). *Probabilistic graphical models: Principles and techniques*. MIT Press.

Kruis, J., & Maris, G. (2016). Three representations of the ising model. *Scientific Reports*, *6*(34175).

Meinshausen, N., & Bühlmann, P. (2006). High-dimensional graphs and variable selection with the lasso. *The Annals of Statistics*, *34*(3), 1436–1462.

Ravikumar, P., Wainwright, M. J., & Lafferty, J. D. (2010). High-dimensional ising model selection using l-regularized logistic regression. *The Annals of Statistics*, *38*(3), 1287–1319.

Roelen, K. (2017). Monetary and multidimensional child poverty: A contradiction in terms? *Development and Change*, *48*(3), 502–533.

Salecker, L., Ahmadov, A. K., & Karimli, L. (2020). Contrasting monetary and multidimensional poverty measures in a low-income sub-saharan african country. *Social Indicators Research*, *151*(2), 547–574.

Santos, M. E., & Villatoro, P. (2018). A multidimensional poverty index for latin america. *Review of Income and Wealth*, *64*(1), 52–82.

Stiglitz, J., Sen, A., & Fitoussi, J.-P. (2009). *The measurement of economic performance and social progress revisited: Reflections and overview* (Sciences Po publications No. 2009-33). Sciences Po.

Suppa, N. (2016). *Comparing monetary and multidimensional poverty in germany* (OPHI Working Papers No. 103). Queen Elizabeth House, University of Oxford.

Tran, V. Q., Alkire, S., & Klasen, S. (2015). Static and dynamic disparities between monetary and multidimensional poverty measurement: Evidence from vietnam. In T. I. Garner & K. S. Short (Eds.), *Measurement of poverty, deprivation, and economic mobility* (Vol. 23, p. 249–281). Emerald Publishing Ltd.

Wainwright, M. J., Jordan, M. I., et al. (2008). Graphical models, exponential families, and variational inference. *Foundations and Trends in Machine Learning*, *1*(1–2), 1–305.

Wang, X., Feng, H., Xia, Q., & Alkire, S. (2016). *On relationship between income poverty and multidimensional poverty in china* (OPHI Working Papers No. 101). Queen Elizabeth House, University of Oxford.