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Juries and Information Aggregation in Dynamic Environments

(Preliminary version)

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Abstract

We study information aggregation through voting in dynamic environments. We show that the voting rule under which an informative vote is a Nash equilibrium entails a time-varying quota, which suggests that efficient information aggregation requires the use of time-varying voting rules. We also show that a time-invariant simple majority quota rule is asymptotically efficient, that is when the size of the committee tends to infinity. We discuss possible applications to the monitoring and managing of natural resources and the environment.

Keywords: Condorcet Jury Theorem - Information aggregation - Partially Observable Markov Decision Processes - Management of natural resources - Environment

JEL codes:

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1 Introduction

Importance of Condorcet's jury theorem (information aggregation: the wisdom of crowds) and its variants and extension in numerous directions (correlation of signals, different degrees of expertise, such as Ben-Yashar & Nitzan (1997). Importance in the design (architecture) of organisations). Relevant applications: juries, expert committees (medicine, monetary policy, regulation, project evaluation, FDA committees). A relevant criticism is that of Austen-Smith & Banks (1996) who show that informative (sincere) voting by all individuals is not a Nash Equilibrium even when the committee members have common preferences. Therefore, particularly important contributions are those that consider strategic voting (Wit 1998, McLennan 1998, Duggan & Martinelli 2001). In general, it is possible to define a quota aggregation rule for which there is a Nash equilibrium in which each member of the committee votes according to their information. If the conditions are met for the informative vote to be a better aggregator than the dictator's vote, then Condorcet's jury theorem holds: as the size of the committee increases, the probability of making a mistake tends to zero because it is possible to design an aggregation rule that encourages the informative vote.

2 The model

In this section, we provide the model which extends jury decision-making to dynamic environments, and we derive the optimal voting rule.

2.1 Setup

I consider a decision maker (DM) whose task is to make decisions regarding the execution of projects, the returns of which depend on a changing environment. Each period a committee of N experts makes a report about the state of the world. $\mathcal{N}$ denotes the set of experts. In each period, there may be two possible states of the world: $x_t = g, b$, and two possible decisions: $D_t = G, B$. The ex-post payoffs in each period is $U(x_t, D_t)$ when state is x_t and decision D_t is made:

$$U(g,G) \quad U(g,B) \quad U(b,G) \quad U(b,B)$$

Assumption 1 $U(g,G) > U(b,G)$

Assumption 2 $U(b,B) > U(g,B)$

Assumption 3 $U(g,G) > U(g,B) \rightarrow \Delta_g > 0$

Assumption 4 $U(b,B) > U(b,G) \rightarrow \Delta_b > 0$

The state of the world follows a Markov chain with transition probabilities

$$\begin{aligned} \lambda_g &= \Pr(x_{t+1} = g | x_t = g) \\ \lambda_b &= \Pr(x_{t+1} = g | x_t = b), \end{aligned}$$

(We assume that it is not affected by the decisions of the DM or of the committee). Let $\alpha \equiv \Pr(x_0 = g)$ denote the prior probability that the state is a good one.

2.2 The optimal voting rule

We proceed as follows: first, we derive the optimal information aggregation rule for centralised decision-making, that is, decision-making is made by a single agent who aggregates the private signals reported truthfully by committee members. Then we show that with decentralised decision-making, if the voting rule replicates the optimal aggregation rule, there exists a Markovian equilibrium in which each juror votes informatively, that is, according to her signal. Suppose that the decision rests on a single decision maker who receives the opinion of N jurors, each of whom reports truthfully his private signal s_t^i. A profile of N signals is denoted $s_t \equiv (s_t^i)_{i \in \mathcal{N}}$. Let $p_t \equiv \Pr(x_t = g | I_t)$ denote the probability that the state be $x_t = g$ conditional on the information available at t, I_t. After the decision is made, the true state of nature is known (because it can be inferred from the perceived utility). Information available for DM at t is $I_t = \{s_t, x_{t-1}, I_{t-1}\}, t \geq 1$, ($I_0 = \varnothing$), but it suffices to consider s_t, x_{t-1} (previous history I_{t-1} does not add information). Provided that s_t and x_{t-1} are known, the posterior of the event $x_t = g$ is

$$p_t = \frac{\Pr(s_t | x_t = g) \Pr(x_t = g | x_{t-1})}{\Pr(s_t | x_t = g) \Pr(x_t = g | x_{t-1}) + \Pr(s_t | x_t = b) \Pr(x_t = b | x_{t-1})}$$

We denote the right hand side with $\Phi(x_{t-1}, s_t)$. Expected value of choosing G given prior p_t is

$$V_G = p_t U(g,G) + (1-p_t) U(b,G) + \beta \mathbf{E}_{x_t, s_{t+1}} V(\Phi(x_t, s_{t+1}))$$

and expected value of choosing B given prior p is

$$V_B = p_t U(g,B) + (1-p_t) U(b,B) + \beta \mathbf{E}_{x_t,s_{t+1}} V(\Phi(x_t, s_{t+1}))$$

where $0 < \beta < 1$ is the discount factor. Then, the Bellman equation for DM's problem can be stated as follows:

$$V \equiv \max\{V_G, V_B\}. \tag{1}$$

The optimal policy is G if

$$\begin{aligned} V_G \geq V_B \quad &\Leftrightarrow \quad p_t U(g,G) + (1-p_t) U(b,G) \geq p_t U(g,B) + (1-p_t) U(b,B) \\ &\Leftrightarrow \quad p_t [U(g,G) - U(g,B)] \geq (1-p_t) [U(b,B) - U(b,G)] \\ &\Leftrightarrow \quad p_t \Delta_g \geq (1-p_t) \Delta_b \\ &\Leftrightarrow \quad p_t \geq \frac{\Delta_b}{\Delta_g + \Delta_b}. \end{aligned}$$

Proposition 1 *The optimal policy is*

$$d(p_t) = \begin{cases} G \text{ if } p_t \geq \Delta_b / (\Delta_g + \Delta_b) \\ B \text{ if } p_t \leq \Delta_b / (\Delta_g + \Delta_b) \end{cases} \tag{2}$$

A stationary policy function $d(p_t)$ associated to the functional equation (1) is a mapping from the set $[0,1]$ where p_t belongs, to an optimal action in $\mathcal{A}$. As information unfolds over time (i.e. signal profiles are learned), a sequence of optimal decisions is generated as follows: (i) s_t is learned; (ii) $p_t = \Phi(x_{t-1}, s_t)$ is calculated; (iii) optimal decision $d(p_t)$ is chosen. Thus, an information aggregation rule f_t^* at period t, which is a mapping from the space of signal profiles to $\mathcal{A}$, is optimal, if it satisfies $f_t^*(s_t) = d(\Phi(x_t, s_{t+1}))$. Note that f_t^* is (in general) time-varying because the information state p_t is also time-varying, even when in the infinite horizon case, $d(p_t)$ is stationary.[1] An optimal information aggregation rule is a sequence $f^* = \{f_t^*\}_{t=1}^{\infty}$ of optimal period information aggregation rules. We say that the rule $f^* = \{f_t^*\}_{t=1}^{\infty}$ aggregates information optimally in the centralized problem if for each t, $f_t^*(s_t) = d(\Phi(x_t, s_{t+1}))$, where d is the policy function

[1] In general, an information aggregation rule should map every available information to the set of actions $\mathcal{A}$; because of the sufficient statistic result, we can restrict wlg to the class of rules mapping signal profiles to actions and ignoring past information I_{t-1}.

associated to (1). An information aggregation rule is associated with a voting aggregation rule in the dynamic voting game with informative voting. An informative profile of voting strategies is a Nash equilibrium of the dynamic voting game of common interest $\Gamma(f)$ if f aggregates information optimally. If f aggregates information optimally, and the strategies of players other than i are informative, then it is the best response for i to vote informatively because the game is of common interest. An informative profile of voting strategies is a Nash equilibrium of the dynamic voting game of common interest $\Gamma(f)$ only if f aggregates information optimally within the class of qualified aggregation rules. If voting is informative, (2) leads to the following optimal aggregation rule (Appendix): choose G if

$$\#\mathbb{A}(\mathbf{v}_t) \geq \lceil \tilde{q}(x_{t-1}) \rceil$$

and choose B otherwise, where $\lceil \tilde{q} \rceil$ is the minimum integer q such that $q \geq \tilde{q}$, for $0 \leq \tilde{q} \leq N$, $\#\mathbb{A}(\mathbf{v}_t)$ is the number of voters voting for $d_t = G$ in the profile $\mathbf{v}_t$ and

$$\tilde{q}(x_{t-1}) \equiv \frac{N}{2} + \frac{\varphi(x_{t-1})}{2\eta} + \frac{\xi}{2\eta} \tag{3}$$

where $\xi \equiv \ln \frac{\upsilon}{\kappa}$, $\varphi(x_{t-1}) \equiv \ln \frac{\lambda(0, x_{t-1})}{\lambda(1, x_{t-1})}$ and $\eta \equiv \ln \frac{\theta}{1-\theta}$. We denote

$$\lceil \tilde{q}(1) \rceil \equiv q_1, \qquad \lceil \tilde{q}(0) \rceil \equiv q_0$$

To develop some intuition from the optimal voting rule, suppose $\xi = 0$. Note that $\theta > 1/2 \Rightarrow \eta > 0$. If in the last period, the mode was $x_{t-1} = 1$ but persistence in this mode is low ($\lambda(1, 1) < 1/2$), then a supermajority is required for the choice $d_t = 1$, and if last period, the mode was $x_{t-1} = 0$, and persistence in the bad state is high ($\lambda(0, 0) > 1/2$), then a supermajority is required for the choice $d_t = 1$. Similarly, if last period mode was $x_{t-1} = 1$ and persistence in this mode is high, less than a simple majority is required in period t for the choice $d_t = 1$. Similarly, if the last period mode was $x_{t-1} = 0$, but persistence in a bad state is low, less than a simple majority is required in period t for the choice $d_t = 1$. Note that the gap between $\min\{q_0, q_1\}$ and $\max\{q_0, q_1\}$ does not depend on N. Thus, the difference in quotas relative to the committee's size goes to zero as N tends to infinity. Thus, in dynamic environments, a constant quota rule with $\bar{q} = (N + 1)/2$ (simple majority) is nearly efficient if the committee is very large.

3 Asymptotic efficiency of constant quota rules

A constant aggregation rule is characterised by the quota $\bar{q} \in \{2, ..., N-1\}$ such that $\forall t$, $d_t = i_1$ if and only if $\#\mathbb{A}(v_t) \geq \bar{q}$. A stationary Markov voting strategy for voter i is a function $v^i : S \times X \rightarrow [0, 1]$ where $S = \{s_0, s_1\}$ is the space of signals and X is the space of states. That is, a Markov strategy maps each private signal s_t^i and each state x_{t-1} to $[0, 1]$ where $v^i(s, x) \equiv \Pr\left(v_t^i = i_1 | s_t^i = s, x_{t-1} = x\right)$ is the probability of voting $v_t^i = i_1$ when the voter privately observes signal $s_t^i = s$ and last period's state was $x_{t-1} = x$. We focus on symmetric equilibria -agents receiving the same signal use the same strategy. Define $v^i(s_1, x) = v_x^1$ and $v_x^0 = 1 - v^i(s_0, x)$, so v_x^1 is the probability of voting $v_t^i = i_1$ when the signal is s_1, and v_x^0 is the probability of voting $v_t^i = i_0$ when the signal s_0. By lemma 2 of Austen-Smith & Banks (1996), informative voting $v_x^1 = 1, v_x^0 = 1, \forall x$ is a Bayesian equilibrium if and only if the aggregation rule aggregates information optimally. Thus, it has to be $\bar{q} = q_1 = q_0$, but this occurs if and only if $\lambda(1, 1) + \lambda(0, 0) = 1$; thus, in general, Bayesian equilibria in stationary Markov strategies are not informative under constant aggregation rules. It also follows (e.g. proposition 2 of Ben-Yashar & Milchtaich (2007)) that constant rules give lower expected utility than the rule that aggregates information optimally, which, as it has been shown above, is, in general, time-varying. This raises the issue of which is the (second) best voting rule (which we denote $\bar{q}$) within the class of constant quota voting rules.

Lemma 1 *(i)* $\min\{q_0, q_1\} \leq \bar{q} \leq \max\{q_0, q_1\}$. *(ii)If* $\lambda(1, 1) + \lambda(0, 0) = 1$*, then* $\bar{q} = q_1 = q_0$. *(iii)* $\lambda(1, 1) = \lambda(0, 0) = 1/2 \Rightarrow \bar{q}^* = (N + 1)/2$.

3.1 Characterization of equilibrium under constant voting rules

It can be shown (Appendix) that if under the constant quota $1 < \bar{q} < N$, a voter mixes when observing s_0 and past state is $x_{t-1} = x$ (that is $0 < v_x^{s_0} < 1$) then she does not mix when she observes s_1, and in particular, $v_x^{s_1} = 1$. Similarly, if the voter mixes when observing s_1, then $v_x^{s_0} = 1$ [2] Given $x_{t-1} = x$ and constant quota $1 < \bar{q} < N$, the following proposition characterises mixed strategies.

Proposition 2 *(i) if* $v_x^{s_1} = 1$ *and* $0 < v_x^{s_0} < 1$*, then*

$$v_x^{s_0} = \frac{1 - \varpi_0(x, \bar{q})}{\theta - (1 - \theta)\varpi_0(x, \bar{q})}$$

[2] Wit (1998) considers the simple majority case. Here, we extend Wit's result to the case of an arbitrary quota $1 < \bar{q} < N$.

where

$$\varpi_0(x,\bar{q}) \equiv \left[\frac{\kappa\lambda(1,x)}{\upsilon\lambda(0,x)}\left(\frac{1-\theta}{\theta}\right)^{N+1-\bar{q}}\right]^{\frac{1}{\bar{q}-1}}.$$

(ii) if $v_x^{s_0} = 1$ *and* $0 < v_x^{s_1} < 1$, *then*

$$v_x^{s_1} = \frac{1-\varpi_1(x,\bar{q})}{(1-\theta)-\theta\varpi_1(x,\bar{q})}$$

where

$$\varpi_1(x,\bar{q}) \equiv \left[\frac{\kappa\lambda(1,x)}{\upsilon\lambda(0,x)}\left(\frac{\theta}{1-\theta}\right)^{\bar{q}}\right]^{\frac{1}{N-\bar{q}}}.$$

Let $v_x^s(\bar{q})$ be the optimal mixed strategy when the constant quota rule is $\bar{q}$, past state is x and signal is s.

Lemma 2 *For each* x, $v_x^{s_1}(\bar{q})$ *(*$v_x^{s_0}(\bar{q})$*) is nondecreasing (non increasing) in* $\bar{q}$.

4 Optimal stopping

In each period, a decision maker (DM) may choose to undertake an irreversible project with an unknown and time-varying payoff (the state of the world) or wait until the following period when new information arrives. A committee of experts provides the information. In each period, each expert receives a signal correlated to the state, which follows a Markov chain. The purpose of this paper is to obtain an optimal rule for aggregating experts' information for each period. I build on Ben-Yashar & Nitzan (1997), who consider a committee whose task is to approve or reject projects. In their model, each expert receives a signal correlated to the project's profitability. Ben-Yashar & Nitzan's (1997) optimal rule is static. Their model can capture situations in which there is no possibility to postpone the execution of the project, either because the opportunity disappears or because, in the future new information will not arrive, making the waiting worthless. Their result is significant to jury decision-making and other political, legal, economic and medical applications. However, there are many situations in which waiting is possible and has value because new information may be coming and because the project's execution is irreversible. For example, oil drilling can be postponed if a rise in prices is likely to happen. Similarly, an entrepreneur facing uncertain demand may prefer to wait before introducing a new product or brand to the market.

Even in medical treatments, a committee of experts may prefer to wait for the appearance of new symptoms before suggesting a risky treatment. A monetary policy committee may prefer to wait for the resolution of some uncertainty (the magnitude of supply or demand shock or the resolution of a wage bargaining round) before changing the policy instrument. In short, these "accept or wait" situations are also pervasive.

4.1 The model

At each period, a committee of N experts reports the state of the world to DM. There are two states of the world: in the good state, the project is profitable, with a positive net present value (NPV) $v = 1$. In the bad state, the project is worthless, with $v = 0$. The state of the world follows a Markov chain with transition probabilities $\lambda_1 = \Pr(v_{t+1} = 1 | v_t = 1)$, $\lambda_0 = \Pr(v_{t+1} = 1 | v_t = 0)$. In the beginning, a prior probability $\alpha = \Pr(v_0 = 1)$ is known. At each t, available actions to DM are "execute" (e) or "wait" (w). Let $\mathcal{A} = \{w, e\}$. In the final period, T the project is undertaken. The state of the process is not observable. Instead, each expert i receives a private signal $s_{i,t}$ related to the NPV of the project if it is undertaken at t. Let $s_{i,t} \in \{-1, 1\}$, $i = 1, \cdots, N$. Denote

$$\theta_{1i} = \Pr\left(s_{i,t} = 1 | v_t = 1\right), \qquad \theta_{0i} = \Pr\left(s_{i,t} = -1 | v_t = 0\right),$$

the precision of the signal for expert i. Expertise is formalized by assuming that $\theta_{0i}, \theta_{1i} > 1/2$ (signals are informative). We may also interpret $s_{i,t}$ as expert i's assessment of the state of the process at t.

Timing

Let $\mathbf{s}_t \equiv \left\{s_{i,t}\right\}_{i=1}^{N}$ denote a report profile at period t, let $\mathcal{X}$ denote the set of possible report profiles and let $\mathbf{h}_t \equiv \{\mathbf{s}_\tau\}_{\tau=1}^{t}$ denote a history of report profiles. The sequence of events after DM selects $a_t = w$ is: (i) the state changes ($v_t \rightarrow v_{t+1}$) following a Markov chain process; (ii) each expert independently observes signal $s_{i,t+1}$ and report it to DM; (iii) DM selects a_{t+1}; if $a_{t+1} = e$ is selected, a reward equal to v_{t+1} is received and no more decisions are made. A decision rule $f_t : \mathcal{X}^t \rightarrow \mathcal{A}$ for DM is a function that maps every report at time t to the action set $\mathcal{A} = \{e, w\}$. Due to the Markov chain assumption, it can be shown that the function $\phi(\mathbf{h}_t) = \mathbf{s}_t$ that picks the last report profile from the history $\mathbf{h}_t$ of report profiles is a sufficient statistic for the process (Degroot 2004). If $p_t = \Pr(v_t = 1 | \mathbf{h}_t)$ is the probability of a good state given the history of profiles, bayesian updating

$p_{t-1} \rightarrow p_t = \Phi(\mathbf{s}_t, p_{t-1})$ requires only the current report profile $\mathbf{s}_t$. This is done using the following formula:

$$\Phi(\mathbf{s}, p) = \frac{r_1(\mathbf{s})\,\varphi(p)}{\gamma(\mathbf{s}, p)}$$

where $\mathbf{s}$ is a report profile,

$$r_j(\mathbf{s}) = \Pr(\mathbf{s}|v = j)$$

is the conditional probability of $\mathbf{s}$ given that state is j,

$$\gamma(\mathbf{s}, p) = r_0(\mathbf{s})\,[1 - \varphi(p)] + r_1(\mathbf{s})\,\varphi(p),$$

is the likelihood of $\mathbf{s}$, and

$$\varphi(p_{t-1}) = \lambda_0(1 - p_{t-1}) + \lambda_1 p_{t-1}$$

is the prior for p_t before observing $\mathbf{s}_t$. At period t, expected value of undertaking the project is p_t. The functional equation associated with DM's problem is

$$\begin{aligned} V_t(p) &= \max\left\{p, \beta \sum_{\mathbf{s}\in\mathcal{X}} \gamma(\mathbf{s}, p)\, V_{t+1}[\Phi(\mathbf{s}, p)]\right\}, t = 1, ..., T-1 \qquad (4) \\ V_T(p) &= p \end{aligned}$$

where $0 < \beta < 1$ is DM's discount factor. A solution to problem (4) maps each possible value of p_t, to an action $a \in \{e, w\}$, for each t.

4.2 Optimal aggregation of information

Let $\mathcal{U}_t$ denote the subset of $[0, 1]$ for which the optimal action at t is $a_t^* = e$:

$$\mathcal{U}_t = \{p \in [0, 1] : V_t(p) = p\}.$$

It can be shown that each $\mathcal{U}_t$ has the form $[p_t^*, 1]$ where p_t^* is non-decreasing in t (Appendix, Proposition 3). Then, there exist threshold values $\{p_t^*\}_{t=1}^T$ such that, for each t, the following policy is optimal:

$$a_t^* = \begin{cases} w & \text{if} \quad p_t < p_t^* \\ e & \text{if} \quad p_t \geq p_t^* \end{cases}$$

Theorem 1 *Given report profile* $\mathbf{s} \equiv \{s_i\}_{i=1}^N$, *the optimal decision rule* f_t^* *is:*

$$f_t^*(\mathbf{s}) = \text{sign}\left(\sum_{i=1}^N \omega_i x_i(\mathbf{s}) + b_t\right)$$

where

$$\text{sign}(a) = \begin{cases} +1 \text{ if } a \geq 0 \\ -1 \text{ if } a < 0 \end{cases},$$

$+1$ *corresponds to e and* -1 *corresponds to w,*

$$\omega_i = \frac{1}{2}\left(\ln \frac{\theta_i^0}{1-\theta_i^0} + \ln \frac{\theta_{1i}}{1-\theta_{1i}}\right)$$

is expert i's weight,

$$x_i(\mathbf{s}) = \begin{cases} 1 \text{ if } s_i = 1 \\ -1 \text{ if } s_i = -1 \end{cases}$$

is expert i^*s *advice,*

$$b_t(\mathbf{s}) = \xi_t + \phi_t + \psi,$$

is a time-varying threshold where

$$\begin{aligned} \xi_t &= \ln \frac{\varphi(p_{t-1})}{1-\varphi(p_{t-1})} \\ \phi_t &= \ln \frac{1-p_t^*}{p_t^*}, \\ \psi &= \frac{1}{2}\sum_{i=1}^N \left(\ln \frac{\theta_i^1}{\theta_i^0} + \ln \frac{1-\theta_i^1}{1-\theta_i^0}\right), \end{aligned}$$

and p_t^* *is computed recursively as follows:*

$$\begin{aligned} p_T^* &= 0, \\ p_{t-1}^* &= \frac{r_1(\mathbf{s}_{t-1})\,\varphi\left(p_{t-2}^*\right)}{r_1(\mathbf{s}_{t-1})\,\varphi\left(p_{t-2}^*\right) + r_0(\mathbf{s}_{t-1})\left[1-\varphi\left(p_{t-2}^*\right)\right]}. \end{aligned}$$

5 Applications to the monitoring and managing of natural resources

(To be completed)

6 Discussion

We show that in time-varying environments with an unobservable state following a binary first order Markov process, a decision maker should use a time-varying information aggregation rule. If decision-making is decentralised to a common interest committee [3] who employs a quota rule to decide, then the committee will be better off using a time-varying quota. If the rule aggregates information optimally, then an equilibrium exists in which each member votes informatively, that is, according to the private signal. (McLennan (1998), Theorem 1). The optimal quota rule can be expressed as a weighted majority rule with a time-varying bias component. If the number of decision makers goes to infinity, then the constant simple majority rule is asymptotically efficient.

In the optimal stopping case, a decision maker should use a time-varying information aggregation rule to determine the optimal period to undertake a project. If the timing decision is decentralised to a common interest committee, the optimal voting rule should have a time-varying quota, which is a weighted majority rule with a time-varying bias component. The idea is that if the rule aggregates information optimally, then there exists a Nash equilibrium in which each member votes according to the private signal (McLennan 1998). Note that the rule is not derived ex-ante. Rather, period $t+1$ rule is defined using $\mathbf{s}_t$ (given that in t the decision was to wait). In this sense, it is a dynamic aggregation rule. Possible applications are managerial decision-making (exercise of a license for oil drilling; stop fishing before a regulated deadline; duration of advertising; selling an item along a finite selling period; replacing a machine before recommended replacement date); environment (Government Advisory Committees, geoengineering); medicine (data monitoring committees for clinical trials; drug quality and therapeutics committee); policymaking (monitoring institutions; monetary policy committees).

[3]That is, a committee in which every member has the same ex-post payoff.

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Appendix 1

Let $\Pr(x_t = x^*|I_{t-1}) = \Pr(x_t = x^*|x_{t-1} = x) = \lambda(x^*, x)$, so $p_t = \dfrac{1}{1+\vartheta(s_t)\Gamma(x_{t-1})}$ where $\vartheta(s_t) \equiv \dfrac{\Pr(s_t|x_t=0)}{\Pr(s_t|x_t=1)}$ and $\Gamma(x_{t-1}) \equiv \dfrac{\lambda(0,x_{t-1})}{\lambda(1,x_{t-1})}$. Thus, optimal choice is $d_t = G$ iff $\dfrac{1}{1+\vartheta(s_t)\Gamma(x_{t-1})} \geq \dfrac{\upsilon}{\kappa+\upsilon} \Leftrightarrow lr(s_t) \geq \xi + \varphi(x_{t-1})$ where $\xi \equiv \ln\dfrac{\upsilon}{\kappa}$, $\varphi(x_{t-1}) \equiv \ln\dfrac{\lambda(0,x_{t-1})}{\lambda(1,x_{t-1})}$, $lr(s) \equiv \dfrac{\Pr(s_t = s|x_t=1)}{\Pr(s_t=s|x_t=0)}$. When comparing the differences in expected profits between voting for G and voting for B, it suffices to consider the function

$$\begin{aligned} w(s_1, x_{t-1}) - w(s_0, x_{t-1}) &= \kappa z_t^2 \Pr\left(\mathbf{piv}_i\left(v^*_{-i}(x_{t-1}), \bar{q}\right)|1\right) \times \lambda(1, x_{t-1})\left[\Pr(s_1|1) - \Pr(s_0|1)\right] \\ &\quad + \upsilon z_t^2 \Pr\left(\mathbf{piv}_i\left(v^*_{-i}(x_{t-1}), \bar{q}\right)|0\right) \times \lambda(1, x_{t-1})\left[\Pr(s_0|0) - \Pr(s_1|0)\right] \end{aligned}$$

Now, note that $\forall x_{t-1}, w(s_1, x_{t-1}) - w(s_0, x_{t-1}) > 0$. So $0 < v_x^{s_0} < 1 \Rightarrow w(s_0, x) = 0 \Rightarrow w(s_1, x) > 0 \Rightarrow v_x^{s_1} = 1$ and $0 < v_x^{s_1} < 1 \Rightarrow w(s_1, x) = 0 \Rightarrow w(s_0, x) < 0 \Rightarrow v_x^{s_0} = 1$. Fix $x_{t-1} = x$. Given that others use the strategy $\left(v_x^{s_0}, v_x^{s_1}\right)$, a voter correctly votes for i_1 in the state 1 with probability

$$\begin{aligned} c_{i_1}\left(v_x^{s_0}, v_x^{s_1}\right) &\equiv \Pr[s_1|1]\Pr\left[v^i = i_1|s_1\right] + \Pr[s_0|1]\Pr\left[v^i = i_1|s_0\right] \\ &= \theta v_x^{s_1} + (1-\theta)\left(1 - v_x^{s_0}\right) \end{aligned}$$

and correctly votes for i_0 in state 0 with probability

$$\begin{aligned} c_{i_0}\left(v_x^{s_0}, v_x^{s_1}\right) &\equiv \Pr[s_1|0]\Pr\left[v^i = i_0|s_1\right] + \Pr[s_0|0]\Pr\left[v^i = i_0|s_0\right] \\ &= (1-\theta)\left(1 - v_x^{s_1}\right) + \theta v_x^{s_0} \end{aligned}$$

With these two definitions, we have

$$\begin{aligned} \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|0\right) &= \Pr\begin{pmatrix} \bar{q}-1 \text{ incorrect votes for } i_1 \\ \text{and } N-\bar{q} \text{ correct votes for } i_0 \end{pmatrix} \\ &= \binom{N-1}{\bar{q}-1}\left(1 - c_{i_0}\left(v_x^{s_0}, v_x^{s_1}\right)\right)^{\bar{q}-1}\left(c_{i_0}\left(v_x^{s_0}, v_x^{s_1}\right)\right)^{N-\bar{q}} \\ \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|1\right) &= \Pr\begin{pmatrix} \bar{q}-1 \text{ correct votes for } i_1 \\ \text{and } N-\bar{q} \text{ incorrect votes for } i_0 \end{pmatrix} \\ &= \binom{N-1}{\bar{q}-1}\left(c_{i_1}\left(v_x^{s_0}, v_x^{s_1}\right)\right)^{\bar{q}-1}\left(1 - c_{i_1}\left(v_x^{s_0}, v_x^{s_1}\right)\right)^{N-\bar{q}} \end{aligned}$$

The equilibrium condition for $v_x^{s_1} = 1$ and $0 < v_x^{s_0} < 1$ is $w(s_0, x) = 0$. That is, upon receiving a signal s_0, the voter is indifferent between voting for G or for B, i.e.

$$\kappa z_t^2 \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|1\right)\Pr(s_0|1)\lambda(1,x) = \upsilon z_t^2 \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|0\right)\Pr(s_0|0)\lambda(0,x)$$

$\Leftrightarrow$

$$v_x^{s_0} = \frac{1 - \varpi_0(x, \bar{q})}{\theta - (1-\theta)\varpi_0(x, \bar{q})}$$

where

$$\varpi_0(x,\bar{q}) \equiv \left[\frac{\kappa\lambda(1,x)}{\upsilon\lambda(0,x)}\left(\frac{1-\theta}{\theta}\right)^{N+1-\bar{q}}\right]^{\dfrac{1}{\bar{q}-1}}.$$

Similarly, the equilibrium condition for $v_x^{s_0} = 1$ and $0 < v_x^{s_1} < 1$ is $w(s_1, x) = 0$. That is, upon receiving a signal s_1, a voter is indifferent between voting for i_1 or for i_0, i.e.

$$\kappa z_t^2 \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|1\right) \Pr(s_1|1)\,\lambda(1,x) = \upsilon z_t^2 \Pr\left(\mathbf{piv}_i\left(v_{-i}(x), \bar{q}\right)|0\right) \Pr(s_1|0)\,\lambda(0,x)$$

$\Leftrightarrow$

$$v_x^{s_1} = \frac{1-\varpi_1(x,\bar{q})}{(1-\theta)-\theta\varpi_1(x,\bar{q})}$$

where

$$\varpi_1(x,\bar{q}) \equiv \left[\frac{\kappa\lambda(1,x)}{\upsilon\lambda(0,x)}\left(\frac{\theta}{1-\theta}\right)^{\bar{q}}\right]^{\dfrac{1}{N-\bar{q}}}.$$

Appendix 2

Proposition 3 *(Smallwood & Sondik 1973, Monahan 1980, Bertsekas 2005, Degroot 2004)*
(i) $1 \in \mathcal{U}_t \forall t$;
(ii) $\mathcal{U}_t$ *is convex;*
(iii) $\mathcal{U}_1 \subset \mathcal{U}_2 \subset \cdots \subset \mathcal{U}_t \subset \cdots \subset \mathcal{U}_T = [0,1]$.

The proof requires the following lemmata.

Lemma 3
(i) $V_t(p) \leq 1$ *for every* t;
(ii) *for every* p, $V_t(p)$ *is non increasing in* t.

Proof. (i) By assumption, the NPV of the project is 1 or 0 and is perceived only in the period in which action e is selected. Thus, $V_t(p) \leq 1$. (ii) Note that

$$\begin{aligned} V_{T-1}(p) = \max\left\{p, \beta \sum_{\mathbf{s}\in\mathcal{X}} \gamma(\mathbf{s},p)\,\Phi(\mathbf{s},p)\right\} = \max\left\{p, \beta \sum_{\mathbf{s}\in\mathcal{X}} r_1(\mathbf{s})\,\varphi(p)\right\} &= \max\{p, \beta\varphi(p)\} \\ &\geq p = V_T(p), \end{aligned}$$

where the first equality follows because optimal action at period T is to undertake the project ($a_T^* = e$) and then, $V_T(p) = p$, the second equality follows from the expressions for $\gamma(\mathbf{s},p)$ and $\Phi(\mathbf{s},p)$ and the third equality follows because

$$\sum_{\mathbf{s}\in\mathcal{X}} r_1(\mathbf{s}) = \sum_{\mathbf{s}\in\mathcal{X}} \Pr(\mathbf{s}|v_t = 1) = 1.$$

Suppose that $V_t(p) \geq V_{t+1}(p)$ for some t and for all $p \in [0,1]$. To complete the proof, note that

$$V_{t-1}(p) = \max\left\{p, \beta \sum_{\mathbf{s}\in\mathcal{X}} \gamma(\mathbf{s},p)\,V_t[\Phi(\mathbf{s},p)]\right\} \geq \max\left\{p, \beta \sum_{\mathbf{s}\in\mathcal{X}} \gamma(\mathbf{s},p)\,V_{t+1}[\Phi(\mathbf{s},p)]\right\} = V_t(p),$$

where the inequality follows from the induction hypothesis. ■

Lemma 4 *Let*

$$V_{t-1}^w(p) = \beta \sum_{\mathbf{s}\in\mathcal{X}} \gamma(\mathbf{s},p)\,V_t[\Phi(\mathbf{s},p)]$$

and suppose that $V_t(p)$ *is convex. Then* $V_{t-1}^w(p)$ *is also convex.*

Proof. Let ξ and $\nu \in [0,1]$ and let

$$p = \mu\xi + (1-\mu)\nu, 0 < \mu < 1;$$

we need to show the following:

$$V_{t-1}^w(p) \leq \mu V_{t-1}^w(\xi) + (1-\mu)\,V_{t-1}^w(\nu).$$

Note that

$$\gamma(\mathbf{s}, p) = \gamma(\mathbf{s}, \mu\xi + (1-\mu)\nu) = \mu\gamma(\mathbf{s}, \xi) + (1-\mu)\gamma(\mathbf{s}, \nu)$$

and

$$\Phi(\mathbf{s}, p) = \frac{r_1(\mathbf{s})\,\varphi(\mu\xi + (1-\mu)\nu)}{\gamma(\mathbf{s}, \mu\xi + (1-\mu)\nu)} = \frac{\mu\gamma(\mathbf{s}, \xi)\,\Phi(\mathbf{s}, \xi)}{\mu\gamma(\mathbf{s}, \xi) + (1-\mu)\gamma(\mathbf{s}, \nu)} + \frac{(1-\mu)\gamma(\mathbf{s}, \nu)\,\Phi(\mathbf{s}, \nu)}{\mu\gamma(\mathbf{s}, \xi) + (1-\mu)\gamma(\mathbf{s}, \nu)}.$$

It follows from the convexity of $V_t(p)$ that

$$\gamma(\mathbf{s}, p)\, V_t[\Phi(\mathbf{s}, p)] \leq \mu\gamma(\mathbf{s}, \xi)\, V_t[\Phi(\mathbf{s}, \xi)] + (1-\mu)\gamma(\mathbf{s}, \nu)\, V_t[\Phi(\mathbf{s}, \nu)].$$

Then

$$\begin{aligned} V_{t-1}^{w}(p) &= \beta\sum_{\mathbf{s}\in\mathcal{X}}\gamma(\mathbf{s}, p)\, V_t[\Phi(\mathbf{s}, p)] \\ &\leq \mu\beta\sum_{\mathbf{s}\in\mathcal{X}}\gamma(\mathbf{s}, \xi)\, V_t[\Phi(\mathbf{s}, \xi)] + (1-\mu)\beta\sum_{\mathbf{s}\in\mathcal{X}}\gamma(\mathbf{s}, \nu)\, V_t[\Phi(\mathbf{s}, \nu)] \\ &= \mu V_{t-1}^{w}(\xi) + (1-\mu)\, V_{t-1}^{w}(\nu). \end{aligned}$$

We conclude that $V_{t-1}^{w}(p)$ is convex. ■

Lemma 5 *$V_t(p)$ is convex for $t = 1, \ldots, T$.*

Proof. We proceed by induction. Suppose that $V_t^{w}(p)$ is convex, $t \leq T-1$. Then, $V_t(p) \equiv \max\{p, V_t^{w}(p)\}$ is convex since it is the maximum of two convex functions, and $V_{t-1}^{w}(p) \equiv \beta\sum_{\mathbf{s}\in\mathcal{X}} V_t[\Phi(p)]$ is also convex (Lemma 4). This implies that $V_{t-1}(p) \equiv \max\{p, V_{t-1}^{w}(p)\}$ is convex in p. To complete the proof, note that $V_T^{w}(p) = 0$ and $V_T^{e}(p) = p$ are linear functions, so $V_T(p) \equiv \max\{p, V_T^{w}(p)\}$ is convex, and by Lemma 4, $V_{T-1}^{w}(p)$ is also convex. ■

Proof of Proposition 3. (i) $1 \in \mathcal{U}_T$ because $V_T(1) = 1$. Then, $V_t(1) = 1\ \forall t < T$ because for each p, $V_t(p)$ is non increasing in t (Lemma 3) and for each t, is bounded above by 1 (Lemma 3). We conclude that $1 \in \mathcal{U}_t \forall t$. (ii) Suppose p and p' are in $\mathcal{U}_t$. Let $p'' = \nu p + (1-\nu)\,p'$ for some $\nu \in (0,1)$. Then, $V_t(p'') \leq \nu V_t(p) + (1-\nu)\, V_t(p') = \nu p + (1-\nu)p' = p''$ where the inequality is a result of convexity of value function (Lemma 5) and the first equality results because p and p' belong to $\mathcal{U}_t$. But, by definition of the value function, $V_t(p'') \geq p''$ so we conclude that $V_t(p'') = p''$ and thus $p'' \in \mathcal{U}_t$. (iii) Suppose $p \in \mathcal{U}_t$. Then $p = V_t(p) \geq V_{t+1}(p)$ (Lemma 3) and by definition of the value function, $V_{t+1}(p) \geq p$. Thus $p = V_{t+1}(p)$, so $p \in \mathcal{U}_{t+1}$. ■

Proof of Theorem 1. DM uses the profile $\mathbf{s}$ to update the state p_{t-1} to $p_t = \Phi(\mathbf{s}, p_{t-1})$. Then, the decision is to undertake if $\Phi(\mathbf{s}, p_{t-1}) \geq p_t^{*}$, that is, if

$$\frac{r_1(\mathbf{s})\,\varphi(p_{t-1})}{r_0(\mathbf{s})\,[1-\varphi(p_{t-1})] + r_1(\mathbf{s})\,\varphi(p_{t-1})} \geq p_t^{*}, \tag{5}$$

which, taking logs and using definitions of ϕ_t and ξ_t above, can be expressed as

$$\ln\frac{r_1(\mathbf{s})}{r_0(\mathbf{s})} + \phi_t + \xi_t \geq 0.$$

Denoting by $\mathbb{I}^+(\mathbf{s})$ the subset of experts that report $\mathbf{s}_i = 1$, and by $\mathbb{I}^-(\mathbf{s})$ the subset of experts that report $\mathbf{s}_i = -1$, when the profile is $\mathbf{s}$, it is straightforward to show that the log likelihood ratio is

$$\ln \frac{r_1(\mathbf{s})}{r_0(\mathbf{s})} = \sum_{i \in \mathbb{I}^+(\mathbf{s})} \ln \frac{\theta_{1i}}{(1-\theta_{0i})} + \sum_{i \in \mathbb{I}^-(\mathbf{s})} \ln \frac{(1-\theta_{1i})}{\theta_{0i}}.$$

Using definitions of ω_i, $x_i(\mathbf{s})$, and ψ above, we get

$$\ln \frac{r_1(\mathbf{s})}{r_0(\mathbf{s})} = \sum_{i=1}^{N} \omega_i x_i(\mathbf{s}) + \psi,$$

so condition (5) becomes

$$\sum_{i=1}^{N} \omega_i x_i(\mathbf{s}) + \psi + \phi_t + \xi_t \geq 0,$$

or equivalently $\operatorname{sign}\left(\sum_{i=1}^{N} \omega_i x_i(\mathbf{s}) + b_t\right) = 1$. By a similar procedure, it is shown that

$$\Phi(\mathbf{s}, p_{t-1}) < p_t^* \Leftrightarrow \operatorname{sign}\left(\sum_{i=1}^{N} \omega_i x_i(\mathbf{s}) + b_t\right) = -1.$$

■