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Fiscal Theory of the Price Level in Small and Open Economies*

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Abstract

A salient feature of many emerging and developing economies is that a substantial fraction of government debt is denominated in foreign currency. We study the implications of the Fiscal Theory of the Price Level (FTPL) in a standard New Keynesian small and open economy model, with an explicit role for the currency denomination of public debt. We show that, while the classical FTPL characterization of equilibrium existence and uniqueness extends largely independently of debt composition, the propagation of shocks does not. The currency denomination of public liabilities alters the effects of monetary and fiscal policy, including the possibility that a monetary tightening leads to a depreciation under active fiscal regimes. More broadly, the interaction between the fiscal-monetary policy mix and the share of foreign-currency debt also plays a central role in shaping the response to external shocks.

JEL codes: E31; E52; E63; F41.

Keywords: Fiscal theory of the price level; inflation; exchange rate; fiscal and monetary policy interactions; currency composition of government debt.

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1 Introduction

The global surge in inflation following the COVID-19 pandemic, coupled with unprecedented fiscal expansions, has reignited the debate over the interaction between fiscal and monetary policy. These concerns have become particularly pressing in light of the sharp increase in public debt worldwide. For instance, IMF (2025) reports that by 2024 nearly 70% of countries carried higher public debt burdens than in 2020. The Fiscal Theory of the Price Level (FTPL) has emerged as a central theoretical framework for analyzing these issues (foundational contributions include Leeper, 1991, Sims, 1994, and Woodford, 1994, with a detailed textbook treatment provided by Cochrane, 2023).

Yet, most of this literature has focused on closed economies. The few open-economy studies (discussed below) neglect a key feature of open economies: a substantial fraction of government debt is issued in foreign currency.[1] This omission is central since currency composition directly determines how inflation and exchange rate adjustments affect debt valuation and sustainability. In this paper, we extend the FTPL to a small and open economy New Keynesian model, with traded and non-traded goods, explicitly incorporating foreign-currency debt. This framework allows us to analyze how the composition of government liabilities shapes the joint dynamics of inflation, exchange rates, and economic activity under alternative fiscal and monetary regimes.

Our main findings are threefold. First, the classical FTPL insights regarding equilibrium existence and uniqueness extend to the open-economy environment, largely independently of debt composition. Second, the transmission of monetary and fiscal shocks crucially depends on both the policy regime and the share of foreign-currency debt: under active fiscal policy, a monetary tightening generates a nominal depreciation rather than the conventional appreciation; consistent with empirical evidence from emerging markets. Third, foreign-currency debt fundamentally alters the propagation of external shocks, as exchange rate adjustments that help dilute local-currency debt simultaneously increase the burden of foreign-currency debt, amplifying inflation and activity responses.

To develop these results, we set up an infinite-horizon small open economy model with incomplete asset markets and a representative household. There is an exogenous endowment of tradables, while non-tradables are produced using labor under price rigidities. Fiscal policy levies lump-sum taxes to finance debt denominated in both domestic and foreign currencies (assuming a constant currency composition), while monetary policy sets a rule for the nominal interest rate. Different combinations of active or passive fiscal and monetary policies modify the dynamics of the economy.

We first study the (local) existence and uniqueness of a stationary equilibrium under interest-rate rules. Results resemble those obtained in closed-economy setups, almost independently of the currency composition of debt. In particular, under a Ricardian/passive fiscal rule, the Taylor principle (i.e., the monetary policy rate reacting more than proportionally to inflation) is enough to guarantee a unique stationary rational-expectations equilibrium; otherwise, multiplicity arises. Instead, if fiscal policy is non-Ricardian/active, the uniqueness of the equilibrium requires a passive monetary policy (i.e., the policy rate moving less than proportionally to inflation).

We then characterize the transmission of monetary and fiscal shocks. Dynamics vary depending on the fiscal and monetary regime, especially those related to exchange rates. For instance, while a monetary tightening produces nominal and real appreciations under a passive fiscal regime, a depreciation emerges when fiscal policy is active. These differences are amplified for larger shares of

[1]This is especially true for emerging and developing economies. According to the Arslanalp & Tsuda (2014b) database, in 2023, on average 30% of government debt in emerging countries was in foreign currency, with a standard deviation of 21%; while for developing economies these values were 52% and 25%, respectively. This is significantly smaller in advanced countries: the average in 2023 was just 2%, with Israel and Canada being exceptions (20 and 10%, respectively), according to Arslanalp & Tsuda (2014a).

public debt denominated in foreign currency. Under an active fiscal regime, a smaller share of domestically denominated debt requires a larger adjustment in the price level to stabilize the government's intertemporal budget, thereby magnifying exchange-rate movements.

We further show that when the share of foreign-currency debt exceeds a certain threshold, the responses to policy shocks may reverse. In the limiting case of fully dollarized government debt, dilution can only occur through a real-exchange-rate appreciation rather than a depreciation. More generally, dynamics are qualitatively different if foreign-currency debt is large enough. Moreover, debt duration interacts non-trivially with its currency composition in shaping equilibrium dynamics.

These findings speak directly to the empirical literature studying the exchange-rate effects of domestic monetary policy shocks across countries. For example, Hnatkovska et al. (2016) and Bolhuis et al. (2024) document that a monetary tightening generally leads to nominal appreciations in advanced economies but depreciations in emerging markets. Our framework offers a novel interpretation of these patterns based on FTPL mechanisms operating in the presence of foreign-currency debt, identifying conditions under which such cross-country differences may arise.

Finally, we analyze how alternative monetary and fiscal regimes shape the propagation of external shocks, such as tradable income or the international cost of borrowing. When public debt is fully denominated in domestic currency and monetary policy responds to inflation —even if the Taylor principle is not satisfied under a non-Ricardian fiscal regime— the qualitative dynamics are largely invariant to the policy configuration. By contrast, a higher share of foreign-currency debt substantially alters the adjustment process under active fiscal policy, particularly in the non-traded sector. Exchange-rate depreciations that dilute domestic-currency liabilities simultaneously increase the real burden of foreign-currency debt, requiring a stronger non-traded inflation response and potentially reversing the initial effects of the underlying real shock.

RELATED LITERATURE This paper contributes to the growing body of work examining how New Keynesian dynamics differ under alternative fiscal and monetary regimes (e.g., Leeper & Leith, 2016, Cochrane, 2023, Caramp & Silva, 2023). We complement these studies by extending the analysis to a small and open economy setting. Our results highlight that, under sticky prices, the FTPL acquires a distinct open-economy dimension, effectively becoming a fiscal theory of the real exchange rate, especially when a portion of government debt is denominated in foreign currency.

A limited number of studies have examined FTPL in open economies, including Loyo (1999), Dupor (2000), Daniel (2001), and more recently Bianchi (2021), Witheridge (2024) and Ferrer (2025). Many assume flexible prices, so purchasing-power parity holds and there is no independent role for the real exchange rate. By contrast, non-traded goods and sticky prices generate non-trivial nominal and real exchange-rate dynamics. Once the currency composition of debt is considered —absent from prior contributions— the real exchange rate becomes central to the government's valuation equation. By combining these three key factors, we uncover exchange-rate dynamics that are absent from both the closed-economy FTPL literature and existing open-economy extensions.

Our work is also related to papers that study the optimal currency composition of government debt, such as Ottonello & Perez (2019) and Engel & Park (2022), analyzing how devaluation risk and sovereign incentives shape the portfolio choices of foreign investors. In their frameworks, monetary and fiscal policies are analyzed in tandem, with monetary policy modeled as directly controlling inflation. By contrast, the FTPL literature emphasizes the institutional separation between monetary and fiscal authorities and examines how fiscal behavior may constrain the outcomes attainable by a central bank under interest rate rules. Our approach is therefore complementary: rather than characterizing the optimal denomination of debt, we take its currency composition as given and analyze how it shapes macroeconomic equilibrium determination and the transmission of both policy and external shocks.

OUTLINE The rest of the paper is organized as follows. Section 2 presents the model and the alternative fiscal and monetary regimes under consideration. Section 3 analyzes the existence and uniqueness of equilibrium. Sections 4 and 5 examine the transmission of monetary and fiscal shocks, respectively. Section 6 discusses extensions and additional exercises, including the role of long-term debt, Fisherian deflation, and the propagation of external shocks under alternative policy configurations. Section 7 concludes. A Technical Appendix contains the proofs of all propositions and supplementary material.

2 The Model

We use a standard New Keynesian model; e.g. Schmitt-Grohé & Uribe (2017, Ch. 9.16). This is an infinite-horizon, small and open economy with discrete time and uncertainty. Domestic households derive utility from final-consumption goods and disutility from labor. Financial markets include one-period, non-contingent bonds denominated in both domestic and foreign currencies (pesos and dollars, for simplicity). There is an exogenous endowment of tradables, with a dollar price determined abroad, satisfying the law of one price. In contrast, non-traded goods have a monopolistically competitive structure, with a continuum of varieties, each produced by a monopolist that demands labor and faces Calvo-pricing frictions. Final-consumption goods are composed of both tradables and non-tradables.

The fiscal authority collects lump-sum taxes and issues debt in both currencies. We assume that local-currency bonds are held exclusively by domestic households, while dollar-denominated debt can also be purchased by foreigners (who also lend to domestic households). Finally, monetary policy follows an interest-rate rule.

2.1 Households

The domestic representative household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(h_t)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where c_t denotes aggregate consumption, h_t hours worked, while $\beta \in (0,1)$ and $\sigma, \varphi, \chi > 0$ are parameters capturing, respectively, the discount factor, risk aversion, inverse-Frisch elasticity of labor supply, and relative dis-utility of hours worked. In period t they face the following budget constraint in domestic currency units:

$$P_t c_t + S_t D_{t-1}^{H*} + \frac{B_t}{R_t} + T_t = W_t h_t + \Sigma_t + B_{t-1} + S_t \frac{D_t^{H*}}{R_t^*}. \quad (1)$$

The left-hand side includes uses of income: final consumption (with price P_t, representing the CPI in this model), repayment of debt obligations in dollars decided at $t-1$ (D_{t-1}^{H*} is the amount in dollars to be repaid, and S_t is the nominal exchange rate), purchases of domestic-currency bonds B_t (with a gross nominal rate R_t), and lump-sum taxes T_t. Available resources on the right-hand side include labor income (W_t is the hourly nominal wage), profits from the ownership of local firms (Σ_t), income from domestic bonds purchased at $t-1$ (B_{t-1}), and new debt in dollars (D_t^{H*}, with a gross rate R_t^*). Households are assumed to face one No-Ponzi-game condition (NPGC) for each financial asset.[2]

The optimality conditions characterizing the solution to the household problem are the budget constraint (1), both NPGC holding with equality (transversality conditions), and

$$w_t(c_t)^{-\sigma} = \chi(h_t)^{\varphi}, \quad (c_t)^{-\sigma} = \beta R_t E_t \left\{ \frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}} \right\}, \quad (c_t)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1})^{-\sigma} \pi_{t+1}^S}{\pi_{t+1}} \right\},$$

with $w_t \equiv W_t/P_t$, $\pi_t \equiv P_t/P_{t-1}$, and $\pi_t^S \equiv S_t/S_{t-1}$. These characterize the trade-offs between, respectively, consumption and hours worked, as well as current and future consumption through either type of financial asset.

2.2 Supply side

Final consumption goods are produced by competitive firms using the technology

$$c_t = \left[\omega^{1/\eta} \left(c_t^N \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(c_t^T \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}},$$

where c_t^N and c_t^T denote, respectively, the consumption of non-tradables and tradables, with prices P_t^N and P_t^T in local currency. The parameters $\eta > 0$ and $\omega \in [0,1]$ are, respectively, the elasticity of substitution between goods and the weight of non-tradables in the final-consumption basket. Profit maximization leads to the following demands,

$$c_t^N = \omega \left(p_t^N \right)^{-\eta} c_t, \quad c_t^T = (1-\omega) \left(p_t^T \right)^{-\eta} c_t,$$

with $p_t^N \equiv P_t^N/P_t$ and $p_t^T \equiv P_t^T/P_t$. The ratio of these two equations captures the intra-temporal trade-off between both goods as a function of their relative price, which we define as $q_t \equiv P_t^T/P_t^N$.

Traded goods are a stochastic endowment y_t^T, sold at an international dollar price P_t^*, with $\pi_t^* \equiv P_t^*/P_{t-1}^*$. The local price satisfies $P_t^T = S_t P_t^*$. Given its previous definition, p_t^T is also the real exchange rate (rer_t) in CPI units in this model, which moves proportionally with the relative price q_t.

Non-tradables are produced in two stages (see Appendix B.1 for details). First, a representative competitive firm combines a continuum of non-tradable varieties using a Dixit-Stiglitz aggregator ($\epsilon_N > 1$ represents the elasticity of substitution across varieties). Second, each variety j is produced by a monopolist using labor according to $y_{jt}^N = (h_{jt})^{\alpha}$, with $\alpha \in (0,1]$, facing a Calvo (1983) problem in setting prices: with a probability of θ_N, it must keep the previous-period price; otherwise, it is free to choose.

2.3 Fiscal and monetary policy

The government budget constraint in period t is,

$$\frac{D_t^G}{R_t} + S_t \frac{D_t^{G*}}{R_t^*} + T_t = D_{t-1}^G + S_t D_{t-1}^{G*}, \tag{2}$$

[2] These, as well as transversality conditions for fiscal policy, are discussed below in section 2.5 and analyzed in depth in Appendix A.

where D_t^G and D_t^{G*} denote the repayment value at $t+1$ of non-contingent debt issued at t in, respectively, domestic and foreign currencies. As we assume that local-currency debt is only held by domestic households, in equilibrium $D_t^G = B_t$. Notice that the interest rate on foreign-currency debt is the same as that for households (R_t^*, we discuss this assumption below). We finally assume the government satisfies a transversality condition for each type of debt, as discussed in Appendix A.

A fiscal policy is defined as *Ricardian* (a.k.a. passive) if instruments are set such that (2) and the transversality conditions hold for any value of predetermined debt (D_{t-1}^G, D_{t-1}^{G*}) as well as for any possible path of the endogenous variables that affect the government's resource constraint (such as S_t, R_t, R_t^*). In turn, fiscal policy is *Non-Ricardian* (a.k.a. active) if this condition is not satisfied.

We consider two alternative policy setups. In the first, fiscal policy targets real lump-sum taxes defined in terms of the CPI (P_t), while monetary policy targets CPI inflation (π_t). The other setup assumes that lump-sum taxes are set in terms of non-tradables (P_t^N), and monetary policy is concerned with fluctuations in non-traded inflation (π_t^N). The former will be our baseline specification for numerical exercises, as it compares more realistically with actual policies. However, a setup centered around non-tradables has the advantage of delivering clean, algebraic results. For future reference, the former is referred to as the CPI-based setup, while the NT-based configuration describes the latter.

Under the CPI-setup, real lump-sum taxes are $\tau_t \equiv T_t/P_t$, and the real value of peso-denominated debt is $d_t^G \equiv D_t^G/P_t$. Instead, under the NT-based setup, these are $\tau_t \equiv T_t/P_t^N$ and $d_t^G \equiv D_t^G/P_t^N$. Thus, the government's budget constraint (2) in real terms for each case is, respectively,

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \quad \text{or} \quad \frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \tag{3}$$

where $d_t^{G*} \equiv D_t^{G*}/P_t^*$ in either case.

Lump-sum taxes in each setup are determined, respectively, by

$$\tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \quad \text{or} \quad \tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \tag{4}$$

where $\overline{\tau}$ is the steady-state value of primary surpluses, and u_t^τ is an exogenous and stationary shock.

In terms of the currency-composition of debt, we assume the government maintains a constant share of debt denominated in dollars $\Omega \in [0,1]$, where this ratio is computed at steady-state relative prices. That is, we assume either

$$\Omega = \frac{\overline{rer}\, d_t^{G*}}{d_t^G + \overline{rer}\, d_t^{G*}}, \quad \text{or} \quad \Omega = \frac{\overline{q}\, d_t^{G*}}{d_t^G + \overline{q}\, d_t^{G*}}, \tag{5}$$

for the CPI- and NT-based setups, respectively; where $\overline{rer}$ and $\overline{q}$ are steady-state values of rer_t and q_t.

Overall, equations (3)-(5) characterize the evolution of the fiscal variables d_t^G, d_t^{G*}, and τ_t. Under a constant share of dollar debt, a Ricardian fiscal policy relies on the parameter ϕ_T. If $\phi_T = 0$ lump-sum taxes are exogenous, then we have a Non-Ricardian configuration. In the other extreme, $\phi_T = 1$ generates a Ricardian policy, while values between 0 and 1 are also possible, as analyzed below.[3]

[3]We do not consider the possibility of outright default on debt, as in most of the FTPL literature. An exception is Uribe (2006), who discusses a tension between price level determination and default in a closed economy model. Bianchi (2021) extends this to open economies under two simplifying assumptions (full international risk sharing and only domestic-currency debt), which we show here have non-trivial implications for studying FTPL. Of course, there is a large literature on sovereign default (e.g., see the survey by Aguiar & Amador, 2014). However, given the computational challenges derived from the global analysis in those models, the combination of monetary and fiscal policy is generally omitted, as is

In terms of monetary policy, we assume a Taylor-type rule for the short-term interest rate in domestic currency, which, in each policy setup, is:

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \quad \text{or} \quad \left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t^N}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \tag{6}$$

where $\overline{R}$ and $\overline{\pi}$ are steady state values for R_t and π_t (in steady state, $\pi^N = \pi$), u_t^R is an exogenous and stationary monetary shock, and $\phi_\pi \geq 0$. Following the definition proposed by Leeper (1991), an active monetary policy is characterized by $\phi_\pi > 1$, while a passive configuration occurs if $\phi_\pi < 1$. A relevant question, explored below, is which combination of parameters ϕ_π, ϕ_T, and Ω delivers a unique stationary equilibrium.

2.4 Rest of the world

The interest rate for debt in dollars R_t^* is determined by

$$R_t^* = R_t^W \exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}. \tag{7}$$

where $d_t^* = d_t^{H*} + d_t^{G*}$ is the consolidated-net-foreign-debt position, R_t^W is an exogenous (stationary) process, and $\psi > 0$. Thus, locals are assumed to pay a premium for borrowing in dollars ($\exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}$) over the international rate (R_t^W). This elastic premium serves as a "closing device" (Schmitt-Grohe & Uribe, 2003), ensuring that dynamics in the linearized, real version of the model are stationary. In turn, as discussed below, assuming a premium that is elastic to the *consolidated* net-foreign debt, in tandem with specific assumptions about preferences, simplifies the dynamics of this model.

2.5 Equilibrium definition, main channels and calibration

A stationary rational-expectations equilibrium is a set of stochastic processes for endogenous allocations and prices, given initial conditions for predetermined variables and stationary stochastic processes for exogenous variables, that satisfy: (i) the optimality conditions of households and firms; (ii) market clearing in all domestic markets; (iii) the government's budget constraint and its transversality conditions (as discussed in Appendix A); and (iv) the determination of the interest rate in dollars (7). Appendix B.2 lists all equilibrium conditions, while Section 3 discusses the conditions for local existence and uniqueness. Below we discuss several equilibrium features that will prove useful for the analysis presented in the following sections.

By forward iteration of the government's budget constraint (3) in the CPI-based setup, we obtain (see Appendix A for details)

$$\left[\frac{(1-\Omega)}{\pi_t} + \frac{rer_t}{\overline{rer}}\frac{\Omega}{\pi_t^*}\right] d_{t-1} = E_t\left\{\sum_{j=0}^{\infty} \frac{\tau_{t+j}}{rr_{t,t+j}}\right\} + h.o.t., \tag{8}$$

where $d_t \equiv d_t^G + \overline{rer} d_t^{G*}$ is total government debt (in domestic consumption units), the real discount rate is $rr_{t,t+J} \equiv \prod_{j=0}^{J-1}(R_{t+j}/\pi_{t+1+j})$, while $h.o.t.$ includes terms (related to deviations from perfect-foresight/complete-markets non-arbitrage conditions.) that are zero under certainty and thus vanish

the currency composition of debt. These issues, while potentially relevant, are beyond the scope of the paper, for reasons we discuss in the concluding section.

up to first order. [4]

Before analyzing the implications of this debt-valuation equation, we briefly discuss the transversality conditions (TVC) behind (8), described at length in Appendix A. For households, TVC follow from no-Ponzi conditions imposed separately for domestic- and foreign-currency assets, as well as from optimality. For the government, the corresponding transversality condition does not arise from optimization but must be imposed as a policy requirement. In closed-economy representative-agent models, the government's TVC coincides with the household's and therefore holds automatically in equilibrium. In contrast, a small and open economy where foreign agents may hold government bonds requires a distinct government's TVC for foreign-currency debt. This amounts to assuming "no-surplus" policies (Daniel, 2001), ruling out schemes that waste resources asymptotically. It turns out that this assumption is a mirror image of requiring balance-of-payments sustainability.

Going back to equation (8), in the absence of dollar-denominated debt ($\Omega = 0$), it is the familiar valuation equation from closed-economy FTPL models. For a given path of the real rate, a change in primary surpluses τ (either contemporaneous or expected) that is not compensated by an offsetting change at some other time (thus changing the net-present value on the right-hand side) needs to be met by an opposite-sign change in π_t on the left-hand side (as d_{t-1} is predetermined). If prices are fully flexible, the real rate is not affected by monetary policy, and current prices are pinned down by fiscal policy alone (monetary policy still determines *expected* inflation). Under sticky prices, an additional channel emerges, as monetary policy affects the real rate. Nevertheless, the degree of accommodation of primary surpluses is key, as governed by the parameter ϕ_T in (4).

As analyzed below, the results from closed economy models are qualitatively similar in this open economy setup when all government debt is denominated in pesos, $\Omega = 0$, although here we can also explore the exchange-rate consequences. When $\Omega > 0$, several differences arise. For given values of d_{t-1} and rer_t, a change on the right-hand side of (8), either through primary surpluses or the real rate, requires a larger absolute-value change in π_t on the left-hand side. Thus, we would expect inflation to become more volatile as the share of dollar-denominated debt increases. This is indeed the case as long as Ω is not too large: we will describe how dynamics differ if the share of debt denominated in foreign currency is sufficiently high.

Other relevant differences arise from the real exchange rate (rer_t) on the left-hand side of (8). First, any shock that induces a real depreciation requires (for a given path of primary surpluses and the real rate) an increase in π_t. Of course, the same shock may also induce a direct effect on prices (both traded and non-traded), and the final effect will depend on the policy configuration as well as the degree of price stickiness. Importantly, this channel is absent in the related literature, as most open-economy FTPL papers rely on one-good models.

In addition, if $\Omega > 0$, a smaller net-present-value of primary surpluses can be accommodated not only by increasing inflation but also by real appreciation (or a combination of both).Therefore, the co-movement between rer_t and π_t may be influenced by FTPL considerations (as does the link between the nominal depreciation, π_t^S, and inflation). In particular, the sign of this correlation is not clear ex ante and is likely to depend on the type of shock affecting the economy. This creates a role for fiscal policy as a determinant of the real exchange rate, different from other channels present in standard models.

To analyze the exchange rate dynamics, note that in the linearized version of the model, the un-

[4] Under the NT-based setup, an analogous expression can be derived, replacing rer with q, and π with π^N.

covered interest rate parity (UIP) holds:

$$\widehat{R}_t = \widehat{R}_t^* + E_t \left\{ \widehat{\pi}_{t+1}^S \right\}. \tag{9}$$

Using the real-exchange-rate definition and iterating forward, in a stationary equilibrium we obtain

$$\widehat{\pi}_t^S = \widehat{\pi}_t - \widehat{\pi}_t^* - \widehat{rer}_{t-1} + \sum_{j=0}^{\infty} E_t \left\{ (\widehat{R}_{t+j}^* - \widehat{\pi}_{t+1+j}^*) - (\widehat{R}_{t+j} - \widehat{\pi}_{t+1+j}) \right\}, \tag{10}$$

As can be seen, all else equal, an increase in the monetary policy rate $\widehat{R}$ (either current or expected) generates a nominal appreciation. The final effect, however, is determined by the endogenous response of the entire inflation path. When it falls, the direct effect is reinforced, and the nominal exchange rate unequivocally appreciates. Conversely, if the inflation path increases, the final effect on $\widehat{\pi}_t^S$ is ambiguous. As we will analyze, whether fiscal policy is Ricardian or not, as well as the share of government debt denominated in dollars, is key to determining the final effect.

To sharpen intuition and obtain cleaner results, we additionally assume $\sigma\eta = 1$ (i.e., equality of the intra- and inter-temporal elasticities of substitution, η and $1/\sigma$, respectively). In tandem with a closing device specified as in (7), c_t^T, d_t^*, and R_t^* are determined solely by the exogenous variables π_t^*, R_t^W, and y_t^T, with no influence from other domestic variables, particularly monetary and fiscal policies (see Appendix B.2.1).

As shown in Appendix C.1, assuming zero steady-state inflation (both domestic and foreign), the log-linearized equilibrium conditions under the NT-based policy setup can be reduced to

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa} \left(\eta \widehat{q}_t + \widehat{c}_t^T \right), \tag{11}$$

$$\widehat{R}_t = \widehat{R}_t^* + E_t \left\{ \widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^* \right\}, \tag{12}$$

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R, \tag{13}$$

$$(1 - \phi_T) \left[\widehat{d}_{t-1} - (1 - \Omega) \widehat{\pi}_t^N + \Omega (\widehat{q}_t - \widehat{\pi}_t^*) \right] = \beta \left[\widehat{d}_t - (1 - \Omega)(\widehat{R}_t) + \Omega (\widehat{q}_t - \widehat{R}_t^*) \right] + \widehat{u}_t^\tau. \tag{14}$$

Equation (11) is the New-Keynesian Phillips curve for non-traded inflation, which, in equilibrium, is driven by movements in $\widehat{q}_t$ and $\widehat{c}_t^T$ (the relevant determinants of non-traded demand); where $\widetilde{\kappa}$ is a function of parameters $\beta, \theta_N, \alpha, \epsilon_N, \sigma, \varphi$. In turn, equation (12) is the UIP condition, expressed in terms of the relative price $\widehat{q}_t$, while (13) is a Taylor rule that reacts only to non-traded inflation. Finally, equation (14) is the combination of the government's budget constraint and the rules for both the currency composition of debt and lump-sum taxes. These equations characterize the dynamics of $\widehat{\pi}_t^N$, $\widehat{q}_t$, $\widehat{d}_t$ and $\widehat{R}_t$, given the exogenous external variables $\widehat{R}_t^W$, $\widehat{y}_t^T$ and $\widehat{\pi}_t^*$ (recall that, under $\sigma\eta = 1$, $\widehat{R}_t^*$ and $\widehat{c}_t^T$ are driven solely by these external variables) and policy shocks $\widehat{u}_t^R$ and $\widehat{u}_t^\tau$.

Notice that when $\Omega = 0$, and in the absence of external shocks, this system resembles that of a closed-economy model, with $\widehat{\pi}_t^N$ and $\widehat{q}_t$ playing the roles of total inflation and aggregate demand, respectively, in the closed-economy counterpart. This implies that, as long as $\Omega = 0$, results from the closed-economy literature hold here as well, reinterpreting them in terms of non-traded inflation and the relative price/real exchange rate. If we also want to understand the behavior of the nominal exchange rate, based on the definition of q_t after log-linearization, we have $\widehat{\pi}_t^S = \widehat{q}_t - \widehat{q}_{t-1} - \widehat{\pi}_t^N + \widehat{\pi}_t^*$. This equivalence between closed- and open-economy setups may help explain why fewer papers analyze the FTPL in an open-economy context. However, to the extent that a non-trivial fraction of government debt is denominated in foreign currency, or if one is interested in the propagation of external shocks, our study offers novel insights.

To conclude the description of the model, Table 1 shows the baseline calibration. Parameters β, σ, φ, η, ω, α, ϵ_N, θ_N and ψ are taken from Schmitt-Grohé & Uribe (2017, Ch. 9.16). In turn, $\overline{R}^*$, $\overline{y}^T$, $\overline{d}^*$, χ, and $\overline{\tau}$ are set endogenously in steady state to match a null country premium, a relative size of non-tradable GDP of 57% (as implicit in the calibration of reference), a real exchange rate normalized to one, as well as shares of primary surplus and trade balance to GDP of 5%.[5] Finally, parameters related to monetary and fiscal policy (ϕ_π, ϕ_T, and Ω) will be set to specific values depending on the particular exercise we implement.

Table 1: Baseline Calibration

β	σ	φ	η	ω	α	ϵ_N	θ_N	ψ	$\overline{\pi}$	$\overline{\pi}^*$	$\frac{\overline{\tau}}{\overline{gdp}}$	$\frac{\overline{tb}}{\overline{gdp}}$	$\frac{\overline{y}^N}{\overline{gdp}}$	$\overline{rer}$
0.9694	2	0.5	0.5	0.6	0.75	6	0.7	0.000034	1	1	0.05	0.05	0.57	1

3 Existence and Uniqueness

In this section, we explore the determination of a locally-stationary equilibrium. We first provide an intuitive characterization based on the NT policy setup. The system (11)-(14), after eliminating the policy rate, contains one predetermined/state variable ($\widehat{d}_t$) and two non-predetermined/jumping variables ($\widehat{q}_t, \widehat{\pi}_t^N$). Thus, the existence of a unique equilibrium requires only one stable eigenvalue. Notice that government debt $\widehat{d}_t$ only features in equation (14), which can be written as

$$(1-\phi_T)\widehat{d}_{t-1} = \beta\widehat{d}_t + \widehat{o}_t,$$

where $\widehat{o}_t$ collects all other terms. Thus, the ratio $(1-\phi_T)/\beta \geq 0$ is an eigenvalue of the system. As long as $\widehat{o}_t$ is stationary, $\widehat{d}_t$ is non-explosive if this eigenvalue is less than one, which occurs if ϕ_T is large enough ($\phi_T > 1-\beta$). However, it might be the case that a continuum of stationary solutions exists, depending on the Taylor rule parameter ϕ_π. Alternatively, for a relatively small ϕ_T (i.e. $\phi_T < 1-\beta$), $\widehat{d}_t$ could still be stationary if the endogenous behavior of variables in $\widehat{o}_t$ generates another stable eigenvalue that offsets the explosive dynamics that $\widehat{d}_t$ would otherwise display. This also depends on ϕ_π; as long as Ω is different from a knife-edge case detailed below.

As it turns out, this intuition also carries over to the CPI-based policy setup. The following proposition, proven in appendix C.2 for the NT-policy setup and in appendix D for the CPI-based setup, provides a formal characterization:

Proposition 1 *Under both the NT- or the CPI-based policy setups, the necessary conditions characterizing existence and uniqueness of the linearized model are:*

- *Unique stable equilibrium: if either $\phi_\pi > 1$ and $\phi_T > 1-\beta$ (active monetary, passive fiscal), or $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary).*
- *No stable equilibrium: if $\phi_\pi > 1$ and $\phi_T < 1-\beta$ (active fiscal and monetary).*

[5]Considering a time period of a quarter, a ratio of primary surplus to GDP of 5% implies a ratio of debt to *quarterly* GDP in steady state equal to $5\%/(1-\beta) \approx 163\%$, equivalent to a share of debt to *annual* GDP of $163\%/4 = 40.75\%$. According to the IMF's Global Debt Database (`https://www.imf.org/external/datamapper/GG_DEBT_GDP@GDD/`), 40% is the 1-third percentile of General Government Debt's cross-country distribution for 2024; implying a conservative calibration.

– *A continuum of stable equilibria: if $\phi_\pi < 1$ and $\phi_T > 1 - \beta$ (passive fiscal and monetary).*

These are also sufficient, except in the case with $\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary), where uniqueness also requires $\Omega \neq \Omega_{cut,1}$ (defined in the appendix), with $\Omega_{cut,1} \in (0,1)$ if $\theta_N \in (0,1)$, while $\Omega_{cut,1} = 1$ if $\theta_N = 0$. If this condition fails, there is no stable equilibrium.

In line with the previous intuition, determinacy of the linearized model arises either with $\phi_T < 1 - \beta$ and $\phi_\pi < 1$ (active fiscal, passive monetary) or $\phi_T > 1 - \beta$ and $\phi_\pi > 1$ (passive fiscal, active monetary). Instead, if $\phi_T < 1 - \beta$ and $\phi_\pi > 1$ (both active), there are no equilibria, while $\phi_T > 1 - \beta$ and $\phi_\pi < 1$ (both passive) generate multiplicity.

The share Ω can potentially play a role only in the case of active fiscal and passive monetary policies ($\phi_T < 1 - \beta$, $\phi_\pi < 1$). If prices are flexible ($\theta_N = 0$) and debt is fully dollarized ($\Omega = 1$), there is no stable equilibrium with this policy mix. When $\phi_T < 1 - \beta$, government debt displays explosive behavior that monetary policy cannot compensate: under flexible prices, the real exchange rate (the only means of affecting the value of outstanding debt if $\Omega = 1$) is independent of monetary policy.

If prices are sticky, there is a knife-edge value for Ω, strictly between zero and one, for which there is no stable equilibrium either. With $\Omega \in (0,1)$, debt can be diluted through either inflation or real appreciation. As we will discuss in detail in the next section, the sign of the dynamics is determined by which of the two means of adjusting the outstanding value of debt dominates. This, in turn, hinges on how Ω compares with $\Omega_{cut,1}$. As responses have different signs, increasing in absolute value on each side of this threshold, a stationary equilibrium ceases to exist when $\Omega = \Omega_{cut,1}$.

Overall, the share of debt denominated in dollars is largely irrelevant in determining the conditions under which an equilibrium exists and is unique. However, in the following sections we show that, provided determinacy, the dynamics are indeed significantly altered by the currency composition of debt in cases where fiscal policy is Non-Ricardian/active.

4 The Monetary Transmission Mechanism

This section analyzes the dynamics generated by an i.i.d. monetary shock, u_t^R in equation (6). We begin by studying the case in which government debt is fully denominated in pesos ($\Omega = 0$), then discussing the role of its currency composition ($\Omega \in (0,1]$). In both cases, we first present the analytical results using the NT-based policy assumptions, followed by numerical responses based on the CPI setup and baseline parameterization, analyzing the intuition behind the results.

4.1 Only domestic currency debt

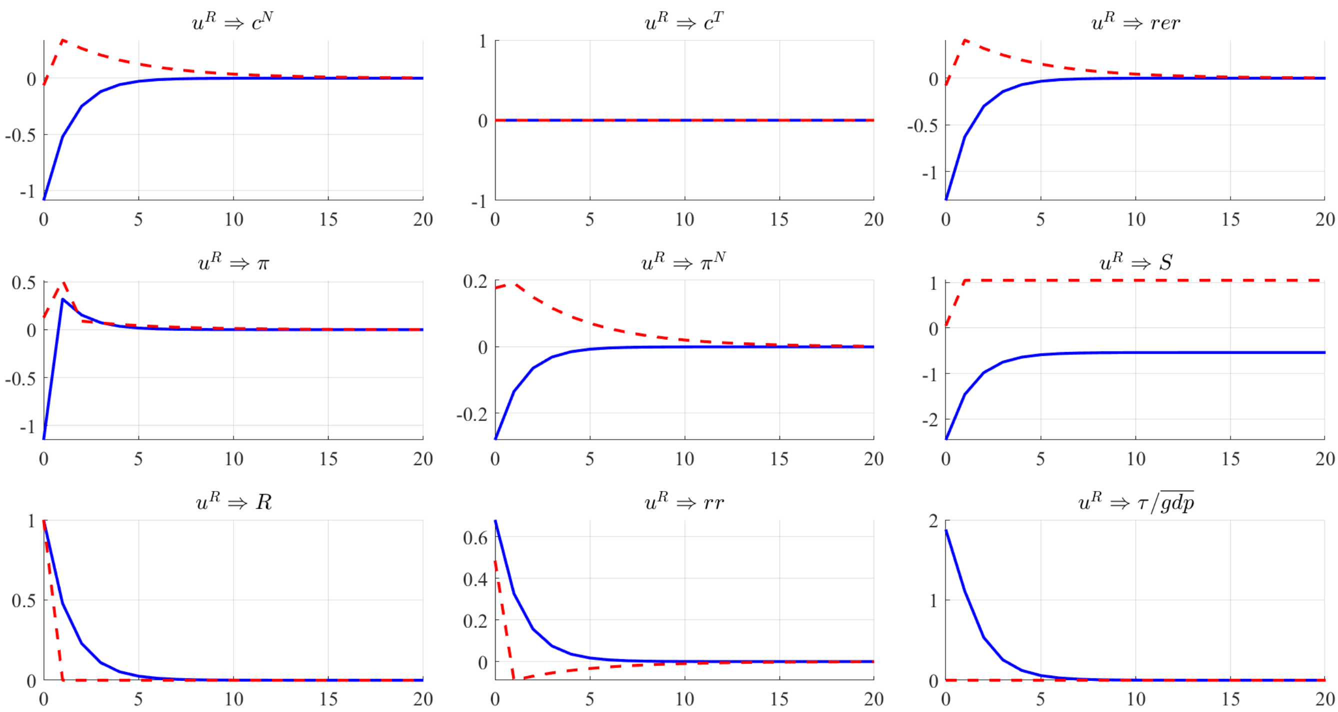
In the NT-based setup, we can analytically characterize the responses of the relevant variables at the moment the shock hits in two polar cases: active monetary and passive fiscal, with $\phi_\pi > 1$ and $\phi_T > 1 - \beta$, and active fiscal and passive monetary, with $\phi_\pi = \phi_T = 0$. These results are derived in Appendix C.3.1 and C.3.2, summarized by the following propositions:

Proposition 2 *Under the NT-based policy setup with $\phi_\pi > 1$ (active monetary) and $\phi_T > 1 - \beta$ (passive fiscal), an i.i.d. positive shock to u_t^R induces a contemporaneous fall in non-traded inflation $\hat{\pi}_t^N$, the relative price of tradables $\hat{q}_t$ and non-traded output $\hat{y}_t^N$, as well as a nominal appreciation ($\hat{\pi}_t^S$ falls). This holds for any value of $\Omega \in [0,1]$.*

Proposition 3 *Under the NT-based policy setup with $\phi_\pi = 0$ (passive monetary), $\phi_T = 0$ (active fiscal) and $\Omega = 0$ (only domestic-currency debt), an i.i.d. positive shock to u_t^R leads to a contemporaneous increase in* *non-traded inflation $\widehat{\pi}_t^N$ and a nominal depreciation ($\widehat{\pi}_t^S$ increases). The relative price of tradables $\widehat{q}_t$ falls if $\widetilde{\kappa}\eta + \beta > 1$, otherwise it increases after the shock.*[6] *Non-traded output $\widehat{y}_t^N$ follows the same pattern as $\widehat{q}_t$.*

The cases analyzed in these propositions are also shown for the CPI-setup and baseline calibration in Figure 1. The blue lines display the case with active monetary policy (with the usual parameter for a Taylor rule of $\phi_\pi = 1.5$) and a fully Ricardian/passive fiscal policy ($\phi_T = 0$). From the household's perspective, the shock induces (*ceteris paribus*, given the same prices) an increase in the real interest rate. This leads to both inter-temporal substitution and negative wealth effects, the latter induced by the fall in the net present value of after-tax real and financial income. Both tend to decrease current aggregate consumption.

Figure 1: Responses to a monetary shock, Ricardian vs. Non-Ricardian.



Notes: non-traded consumption (c^N, equal to output y^N), traded consumption (c^T), real exchange rate (rer), total inflation (π), non-traded inflation (π^N), nominal exchange rate (S), monetary-policy rate (R), ex-ante real interest rate (rr), and lump-sum taxes as a share of steady-state GDP ($\tau/\overline{gdp}$). The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue lines correspond to the case with $\phi_\pi = 1.5$ and $\phi_T = 1$, while dashed-red lines assume $\phi_\pi = \phi_T = 0$; all with $\Omega = 0$. All responses are in percentage-points deviations relative to the steady state.

As analyzed in Caramp & Silva (2023) for a closed economy setup, the size of the wealth effect heavily depends on the fiscal policy response to the shock. From the perspective of the lifetime government budget constraint (8), the increase in the real rate induces a reduction in the net present value of primary surpluses. If fiscal policy is Ricardian ($\phi_T = 1$), lump-sum taxes need to increase to counteract this effect. In turn, higher taxes induce an even deeper negative income effect for households, further decreasing desired consumption. According to Caramp & Silva (2023), this extra wealth effect can account for almost the totality of the contraction in consumption induced by the higher policy rate in closed-economy models.

In the open economy, the required change in taxes when $\phi_T = 1$ is also present; as shown in the

[6] Under our baseline calibration, $\widetilde{\kappa}\eta + \beta > 1$.

figure. This translates into a reduction in desired demand for both traded and non-traded goods (provided homothetic preferences). The contraction in c^N induces a fall in non-traded inflation (π^N). In turn, by (10), the increase in the policy rate induces a nominal appreciation, as long as it is not compensated by an increase in inflation (indeed, π^N falls). Overall, *rer* appreciates. Instead, c^T does not move in equilibrium under $\sigma\eta = 1$.

After the initial period, the nominal appreciation is partially offset, as the UIP condition (9) requires a depreciation going forward if R increases today (i.e., the shock induces exchange-rate overshooting).[7] Overall, the shock contracts activity in the non-traded sector and reduces both traded and non-traded inflation.

In turn, as anticipated by Proposition 3, dynamics are quite different if fiscal policy is Non-Ricardian/active ($\phi_T = 0$) and the interest rate is not responsive to inflation ($\phi_\pi = 0$), as shown by the dashed-red lines in Figure 1. In this case, while the real rate increases when the shock hits, lump-sum taxes remain constant. This has two important consequences. First, the negative wealth effect brought about by a Ricardian policy (through tax increases, as previously discussed) is absent; therefore, c^N only marginally drops at the moment the shock is realized. This extends the result in Caramp & Silva (2023) to this small and open economy setup.

Second, by the FTPL mechanics, the price level needs to increase to satisfy the government lifetime constraint equation (8), which leads to an increase in both non-traded prices and the nominal exchange rate. In terms of S, the initial-period response is only marginally positive. But we also know from UIP (9) that further depreciation is expected if R increases today, so the new higher level for S is achieved with a delay; eliminating the overshooting dynamics. As discussed in the introduction, this could rationalize the empirical finding in the literature that, for emerging countries, an increase in the policy rate tends to be followed by a nominal depreciation instead of an appreciation.

In terms of π^N, there are two channels at play. On the one hand, the fall in c^N pushes non-traded prices downward; though this channel should be small, as the negative wealth effect induced by taxes is absent. On the other hand, the FTPL channel requires prices to increase. This would materialize instantaneously if prices were fully flexible. Under nominal rigidities, this builds up over time; leading to additional non-traded inflation in the future. Therefore, from the perspective of the New-Keynesian Phillips curve, the initial contraction in demand for non-tradables is compensated by the forward-looking channel that anticipates higher future inflation. As a result, π^N rises from the moment the shock hits, with an additional increase in the subsequent period.

These dynamics for π^N and S also induce a different behavior for the real exchange rate: it initially decreases and, in the following periods, increases above the steady state, as S jumps even further. This leads to an additional substitution effect in favor of c^N (expenditure switching), as traded goods become relatively more expensive if *rer* increases.[8]

The dynamics of c^N can also be understood from the behavior of the ex-ante real rate *rr*. In the Ricardian case (solid-blue line), the real-rate path converges back to the steady state monotonically from above, explaining why consumption displays a similar path back to the steady state with the opposite sign (due to intertemporal substitution). In turn, in the Non-Ricardian case (dashed-red line), the real rate falls below its steady state value after the initial increase due to the expected higher

[7] This also explains why, even though the shock is i.i.d., the policy rate slowly converges back to the steady state and not instantaneously. Recall that the Taylor rule (6) for the CPI-based setup is specified in terms of π, not just π^N. Thus, the overshooting behavior of S generates total inflation that is above the steady state even after the shock materializes, leading to analogous dynamics for R in the periods following the realization of the shock. This adds an extra push downwards to c^N (via the same channels previously discussed), and the effect of the shock does not simply vanish after the initial period (as would be the case in the closed-economy version).

[8] Again, c^T is not affected in equilibrium, as the several effects offset each other if $\sigma\eta = 1$.

inflation, converging to the steady state from below. This is consistent with the dynamics displayed by c^N in the dashed-red lines. Of course, explanations based on *rer* or *rr* are two sides of the same coin, as they are linked in equilibrium by the UIP condition expressed in real terms.

In the Appendix F.1, we explore the role of the monetary response parameter ϕ_π for a given fiscal regime when $\Omega = 0$. In the Ricardian case ($\phi_T = 1$), only marginal differences arise when comparing the baseline $\phi_\pi = 1.5$ to a value just above the determinacy region ($\phi_\pi = 1.01$), with responses being relatively muted for lower values of ϕ_π. Instead, in the Non-Ricardian regime ($\phi_T = 0$), a monetary response parameter marginally below that required for determinacy ($\phi_\pi = 0.99$) significantly alters the dynamics. In particular, the policy rate remains above the steady state for much longer, despite the shock being i.i.d. This, in turn, implies a persistently higher path for the real rate which, through the FTPL channel, induces additional (and more persistent) non-traded inflation, while the nominal depreciation continues even after the initial periods. At the same time, the initial response of c^N and *rer* is negative, explained by the more contractionary real-rate path. However, their convergence back to the steady state, albeit much slower, is still from above.

Overall, under this alternative policy configuration, an increase in the nominal rate leads to an increase in inflation and an expansion in output, similar to the effects shown, for instance, by Cochrane (2023) in a closed-economy setting. Our analysis extends these previous results to open economies, leading to a better understanding of the dynamics of both the nominal and real exchange rates.

4.2 The role of foreign currency debt

We next turn to cases with a positive share of dollar-denominated government debt. The following proposition is developed in Appendix C.3.2:

Proposition 4 *Under the NT-based policy setup with $\phi_\pi = \phi_T = 0$ (passive monetary, active fiscal), following an i.i.d. positive shock to u_t^R, the sign of the contemporaneous response of the variables of interest depends on the value of Ω relative to two cutoff values ($\Omega_{\text{cut},1} > \Omega_{\text{cut},2}$), as detailed in columns one to five in Table 2. Within each range defined by these cutoffs, a marginally larger value of Ω increases these responses, as seen in the last column of Table 2. Finally, if $\Omega = \Omega_{\text{cut},1}$ there is no stationary solution.*

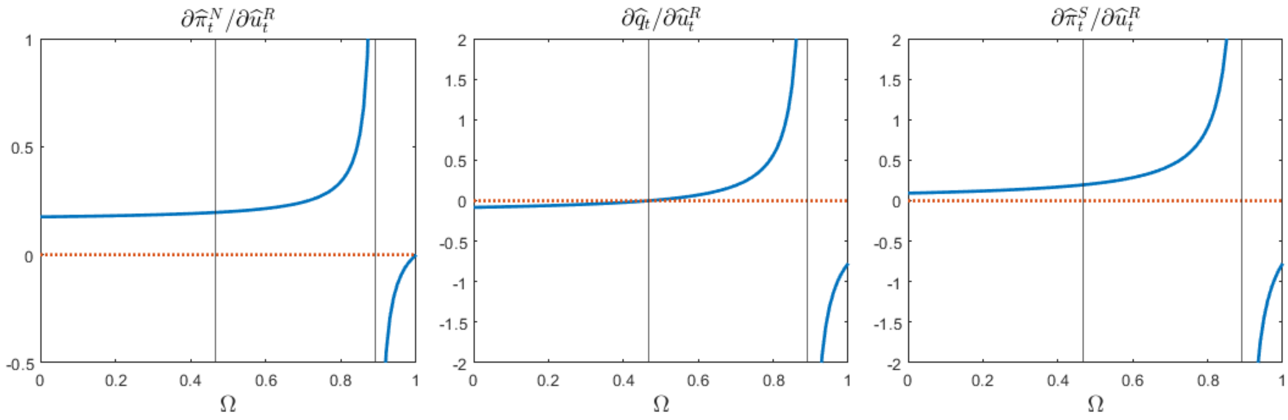
Table 2: The impact of monetary shocks in the NT-based setup, with $\phi_T = \phi_\pi = 0$, and the role of Ω

	Values of Ω				Derivative
Response of	$0 \leq \Omega < \Omega_{\text{cut},2}$	$\Omega_{\text{cut},2} < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega < 1$	$\Omega = 1$	w.r.t. Ω
$\widehat{\pi}_t^N$	+	+	-	0	+
$\widehat{q}_t$	-	+	-	-	+
$\widehat{\pi}_t^S$	+	+	-	-	+
$\widehat{y}_t^N$	-	+	-	-	+

Notes: Columns two to five display the sign of the contemporaneous response of the variable indicated in the first column after a positive shock to $\widehat{u}_t^R$, for different ranges of Ω. The last column indicates how the response of that specific variable is modified for a marginally larger value of Ω. The table assumes $\widetilde{\kappa}\eta + \beta > 1$, which is only relevant to determine the effect on $\widehat{q}_t$ and $\widehat{y}_t^N$ in the case with $0 \leq \Omega < \Omega_{\text{cut},2}$ (if instead $\widetilde{\kappa}\eta + \beta \leq 1$, these signs are the opposite, see Appendix C.3.2).

Figure 2 helps visualize these results by plotting the contemporaneous effect of $\widehat{u}_t^R$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$, and $\widehat{\pi}_t^S$ for different values of Ω, using the baseline parameterization. As can be seen, for a sufficiently low ratio of dollar denominated debt ($\Omega < \Omega_{\text{cut},1}$), the contemporaneous responses of the four variables display qualitatively the same sign as those analyzed under $\Omega = 0$. As Ω increases, the initial responses of non-traded inflation and nominal depreciation become larger, provided that $\Omega < \Omega_{\text{cut},1}$. The same can be observed for the relative price: its response is less negative for larger values of Ω, and it eventually turns positive in cases with $\Omega_{\text{cut},2} < \Omega < \Omega_{\text{cut},1}$.

Figure 2: The contemporaneous effect of $\widehat{u}_t^R$ under the N-policy setup depending on Ω



Note: Each line plots the contemporaneous effect of an **increase** in $\widehat{u}_t^R$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$ and $\widehat{\pi}_t^S$, respectively, as a function of Ω (recall $\widehat{y}_t^N$ is proportional to $\widehat{q}_t$). Vertical lines indicate the cutoff values $\Omega_{\text{cut},1}$ and $\Omega_{\text{cut},2}$ (recall $\Omega_{\text{cut},2} < \Omega_{\text{cut},1}$). These are computed under the baseline parametrization, which generates $\Omega_{\text{cut},1} = 0.8921$ and $\Omega_{\text{cut},2} = 0.4663$.

These results are in line with the discussion of the FTPL equation (8) in section 2.5: whenever there is a need to compensate for a change in the net-present value of primary surpluses, a smaller fraction of debt denominated in pesos requires a larger rise in inflation to dilute its value. This also pushes the exchange rate upward by the UIP condition (10). As a result, the initial fall in the relative price and non-traded output observed when $\Omega = 0$ is compensated, and it may even be positive for relatively larger shares of dollar-denominated debt. In turn, according to the FTPL equation (8), this real-exchange-rate behavior increases the burden of outstanding dollar debt, which requires even more non-traded inflation under passive fiscal policy. This reinforcement explains why the responses in Figure 2 are increasing in Ω, as it approaches $\Omega_{cut,1}$ from the left.

However, this is not monotonic: if $\Omega > \Omega_{\text{cut},1}$, all initial responses flip their signs.[9] This can be intuitively understood by first considering the case of $\Omega = 1$. From the left-hand side of the FTPL equation (8), a real appreciation is required to compensate for the drop in the net-present value of surpluses triggered by the shock. As non-traded prices are sticky, this does not materialize through an increase in π^N; an initial nominal appreciation is induced. When $\Omega = 1$, the initial response of π^N is, in fact, exactly zero. However, for relatively smaller values of Ω, the real appreciation exerts downward pressure on non-traded inflation (as the real exchange rate is a demand shifter in the Phillips curve (11)). In turn, less inflation increases the burden of outstanding local-currency debt, requiring an even larger real appreciation for the FTPL equation to hold. This reinforcement is behind the responses in Figure 2 getting smaller as Ω approaches $\Omega_{\text{cut},1}$ from the right.

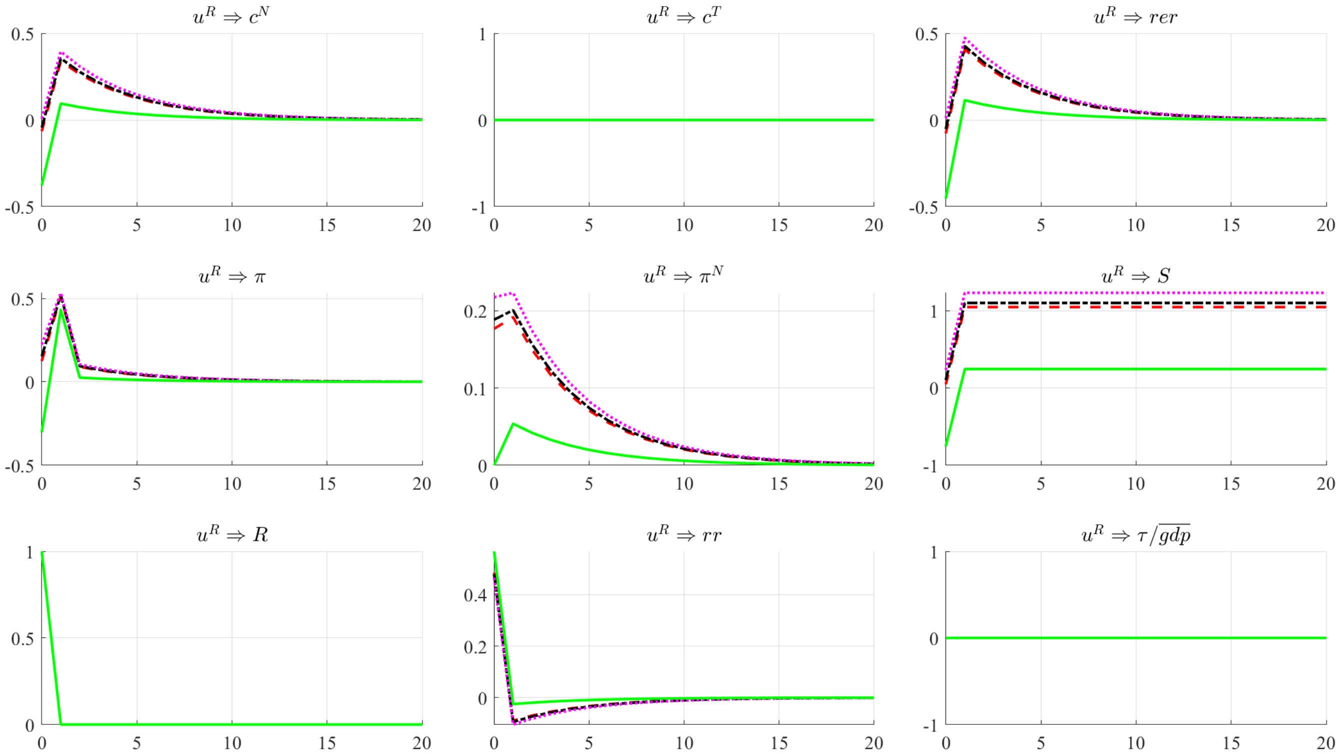
Overall, there are two forces behind how Ω affects the dynamics. On the one hand, a relatively smaller share of debt denominated in pesos induces a larger inflation reaction (non-traded in the NT-based setup, aggregate inflation in the CPI-based configuration), also inducing a rise in both exchange rates. These effects reinforce each other through the FTPL mechanism. On the other hand, for a relatively large Ω, the possibility of diluting debt through real appreciation takes center stage;

[9]The vertical asymptote in $\Omega = \Omega_{\text{cut},1}$ (i.e., the initial response tends to infinity when approaching $\Omega_{\text{cut},1}$ from the left and to minus infinity from the right) reflects that a stationary equilibrium does not exist in this knife-edge case.

requiring a fall in the nominal exchange rate, as non-traded prices are sticky. The real appreciation also pushes non-traded inflation downwards through the Phillips curve, reinforcing each other via the FTPL equation. As these two forces induce a different sign in the responses, and the reinforcement is larger when Ω is closer to $\Omega_{\text{cut},1}$, a stationary equilibrium fails to exist when $\Omega = \Omega_{\text{cut},1}$.

To conclude the analysis, Figure 3 compares the responses in the CPI-based policy setup for alternative values of the share of dollar-denominated debt.[10] All cases displayed assume $\phi_\pi = \phi_T = 0$, considering alternative values for Ω equal to 0, 0.5, 0.75, and 1. Except for the case of $\Omega = 1$, we can see that a higher share of dollar-denominated debt yields larger responses for both non-traded inflation and the nominal exchange rate at the moment the shock hits, in line with the results in Proposition 4 for cases with $\Omega < \Omega_{\text{cut},1}$. We can also see the real exchange rate and non-traded consumption increasing with larger shares of dollar-denominated debt, starting from initially negative values for relatively smaller values of Ω. The figure also allows us to observe that these differences are magnified in the periods following the impact of the shocks.

Figure 3: Responses to a monetary shock, Non-Ricardian, different values of Ω.



Notes: The figure is analogous to Figure 1, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.9$, and in solid-green $\Omega = 1$.

Additionally, we see that for $\Omega = 1$ the initial responses of both the nominal and real exchange rates (as well as non-trade activity) change sign, in line with Proposition 4. However, we see that in this version of the model, this change in sign only materializes during the impact's period. Afterwards, the responses have the same sign as those displayed for smaller values of Ω, albeit with a smaller magnitude, even smaller than in the case with $\Omega = 0$. Once again, these results emphasize the importance of accounting for the currency composition of debt in understanding the effects of

[10] While we are unable to find an algebraic expression for the cutoff values $\Omega_{\text{cut},1}, \Omega_{\text{cut},2}$ in the CPI-based policy setup, we can approximate them numerically to $\Omega_{\text{cut},1} = 0.945$ and $\Omega_{\text{cut},2} = 0.735$.

monetary policy innovations under active fiscal policy.

5 The Effects of Fiscal Shocks

We next turn to the dynamics generated by a fiscal shock, u^τ in the policy rule (4). Under a Ricardian/passive fiscal policy, a change in taxes today induces an opposite-sign modification in future taxes, such that the net-present value on the right-hand side of (8) is unaltered; therefore, it has zero effects. Instead, the shock induces non-trivial aggregate dynamics under non-Ricardian/active fiscal policy. For the NT-based policy, Appendix C.3.2 derives the following results:

Proposition 5 *Under the NT-based policy setup with $\phi_\pi = \phi_T = 0$ (passive monetary, active fiscal), following an i.i.d.* ***negative*** *shock to u_t^T, the sign of the contemporaneous response of the variables of interest depends on the value of Ω relative to the cutoff value $\Omega_{\text{cut},1}$, as detailed in Table 3. Moreover, a marginally larger value of Ω increases these responses, as seen in the last column of Table 3.*

Table 3: The impact of fiscal shocks in the NT-based setup, with $\phi_T = \phi_\pi = 0$, and the role of Ω

	Values of Ω		Derivative
Response of	$0 \leq \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq 1$	w.r.t. Ω
$\widehat{\pi}_t^N$	+	-	+
$\widehat{q}_t$	+	-	+
$\widehat{\pi}_t^S$	+	-	+
$\widehat{y}_t^N$	+	-	+

Notes: This is analogous to Table 2, for the case of a **negative** shock to $\widehat{u}_t^\tau$.

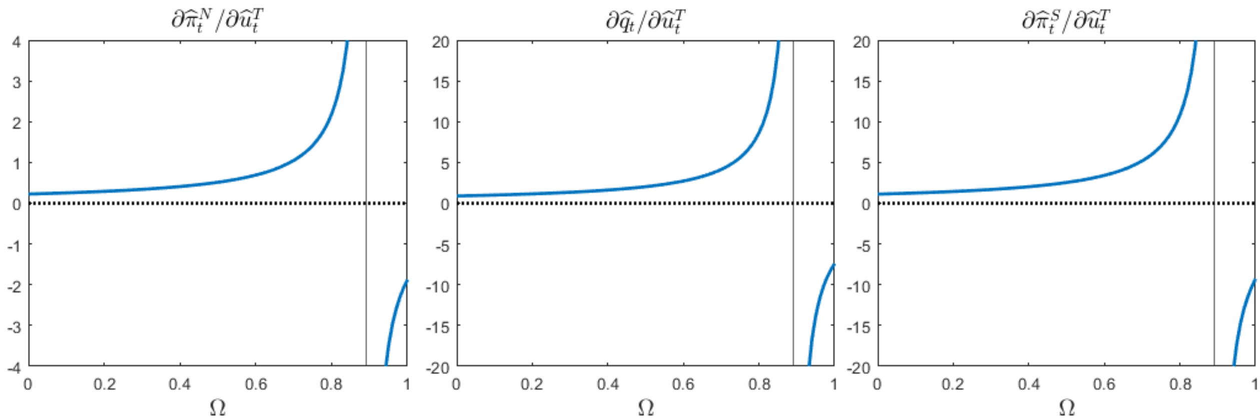
Figure 4 helps visualize these results by plotting the contemporaneous effect of $\widehat{u}_t^\tau$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$, and $\widehat{\pi}_t^S$ for different values of Ω, using the baseline parameterization. Beginning with the case of only peso-denominated debt ($\Omega = 0$), if the policy rate is fixed, the FTPL equation (8) indicates that a surprise reduction in primary surpluses requires either an increase in the price level today, an increase in expected inflation, or a combination of both. This is achieved by both a nominal depreciation and an increase in π^N, the latter being quantitatively smaller and spread over time due to sticky prices. As a result, a real depreciation materializes.

This real depreciation leads to an expansion in non-traded activity through intra-temporal substitution, as non-traded goods become relatively cheaper. Moreover, an additional expansionary channel arises from the positive wealth effect that the tax reduction generates. While the real interest rate is reduced (as expected inflation rises), this is not enough to compensate for the increase in the net present value of after-tax income. Overall, total output rises in equilibrium.

Turning to cases where a fraction of the debt is denominated in dollars, as long as $\Omega < \Omega_{\text{cut},1}$, the dynamics are magnified by a larger share Ω. Again, this can be explained by the additional inflation that is required to dilute the relatively smaller outstanding amount of peso-denominated debt. In such a process, the nominal depreciation and the increase in non-traded inflation reinforce each other: the real exchange rate depreciates, which puts upward pressure on π^N according to the Phillips curve, while the additional inflation also increases the nominal exchange rate through the UIP condition. This, in turn, explains the vertical asymptote in $\Omega = \Omega_{\text{cut},1}$.

Instead, when $\Omega > \Omega_{\text{cut},1}$, as we previously discussed, the other potential way of diluting debt obligations dominates: a real appreciation helps reduce the real burden of dollar denominated debt.

Figure 4: The contemporaneous effect of $\widehat{u}_t^T$ under the N-policy setup depending on Ω



Note: Each line plots the contemporaneous effect of a **drop** in $\widehat{u}_t^T$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$ and $\widehat{\pi}_t^S$, respectively, as a function of Ω (recall $\widehat{y}_t^N$ is proportional to $\widehat{q}_t$). Vertical lines indicate the cutoff value $\Omega_{\text{cut},1}$ ($\Omega_{\text{cut},2}$ is not relevant for the fiscal shock). This is computed under the baseline parametrization, which generates $\Omega_{\text{cut},1} = 0.8921$.

This triggers a nominal appreciation and a fall in non-traded inflation that, again, reinforce each other for values of Ω approaching $\Omega_{\text{cut},1}$ from above.

Figure (5) shows the effects of a fall in lump-sum taxes, normalized to represent 1% of steady-state GDP (with an autocorrelation of 0.7), for different values of Ω under the CPI-policy framework and the baseline parameterization. In all cases, the policy rate is kept fixed ($\phi_\pi = 0$) and $\phi_T = 0$.[11] The results for the initial period replicate those described in Proposition 5. Here we can also see that these initial effects are quite persistent, except for the behavior of the nominal exchange rate S, which directly jumps to its new higher level without further movements.

In summary, the currency composition of government debt is also relevant in shaping the equilibrium dynamics induced by fiscal shocks. In particular, a larger share of dollar-denominated debt magnifies the effects of both shocks (as long as it is not large enough). Moreover, in tandem with those related to monetary policy shocks, these results provide an interesting testable implication that future work can use to empirically analyze the FTPL channels from cross-country variation in the share of dollar-denominated government debt.

6 Extensions

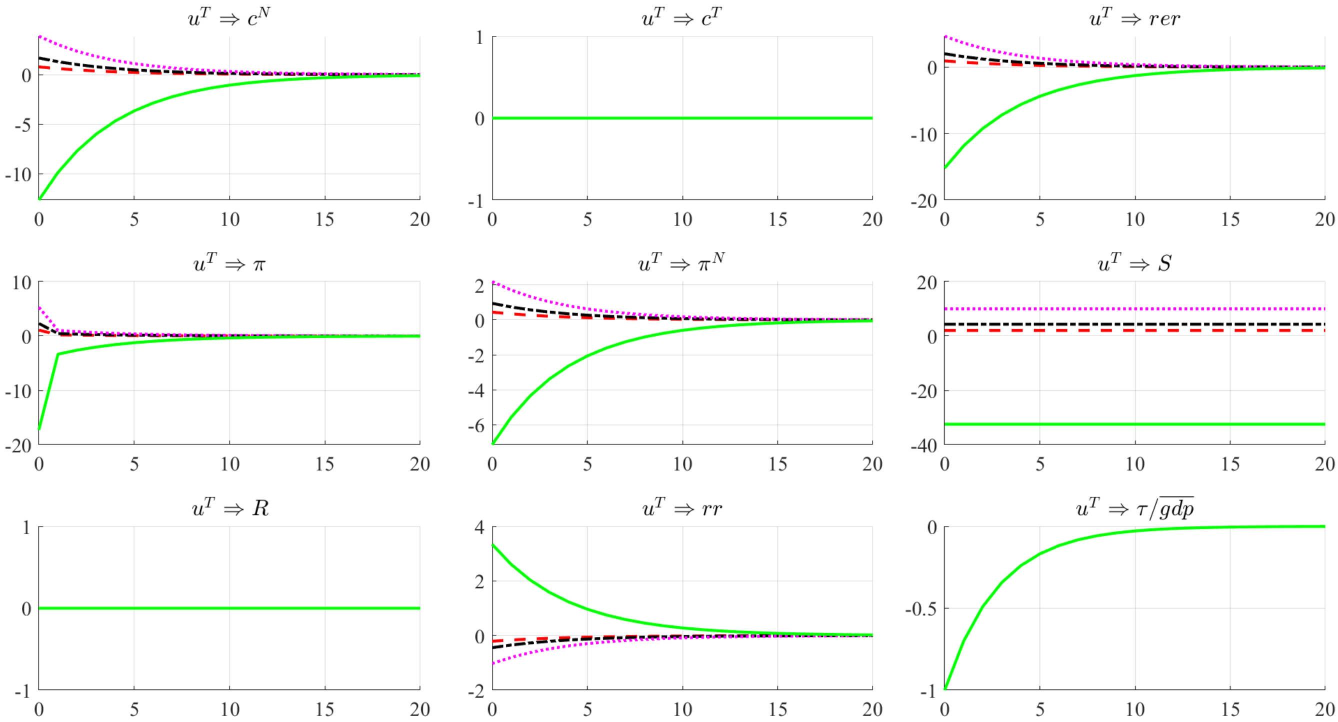
In this section, we first include the possibility of long-term debt. We then consider the role of Fisherian deflation. Finally, we study how the propagation of real shocks is affected by the policy setup. These are developed in detail in the Technical Appendix, while here we discuss the main lessons.

6.1 Long-term debt

How does the presence of long-term debt alter the results in the previous sections? We consider a setup following Woodford (2001): a domestic-currency bond promises to pay $x\delta^h$ per unit, h periods

[11] In the robustness appendix F.1, we see that if $\phi_T = 0$, but the monetary-response parameter is close to one, non-trivial differences arise; similar to those described before in the case of the monetary shock. In particular, if $\phi_\pi = 0.99$, the policy rate remains high for a longer period, increasing non-traded inflation and the nominal exchange rate even further, resulting in the policy rate rising even more persistently. At the same time, the initial increases in both c^N and rer are smaller than with $\phi_\pi = 0$, but its convergence back to the steady state is much slower.

Figure 5: Responses to a fisical shock, Non-Ricardian, different values of Ω.



Notes: The figure is analogous to Figure 3, except that it shows responses to a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.75$, and in solid-green $\Omega = 1$.

after it was issued; where δ governs the maturity of the bond and x scales the coupon.[12] As shown in Appendix E, the FTPL equation comparable to (8) is

$$\left[(x + \delta Z_t)\frac{(1-\Omega)}{\pi_t} + (x^* + \delta^* Z_t^*)\frac{rer_t}{\overline{rer}}\frac{\Omega}{\pi_t^*}\right]\widetilde{d}_{t-1} = E_t\left\{\sum_{j=0}^{\infty}\frac{\tau_{t+j}}{rr_{t,t+j}}\right\} + h.o.t., \tag{15}$$

where $\widetilde{d}_{t-1}$ denotes the outstanding stock of government debt in CPI units, while Z_t and Z_t^* are, respectively, the prices of domestic and foreign currency bonds.

As shown in the appendix, the main difference in this alternative setup relates to the propagation of monetary policy shocks whenever fiscal policy is active. Consider first the case of $\Omega = 0$. A positive shock to the policy rate has two effects. On one hand, as with one-period debt, it reduces the net-present value of primary surpluses (discounting channel). On the other hand, with long-term bonds, the shock also lowers the value of outstanding nominal debt by lowering its price Z_t (valuation effect). As both sides of (15) are reduced, the consequences for inflation hinge on which one dominates. If the discounting channel outweighs the valuation effect, qualitatively the results are similar to those in the short-term debt case (i.e., inflation must increase to make the FTPL equation hold), although the impact is quantitatively milder.[13] Otherwise, if the valuation effect dominates, inflation must fall to compensate; this materializes for large enough δ. These differences also arise in

[12]For foreign-currency bonds, the analogous parameters are x^* and δ^*.

[13]In the appendix, we show that this occurs as long as δ is relatively small. In the NT-policy setup and the baseline calibration, the threshold value is near 0.8; an average duration close to 4.4 quarters.

closed-economy setups (as shown, for instance, in Cochrane, 2023).

The nominal exchange rate follows a qualitatively similar pattern. Recall, from the iterated UIP condition (10), that $\widehat{\pi}_t^S$ depends not only negatively on the monetary policy rate but also positively on inflation's expected path. Thus, qualitatively, $\widehat{\pi}_t^S$ follows a similar pattern to $\widehat{\pi}_t$; however, quantitatively, the threshold value for δ to flip from depreciation to appreciation is smaller than that for inflation. Finally, the real exchange rate appreciates in the baseline calibration, and non-tradable output falls; an effect that intensifies for longer debt durations. Overall, the exchange-rate effect of a monetary tightening crucially depends on the domestic-currency debt duration: for countries with relatively shorter maturities, depreciation is likely to occur, while appreciation is expected with a longer maturity profile.

The fraction of debt denominated in foreign currency Ω interacts non-trivially with debt's duration δ. As previously discussed, when $\delta = 0$ the response of inflation is positive (and increasing) as long as Ω is below a certain threshold; otherwise, debt-dilution can also materialize through a real appreciation if Ω is large enough. In the appendix, we show that this result qualitatively extends to cases in which δ is relatively small, such that the aforementioned valuation effect is not as large. However, for longer durations, this additional channel dominates, so the relationship with Ω flips signs: when δ is large, a relatively small share of dollar-denominated debt induces a fall in inflation after a monetary tightening, while it increases after the same shock if Ω is relatively large. Moreover, as explored in the appendix, the dynamics of both the nominal and the real exchange rate also depend on the particular combination of δ and Ω. Once again, we conclude that the presence of dollar denominated debt alters the dynamics driven by FTPL considerations.

6.2 Fisherian deflation

As we have analyzed, in a non-Ricardian regime a monetary tightening might induce both inflation and an expansion in activity; as well as a real depreciation. At the same time, a large literature has emphasized that under financial frictions, coupled with currency mismatches, real depreciations can be contractionary. The main channel is Fisherian deflation: the real depreciation tightens credit costs through financial frictions, contracting aggregate demand, which in turn amplifies the initial depreciation, increases credit costs even further, and induces a larger contraction (see, for instance, the survey by Bianchi & Mendoza, 2020). To consider this possibility, albeit in a reduced-form manner, we change the equation for the external interest rate to

$$R_t^* = R_t^W \exp\left\{\psi\left(\frac{d_t^*}{y_t^Y + y_t^N/q_t} - \bar{d}^*\right)\right\}. \tag{16}$$

Here, the premium depends on foreign aggregate debt relative to GDP, measured in terms of tradables. Crucially, a real depreciation (a rise in q_t) increases the debt-to-GDP ratio, raising the foreign cost of borrowing. For this channel to be active, the coefficient ψ needs to be relatively large; under the baseline calibration ($\psi = 0.000034$), either the original specification or (16) delivers virtually identical dynamics. In such a case, the consumption of tradables is no longer independent of policy variables, even though we still have $\eta = 1/\sigma$.

Appendix F.2 reports the responses following both monetary and fiscal shocks, under alternative policy configurations, based on the CPI-based setup. Under an active monetary and passive fiscal setup, the real appreciation induced by the monetary tightening reduces the premium charged for external borrowing. This expansionary channel induces a further real appreciation and an increase in c^T. While this channel should also increase c^N, an additional mechanism is at play: the stronger real appreciation induces expenditure switching away from non-tradables. In equilibrium, the latter channel dominates, leading to a fall in c^N that is stronger the larger the elasticity ψ. Moreover, non-traded inflation falls even further, while the nominal appreciation is larger to allow for the required decrease in the real exchange rate.

Under an active fiscal and passive monetary setup, with $\Omega = 0$, the induced real depreciation triggers a contractionary Fisherian deflation, leading to a drop in c^T. While non-traded consumption is also affected by this contractionary channel, the expenditure switching effect compensates. In this case, non-traded activity is quantitatively similar to that in the case with a small value for ψ, although tradable absorption falls. Without an effect on non-traded demand, the additional depreciation is mainly generated by a larger increase in the nominal exchange rate.

In turn, with a positive share of dollar denominated debt, as long as the monetary shock induces a real depreciation (which, recall, materializes if Ω is not as large), the previously described dynamics are amplified. In particular, the expenditure switching further expands non-traded consumption (relative to the case with a small elasticity ψ), leading to a larger increase in non-traded inflation and an additional nominal depreciation. The figures in the appendix reporting the responses to a fiscal surprise yield conclusions similar to those for the monetary shock.

Overall, despite Fisherian deflation generating a contraction in tradable demand whenever fiscal policy is active, this is not strong enough to overturn the effect on non-traded consumption brought about by the FTPL mechanism, especially under dollar-denominated debt.

6.3 The effect of real shocks

This section compares the transmission of two expansionary real disturbances —a positive shock to traded output y^T and a decline in the world interest rate R^W— under alternative fiscal-monetary configurations and different currency compositions of public debt. These are described in detail in Appendix G. Under flexible prices, both shocks produce qualitatively similar real effects. A positive y^T shock triggers a positive wealth effect, raising consumption of both types of goods. Stronger demand for non-tradables increases their relative price, implying a real appreciation. A decline in R^W also generates a positive wealth effect in a net-debtor economy, as the cost of servicing external liabilities falls, along with intertemporal substitution toward present consumption. Non-traded demand also rises, again leading to a real appreciation. Overall, the flexible-price implications of both shocks are qualitatively similar: an expansion in activity and real appreciation.

When fiscal policy is Ricardian and there is an active Taylor rule, monetary policy reacts to inflation and shapes the path of the real interest rate and the nominal exchange rate via the UIP condition. Following a positive y^T shock, the central bank reduces the policy rate in response to declining inflation, reinforcing the expansion through a lower real rate and further nominal appreciation. In turn, following a fall in R^W, the domestic real rate declines due to both the external shock and monetary feedback. The expansion in non-traded consumption is therefore supported by intertemporal substitution and by the relative-price effect associated with the real appreciation. In both cases, the interaction between monetary policy and exchange-rate expectations determines the persistence of the appreciation and the dynamic profile of non-traded consumption. Fiscal policy only plays a stabilizing role through tax adjustments that ensure the intertemporal budget constraint holds.

Under Non-Ricardian fiscal policy with a constant policy rate, the adjustment mechanism changes. The FTPL's valuation equation must be satisfied through movements in prices and exchange rates. With $\Omega = 0$, however, the qualitative responses to both shocks remain expansionary and appreciatory, as in the Ricardian regime. The main differences are quantitative and stem from the absence of monetary feedback: if the policy rate does not adjust, the real rate falls by less, dampening intertemporal substitution relative to the active-monetary case. Nevertheless, both shocks still increase non-traded demand and generate real appreciation. In this sense, when debt is entirely domestic-currency-denominated, fiscal regime differences mainly influence the magnitude and persistence of responses rather than their direction.

The most important differences emerge when a fraction of public debt is denominated in foreign currency. Once $\Omega > 0$, exchange-rate movements alter the domestic value of outstanding liabilities, and valuation effects become central to the transmission mechanism. In the case of a positive y^T shock, the flexible-price benchmark implies real appreciation. When debt is partly dollar-denominated, this appreciation reduces the domestic-currency value of government liabilities. With fixed taxes, satisfying the valuation equation requires offsetting adjustments, either through a higher real rate or lower non-traded inflation that increases the real value of domestic-currency debt. As a result, the expansion in non-traded consumption may be dampened, and for z sufficiently large Ω, the equilibrium adjustment can even overturn the flexible-price outcome and induce a contraction in the non-traded sector despite the positive real shock.

A related but distinct mechanism operates after a decline in R^W. The flexible-price response again features real appreciation, which reduces the domestic value of foreign-currency debt. In contrast to the y^T shock, however, the fall in the world rate also directly lowers the relevant real discount rate in the government's intertemporal constraint, increasing the present value of primary surpluses. If $\Omega = 0$, this requires a fall in inflation. With $\Omega > 0$, these combined effects may require a further decline in inflation or a stronger real appreciation to make the FTPL condition hold. As the share of foreign-currency debt increases, valuation effects become stronger, and the appreciation can be amplified. For sufficiently large Ω, the need to reconcile exchange-rate movements with fiscal solvency can again generate non-standard dynamics, potentially attenuating or reversing the expansionary effects on the non-traded sector.

The comparison, therefore, shows that differences across fiscal regimes with $\Omega = 0$ mainly reflect the degree of monetary accommodation rather than fundamentally distinct mechanisms. However, once foreign-currency debt is introduced, exchange-rate movements feed back into the government's budget constraint. These valuation effects can substantially modify the strength of the expansion and, for sufficiently high Ω, even overturn the flexible-price dynamics. The currency composition of public debt thus remains the decisive factor determining whether expansionary real shocks propagate in a conventional manner or give rise to unconventional dynamics driven by FTPL channels.

7 Conclusions

This paper studies the implications of the Fiscal Theory of the Price Level in small and open economies. While the FTPL has been extensively analyzed in closed-economy settings, its open-economy implications remain comparatively underexplored. In particular, the existing literature largely abstracts from a defining feature of many economies, especially emerging markets: the substantial share of government debt denominated in foreign currency. We incorporate these considerations into a standard small open economy New Keynesian framework with traded and non-traded goods and foreign-currency-denominated public debt.

Our analysis shows that macroeconomic dynamics following both policy and real shocks depend not only on the monetary–fiscal mix but also critically on the currency composition of public debt. Under an active fiscal regime, a monetary tightening may generate a nominal depreciation rather than the conventional appreciation. Moreover, these effects are amplified as the share of foreign-currency debt increases—up to a threshold beyond which qualitative responses reverse. Debt composition therefore plays a central role in shaping exchange-rate and inflation dynamics under the FTPL in open economies.

These findings have important empirical implications. Cross-country heterogeneity provides a natural testing ground for FTPL predictions. However, interpreting such evidence through a theoretical framework that abstracts from the currency composition of public debt may lead to incorrect interpretations about the nature of fiscal–monetary interactions. A promising direction for future research is to study the cross-country propagation of well-identified shocks —such as policy surprises or common external disturbances— to assess whether variation in responses is consistent with the mechanisms highlighted here.

Throughout the paper, we treat the currency composition of public debt as exogenous and constant. This choice allows us to isolate how a given liability structure shapes equilibrium determination and policy transmission within a standard linearized FTPL framework, facilitating comparison with the existing literature. In practice, the currency denomination of sovereign debt reflects interactions between government policy and investor demand. Incorporating endogenous debt composition within an FTPL environment would allow for the joint study of the determination of liability structure and its macroeconomic consequences, an important avenue for future research.

Finally, the FTPL is likely to interact in non-trivial ways with the possibility of sovereign default, as both mechanisms affect the real burden of government liabilities. We abstract from default for three reasons. First, doing so allows for a clean comparison with the closed-economy FTPL literature and clarifies how its core insights extend to open-economy settings. Second, a no-default framework may remain empirically relevant for many —particularly advanced– economies where default risk is limited, but fiscal–monetary interactions remain central. Third, introducing default would substantially alter the computational solution of the model, complicating comparison with existing results. Exploring the joint linkages between inflation, exchange rates, and default risk within an FTPL framework is therefore an important direction for future research, especially given the central role of foreign-currency debt documented here.

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Fiscal Theory of the Price Level
in Small and Open Economies

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Supplementary Appendix

(Not for publication)

A Budget constraints and transversality conditions

Define $rr_{t,t+J}$ such that $rr_{t,t} \equiv 1$ and $rr_{t,t+J} \equiv \prod_{j=0}^{J-1} \frac{R_{t+j}}{\pi_{t+1+j}}$ for $J \geq 1$. Under perfect foresight, this would be the cumulative (up to period J) ex-ante real rate from the perspective of time t. In a stochastic world, this is the cumulative rate for a given history of realizations from t to $t+J$.

Let the household's real assets be $b_t \equiv B_t/P_t$ and $d_t^{H*} \equiv D_t^{H*}/P_t^*$. We assume they face two No-Ponzi-game conditions (NPGC) for each financial asset,

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} \right\} \leq 0, \quad \lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \frac{b_{t-1+J}}{\pi_{t+J}} \right\} \geq 0. \tag{A.1}$$

The discussion below clarifies why we impose one for each asset instead of one for total financial wealth.

In real terms, the household's budget constraint (1) is

$$c_t + rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} + \frac{b_t}{R_t} + \tau_t = w_t h_t + \sigma_t + \frac{b_{t-1}}{\pi_t} + rer_t \frac{d_t^{H*}}{R_t^*}, \tag{A.2}$$

where $\sigma_t \equiv \Sigma_t/P_t$. Defining $sav_t \equiv w_t h_t + \sigma_t - c_t - \tau_t$, the previous can be written as,

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = rer_t \frac{d_t^{H*}}{R_t^*} - \frac{b_t}{R_t} + sav_t.$$

From the definition of the real exchange rate, $rer_t = rer_{t+1} \frac{\pi_{t+1}}{\pi_{t+1}^S \pi_{t+1}^*}$. Replacing this in the first-term on the right-hand side, and then multiplying and dividing the second-term by π_{t+1}, yields

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{\pi_{t+1}}{R_t} \frac{b_t}{\pi_{t+1}} + sav_t,$$

Adding and subtracting $\frac{\pi_{t+1}}{R_t} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{R_t} \left(rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{b_t}{\pi_{t+1}} \right) + sav_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^H \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$. The expectation $E_t\{so_{t+j}^H\}$ captures expected valuation changes due to deviations from perfect-foresight (or complete markets in a stochastic setting) uncovered interest rate parity, which, in a non-linear stochastic model, generally does not hold because of covariance/premium terms. Notice also that this term arises only because we have more than one non-contingent asset and financial markets are incomplete; otherwise, perfect-foresight non-arbitrage relationships would hold and $so_{t+1} = 0$. However, this term is zero up to a first order of approximation, for in the non-stochastic steady state, valuation effects are nil.

Let $a_t^H \equiv rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Thus, we can write

$$a_t^H = \frac{a_{t+1}^H}{rr_{t,t+1}} + sav_t + so_{t+1}^H.$$

Replacing forward a_{t+1}^H yields

$$a_t^H = \frac{1}{rr_{t,t+1}} \left(\frac{a_{t+2}^H}{rr_{t+1,t+2}} + sav_{t+1} + so_{t+2}^H \right) + sav_t + so_{t+1}^H = \frac{a_{t+1}^H}{rr_{t,t+2}} + \sum_{j=0}^{1} \frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}},$$

where we have used the property $rr_{t,t+J} = rr_{t,t+N} \cdot rr_{t+N+1,t+J}$ for $0 \leq N < J$. Continuing forward up to an arbitrary period J, we get

$$a_t^H = \frac{a_{t+J}^H}{rr_{t,t+J}} + \sum_{j=0}^{J-1} \frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on time t's information set on both sides and then taking the limit for J as it approaches infinity, we obtain

$$a_t^H = \lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^H}{rr_{t,t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}} \right\}$$

Recall that $a_t^H = rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Given the NPGC (A.1), the household optimal plan needs to satisfy the transversality condition

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \left(rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} - \frac{b_{t-1+J}}{\pi_{t+J}} \right) \right\} = 0, \tag{A.3}$$

otherwise, if this limit were negative, consumption could be increased in every period, thereby increasing utility. Thus, it is not optimal to choose a plan in which this limit in (A.3) is not equal to zero. For this condition to hold, and given that the NPGC are imposed *individually* for each asset, both of the following transversality conditions need to hold:

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} \right\} = 0, \quad \lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \frac{b_{t-1+J}}{\pi_{t+J}} \right\} = 0. \tag{A.4}$$

Regarding the government, its period t resource constraint in real terms is,

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + sp_t, \tag{A.5}$$

where sp_t denotes the real primary surplus (equal simply to τ_t). Following similar steps as in the household case, this can be written as

$$\frac{d_{t-1}^{G}}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{\pi_{t+1}}{R_t}\frac{d_t^G}{\pi_{t+1}} + \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*} + sp_t$$

Adding and subtracting $\frac{1}{rr_{t,t+1}} rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$\frac{d_{t-1}^{G}}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{1}{rr_{t,t+1}} \left(\frac{d_t^G}{\pi_{t+1}} + rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*} \right) + sp_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^G \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$. Similar to so_t^H, this represents valuation effects due to market incompleteness. Defining also $a_t^G \equiv \frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*}$, we get

$$a_t^G = \frac{1}{rr_{t,t+1}} a_{t+1}^G + so_{t+1}^G + sp_t$$

Replacing forward a_{t+1}^G until an arbitrary period J , we get

$$a_t^G = \frac{a_{t+J}^G}{rr_{t,t+J}} + \sum_{j=0}^{J-1} \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on information at time t on both sides, and then taking the limit for J as it approaches infinity,

$$a_t^G = \lim_{J\to\infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}} \right\}$$

It follows that we need to impose the condition $\lim_{J\to\infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} = 0$, which, from the definition of a_t^G, is equivalent to

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \left(\frac{d_{t+J-1}^G}{\pi_{t+J}} + rer_{t+J} \frac{d_{t-1+J}^{G*}}{\pi_{t+J}^*} \right) \right\} = 0. \tag{A.6}$$

While the household's TVC (A.4) follows from optimization, as we have discussed, the government's TVC does not. For that reason, the transversality condition on government debt is part of the controversy surrounding FTPL in the literature.

In closed economy models with representative agents, the transversality condition for the government is not an extra requirement, for it coincides with the household's transversality condition, which is an optimality condition. Here, as in equilibrium $b_t = d_t^H$ (assuming that only domestic households hold peso bonds), the part of (A.6) corresponding to the debt in pesos is equivalent to the household's TVC for its holding of domestic assets (i.e., the second equation in (A.4)), which is again an optimality condition given our assumptions.

However, if in an open economy we allow *both* domestic and foreign agents to hold government bonds, the household's TVC related to foreign debt is not the same as that of the government's. This is discussed by earlier open-economy contributions such as Dupor (2000) and Daniel (2001).

In particular, notice that a government not satisfying the TVC, but which is still facing an NPGC condition (which is natural to assume), is wasting resources that could be used to either finance additional spending or reduce taxes at some point. Therefore, welfare could be improved if policy is constrained to satisfy the transversality condition with equality. As such, Daniel (2001) introduces the concept of "no-surplus fiscal policy" to describe schemes in which governments in a multi-country model do not waste resources in this manner.

In our case, assuming that

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d^{G*}_{t-1+J}}{\pi^*_{t+J}} \right\} = 0, \tag{A.7}$$

in tandem with the household's TVC in pesos, as previously discussed, it is implied that we focus on "no-surplus fiscal policies."

After imposing these transversality conditions, the lifetime government budget constraint is

$$\frac{d^G_{t-1}}{\pi_t} + rer_t \frac{d^{G*}_{t-1}}{\pi^*_t} = \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sp_{t+j} + so^G_{t+1+j}\right)}{rr_{t,t+j}} \right\}$$

Using the definition of the share of government debt in dollars Ω_t in (5), plus the constant-currency composition rule ($\Omega_t = \Omega$), we obtain

$$\left[\frac{1-\Omega}{\pi_t} + \frac{rer_t}{\overline{rer}} \frac{\Omega}{\pi^*_t}\right] d_{t-1} = E_t \left\{ \sum_{j=0}^{\infty} \frac{sp_{t+j}}{rr_{t,t+j}} \right\} + h.o.t.,$$

where $d_t \equiv d^G_t + \overline{rer} d^{G*}_t$, and also $h.o.t. \equiv \sum_{j=0}^{\infty} E_t \left\{ so^G_{t+1+j}/rr_{t,t+j} \right\}$. This is equation (8) in the text.

Finally, it is important to note that the transversality condition for government debt in dollars (A.7) is not only relevant for describing the government problem but also for guaranteeing that the balance of payments (BoP) is sustainable. Consolidating the household and government's budget constraints (A.2) and (A.5), using the market clearing condition for non-traded goods, the following represents the balance of payments in this model, expressed in domestic-consumption units,

$$rer_t \frac{d^*_{t-1}}{\pi^*_t} = rer_t \frac{d^*_t}{R^*_t} + tb_t$$

where $d^*_t \equiv d^{G*}_t + d^{H*}_t$ is the countries' net-foreign lending position, and $tb_t \equiv rer_t(y^T_t - c^T_t)$ is the trade balance. As before, this can be written as

$$rer_t \frac{d^*_{t-1}}{\pi^*_t} = \frac{\pi_{t+1}}{\pi^S_{t+1} R^*_t} rer_{t+1} \frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

or,

$$rer_t \frac{d^*_{t-1}}{\pi^*_t} = \frac{1}{rr_{t,t+1}} rer_{t+1} \frac{d^*_t}{\pi^*_{t+1}} + \left(\frac{\pi_{t+1}}{\pi^S_{t+1} R^*_t} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

Defining $so_{t+1} \equiv \left(\frac{\pi_{t+1}}{\pi^S_{t+1} R^*_t} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d^*_t}{\pi^*_{t+1}}$, iterating forward up to the period J, applying expectations conditional on information at time t on both sides, and taking the limit for J as it approaches infinity, we obtain

$$rer_t \frac{d^*_{t-1}}{\pi^*_t} = \lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d^*_{t-1+J}}{\pi^*_{t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{(tb_{t+j} + so_{t+1+j})}{rr_{t,t+j}} \right\}$$

It follows that, for the BoP to be sustainable, we need,

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d^*_{t-1+J}}{\pi^*_{t+J}} \right\} = 0,$$

Therefore, as $d^*_t \equiv d^{G*}_t + d^{H*}_t$, the households optimal TVC for dollar assets, along with the assumption of the "no-surplus" policy previously discussed, is equivalent to requiring the sustainability of the BoP. In other words, in the open economy, arguing about whether the TVC condition for the government's debt needs to hold is equivalent to discussing the sustainability of the BoP.

Still, it should be highlighted that this discussion is more pertinent to a **global** equilibrium analysis, which previous open-economy papers have only tackled in deterministic models. Instead, in this paper, as we approximate the solution by linearization around a non-stochastic steady state, the **local** existence and uniqueness requirements implicitly assume that these transversality conditions hold. Moreover, the valuation terms previously described vanish up to a first order of approximation. Future research could be devoted to studying the requirements for global existence and uniqueness in stochastic small and open economy models.

B Equilibrium characterization

B.1 Production of non-tradable varieties and Calvo pricing

A representative competitive firm combines non-tradable varieties with the Dixit-Stiglitz aggregator

$$y^N_t = \left[\int_0^1 \left(y^N_{jt} \right)^{\frac{\epsilon_N - 1}{\epsilon_N}} \mathrm{d}j \right]^{\frac{\epsilon_N}{\epsilon_N - 1}},$$

where y^N_{jt} is the demand for variety j. From profit maximization,

$$y^N_{jt} = \left(\frac{P^N_{jt}}{P^N_t} \right)^{-\epsilon_N} y^N_t, \quad \text{for all } j.$$

A monopolist producing the variety j uses labor according to $y^N_{jt} = (h_{jt})^{\alpha}$, with $\alpha \in (0, 1]$, and internalizes previous demand for j. In addition, it faces a Calvo problem in choosing its price P^N_{jt}: with a probability θ_N, it is forced to set $P^N_{jt} = P^N_{jt-1}$, while with a probability $1 - \theta_N$, it can freely choose a price $\widetilde{P}^N_{jt}$. Using well-known aggregation results (e.g. Schmitt-Grohé & Uribe, 2017, Ch. 9.16), in particular that all firms able to choose set the same price $\widetilde{P}^N_t$, the following characterize the dynamics of non-traded prices

$$p^N_t mc^N_t = \frac{w_t}{\alpha} (y^N_t)^{\left(\frac{1}{\alpha} - 1\right)}, \tag{B.1}$$

$$f_t = (\widetilde{p}_t^N)^{1-\epsilon_N} y_t^N \frac{(\epsilon_N - 1)}{\epsilon_N} + \theta_N E_t \left\{ \chi_{t,t+1} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{1-\epsilon_N} (\pi_{t+1}^N)^{\epsilon_N} f_{t+1} \right\}, \tag{B.2}$$

$$f_t = (\widetilde{p}_t^N)^{-\epsilon_N} y_t^N mc_t^N + \theta_N E_t \left\{ \chi_{t,t+1} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{-\epsilon_N} (\pi_{t+1}^N)^{1+\epsilon_N} f_{t+1} \right\}, \tag{B.3}$$

$$1 = \theta_N (\pi_t^N)^{\epsilon_N - 1} + (1 - \theta_N)(\widetilde{p}_t^N)^{1-\epsilon_N}. \tag{B.4}$$

Equation (B.1) equates marginal costs to the ratio between wages and the marginal product of labor, where mc_t^N represents the real marginal cost in non-traded units. Equations (B.2)-(B.3) provide a recursive representation of the optimal choice of price $\widetilde{P}_t^N$ for those allowed to choose (where $\widetilde{p}_t^N \equiv \widetilde{P}_t^N / P_t^N$), equating the net present value of marginal revenues in those states in which the price $\widetilde{P}_t^N$ still holds (called f_t in this notation) to that of marginal costs.[14] The last condition (B.4) relates the optimal price chosen by those allowed to do so and non-traded inflation $\pi_t^N \equiv P_t^N / P_{t-1}^N$.

These equations can be log-linearized around the zero-inflation steady state (see Schmitt-Grohé & Uribe, 2017, Ch. 9.16) to obtain,

$$\widehat{\pi}_t^N = \beta E_t \{ \widehat{\pi}_{t+1}^N \} + \kappa \cdot \widehat{mc}_t^N,$$

where $\widehat{\cdot}$ denotes the log-linear approximation, and $\kappa = \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N \left(\frac{\epsilon_N}{\alpha} + 1 - \epsilon_N \right)}$.

B.2 Equilibrium conditions

Besides the transversality conditions in Appendix A, the following characterize the equilibrium:

$$w_t (c_t)^{-\sigma} = \chi (h_t)^{\varphi}, \tag{Eq.1}$$

$$(c_t)^{-\sigma} = \beta R_t E_t \left\{ \frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}} \right\}, \tag{Eq.2}$$

$$(c_t)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1})^{-\sigma} \pi_{t+1}^S}{\pi_{t+1}} \right\}, \tag{Eq.3}$$

$$c_t = \left[\omega^{1/\eta} \left(c_t^N \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(c_t^T \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}}, \tag{Eq.4}$$

$$c_t^N = \omega \left(p_t^N \right)^{-\eta} c_t. \tag{Eq.5}$$

$$c_t^T = (1-\omega) \, (rer_t)^{-\eta} \, c_t, \tag{Eq.6}$$

$$p_t^N mc_t^N = \frac{w_t}{\alpha} (y_t^N)^{\left(\frac{1}{\alpha} - 1\right)}, \tag{Eq.7}$$

$$f_t = (\widetilde{p}_t^N)^{1-\epsilon_N} y_t^N \frac{(\epsilon_N - 1)}{\epsilon_N} + \theta_N E_t \left\{ \beta \frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma} \pi_{t+1}} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{1-\epsilon_N} (\pi_{t+1}^N)^{\epsilon_N} f_{t+1} \right\}, \tag{Eq.8}$$

$$f_t = (\widetilde{p}_t^N)^{-\epsilon_N} y_t^N mc_t^N + \theta_N E_t \left\{ \beta \frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma} \pi_{t+1}} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{-\epsilon_N} (\pi_{t+1}^N)^{1+\epsilon_N} f_{t+1} \right\}, \tag{Eq.9}$$

[14] In these, $\chi_{t,t+h} \equiv \beta^h \frac{(c_{t+h})^{-\sigma} P_t}{(c_t)^{-\sigma} P_{t+h}}$ is the households' stochastic discount factor for nominal claims.

$$1 = \theta_N (\pi_t^N)^{\epsilon_N - 1} + (1-\theta_N)(\widetilde{p}_t^N)^{1-\epsilon_N}. \tag{Eq.10}$$

$$R_t^* = R_t^W \exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}, \tag{Eq.11}$$

$$d_t^* = d_t^{*G} + d_t^{*H}, \tag{Eq.12}$$

$$y_t^N = (h_t)^\alpha, \tag{Eq.13}$$

$$y_t^N = (\Delta_t)^\alpha c_t^N, \tag{Eq.14}$$

$$\Delta_t = \theta_N \Delta_t (\pi_t^N)^{\frac{\epsilon_N}{\alpha}} + (1-\theta_N)(\widetilde{p}_t^N)^{-\frac{\epsilon_N}{\alpha}}, \tag{Eq.15}$$

$$q_t = rer_t / p_t^N, \tag{Eq.16}$$

$$rer_t = rer_{t-1}\frac{\pi_t^S \pi_t^*}{\pi_t}, \tag{Eq.17}$$

$$p_t^N = p_{t-1}^N \frac{\pi_t^N}{\pi_t}, \tag{Eq.18}$$

$$\frac{d_{t-1}^*}{\pi_t^*} = \frac{d_t^*}{R_t^*} + (y_t^T - c_t^T). \tag{Eq.19}$$

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \quad \text{or} \quad \frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \tag{Eq.20}$$

$$\tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta}\right] + u_t^\tau, \quad \text{or} \quad \tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta}\right] + u_t^\tau, \tag{Eq.21}$$

$$\Omega = \frac{\overline{rer} d_t^{G*}}{d_t^G + \overline{rer} d_t^{G*}}, \quad \text{or} \quad \Omega = \frac{\overline{q} d_t^{G*}}{d_t^G + \overline{q} d_t^{G*}}, \tag{Eq.22}$$

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \quad \text{or} \quad \left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t^N}{\overline{\pi}^N}\right)^{\phi_\pi} u_t^R, \tag{Eq.23}$$

All variables were described previously, except for Δ_t, which captures the potential inefficiency induced by price dispersion in the presence of sticky prices. However, as we assume zero steady-state inflation, it is not relevant up to first order.

Overall, we have 23 endogenous variables:

$$\begin{array}{cccccccccccc} w_t & c_t & h_t & R_t & \pi_t & R_t^* & \pi_t^S & c_t^N & c_t^T & p_t & p_t^T & rer_t \\ mc_t^N & y_t^N & f_t & \widetilde{p}_t^N & \pi_t^N & d_t^G & d_t^{G*} & \tau_t & d_t^* & d_t^{*H} & \Delta_t & \end{array}$$

and 5 exogenous variables:

$$y_t^T \quad \pi_t^* \quad R_t^W \quad u_t^\tau \quad u_t^R$$

For $x_t = \{y_t^T, \pi_t^*, R_t^W, u_t^R\}$ we assume the following AR(1) process in logs

$$\log\left(\frac{x_t}{\overline{x}}\right) = \rho_x \log\left(\frac{x_{t-1}}{\overline{x}}\right) + \sigma_x \varepsilon_t^x,$$

where ε_t^x is an i.i.d. shock, $\rho_x \in [0,1)$ and $\sigma_x > 0$. Instead, for u_t^τ we assume

$$\frac{(u_t^\tau - \overline{u}^\tau)}{\overline{gdp}} = \rho_\tau \frac{(u_{t-1}^\tau - \overline{u}^\tau)}{\overline{gdp}} + \sigma_\tau \varepsilon_t^\tau,$$

where $\overline{gdp} = \overline{y}^N + \overline{y}^T$ is real GDP in steady state, $\rho_\tau \in [0,1)$, and $\sigma_\tau > 0$.

B.2.1 Tradable block

Combining equations (Eq.3), (Eq.6), (Eq.16), and (Eq.17), we get

$$(c_t^T)^{-\sigma}(rer_t)^{-\sigma\eta} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma}(rer_{t+1})^{-\sigma\eta} rer_{t+1}}{\pi_{t+1}^* rer_t} \right\},$$

If, in addition, we assume $\eta\sigma = 1$, this yields

$$(c_t^T)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma}}{\pi_{t+1}^*} \right\}, \tag{Eq.24}$$

Equations (Eq.11), (Eq.19) and (Eq.24) form a system for the endogenous variables c_t^T, d_t^*, and R_t^*, which can be solved as a function of the exogenous variables π_t^*, R_t^W, y_t^T alone. Thus, in this setup, the tradable block is isolated from non-tradables, as well as from monetary and fiscal policies. The intuitive reason for this result is that any effect on the real exchange rate (rer_t), originating from exogenous variables different from π_t^*, R_t^W, y_t^T, induces both intra- and inter-temporal substitution effects. By the former, an increase in rer_t produces (*ceteris paribus*) a desire to substitute away from tradables, as these become more expensive; an effect determined by the intra-temporal elasticity of substitution η. By the latter, a temporary increase in rer_t produces (*ceteris paribus*) a fall in the real return of saving in tradables, increasing desired tradable consumption today (an effect determined by the intra-temporal substitution elasticity $1/\sigma$). Thus, if $\eta\sigma = 1$ the two offset each other.

C Model under the NT policy setup

This section derives analytical results based on the NT-policy setup. The first part derives the log-linearized version and its minimalist representation. The second discusses the conditions for the (local) existence and uniqueness of a stationary equilibrium. The third characterizes the solution and shows how both policy shocks affect the economy and how those results depend on the fiscal/monetary policy setup.

Relative to the general model with equilibrium conditions listed in Appendix B.2, we additionally assume equal inter- and intra-temporal elasticity of substitution: $\eta = 1/\sigma$, and zero inflation in steady state ($\overline{\pi} = \overline{\pi}^N = \overline{\pi}^* = \overline{\pi}^S = 1$).

C.1 Log-linearization

Recall from section B.2.1, c_t^T, d_t^*, and R_t^* are a function of the exogenous variables π_t^*, R_t^W, y_t^T alone. Thus, in what follows, we treat them as given in accounting for the dynamics of the other variables.

C.1.1 Fiscal and monetary policy

Assuming a rule for real lump-sum taxes defined as $\tau_t \equiv T_t/P_t^N$, the government budget constraint in terms of non-tradables is

$$\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t,$$

where $d_t^G \equiv D_t^G / P_t^N$ and $d_t^{G*} \equiv D_t^{G*} / P_t^*$. Considering the rule for the currency composition of debt.

$$\Omega = \frac{\overline{q} d_t^{G*}}{d_t^G + \overline{q} d_t^{G*}},$$

and defining $d_t \equiv d_t^G + \overline{q} d_t^{G*}$, the budget constraint is

$$\left[\frac{(1-\Omega)}{\pi_t^N} + \frac{\Omega}{\pi_t^*} \frac{q_t}{\overline{q}} \right] d_{t-1} = \left[\frac{(1-\Omega)}{R_t} + \frac{\Omega}{R_t^*} \frac{q_t}{\overline{q}} \right] d_t + \tau_t$$

To log-linearize, define $A_t \equiv \left[\frac{(1-\Omega)}{\pi_t^N} + \frac{q_t}{\overline{q}} \frac{\Omega}{\pi_t^*} \right]$, $B_t \equiv \left[\frac{(1-\Omega)}{R_t} + \frac{q_t}{\overline{q}} \frac{\Omega}{R_t^*} \right]$, $\widehat{A}_t \equiv \log(A_t / \overline{A})$, and $\widehat{B}_t \equiv \log(B_t / \overline{B})$. Also, as we focus on cases with $\overline{d}, \overline{\tau} > 0$, let $\widehat{d}_t \equiv \log(d_t / \overline{d})$ and $\widehat{\tau}_t \equiv \log(\tau_t / \overline{\tau})$. Thus, applying the change of variables, the constraint is

$$\overline{A} e^{\widehat{A}_t} \overline{d} e^{\widehat{d}_{t-1}} = \overline{B} e^{\widehat{B}_t} \overline{d} e^{\widehat{d}_t} + \overline{\tau} e^{\widehat{\tau}_t}.$$

Taking a first-order approximation

$$\overline{A}\overline{d}(\widehat{A}_t + \widehat{d}_{t-1}) = \overline{B}\overline{d}(\widehat{B}_t + \widehat{d}_t) + \overline{\tau}\widehat{\tau}_t.$$

In a steady state with $\overline{\pi} = \overline{\pi}^* = \overline{\pi}^S = 1$, then $R = R^* = 1/\beta$, so we have

$$\overline{A} = 1, \quad \overline{B} = \beta, \quad \overline{d}(1-\beta) = \overline{\tau}.$$

Therefore, the first-order approximation simplifies to

$$\widehat{A}_t + \widehat{d}_{t-1} = \beta(\widehat{B}_t + \widehat{d}_t) + (1-\beta)\widehat{\tau}_t. \tag{C.1}$$

Similarly, the rule for lump-sum taxes can be written as

$$\tau_t - \overline{\tau} = \phi_T \left(A_t d_{t-1} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau \quad \Rightarrow \quad \overline{\tau} e^{\widehat{\tau}_t} - \overline{\tau} = \phi_T \left(\overline{A} e^{\widehat{A}_t} \overline{d} e^{\widehat{d}_{t-1}} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau,$$

or

$$e^{\widehat{\tau}_t} - 1 = \phi_T \left(\frac{1}{(1-\beta)} e^{\widehat{A}_t + \widehat{d}_{t-1}} - \frac{1}{1-\beta} \right) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau,$$

where $\widehat{u}_t^\tau \equiv u_t^\tau (1-\beta)/\overline{\tau}$. Taking a first order approximation,

$$\widehat{\tau}_t = \phi_T \frac{1}{(1-\beta)} (\widehat{A}_t + \widehat{d}_{t-1}) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau. \tag{C.2}$$

Thus, combining (C.1) and (C.2), we can write

$$(1-\phi_T)(\widehat{d}_{t-1} + \widehat{A}_t) = \beta(\widehat{d}_t + \widehat{B}_t) + \widehat{u}_t^\tau.$$

Moreover, from the definitions of A_t and B_t, we can log-linearize to obtain,

$$\begin{aligned} \widehat{A}_t &= -(1-\Omega)\widehat{\pi}_t^N + \Omega(\widehat{q}_t - \widehat{\pi}_t^*) = -\widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \widehat{\pi}_t^*), \\ \widehat{B}_t &= -(1-\Omega)\widehat{R}_t + \Omega(\widehat{q}_t - \widehat{R}_t^*) = -\widehat{R}_t + \Omega(\widehat{R}_t + \widehat{q}_t - \widehat{R}_t^*). \end{aligned}$$

Thus, the log-linearized version of the government debt's equation is

$$(1-\phi_T)\left[\widehat{d}_{t-1}-\widehat{\pi}_t^N+\Omega(\widehat{q}_t+\widehat{\pi}_t^N-\widehat{\pi}_t^*)\right]=\beta\left[\widehat{d}_t-\widehat{R}_t+\Omega(\widehat{R}_t+\widehat{q}_t-\widehat{R}_t^*)\right]+\widehat{u}_t^T, \tag{C.3}$$

Finally, log-linearizing the Taylor rule,

$$\widehat{R}_t=\phi_\pi\widehat{\pi}_t^N+\widehat{u}_t^R. \tag{C.4}$$

Overall, equations (C.3) and (C.4) charcaterize the policy block.

C.1.2 The rest of the model

The log-linearized version of the equations not related to policy or traded variables is

$$\widehat{w}_t-\widehat{p}_t^N-\sigma\widehat{c}_t^N=\varphi\widehat{h}_t,\quad \widehat{p}_t^N+\widehat{mc}_t^N=\widehat{w}_t+\left(\frac{1}{\alpha}-1\right)\widehat{y}_t^N,\quad \widehat{y}_t^N=\alpha\widehat{h}_t,\quad \widehat{y}_t^N=\widehat{c}_t^N,\quad \widehat{c}_t^N=\eta\widehat{q}_t+\widehat{c}_t^T,$$

$$0=\omega\widehat{p}_t^N+(1-\omega)\,\widehat{rer}_t,\quad \widehat{q}_t=\widehat{rer}_t-\widehat{p}_t^N,\quad \widehat{p}_t^N=\widehat{p}_{t-1}^N+\widehat{\pi}_t^N-\widehat{\pi}_t,$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*-\widehat{\pi}_t,\quad \widehat{R}_t=\widehat{R}_t^*+E_t\left\{\widehat{\pi}_{t+1}^S\right\},\quad \widehat{\pi}_t^N=\beta E_t\{\widehat{\pi}_{t+1}^N\}+\kappa\cdot\widehat{mc}_t^N.$$

The first four can be used to write marginal costs as

$$\widehat{mc}_t^N=\left(\frac{1+\varphi}{\alpha}-1+\sigma\right)\widehat{c}_t^N,$$

In addition, using $\widehat{c}_t^N=\eta\widehat{q}_t+\widehat{c}_t^T$, the Phillips curve for non-tradables can be written as

$$\widehat{\pi}_t^N=\beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}+\widetilde{\kappa}\left(\eta\widehat{q}_t+\widehat{c}_t^T\right), \tag{C.5}$$

where $\widetilde{\kappa}\equiv\kappa\left(\frac{(1+\varphi)}{\alpha}-1+\sigma\right)$, which is positive if we consider $\sigma>1$.

From $0=\omega\widehat{p}_t^N+(1-\omega)\,\widehat{rer}_t$ and the equations describing the evolution of $\widehat{p}_t^N$ and $\widehat{rer}_t$, we can write inflation as

$$\widehat{\pi}_t=\omega(\widehat{p}_{t-1}^N+\widehat{\pi}_t^N)+(1-\omega)(\widehat{rer}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*)=\omega(\widehat{\pi}_t^N)+(1-\omega)(\widehat{\pi}_t^S+\widehat{\pi}_t^*),$$

where the last equality follows from $0=\omega\widehat{p}_{t-1}^N+(1-\omega)\,\widehat{rer}_{t-1}$.

In addition, from $\widehat{q}_t=\widehat{rer}_t-\widehat{p}_t^N$ and the equations describing the evolution of $\widehat{p}_t^N$ and $\widehat{rer}_t$,

$$\widehat{q}_t=\widehat{q}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*-\widehat{\pi}_t^N.$$

In turn, this can be used to re-write the UIP condition as

$$\widehat{R}_t=\widehat{R}_t^*+E_t\left\{\widehat{q}_{t+1}-\widehat{q}_t+\widehat{\pi}_{t+1}^N-\widehat{\pi}_{t+1}^*\right\}. \tag{C.6}$$

C.1.3 Summary

Equations (C.3), (C.4), (C.5), (C.6) can be further combined to eliminate $\widehat{R}_t$

$$\widehat{\pi}_t^N=\beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}+\widetilde{\kappa}\left(\eta\widehat{q}_t+\widehat{c}_t^T\right), \tag{C.7}$$

$$\phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R = \widehat{R}_t^* + E_t \left\{ \widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^* \right\}, \tag{C.8}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \widehat{\pi}_t^*)\right] = \beta\left[\widehat{d}_t - (1-\Omega)(\phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R) + \Omega(\widehat{q}_t - \widehat{R}_t^*)\right] + \widehat{u}_t^\tau. \tag{C.9}$$

This system characterizes the dynamics of the three endogenous variables $\widehat{\pi}_t^N, \widehat{q}_t, \widehat{d}_t$, given the exogenous external variables $\widehat{R}_t^W, \widehat{y}_t^T, \widehat{\pi}_t^*$ (recall from section B.2.1 that $\widehat{R}_t^*$ and $\widehat{c}_t^T$ are driven solely by these external variables) and policy shocks $\widehat{u}_t^R$ and $\widehat{u}_t^\tau$.

C.2 Existence and uniqueness

To study the determination of a stationary equilibrium in the linearized system (C.7)-(C.9), it is enough to consider the dynamics in the absence of shocks. These can be re-arranged as

$$\widehat{\pi}_{t+1}^N = \frac{1}{\beta}\widehat{\pi}_t^N - \frac{\widetilde{\kappa}\eta}{\beta}\widehat{q}_t,$$

$$\widehat{q}_{t+1} = \left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t^N + \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right)\widehat{q}_t,$$

$$\widehat{d}_t = (\Omega - 1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right)\widehat{\pi}_t^N + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\widehat{q}_t + \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}.$$

Defining $w_t \equiv [\widehat{\pi}_t^N, \widehat{q}_t, \widehat{d}_{t-1}]'$, the system can be written as

$$w_{t+1} = \Psi w_t$$

where Ψ has the form

$$\Psi = \begin{pmatrix} \frac{1}{\beta} & -\frac{\widetilde{\kappa}\eta}{\beta} & 0 \\ \left(\phi_\pi - \frac{1}{\beta}\right) & \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right) & 0 \\ (\Omega - 1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right) & \Omega\left(\frac{1-\phi_T}{\beta} - 1\right) & \frac{(1-\phi_T)}{\beta} \end{pmatrix}$$

Here, $\widehat{d}_t$ is a predetermined/state variable, while $\widehat{\pi}_t^N$ and $\widehat{q}_t$ are non-predetermined/jumping. A necessary condition for local equilibrium uniqueness requires one stable and two non-stable eigenvalues of Ψ. With more than one stable eigenvalue, there are multiple local equilibria. Otherwise, there is no locally stationary equilibrium.

Notice that Ψ has a block lower triangular structure of the form

$$\Psi = \begin{pmatrix} \Psi_{11} & 0 \\ \Psi_{21} & \Psi_{22} \end{pmatrix}, \tag{C.10}$$

and thus $det(\Psi - \lambda I) = det(\Psi_{11} - \lambda I) \times det(\Psi_{22} - \lambda I)$. This means that we can analyze the eigenvalues of Ψ_{11} and Ψ_{22} separately.

The eigenvalue of Ψ_{22} is simply equal to $\frac{1-\phi_T}{\beta}$, which is less than one if $1 - \beta < \phi_T$. In turn, those of Ψ_{11} solve $det(\Psi_{11} - \lambda I) = 0$:

$$det(\Psi_{11} - \lambda I) = \left(\frac{1}{\beta} - \lambda\right)\left(1 + \frac{\widetilde{\kappa}\eta}{\beta} - \lambda\right) + \left(\phi_\pi - \frac{1}{\beta}\right)\frac{\widetilde{\kappa}\eta}{\beta} = 0$$

$$\implies \beta\lambda^2 - (1+\beta+\widetilde{\kappa}\eta)\,\lambda + (1+\phi_\pi\widetilde{\kappa}\eta) = 0 \tag{C.11}$$

The eigenvalues of Ψ_{11} are the solutions to the quadratic polynomial:

$$\begin{aligned} \lambda_1 &= \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\phi_\pi\widetilde{\kappa}\eta)}}{2\beta}, \\ \lambda_2 &= \frac{(1+\beta+\widetilde{\kappa}\eta) + \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\phi_\pi\widetilde{\kappa}\eta)}}{2\beta}. \end{aligned} \tag{C.12}$$

By Vieta's formulas, it follows that:

$$\lambda_1\lambda_2 = \frac{1+\phi_\pi\widetilde{\kappa}\eta}{\beta}$$

Since $\beta \in (0,1)$, $\widetilde{\kappa}\eta > 0$, and $\phi_\pi \geq 0$, the product $\lambda_1\lambda_2 > 1$. Also, as $\lambda_2 > 0$, so is $\lambda_1 > 0$. In addition, as $\lambda_2 > \lambda_1$ by definition, then $\lambda_2 > 1$. Thus, at least one of the eigenvalues of Ψ_{11} is greater than 1.

Regarding the first eigenvalue, notice that

$$\lambda_1 = \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\phi_\pi\widetilde{\kappa}\eta)}}{2\beta}$$

$$= \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\widetilde{\kappa}\eta) - 4\beta\widetilde{\kappa}\eta(\phi_\pi - 1)}}{2\beta}$$

If $\phi_\pi > 1$:

$$\lambda_1 > \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\widetilde{\kappa}\eta)}}{2\beta} = \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1-\beta+\widetilde{\kappa}\eta)^2}}{2\beta} = 1,$$

so $\lambda_1 > 1$ if $\phi_\pi > 1$. Conversely, if $\phi_\pi < 1$

$$\lambda_1 < \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta(1+\widetilde{\kappa}\eta)}}{2\beta} = \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1-\beta+\widetilde{\kappa}\eta)^2}}{2\beta} = 1$$

thus, $\lambda_1 < 1$ when $\phi_\pi < 1$. And recall that we have already shown that $\lambda_1 > 0$.

Overall, of the three eigenvalues of Ψ, one of them must be greater than one, another is greater than one if $\phi_T < 1-\beta$, and the last is greater than one if $\phi_\pi > 1$. Therefore, we have

- Unique stable equilibrium: either $\phi_\pi > 1$ and $\phi_T > 1-\beta$ (active monetary, passive fiscal) or $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary).
- No stable equilibrium: $\phi_\pi > 1$ and $\phi_T < 1-\beta$ (active fiscal and monetary).
- Continuum of stable equilibria: $\phi_\pi < 1$ and $\phi_T > 1-\beta$ (passive fiscal and monetary).

These results are generally independent of Ω; although there is one exception. In the case in which $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary), there is an additional condition that must hold for a unique stationary equilibrium to exist. Intuitively, as in this case the debt equation has an unstable eigenvalue $(1-\phi_T)/\beta > 1$, the existence of a stationary equilibrium requires the dynamics of $\widehat{\pi}_t^N$ and $\widehat{q}_t$ to offset such explosive behavior, which in turn depends on Ω.

More formally, given that Ψ has a block-triangular structure, the predetermined $\widehat{d}_{t-1}$ does not directly affect the jumping variables $\widehat{\pi}_t^N$ and $\widehat{q}_t$. Thus, the stable eigenvalue $|\lambda_1| < 1$ (coming from the jumping block) is required to have an associated eigenvector that has a nonzero component in the predetermined variable. Letting the eigenvector be $v \equiv [v_j{}', v_p]'$, where $v_j \equiv [v_j^1, v_j^2]'$, and using the notation in (C.10), if λ_1 is an eigenvalue of Ψ it holds that

$$\Psi v = \lambda_1 v \Longrightarrow \begin{cases} \Psi_{11} v_j = \lambda_1 v_j \\ \Psi_{21} v_j + \Psi_{22} v_p = \lambda_1 v_p \end{cases} \Longrightarrow \begin{cases} \Psi_{11} v_j = \lambda_1 v_j \\ v_p = (\lambda_1 - \Psi_{22})^{-1} \Psi_{21} v_j. \end{cases}$$

From the second condition, $v_p \neq 0$ requires $\Psi_{21} v_j \neq 0$;[15] otherwise, the solution fails to be stable.

To find $v_j \equiv [v_j^1, v_j^2]'$, we use the first condition. Taking the first row, this is

$$\left(\frac{1}{\beta} - \lambda_1\right) v_j^1 - \frac{\widetilde{\kappa}\eta}{\beta} v_j^2 = 0$$

Normalizing $v_j^1 = 1$, we have $v_j^2 = \frac{1-\beta\lambda_1}{\widetilde{\kappa}\eta}$. Then, using the values for Ψ_{21}, the condition $\Psi_{21} v_j = 0$ is

$$(\Omega - 1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right) + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} = 0$$

Thus, we can define

$$\Omega_{cut,1} \equiv \frac{(1-\phi_T-\beta\phi_\pi)}{(1-\phi_T-\beta\phi_\pi) + (1-\phi_T-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}}, \tag{C.13}$$

such that if $\Omega = \Omega_{cut,1}$, a stationary equilibrium fails to exist in the case of $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary).

Notice that $0 < \Omega_{cut,1} \leq 1$, with equality only if $\frac{1}{\widetilde{\kappa}\eta} = 0$ (as $(1-\beta\lambda_1) > 0$ if $\phi_\pi < 1$). This has several implications. First, in the analogous closed-economy setup with $\Omega = 0$, this extra rank condition is always satisfied, which explains why $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary) are sufficient for uniqueness in the related literature. Second, the case with $\frac{1}{\widetilde{\kappa}\eta} = 0$ holds when prices are flexible ($\widetilde{\kappa}^{-1} = 0$ if $\theta_N = 0$). Thus, as long as prices are sticky, there is a value of $\Omega \in (0,1)$ for which there is no stable equilibrium. Instead, with flexible prices, $\Omega = 1$ (fully dollarized debt) prevents a stable equilibrium.

The results from this subsection are summarized by the following proposition:

Proposition *Under the NT-policy setup, the necessary conditions characterizing existence and uniqueness of the linearized model are:*

- *Unique stable equilibrium: if either $\phi_\pi > 1$ and $\phi_T > 1-\beta$ (active monetary, passive fiscal), or $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary).*
- *No stable equilibrium: if $\phi_\pi > 1$ and $\phi_T < 1-\beta$ (active fiscal and monetary).*
- *A continuum of stable equilibria: if $\phi_\pi < 1$ and $\phi_T > 1-\beta$ (passive fiscal and monetary).*

These conditions are also sufficient, except in the case with $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary), where uniqueness also requires $\Omega \neq \Omega_{cut,1}$, with $\Omega_{cut,1} \in (0,1)$ if $\theta_N > 0$, while $\Omega_{cut,1} = 1$ if $\theta_N = 0$. If this condition fails, the case with $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary) features no stable equilibrium.

[15] Notice that $\lambda_1 \neq \Psi_{22}$, as $0 < \lambda_1 < 1$ and $\Psi_{22} > 1$ in this case with $\phi_\pi < 1$ and $\phi_T < 1-\beta$.

C.3 The effect of policy shocks in two special cases

In this sub-section, we characterize the solution of the linearized model (C.7)-(C.9) conditional on uniqueness, assuming only i.i.d. policy shocks, in two polar cases: Active monetary and passive fiscal, with $\phi_\pi > 1$ and $\phi_T > 1-\beta$, and active fiscal and passive monetary, with $\phi_\pi = \phi_T = 0$. The goal is to characterize the derivatives of $\widehat{\pi}_t^N, \widehat{q}_t$ with respect to $\widehat{u}_t^R, \widehat{u}_t^T$, and how these depend on Ω.

C.3.1 Solution when $\phi_\pi > 1$ and $\phi_T > 1-\beta$

In this case, equations (C.7) and (C.8) are enough to characterize the evolution of $\widehat{\pi}_t^N$ and $\widehat{q}_t$ (as in standard NK models with passive fiscal policy). Excluding non-policy shocks, these are

$$\widehat{\pi}_t^N = \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\} + \widetilde{\kappa}\eta\widehat{q}_t, \tag{C.14}$$

$$\phi_\pi\widehat{\pi}_t^N + \widehat{u}_t^R + \widehat{q}_t = E_t\left\{\widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N\right\}. \tag{C.15}$$

Clearly, fiscal shocks and the currency composition of debt are irrelevant for the equilibrium behavior of these variables. Thus,

$$\frac{\partial\widehat{\pi}_t^N}{\partial u_t^T} = 0, \quad \frac{\partial\widehat{q}_t}{\partial u_t^T} = 0.$$

As both variables are purely forward looking, the solution takes the form

$$\pi_t^N = Nu_t^R, \quad q_t = Mu_t^R.$$

A shocks are assumed to be i.i.d., $E_t\left\{\widehat{\pi}_{t+1}^N\right\} = E_t\left\{\widehat{q}_{t+1}\right\} = 0$ in the proposed solution. Replacing in (C.14) and (C.15),

$$Nu_t^R = \widetilde{\kappa}\eta Mu_t^R, \tag{C.16}$$

$$\phi_\pi Nu_t^R + \widehat{u}_t^R + Mu_t^R = 0. \tag{C.17}$$

Thus, N and M need to solve

$$\begin{cases} N = \widetilde{\kappa}\eta M, \\ 0 = \phi_\pi N + 1 + M, \end{cases} \implies \begin{cases} M = -\frac{1}{\phi_\pi\widetilde{\kappa}\eta+1} < 0, \\ N = -\frac{\widetilde{\kappa}\eta}{\phi_\pi\widetilde{\kappa}\eta+1} < 0, \end{cases}$$

Recall also that the nominal depreciation is $\widehat{\pi}_t^S = \widehat{q}_t + \widehat{\pi}_t^N - \widehat{q}_{t-1}$. Overall, we have

$$\frac{\partial\widehat{\pi}_t^N}{\partial u_t^R} < 0, \quad \frac{\partial\widehat{q}_t}{\partial u_t^R} < 0, \quad \frac{\partial\widehat{\pi}_t^S}{\partial u_t^R} < 0.$$

Moreover, recall that non-traded consumption $\widehat{c}_t^N$ and output $\widehat{y}_t^N$ are proportional to $\widehat{q}_t$, so any conclusions regarding the latter translate into the former as well. In addition, as total inflation is a weighted average of $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$, and both react with the same sign to the monetary shock, aggregate inflation also falls after a monetary tightening.

The following proposition summarizes the results for the monetary shock:

Proposition 2 *Under the NT-based policy setup with $\phi_\pi > 1$ (active monetary) and $\phi_T > 1-\beta$ (passive fiscal), an i.i.d. positive shock to u_t^R induces a contemporaneous fall in non-traded inflation $\widehat{\pi}_t^N$, the relative price of tradables $\widehat{q}_t$ and non-traded output $\widehat{y}_t^N$, as well as a nominal appreciation ($\widehat{\pi}_t^S$ falls). This holds for any value of $\Omega \in [0,1]$.*

C.3.2 Solution when $\phi_\pi = \phi_T = 0$

Equations (C.7)-(C.9) without external shocks and imposing $\phi_\pi = \phi_T = 0$ can be written as

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa}\eta \widehat{q}_t, \tag{C.18}$$

$$\widehat{q}_t + \widehat{u}_t^R = E_t \left\{ \widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N \right\}, \tag{C.19}$$

$$\widehat{d}_{t-1} - \widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N) = \beta \left[\widehat{d}_t - \widehat{u}_t^R + \Omega(\widehat{q}_t + \widehat{u}_t^R) \right] + \widehat{u}_t^\tau. \tag{C.20}$$

We seek the solution of $\widehat{\pi}_t^N, \widehat{q}_{t+j}$ as a function of $\widehat{u}_t^R, \widehat{u}_t^\tau, \widehat{d}_{t-1}$. We proceed with the following steps:

- Combine the (C.18)-(C.19) to obtain a second-order difference equation for $\widehat{\pi}_t^N$. Find the solution for $E_t\{\widehat{\pi}_{t+j}^N\}$ for $j > 0$, as a function of $\widehat{\pi}_t^N$ and $\widehat{u}_t^R$.
- Characterize the solution for $\widehat{q}_t$ as a function of $\widehat{\pi}_t^N$ and $\widehat{u}_t^R$.
- Solve forward the government budget constraint (C.20) and use the previous expectations to replace the relevant terms. This yields the desired solution for $\widehat{\pi}_t^N$.
- Based on the previous result, find also the solution for $\widehat{q}_t$ and $\widehat{\pi}_t^S$.

This strategy is similar to that in Leeper & Leith (2016), although here the algebra is more tedious when $\Omega > 0$, which is not present in their closed-economy setup.

Obtaining $E_t\{\widehat{\pi}_{t+j}^N\}$ for $j > 0$

Using (C.18) to eliminate $\widehat{q}_t$ and $\widehat{q}_{t+1}$ in (C.19) yields

$$\frac{\widehat{\pi}_t^N - \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\}}{\widetilde{\kappa}\eta} + \widehat{u}_t^R = E_t \left\{ \frac{\widehat{\pi}_{t+1}^N - \beta \widehat{\pi}_{t+2}^N}{\widetilde{\kappa}\eta} \right\} + E_t \left\{ \widehat{\pi}_{t+1}^N \right\}.$$

Rearranging,

$$\beta E_t \left\{ \widehat{\pi}_{t+2}^N \right\} - (1 + \beta + \widetilde{\kappa}\eta) E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widehat{\pi}_t^N + \widetilde{\kappa}\eta \widehat{u}_t^R = 0. \tag{C.21}$$

This is a second-order difference equation. We look for solutions for $E_t \left\{ \widehat{\pi}_{t+1}^N \right\}$ of the form

$$E_t \left\{ \widehat{\pi}_{t+1}^N \right\} = \lambda \widehat{\pi}_t^N + \Lambda \widehat{u}_t^R,$$

where λ, Λ are unknown scalars that we need to find. With this solution, using the law of iterated expectations (LIE) and shocks being i.i.d.,

$$E_t \left\{ \widehat{\pi}_{t+2}^N \right\} = E_t \left\{ \lambda \widehat{\pi}_{t+1}^N + \Lambda \widehat{u}_{t+1}^R \right\} = \lambda E_t \left\{ \widehat{\pi}_{t+1}^N \right\} = \lambda \left(\lambda \widehat{\pi}_t^N + \Lambda \widehat{u}_t^R \right) = \lambda^2 \widehat{\pi}_t^N + \lambda \Lambda \widehat{u}_t^R$$

Replacing both expectations in (C.21)

$$\beta(\lambda^2 \widehat{\pi}_t^N + \lambda \Lambda \widehat{u}_t^R) - (1 + \beta + \widetilde{\kappa}\eta)(\lambda \widehat{\pi}_t^N + \Lambda \widehat{u}_t^R) + \widehat{\pi}_t^N + \widetilde{\kappa}\eta \widehat{u}_t^R = 0.$$

Thus, λ and Λ need to satisfy

$$\beta\lambda^2 - (1 + \beta + \widetilde{\kappa}\eta)\lambda + 1 = 0, \quad \beta\lambda\Lambda - (1 + \beta + \widetilde{\kappa}\eta)\Lambda + \widetilde{\kappa}\eta = 0$$

Notice that the first equation is precisely the quadratic polynomial (C.11), under the assumption $\phi_\pi = 0$. Thus, the solution λ_1 in (C.12) satisfies the requirement $|\lambda_1| < 1$ that is needed for a stationary solution in this case. A related result is that, as λ_1 solves the quadratic equation,

$$(1+\beta+\widetilde{\kappa}\eta)\lambda_1 = \beta(\lambda_1)^2 + 1 \quad \Longrightarrow \quad \widetilde{\kappa}\eta = \frac{(1-\lambda_1)(1-\beta\lambda_1)}{\lambda_1}.$$

To compute Λ,

$$\beta\lambda\Lambda - (1+\beta+\widetilde{\kappa}\eta)\Lambda + \widetilde{\kappa}\eta = 0 \implies \Lambda = \frac{\widetilde{\kappa}\eta}{(1+\beta+\widetilde{\kappa}\eta) - \beta\lambda_1} = \dots$$
$$\frac{\widetilde{\kappa}\eta}{(1+\beta+\widetilde{\kappa}\eta) - \frac{(1+\beta+\widetilde{\kappa}\eta)-\sqrt{(1+\beta+\widetilde{\kappa}\eta)^2-4\beta}}{2}} = \frac{\widetilde{\kappa}\eta}{\frac{(1+\beta+\widetilde{\kappa}\eta)}{2} + \frac{\sqrt{(1+\beta+\widetilde{\kappa}\eta)^2-4\beta}}{2}} = \frac{\widetilde{\kappa}\eta}{\beta\lambda_2} = \widetilde{\kappa}\eta\lambda_1$$

where the last equality uses Vieta's formula $\lambda_1\lambda_2 = \frac{1}{\beta}$. Putting all together, the solution for the expectation $E_t\left\{\widehat{\pi}^N_{t+1}\right\}$ is

$$E_t\left\{\widehat{\pi}^N_{t+1}\right\} = \lambda_1\left(\widehat{\pi}^N_t + \widetilde{\kappa}\eta\widehat{u}^R_t\right). \tag{C.22}$$

From this,

$$E_t\left\{\widehat{\pi}^N_{t+j}\right\} = E_t\left\{E_{t+j-1}\left\{\widehat{\pi}^N_{t+j}\right\}\right\} = E_t\left\{\lambda_1\left(\widehat{\pi}^N_{t+j-1} + \widetilde{\kappa}\eta\widehat{u}^R_{t+j-1}\right)\right\} =$$
$$\lambda_1 E_t\left\{\widehat{\pi}^N_{t+j-1}\right\} = \dots = (\lambda_1)^{j-1}E_t\left\{\widehat{\pi}^N_{t+1}\right\} = (\lambda_1)^j\left(\widehat{\pi}^N_t + \widetilde{\kappa}\eta\widehat{u}^R_t\right), \tag{C.23}$$

where the first equality follows from the LIE, the second from (C.22) evaluated at $t+j-1$, the third from $\widehat{u}^R_t$ being i.i.d., the fourth repeats the procedure up to $t+1$, and the last uses (C.22) directly.

Obtaining $\widehat{q}_t$

From the Phillips curve (C.18)

$$\widehat{q}_t = \frac{\widehat{\pi}^N_t - \beta E_t\left\{\widehat{\pi}^N_{t+1}\right\}}{\widetilde{\kappa}\eta} = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\widehat{\pi}^N_t - \beta\lambda_1\widehat{u}^R_t. \tag{C.24}$$

Using the government budget constraint

Equation (C.20) can be rearranged as

$$\widehat{d}_{t-1} = \beta\widehat{d}_t + aux_t,$$

where $aux_t \equiv \widehat{\pi}^N_t(1-\Omega) - \widehat{q}_t\Omega(1-\beta) - \widehat{u}^R_t\beta(1-\Omega) + \widehat{u}^\tau_t$. Iterating the equation forward, taking expectations and imposing the transversality condition yields

$$\widehat{d}_{t-1} = aux_t + \sum_{j=1}^{\infty}\beta^j E_t\{aux_{t+j}\}. \tag{C.25}$$

Notice that, by its definition and (C.24), aux_t can be written as

$$aux_t = \widehat{\pi}_t^N(1-\Omega) - \left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\widehat{\pi}_t^N - \beta\lambda_1\widehat{u}_t^R\right]\Omega(1-\beta) - \widehat{u}_t^R\beta(1-\Omega) + \widehat{u}_t^\tau = ...$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N - \beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R + \widehat{u}_t^\tau.$$

Also, as shocks are i.i.d., using (C.23) we can compute

$$E_t\{aux_{t+j}\} = \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]E_t\{\widehat{\pi}_{t+j}^N\} = ...$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right](\lambda_1)^j\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right),$$

for $j > 0$. Thus

$$\sum_{j=1}^{\infty}\beta^j E_t\{aux_{t+j}\} = \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right)\sum_{j=1}^{\infty}(\lambda_1\beta)^j = ...$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right)$$

Plugging these back into (C.25),

$$\begin{aligned}
\widehat{d}_{t-1} &= \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N - \beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R + \widehat{u}_t^\tau + \\
&\quad \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right) \\
&= \underbrace{\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{1}{(1-\beta\lambda_1)}}_{\equiv A}\widehat{\pi}_t^N + \widehat{u}_t^\tau \\
&\quad \underbrace{-\left\{\beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right] - \widetilde{\kappa}\eta\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\right\}}_{\equiv B}\widehat{u}_t^R
\end{aligned}$$

Thus, provided $A \neq 0$, the solution for $\widehat{\pi}_t^N$ is

$$\widehat{\pi}_t^N = \frac{1}{A}(\widehat{d}_{t-1} - \widehat{u}_t^\tau) + \frac{B}{A}\widehat{u}_t^R. \tag{C.26}$$

If $A = 0$, a stationary solution does not exist. In turn, this happens if $\Omega = \Omega_{cut,1}$: the term in brackets in A's definition is the same as the equation that defines $\Omega_{cut,1}$ in (C.13), evaluated at $\phi_\pi = \phi_T = 0$.

Provided $A \neq 0$, from (C.24) we have

$$\begin{aligned}
\widehat{q}_t &= \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\left(\frac{1}{A}\widehat{d}_{t-1} + \frac{B}{A}\widehat{u}_t^R - \frac{1}{A}\widehat{u}_t^\tau\right) - \beta\lambda_1\widehat{u}_t^R \\
&= \underbrace{\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A}}_{\equiv C}\left(\widehat{d}_{t-1} - \widehat{u}_t^\tau\right) + \underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A} - \beta\lambda_1\right]}_{\equiv D}\widehat{u}_t^R.
\end{aligned} \tag{C.27}$$

Moreover, recall that non-traded consumption $\widehat{c}_t^N$ and output $\widehat{y}_t^N$ are proportional to $\widehat{q}_t$, so any conclusions regarding the latter also translate into the former.

Finally, the nominal depreciation is $\widehat{\pi}_t^S = \widehat{q}_t + \widehat{\pi}_t^N - \widehat{q}_{t-1}$, thus

$$\widehat{\pi}_t^S = \underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right]\frac{1}{A}}_{\equiv F}\left(\widehat{d}_{t-1} - \widehat{u}_t^{\tau}\right) + \underbrace{\left[\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B}{A} - \beta\lambda_1\right]}_{\equiv G}\widehat{u}_t^R - \widehat{q}_{t-1} \tag{C.28}$$

Given these results, we next explore the role of Ω. First, we characterize the solution when $\Omega = 0$, which is analogous to a closed economy setup under active fiscal policy, such as Cochrane (2023), Leeper & Leith (2016), among others. Then, we study the case with $\Omega > 0$; our novel contribution.

Case with $\Omega = 0$

The solution simplifies considerably when $\Omega = 0$. For instance,

$$\frac{1}{A} = (1-\beta\lambda_1) > 0$$

where the inequality holds as $\beta, \lambda_1 \in (0,1)$. Also, when $\Omega = 0$, B simplifies to

$$B = \beta - \widetilde{\kappa}\eta\frac{\beta\lambda_1}{1-\beta\lambda_1} = \beta\left(1 - \frac{\widetilde{\kappa}\eta\lambda_1}{1-\beta\lambda_1}\right) = \beta\lambda_1 > 0.$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$ from the solution of the quadratic equation. Therefore,

$$\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R} = \frac{B}{A} > 0, \qquad \frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^{\tau}} = -\frac{1}{A} < 0.$$

In terms of the relative price, $\widehat{q}_t$,

$$C = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A} = \frac{(1-\beta\lambda_1)^2}{\widetilde{\kappa}\eta} > 0$$

$$D = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A} - \beta\lambda_1 = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}(1-\beta\lambda_1)\beta\lambda_1 - \beta\lambda_1 = \frac{\lambda_1}{(1-\lambda_1)}(1-\beta\lambda_1)\beta\lambda_1 - \beta\lambda_1$$

where we used $\widetilde{\kappa}\eta = \frac{(1-\lambda_1)(1-\beta\lambda_1)}{\lambda_1}$ (as λ_1 solves the quadratic equation) and the previously found A and B. This can be further simplified to

$$D = \frac{\beta\lambda_1}{(1-\lambda_1)}\left[\lambda_1(1-\beta\lambda_1) - 1 + \lambda_1\right] = \frac{\beta\lambda_1^2}{(1-\lambda_1)}(1-\beta-\widetilde{\kappa}\eta)$$

where in the last step we have used again the quadratic formula $\beta(\lambda_1)^2 = (1+\beta+\widetilde{\kappa}\eta)\lambda_1 - 1$. Thus, $D < 0$ as long as $\widetilde{\kappa}\eta + \beta > 1$ (which holds in our baseline calibration). Summarizing, we have

$$\begin{aligned} &\frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} < 0 \quad \text{if} \quad \widetilde{\kappa}\eta + \beta > 1, \\ &\frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} \geq 0 \quad \text{if} \quad \widetilde{\kappa}\eta + \beta \leq 1, \end{aligned} \qquad \frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^{\tau}} < 0.$$

In terms of the nominal depreciation, $\widehat{\pi}_t^S$,

$$F = \left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right]\frac{1}{A} > 0$$

$$G = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B}{A} - \beta\lambda_1 = \left(\frac{\lambda_1}{(1-\lambda_1)} + 1\right)\frac{B}{A} - \beta\lambda_1 = \frac{1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1$$

With this, using the solution for B/A in this case, $G > 0$ if

$$(1-\beta\lambda_1)\beta\lambda_1 > \lambda_1\beta(1-\lambda_1) \iff 1-\beta\lambda_1 > 1-\lambda_1 \iff 1 > \beta,$$

which always holds. Summarizing, we have

$$\frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^R} > 0, \qquad \frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^\tau} < 0.$$

The following proposition summarizes the results for monetary shocks

Proposition 3 *Under the NT-based policy setup with $\phi_\pi = 0$ (passive monetary) and $\phi_T = 0$ (active fiscal) and $\Omega = 0$ (only domestic-currency debt), an i.i.d. positive shock to u_t^R leads to a contemporaneous increase in non-traded inflation $\widehat{\pi}_t^N$ and a nominal depreciation $\widehat{\pi}_t^S$. The relative price of tradables $\widehat{q}_t$ falls if $\widetilde{\kappa}\eta + \beta > 1$, otherwise it increases after the shock. Non-traded output $\widehat{y}_t^N$ follows the same pattern as $\widehat{q}_t$.*

Case with $0 < \Omega \leq 1$

The dynamics of non-traded inflation $\widehat{\pi}_t^N$ in the general case are given by,

$$A = \left[1 - \Omega\left(1 + (1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right](1-\beta\lambda_1)^{-1}.$$

Using $\widetilde{\kappa}\eta = (1-\lambda_1)(1-\beta\lambda_1)/\lambda_1$ from the quadratic equation,

$$A = \left[1 - \Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1}.$$

Thus, $A > 0$ as long as

$$\Omega < \frac{1-\lambda_1}{1-\beta\lambda_1} = \Omega_{\text{cut},1}.$$

This is the same threshold value from (C.13), evaluated at $\phi_T = \phi_\pi = 0$. Otherwise, $A < 0$ if $\Omega > \Omega_{\text{cut},1}$. In the cutoff case with $\Omega = \Omega_{\text{cut},1}$, $A = 0$.

Now consider B,

$$B = \beta\Big[1 - \Omega\big(1 + (1-\beta)\lambda_1\big)\Big] - \widetilde{\kappa}\eta\Big[1 - \Omega\Big(1 + (1-\beta)\frac{1-\beta\lambda_1}{\widetilde{\kappa}\eta}\Big)\Big]\frac{\beta\lambda_1}{1-\beta\lambda_1}.$$

The second term can be simplified to

$$\widetilde{\kappa}\eta\frac{\beta\lambda_1}{1-\beta\lambda_1}\Big[1 - \Omega - \Omega(1-\beta)\frac{1-\beta\lambda_1}{\widetilde{\kappa}\eta}\Big] = \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) - \beta\lambda_1\Omega(1-\beta).$$

Substituting back in B,

$$\begin{aligned} B &= \beta\big[1-\Omega(1+(1-\beta)\lambda_1)\big] - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) + \beta\lambda_1\Omega(1-\beta) \\ &= \beta(1-\Omega) - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) = (1-\Omega)\beta\left[1-\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}\right] = (1-\Omega)\beta\lambda_1 \end{aligned}$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$ from the solution of the quadratic equation. Thus, $B \geq 0$, with strict equality if $\Omega = 1$. These results about A and B are enough to characterize $\dfrac{-1}{A}$ and $\dfrac{B}{A}$, which determine the effects of both shocks on $\widehat{\pi}_t^N$. We present below a summary table.

Besides the signs of the effects of both shocks, we also study how these change for different values of Ω. For that, we need $\partial(B/A)/\partial\Omega$ and $\partial(-1/A)/\partial\Omega$. The former is

$$\frac{\partial B/A}{\partial\Omega} = \frac{A\frac{\partial B}{\partial\Omega} - B\frac{\partial A}{\partial\Omega}}{A^2}$$

whose sign is determined by the numerator's

$$A\frac{\partial B}{\partial\Omega} - B\frac{\partial A}{\partial\Omega} = -\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1}\beta\lambda_1 + (1-\Omega)\beta\lambda_1\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)(1-\beta\lambda_1)^{-1}$$

$$= \frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left[-1+\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right) + (1-\Omega)\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right] = \frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left[-1+\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right]$$

$$= \frac{\beta\lambda_1}{(1-\beta\lambda_1)}\frac{(1-\beta)\lambda_1}{(1-\lambda_1)} > 0$$

In turn, the sign of $\frac{\partial(-1/A)}{\partial\Omega}$ depends upon that of

$$\frac{\partial A}{\partial\Omega} = -\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)(1-\beta\lambda_1)^{-1} < 0$$

Finally, notice that these two derivatives are independent of the value of Ω, so they hold regardless of whether Ω is above or below the cutoff value $\Omega_{\text{cut},1}$.

Overall, the response of $\widehat{\pi}_t^N$ to both shocks, and the role of Ω, can be summarized as:

Effects on $\widehat{\pi}_t^N$

	Values of Ω			Derivative
Effect of	$0 \leq \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega < 1$	$\Omega = 1$	w.r.t. Ω
$\widehat{u}_t^R$	+	-	0	+
$\widehat{u}_t^T$	-	+	+	-

Next, regarding $\widehat{q}_t$ we have

$$C = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A},$$

whose sign equals that of $1/A$. In addition,

$$D = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A} - \beta\lambda_1 = \frac{\lambda_1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1.$$

As $B \geq 0$, $D < 0$ if $A < 0$ (i.e. $\Omega > \Omega_{\text{cut},1}$). Instead, if $A > 0$ (i.e. $\Omega < \Omega_{\text{cut},1}$), $D < 0$ if

$$B < \beta(1-\lambda_1)A \iff (1-\Omega)\beta\lambda_1 < \beta(1-\lambda_1)\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1} \iff$$

$$(1-\Omega)\lambda_1(1-\beta\lambda_1) < 1-\lambda_1-\Omega(1-\beta\lambda_1) \iff \Omega(1-\beta\lambda_1)(1-\lambda_1) < 1-\lambda_1-\lambda_1+\beta(\lambda_1)^2.$$

From the quadratic formula $\beta(\lambda_1)^2-\lambda_1+1=\lambda_1(\beta+\widetilde{\kappa}\eta)$, so $D<0$ if

$$\Omega(1-\beta\lambda_1)(1-\lambda_1) < \lambda_1(\beta+\widetilde{\kappa}\eta-1).$$

Thus, if $\widetilde{\kappa}\eta+\beta>1$ (as it holds in our baseline calibration), $D<0$ if

$$\Omega < \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)}{(1-\beta\lambda_1)(1-\lambda_1)} \equiv \Omega_{\text{cut},2}$$

Instead, when $\widetilde{\kappa}\eta+\beta\leq 1$, we have $D<0$ if

$$\Omega < \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)}{(1-\beta\lambda_1)(1-\lambda_1)} \leq 0$$

which does not hold, as we only consider $\Omega\in[0,1]$. Thus $D>0$ if $\Omega<\Omega_{\text{cut},1}$ and $\widetilde{\kappa}\eta+\beta\leq 1$.

We next establish that $\Omega_{\text{cut},2}<\Omega_{\text{cut},1}$:

$$\Omega_{\text{cut},2}-\Omega_{\text{cut},1} = \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)}{(1-\beta\lambda_1)(1-\lambda_1)} - \frac{1-\lambda_1}{1-\beta\lambda_1} = \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)-(1-\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} = \ldots$$
$$\frac{\beta(\lambda_1)^2-2\lambda_1+1-1+2\lambda_1-(\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} = \frac{(\beta-1)(\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} < 0.$$

Overall, we have obtained

$$\begin{array}{llllll} & & & D<0 & \text{if} & \Omega_{\text{cut},1}<\Omega \\ C>0 & \text{if} & \Omega<\Omega_{\text{cut},1} & D>0 & \text{if} & \Omega_{\text{cut},2}<\Omega<\Omega_{\text{cut},1} \text{ and } \widetilde{\kappa}\eta+\beta>1 \\ C<0 & \text{if} & \Omega>\Omega_{\text{cut},1} & D<0 & \text{if} & \Omega<\Omega_{\text{cut},2} \text{ and } \widetilde{\kappa}\eta+\beta>1 \\ & & & D>0 & \text{if} & \Omega<\Omega_{\text{cut},1} \text{ and } \widetilde{\kappa}\eta+\beta\leq 1 \end{array}$$

Finally, the derivatives of both C and D with respect to Ω are those of $\frac{1}{A}$ and $\frac{B}{A}$, respectively, which we have already characterized. We can then summarize the responses of $\widehat{q}_t$ as follows:

Effects on $\widehat{q}_t$

	Values of Ω and $\widetilde{\kappa}\eta+\beta$				
	$0\leq\Omega<\Omega_{\text{cut},2}$				Derivative
Effect of	$\widetilde{\kappa}\eta+\beta>1$	$\widetilde{\kappa}\eta+\beta\leq 1$	$\Omega_{\text{cut},2}<\Omega<\Omega_{\text{cut},1}$	$\Omega_{\text{cut},1}<\Omega$	w.r.t. Ω
$\widehat{u}_t^R$	-	+	+	-	+
$\widehat{u}_t^T$	-	-	-	+	-

In terms of $\widehat{\pi}_t^S$, as the sign of both $\widehat{\pi}_t^N$ and $\widehat{q}_t$ following a fiscal shock $\widehat{u}_t^T$ is the same (independent of Ω), $\widehat{\pi}_t^S$ shares the same sign. The effect of the monetary shock $\widehat{u}_t^R$ also has the same sign on both $\widehat{\pi}_t^N$ and $\widehat{q}_t$ when $\Omega_{\text{cut},1}<\Omega$, the sign of the effect in $\widehat{\pi}_t^S$ is pinned down for those values of Ω. In the case with $\Omega < \Omega_{\text{cut},1}$, recall G can be written as

$$G = \frac{1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1.$$

With this, $G > 0$ if

$$B > \lambda_1\beta(1-\lambda_1)A \iff (1-\Omega)\beta\lambda_1 > \lambda_1\beta(1-\lambda_1)\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1} \iff$$

$$(1-\Omega)(1-\beta\lambda_1) > 1-\lambda_1-\Omega(1-\beta\lambda_1) \iff 1-\beta\lambda_1 > 1-\lambda_1 \iff \lambda_1(1-\beta) > 0,$$

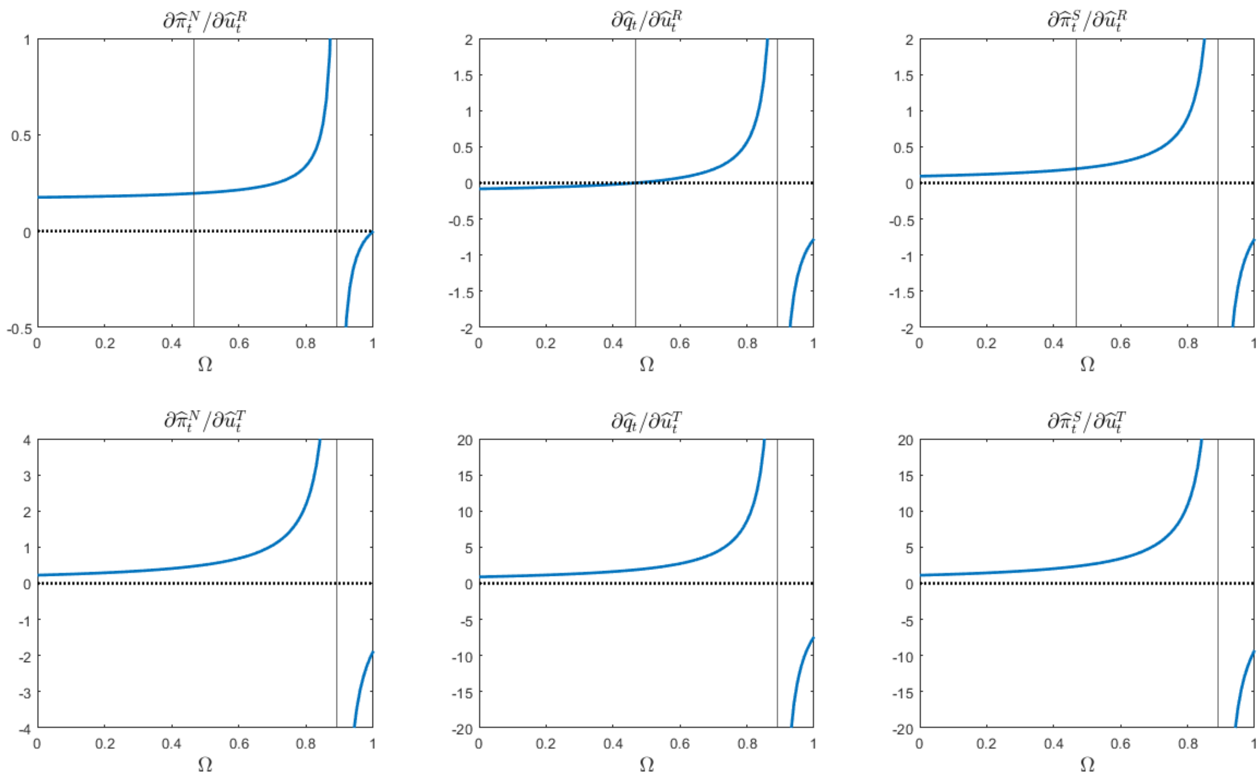
which always holds. Thus, the cutoff $\Omega_{\text{cut},2}$ is irrelevant for $\widehat{\pi}_t^S$. In addition, as with C and D, F and G vary with Ω as $\frac{1}{A}$ and $\frac{B}{A}$ do. Summarizing,

Effects on $\widehat{\pi}_t^S$

	Values of Ω		Derivative
Effect of	$0 \leq \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq 1$	w.r.t. Ω
$\widehat{u}_t^R$	+	-	+
$\widehat{u}_t^T$	-	+	-

Figure 6 below provides a visual summary of the results using the baseline calibration. Finally, as total inflation is a weighted average of $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$, and the sign of the response after either shock on both is the same, aggregate inflation displays the same pattern.

Figure 6: The effect of policy shocks under the simplifying assumption



Note: vertical lines indicate the cutoff values $\Omega_{\text{cut},1}$ and $\Omega_{\text{cut},2}$ (recall $\Omega_{\text{cut},2} < \Omega_{\text{cut},1}$).

D Existence and uniqueness in the CPI-based policy setup

In the baseline setup, the equivalent linearized equations for the government budget constraint (C.3) and the Taylor rule (C.4) are

$$(1-\phi_T)\left[\widehat{d}_{t-1}-\widehat{\pi}_t+\Omega(\widehat{rer}_t+\widehat{\pi}_t-\widehat{\pi}_t^*)\right]=\beta\left[\widehat{d}_t-\widehat{R}_t+\Omega(\widehat{R}_t+\widehat{rer}_t-\widehat{R}_t^*)\right]+\widehat{u}_t^{\tau},$$

$$\widehat{R}_t=\phi_\pi\widehat{\pi}_t+\widehat{u}_t^R.$$

Thus, in the absence of shocks, the relevant system of equations is:

$$\widehat{\pi}_t^N=\beta\widehat{\pi}_{t+1}^N+K\widehat{rer}_t, \tag{D.1}$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\omega(\widehat{\pi}_t^S-\widehat{\pi}_t^N), \tag{D.2}$$

$$\widehat{\pi}_t=\omega(\widehat{\pi}_t^N)+(1-\omega)(\widehat{\pi}_t^S), \tag{D.3}$$

$$\widehat{R}_t=\widehat{\pi}_{t+1}^S, \tag{D.4}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1}-(1-\Omega)\widehat{\pi}_t+\Omega\widehat{rer}_t\right]=\beta\left[\widehat{d}_t-(1-\Omega)\widehat{R}_t+\Omega\widehat{rer}_t\right], \tag{D.5}$$

$$\widehat{R}_t=\phi_\pi\widehat{\pi}_t, \tag{D.6}$$

where in the Phillips curve (D.1) we have defined $K\equiv\widetilde{\kappa}\frac{\eta}{\omega}$.

Combining the equations (D.1)-(D.6) to eliminate $\widehat{\pi}_t$ and $\widehat{R}_t$, we get the following equations characterizing the dynamics of $\widehat{\pi}_t^N, \widehat{rer}_t, \widehat{\pi}_t^S, \widehat{d}_t$:

$$\widehat{\pi}_t^N=\beta\widehat{\pi}_{t+1}^N+K\widehat{rer}_t, \tag{D.7}$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\omega(\widehat{\pi}_t^S-\widehat{\pi}_t^N), \tag{D.8}$$

$$\phi_\pi\left[\omega\widehat{\pi}_t^N+(1-\omega)\widehat{\pi}_t^S\right]=\widehat{\pi}_{t+1}^S, \tag{D.9}$$

$$\widehat{d}_t=\frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}-(1-\Omega)\left[\frac{(1-\phi_T)}{\beta}-\phi_\pi\right]\left[\omega\widehat{\pi}_t^N+(1-\omega)\widehat{\pi}_t^S\right]+\Omega\left[\frac{(1-\phi_T)}{\beta}-1\right]\widehat{rer}_t, \tag{D.10}$$

Here, $\widehat{rer}_t$ and $\widehat{d}_t$ are predetermined/state endogenous variables, while $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$ are non-predetermined/jumping. Local equilibrium uniqueness thus requires 2 stable and 2 non-stable eigenvalues. If there are more than 2 stable eigenvalues, there are multiple local equilibria. Otherwise, there is no locally stationary equilibrium.

These equations can be re-arranged as

$$\widehat{\pi}_{t+1}^S=\phi_\pi\omega\widehat{\pi}_t^N+\phi_\pi(1-\omega)\widehat{\pi}_t^S \tag{D.11}$$

$$\widehat{\pi}_{t+1}^N=\frac{1}{\beta}(1+\omega K)\widehat{\pi}_t^N-\frac{1}{\beta}K\widehat{rer}_{t-1}-\frac{1}{\beta}\omega K\widehat{\pi}_t^S, \tag{D.12}$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\omega\widehat{\pi}_t^S-\omega\widehat{\pi}_t^N, \tag{D.13}$$

$$\widehat{d}_t = \frac{1-\phi_T}{\beta}\widehat{d}_{t-1} + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\widehat{rer}_{t-1} - \omega\left[\frac{1-\phi_T}{\beta} - (1-\Omega)\phi_\pi - \Omega\right]\widehat{\pi}_t^N - ...$$
$$- \left[(1-\Omega-\omega)\frac{1-\phi_T}{\beta} - (1-\Omega)\phi_\pi(1-\omega) + \Omega\omega\right]\widehat{\pi}_t^S \quad \text{(D.14)}$$

Defining $w_t \equiv [\widehat{\pi}_t^S, \widehat{\pi}_t^N, \widehat{rer}_{t-1}, \widehat{d}_{t-1}]'$, the system can be written as

$$w_{t+1} = \Psi w_t$$

where

$$\Psi = \begin{pmatrix} \Psi_{\pi^S\pi^S} & \Psi_{\pi^S\pi^N} & 0 & 0 \\ \Psi_{\pi^N\pi^S} & \Psi_{\pi^N\pi^N} & \Psi_{\pi^N rer} & 0 \\ \Psi_{rer\pi^S} & \Psi_{rer\pi^N} & \Psi_{rerrer} & 0 \\ \Psi_{d\pi^S} & \Psi_{d\pi^N} & \Psi_{drer} & \Psi_{dd} \end{pmatrix}$$

Given that the matrix Ψ is a block lower triangular matrix of the form

$$\Psi = \begin{pmatrix} \Psi_{11} & 0 \\ \Psi_{21} & \Psi_{22} \end{pmatrix},$$

and thus $det(\Psi - \lambda I) = det(\Psi_{11} - \lambda I) \times det(\Psi_{22} - \lambda I)$. This means that we can analyze the eigenvalues of Ψ_{11} and Ψ_{22} separately. The eigenvalue of Ψ_{22} is simply equal to $\frac{1-\phi_T}{\beta}$, which is less than one if $1 - \beta < \phi_T$.

In terms of Ψ_{11}, notice that

$$det(\Psi_{11} - \lambda I) = \begin{vmatrix} \Psi_{\pi^S\pi^S} - \lambda & \Psi_{\pi^S\pi^N} & 0 \\ \Psi_{\pi^N\pi^S} & \Psi_{\pi^N\pi^N} - \lambda & \Psi_{\pi^N rer} \\ \Psi_{rer\pi^S} & \Psi_{rer\pi^N} & \Psi_{rerrer} - \lambda \end{vmatrix} =$$

$$(\Psi_{\pi^S\pi^S} - \lambda)\left[(\Psi_{\pi^N\pi^N} - \lambda)(\Psi_{rerrer} - \lambda) - \Psi_{rer\pi^N}\Psi_{\pi^N rer}\right] - (\Psi_{\pi^S\pi^N})\left[\Psi_{\pi^N\pi^S}(\Psi_{rerrer} - \lambda) - \Psi_{rer\pi^S}\Psi_{\pi^N rer}\right] =$$

Thus, we need to compute the solutions to,

$$\lambda^3 \underbrace{- \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)}_{\equiv A_2}\lambda^2 + \underbrace{\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi\frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right)}_{\equiv A_1}\lambda \underbrace{-\phi_\pi\frac{(1-\omega)}{\beta}}_{\equiv A_0} = 0$$

In other words, the characteristic equation of matrix Ψ_{11} takes the following form:

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

We are interested in determining *sufficient conditions* under which this polynomial has either:

- Exactly **two roots are outside** the unit circle (i.e., $|\lambda| > 1$), and **one is inside** ($|\lambda| < 1$).
- Exactly **two roots are inside** the unit circle, and **one is outside**.

To determine these conditions, we follow Woodford (2003, App. C).

Sufficient Conditions for 2 Roots Outside and 1 Root Inside

Any one of the following sets of conditions is sufficient to ensure this root configuration:

Condition Set 1:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set 2:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set 3:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

Sufficient Conditions for 2 Roots Inside and 1 Root Outside

Any one of the following sets of conditions is sufficient to ensure this alternative configuration:

Condition Set A:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set B:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set C:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

D.1 Case I: $\phi_\pi > 1$

We need 2 roots outside and 1 root inside the unit circle. First, we examine the first condition of each set:

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\,\frac{\omega K}{\beta} > 0,$$

if $\phi_\pi > 1$. Thus, we now need to prove that:

$$-1 + A_2 - A_1 + A_0 < 0$$

where

$$-1 + A_2 - A_1 + A_0 = -2\left(1 + \frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\phi_\pi(1-\omega)}{\beta}\right) - (1+\phi_\pi)\frac{\omega K}{\beta} < 0$$

which is negative if for $\phi_\pi > 0$.

Finally, to show that the sufficient conditions for the existence of two roots outside and one root inside the unit circle are satisfied, we need to prove that:

$$A_2^2 - A_0A_2 + A_1 - 1 > 0,$$

or,

$$A_2^2 - A_0A_2 + A_1 - 1 < 0, \text{ and } |A_2| > 3.$$

We have

$$A_2^2 - A_0A_2 + A_1 - 1 = \left[-\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)\right]^2 - \phi_\pi \frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)$$
$$+ \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi \frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right) - 1.$$

Therefore, $A_2^2 - A_0A_2 + A_1 - 1$ takes the form of a quadratic equation in ϕ_π:

$$A_2^2 - A_0A_2 + A_1 - 1 = a\phi_\pi^2 + b\phi_\pi + c,$$

where $a = (1-\omega)^2(1-\frac{1}{\beta}) < 0$. In particular, this means that this quadratic equation is strictly concave in ϕ_π; a result that we will use momentarily.

Given the alternative conditions listed above, it is sufficient to show that there is no $\phi_\pi > 1$ such that both $A_2^2 - A_0A_2 + A_1 - 1 < 0$ and $|A_2| \leq 3$ hold simultaneously. This can be shown as follows. First, notice that

$$|A_2| = \frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1.$$

Since $\beta \in (0,1)$, $1 + \frac{1}{\beta} > 2$, thus to prove $|A_2| > 3$, it is sufficient to show that:

$$(1-\omega)\phi_\pi + \frac{\omega K}{\beta} \geq 1$$

which holds if

$$\phi_\pi \geq \frac{1-\omega\frac{K}{\beta}}{1-\omega}.$$

Instead, to obtain $A_2^2 - A_0A_2 + A_1 - 1 > 0$, we must show that:

$$\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)^2 - \phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right) + ...$$
$$+ \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi\frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right) - 1 > 0$$

Which can equivalently be written as:

$$\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)^2 - \phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right) + ...$$
$$+ \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right) - \frac{\omega K}{\beta} - 1 + \phi_\pi\frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta} - 1 > 0$$

Upon dividing by A_2, the expression can be reexpressed as:

$$\left(\frac{1}{\beta}+(1-\omega)\phi_\pi+\frac{\omega K}{\beta}+1\right)-\phi_\pi\frac{(1-\omega)}{\beta}+1+\frac{(\phi_\pi-1)\frac{\omega K}{\beta}+\phi_\pi\frac{1}{\beta}(1-\omega)}{\frac{1}{\beta}+(1-\omega)\phi_\pi+\frac{\omega K}{\beta}+1}>\frac{2}{\frac{1}{\beta}+(1-\omega)\phi_\pi+\frac{\omega K}{\beta}+1}$$

As the right-hand side is strictly less than 1, it is sufficient to prove that:

$$\left(\frac{1}{\beta}+(1-\omega)\phi_\pi+\frac{\omega K}{\beta}+1\right)^2-\phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta}+(1-\omega)\phi_\pi+\frac{\omega K}{\beta}+1\right)+\ldots$$
$$(\phi_\pi-1)\frac{\omega K}{\beta}+\frac{\phi_\pi}{\beta}(1-\omega) \quad \text{(D.15)}$$

is positive.

For cases in which $\frac{1-\frac{\omega K}{\beta}}{1-\omega}<1$, then

$$|A_2|>3 \quad \forall\, \phi_\pi\in(1,+\infty)$$

In contrast, for cases with $\phi_\pi>1$ and $\frac{1-\frac{\omega K}{\beta}}{1-\omega}\geq 1$, we know that for $\phi_\pi\in\left(1,\frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, it is not guaranteed that $|A_2|>3$. Thus, we must show that within this interval, the condition $A_2^2-A_0A_2+A_1-1>0$ holds (i.e. that (D.15) is positive). Notice that, if $\phi_\pi=1$,

$$\left(\frac{1}{\beta}+(1-\omega)+\frac{\omega K}{\beta}+1\right)^2-\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta}+(1-\omega)+\frac{\omega K}{\beta}+1\right)+\frac{1}{\beta}(1-\omega)=$$
$$=(1-\omega)+\frac{\omega K}{\beta}+1+\frac{\omega}{\beta}+\frac{1}{\beta}\frac{(1-\omega)}{\frac{1}{\beta}+(1-\omega)+\frac{\omega K}{\beta}+1}>0.$$

Instead, if $\phi_\pi=\frac{1-\frac{\omega K}{\beta}}{1-\omega}$,

$$\left(\frac{1}{\beta}+(1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega}+\frac{\omega K}{\beta}+1\right)^2-\frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta}+(1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega}+\frac{\omega K}{\beta}+1\right)$$
$$+\left(\frac{1-\frac{\omega K}{\beta}}{1-\omega}-1\right)\frac{\omega K}{\beta}+\frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}=$$
$$=2+\frac{\omega K}{\beta^2}+\left[\left(\frac{1-\frac{\omega K}{\beta}}{1-\omega}-1\right)\frac{\omega K}{\beta}+\frac{1}{\beta}-\frac{\omega K}{\beta^2}\right]\frac{1}{\frac{1}{\beta}+2}>0$$

We can now establish the stability conditions for all values of $\phi_\pi>1$. The expression $A_2^2-A_0A_2+A_1-1$ has been shown to be a strictly concave function of ϕ_π (i.e., a downward-opening parabola). Also, on the interval where $\phi_\pi\in\left(1,\frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, which is the region where the condition $|A_2|>3$ is not guaranteed, the function equals a strictly positive value at the boundary points of this interval (i.e., at $\phi_\pi=1$ and $\phi_\pi=\frac{1-\frac{\omega K}{\beta}}{1-\omega}$). A fundamental property of strictly concave functions dictates that:

Lemma D.1 *Let f be a strictly concave function on the closed interval $[a,b]$. If $f(a)>0$ and $f(b)>0$, then $f(x)>0$ for all x in the open interval (a,b).*

Then, it follows that $A_2^2 - A_0 A_2 + A_1 - 1 > 0$ holds for all ϕ_π in the specified range.

We have shown that, for the case where $\phi_\pi > 1$, the sufficient conditions are satisfied to be in a scenario with two roots outside the unit circle and one inside.[16] Given that $\widehat{d}_t$ and $\widehat{rer}_t$ are predetermined variables, and $\widehat{\pi}_t^S$ and $\widehat{\pi}_t^N$ are jumping variables, we require two eigenvalues with a modulus of less than one and two with a modulus greater than one in order to ensure the *local uniqueness of equilibrium*. This implies that the remaining eigenvalue must lie inside the unit circle:

$$det(\Psi_{22} - \lambda I) = |\Psi_{dd} - \lambda|$$

Thus, $\lambda_4 = \Psi_{dd} = \frac{(1-\phi_T)}{\beta}$ is in modulus less than one if $\phi_T > 1 - \beta$.

D.2 Case II: $\phi_\pi < 1$

Now, we need 2 roots inside and 1 root outside the unit circle. In contrast to the previous case, we now have that:

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\frac{\widetilde{\kappa}}{\beta}\frac{\eta}{\omega} < 0$$

As previously stated, for $\phi_\pi > 0$ we know that

$$-1 + A_2 - A_1 + A_0 < 0$$

Now, for the case where $\phi_\pi \in (0, 1)$, it is sufficient to prove that for all the intervals

$$A_2^2 - A_0 A_2 + A_1 - 1 > 0.$$

If $\phi_\pi = 0$, this is

$$\left(\frac{1}{\beta} + \frac{\omega K}{\beta} + 1\right)^2 + \frac{1}{\beta} - 1 > 0,$$

since $\beta < 1$.

We previously proved that the quadratic function is positive when ϕ_π is valued at 1. Therefore, given that the quadratic function is a strictly concave function, we can state that:

$$A_2^2 - A_0 A_2 + A_1 - 1 > 0 \quad \forall\, \phi_\pi \in (0, 1)$$

Therefore, we have shown that when $\phi_\pi < 1$, the sufficient conditions are met to establish the existence of two roots inside the unit circle and one outside. Accordingly, the remaining eigenvalue must lie outside the unit circle, which holds if $\phi_T < 1 - \beta$.

D.3 Summary

Overall, we have shown that the same necessary conditions for existence and uniqueness identified for the NT-based policy setup also hold under the CPI-based setup. However, deriving an algebraic cutoff value equivalent to $\Omega_{cut,1}$ in (C.13) when $\theta > 0$ is more cumbersome in this case. Still, one can numerically verify the existence of such a unique knife-edge case, relevant when $\phi_T < 1 - \beta$ and $\phi_\pi < 1$, satisfying $\Omega_{cut,1} \in (0, 1)$. Instead, if $\theta_N = 0$, $\Omega_{cut,1} = 1$ still holds. Under flexible prices,

[16] This holds whether $A_2^2 - A_0 A_2 + A_1 - 1 > 0$ or $A_2^2 - A_0 A_2 + A_1 - 1 < 0$; we avoid discussion of certain non-generic boundary cases.

$\widehat{rer}_t = 0$ and $\widehat{\pi}_t^N = \widehat{\pi}_t^S$, we are left with

$$\widehat{\pi}_{t+1}^N = \phi_\pi \widehat{\pi}_t^N, \tag{D.16}$$

$$\widehat{d}_t = (1-\Omega)\left[\phi_\pi - \frac{(1-\phi_T)}{\beta}\right]\pi_t^N + \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}, \tag{D.17}$$

As these are analogous to those from the NT-based policy setup when $\theta_N = 0$, the same conclusions hold. In particular, if $\Omega = 1$ the solution is not stable if $\phi_T < 1-\beta$ and $\phi_\pi < 1$.

E Model with long-term debt

Following Woodford (2001), we consider a bond in local currency that promises to pay coupons for the rest of its life at a declining rate. In particular, a bond issued at time t promises the payment sequence $x,\ x\delta,\ x\delta^2, \ldots$ for, respectively, periods $t+1, t+2, t+3, \ldots$, with the duration parameter $0 \le \delta < \beta^{-1}$ and the scale parameter $x > 0$.[17] Under this setup, the stock of bond holdings by households at the end of period t can be summarized by a single variable, called $\widetilde{B}_t$, and let Z_t be the price of new debt issued at t.[18] For dollar-denominated debt, we use an analogous structure, with parameters x^* and δ^*, stock of households' debt $\widetilde{D}_t^{H*}$, and price Z_t^*.

If we consider both long-term and one-period bonds, the household's budget constraint reads,

$$P_t c_t + S_t\left[D_{t-1}^{H*} + (x^* + \delta^* Z_t^*)\widetilde{D}_{t-1}^{H*}\right] + \frac{B_t}{R_t} + Z_t\widetilde{B}_t + T_t = \ldots$$
$$W_t h_t + \Sigma_t + B_{t-1} + (x+\delta Z_t)\widetilde{B}_{t-1} + S_t\left(\frac{D_t^{H*}}{R_t^*} + Z_t^*\widetilde{D}_t^{H*}\right).$$

The optimality conditions related to financial assets include,

$$(c_t)^{-\sigma} = \beta R_t E_t\left\{\frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}}\right\}, \qquad (c_t)^{-\sigma} = \beta R_t^* E_t\left\{\frac{(c_{t+1})^{-\sigma}\pi_{t+1}^S}{\pi_{t+1}}\right\},$$

$$(c_t)^{-\sigma} = \beta\frac{1}{Z_t}E_t\left\{(x+\delta Z_{t+1})\frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}}\right\}, \qquad (c_t)^{-\sigma} = \beta\frac{1}{Z_t^*}E_t\left\{(x^*+\delta^* Z_{t+1}^*)\frac{(c_{t+1})^{-\sigma}\pi_{t+1}^S}{\pi_{t+1}}\right\}.$$

In steady state, we have $R = \frac{x+\delta Z}{Z}$, $R^* = \frac{x^*+\delta^* Z^*}{Z^*}$.[19] In what follows, we assume there are no short term assets in equilibrium (i.e. $B_t = D_t^{H*} = 0$).

Using analogous notation, the government's budget constraint reads:

$$(x+\delta Z_t)\widetilde{D}_{t-1}^G + S_t(x^*+\delta^* Z_t^*)\widetilde{D}_{t-1}^{G*} = Z_t\widetilde{D}_t^G + S_t Z_t^*\widetilde{D}_t^{G*} + T_t$$

In equilibrium, $\widetilde{D}_t^G = \widetilde{B}_t$. Using both budget constraints along with other market clearing conditions and defining $\widetilde{D}_t^* \equiv \widetilde{D}_t^{H*} + \widetilde{D}_t^{G*}$, we are left with the following balance of payments evolution, in real terms,

$$c_t^T + (x^*+\delta^* Z_t^*)\frac{\widetilde{d}_{t-1}^*}{\pi_t^*} = y_t^T + Z_t^*\widetilde{d}_t^*,$$

[17] Note that $\delta = 0, x = 1$ corresponds to a one-period bond. More generally, under a constant one-period real interest rate r, duration equals $\frac{1+r}{1+r-\delta}$, increasing in δ.

[18] The stock $\widetilde{B}_t$ evolves over time as $\widetilde{B}_t = \delta\widetilde{B}_{t-1} + N_t$: the sum of the amount that would carry over to the following period from the previous-period holdings ($\delta\widetilde{B}_{t-1}$), plus the new purchases of long-term bonds N_t.

[19] We calibrate $x = (R-\delta)/R$ and $x^* = (R^*-\delta^*)/R^*$ to get $Z = 1/R$, $Z^* = 1/R^*$, $x+\delta Z = 1$, and $x^*+\delta^* Z^* = 1$.

where $\widetilde{d}_t^* \equiv \widetilde{D}_t^*/P_t^*$. We specify the premium/closing-device as a function of $\widetilde{d}_t^*$.

Under the CPI-policy setup, let $\widetilde{d}_t^G \equiv \widetilde{D}_t^G/P_t$ and $\widetilde{d}_t^{G*} \equiv \widetilde{D}_t^{G*}/P_t^*$. The government's budget constraint is

$$(x+\delta Z_t)\frac{\widetilde{d}_{t-1}^G}{\pi_t} + rer_t(x^*+\delta^* Z_t^*)\frac{\widetilde{d}_{t-1}^{G*}}{\pi_t^*} = Z_t\widetilde{d}_t^G + rer_t Z_t^* \widetilde{d}_t^{G*} + \tau_t. \tag{D.18}$$

Letting $\Omega = \frac{\overline{rer}\ \widetilde{d}_t^{G*}}{\widetilde{d}_t^G + \overline{rer}\ \widetilde{d}_t^{G*}}$ and $\widetilde{d}_t \equiv \widetilde{d}_t^G + \overline{rer}\ \widetilde{d}_t^{G*}$, the constraint is

$$\left[(x+\delta Z_t)\frac{(1-\Omega)}{\pi_t} + (x^*+\delta^* Z_t^*)\frac{rer_t}{\overline{rer}}\frac{\Omega}{\pi_t^*}\right]\widetilde{d}_{t-1} = \left[Z_t(1-\Omega) + Z_t^*\frac{rer_t}{\overline{rer}}\Omega\right]\widetilde{d}_t + \tau_t.$$

Instead, under the NT-based policy setup, defining $\widetilde{d}_t^G \equiv \widetilde{D}_t^G/P_t^N$, the constraint is

$$(x+\delta Z_t)\frac{\widetilde{d}_{t-1}^G}{\pi_t^N} + q_t(x^*+\delta^* Z_t^*)\frac{\widetilde{d}_{t-1}^{G*}}{\pi_t^*} = Z_t\widetilde{d}_t^G + q_t Z_t^* \widetilde{d}_t^{G*} + \tau_t,$$

or, using $\Omega = \frac{\bar{q}\widetilde{d}_t^{G*}}{\widetilde{d}_t^G + \bar{q}\widetilde{d}_t^{G*}}$, and $\widetilde{d}_t \equiv \widetilde{d}_t^G + \bar{q}\ \widetilde{d}_t^{G*}$,

$$\left[(x+\delta Z_t)\frac{(1-\Omega)}{\pi_t^N} + (x^*+\delta^* Z_t^*)\frac{q_t}{\bar{q}}\frac{\Omega}{\pi_t^*}\right]\widetilde{d}_{t-1} = \left[Z_t(1-\Omega) + Z_t^*\frac{q_t}{\bar{q}}\Omega\right]\widetilde{d}_t + \tau_t.$$

E.1 FTPL equation

Here we derive the life-time government constraint. Starting from (D.18), add and subtract

$$\frac{1}{rr_{t,t+1}}\left[(x+\delta Z_{t+1})\frac{\widetilde{d}_t^G}{\pi_{t+1}} + rer_{t+1}(x^*+\delta^* Z_{t+1}^*)\frac{\widetilde{d}_t^{G*}}{\pi_{t+1}^*}\right],$$

where $rr_{t,t+1}$ is defined as in the short-term case. Letting $a_t^G \equiv (x+\delta Z_t)\frac{\widetilde{d}_{t-1}^G}{\pi_t} + rer_t(x^*+\delta^* Z_t^*)\frac{\widetilde{d}_{t-1}^{G*}}{\pi_t^*}$, (D.18) can be written as

$$a_t^G = \frac{a_{t+1}^G}{rr_{t,t+1}} + \tau_t + so_{t+1},$$

where $so_{t+1} \equiv \left[Z_t - \frac{(x+\delta Z_{t+1})}{rr_{t,t+1}\pi_{t+1}}\right]\widetilde{d}_t^G + \left[Z_t^* rer_t - \frac{(x^*+\delta^* Z_{t+1}^*)rer_{t+1}}{\pi_{t+1}^* rr_{t,t+1}}\right]\widetilde{d}_t^{G*}$ captures higher-order terms related to deviations from perfect-foresight non-arbitrage conditions. This has the same form as that with one-period debt, so we can iterate it forward in a similar fashion to obtain (replacing back a_t^G)

$$(x+\delta Z_t)\frac{\widetilde{d}_{t-1}^G}{\pi_t} + rer_t(x^*+\delta^* Z_t^*)\frac{\widetilde{d}_{t-1}^{G*}}{\pi_t^*} = E_t\left\{\sum_{j=0}^{\infty}\frac{\tau_{t+j}}{rr_{t,t+j}}\right\} + \text{h.o.t.}$$

Finally, using the definition for Ω, we obtain equation (15) in the main text.

E.2 Log-linearization under the NT-setup

To log-linearize, define $A_t \equiv \left[(x+\delta Z_t)\frac{(1-\Omega)}{\pi_t^N} + (x^*+\delta^* Z_t^*)\frac{q_t}{\bar{q}}\frac{\Omega}{\pi_t^*}\right]$, $B_t \equiv \left[Z_t(1-\Omega) + Z_t^*\frac{q_t}{\bar{q}}\Omega\right]$, $\widehat{A}_t \equiv \log(A_t/\overline{A})$, and $\widehat{B}_t \equiv \log(B_t/\overline{B})$. Notice that, in a steady state with $\overline{\pi} = \overline{\pi}^* = \overline{\pi}^S = 1$, then $R = R^* =$

$1/\beta$, and recall in our calibration $Z = Z^* = 1/R$ and $x + \delta Z = x^* + \delta^* Z^*$, so we have

$$\overline{A} = 1, \quad \overline{B} = \beta, \quad \overline{d}(1-\beta) = \overline{\tau}.$$

This leads us to the same log-linearized budget constraint as in the short-term debt case:

$$\widehat{A}_t + \widehat{d}_{t-1} = \beta(\widehat{B}_t + \widehat{d}_t) + (1-\beta)\widehat{\tau}_t,$$

and, using the rule for lump-sum taxes, we get

$$(1-\phi_T)(\widehat{d}_{t-1} + \widehat{A}_t) = \beta(\widehat{d}_t + \widehat{B}_t) + \widehat{u}_t^{\tau}.$$

The difference is, of course, the expressions for $\widehat{A}_t$ and $\widehat{B}_t$, which in this case are:

$$\begin{aligned}
\widehat{A}_t = &\quad (1-\Omega)(-\widehat{\pi}_t^N + \delta\beta\widehat{Z}_t) + \Omega(\widehat{q}_t - \widehat{\pi}_t^* + \delta^*\beta\widehat{Z}_t^*), \\
\widehat{B}_t = &\quad (1-\Omega)\widehat{Z}_t + \Omega(\widehat{q}_t + \widehat{Z}_t^*).
\end{aligned}$$

Thus, the log-linearized version of the government debt's evolution can be written as:

$$\begin{aligned}
(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \delta\beta\widehat{Z}_t + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \delta\beta\widehat{Z}_t - \widehat{\pi}_t^* + \delta^*\beta\widehat{Z}_t^*)\right] = ... \\
\beta\left[\widehat{d}_t + \widehat{Z}_t + \Omega(-\widehat{Z}_t + \widehat{q}_t + \widehat{Z}_t^*)\right] + \widehat{u}_t^{\tau},
\end{aligned}$$

Besides this, the UIP for short-term rates still holds

$$\widehat{R}_t = \widehat{R}_t^* + E_t\left\{\widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^*\right\}.$$

In addition, from the household's optimality conditions, we obtain the log-linear expressions

$$\widehat{R}_t = \delta\beta E_t\{\widehat{Z}_{t+1}\} - \widehat{Z}_t, \quad \widehat{R}_t^* = \delta^*\beta E_t\{\widehat{Z}_{t+1}^*\} - \widehat{Z}_t^*.$$

Overall, adding the Taylor rule, the linearized non-tradable and policy blocks with long-term debt under the NT-policy setup is reduced to the following system of equations:

$$\begin{aligned}
\widehat{\pi}_t^N &= \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\} + \widetilde{\kappa}\left(\eta\widehat{q}_t + \widehat{c}_t^T\right), \\
\widehat{R}_t &= \widehat{R}_t^* + E_t\left\{\widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^*\right\}, \\
\widehat{R}_t &= \delta\beta E_t\{\widehat{Z}_{t+1}\} - \widehat{Z}_t, \\
\widehat{R}_t &= \phi_\pi\widehat{\pi}_t^N + \widehat{u}_t^R. \\
(1-\phi_T)&\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \delta\beta\widehat{Z}_t + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \delta\beta\widehat{Z}_t - \widehat{\pi}_t^* + \delta^*\beta\widehat{Z}_t^*)\right] = ... \\
&\beta\left[\widehat{d}_t + \widehat{Z}_t + \Omega(-\widehat{Z}_t + \widehat{q}_t + \widehat{Z}_t^*)\right] + \widehat{u}_t^{\tau},
\end{aligned}$$

Finally, notice that, as in the short-term debt case, tradable-related variables $c_t^T, \widetilde{d}_t^*, R_t^*, Z_t^*$ are still determined by a system of equations that only depends on external exogenous variables, independent from the non-traded sector, as well as from monetary and fiscal policy. The only difference is the evolution of foreign debt previously derived, plus the equation linking Z_t^* and R_t^*.

E.3 Existence and uniqueness

The determination of a (locally) stationary equilibrium in the version with long-term debt is governed by the equations characterizing the deterministic evolution of the system:

$$\widehat{\pi}_t^N = \beta\widehat{\pi}_{t+1}^N + \widetilde{\kappa}\eta\widehat{q}_t,$$

$$\phi_\pi\widehat{\pi}_t^N = \widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N,$$

$$\phi_\pi\widehat{\pi}_t^N = \delta\beta\widehat{Z}_{t+1} - \widehat{Z}_t,$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \delta\beta\widehat{Z}_t + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \delta\beta\widehat{Z}_t)\right] = \beta\left[\widehat{d}_t + \widehat{Z}_t + \Omega(-\widehat{Z}_t + \widehat{q}_t)\right],$$

These can be re-arranged as

$$\widehat{\pi}_{t+1}^N = \frac{1}{\beta}\widehat{\pi}_t^N - \frac{\widetilde{\kappa}\eta}{\beta}\widehat{q}_t, \qquad \widehat{q}_{t+1} = \left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t^N + \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right)\widehat{q}_t, \qquad \widehat{Z}_{t+1} = \frac{\phi_\pi}{\delta\beta}\widehat{\pi}_t^N + \frac{1}{\delta\beta}\widehat{Z}_t,$$

$$\widehat{d}_t = (\Omega-1)\frac{1-\phi_T}{\beta}\widehat{\pi}_t^N + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\widehat{q}_t + (1-\Omega)\left[(1-\phi_T)\delta - 1\right]\widehat{Z}_t + \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}.$$

Defining $w_t \equiv [\widehat{\pi}_t^N, \widehat{q}_t, \widehat{Z}_t, \widehat{d}_{t-1}]'$, this system can be written as

$$w_{t+1} = \Phi w_t$$

where Φ has the form

$$\Phi \equiv \begin{pmatrix} \frac{1}{\beta} & -\frac{\widetilde{\kappa}\eta}{\beta} & 0 & 0 \\ \left(\phi_\pi - \frac{1}{\beta}\right) & \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right) & 0 & 0 \\ \phi_\pi(\delta\beta)^{-1} & 0 & (\delta\beta)^{-1} & 0 \\ (\Omega-1)\frac{1-\phi_T}{\beta} & \Omega\left(\frac{1-\phi_T-\beta}{\beta}\right) & (1-\Omega)\left[(1-\phi_T)\delta - 1\right] & \frac{(1-\phi_T)}{\beta} \end{pmatrix}$$

It should be noted that Φ has a block lower-triangular structure of the form:

$$\Phi = \begin{pmatrix} \Phi_{11} & 0 \\ \Phi_{21} & \Phi_{22} \end{pmatrix}.$$

This implies that the eigenvalues of Φ_{11} and Φ_{22} can be examined independently. That of Φ_{22} is simply given by $\frac{1-\phi_T}{\beta}$, which is less than one if $1-\beta < \phi_T$, as in the one-period debt case.

The matrix Φ_{11} also features a block lower-triangular structure:

$$\Phi_{11} = \begin{pmatrix} \Phi_{AA} & 0 \\ \Phi_{BA} & \Phi_{BB} \end{pmatrix},$$

where the eigenvalue of Φ_{BB} is $(\delta\beta)^{-1}$. As we assume $0 \leq \delta < \beta^{-1}$, so $\Phi_{BB} > 1$; this corresponds to the additional jumping variable $\widehat{Z}_t$. Moreover, Φ_{AA} is identical to the matrix Ψ_{11} from Appendix C.2. This also implies that the root λ_1 that solves the relevant quadratic equation is the same in this version with long-term debt.

Therefore, the analysis of local equilibrium existence and uniqueness conducted for the short-term debt case holds here as well. In particular, we have a knife-edge value of Ω for which a stationary equilibrium does not exist. In fact, the specific value is independent of debt duration, as we show below.

E.4 The effects of policy shocks

The dynamics with long-term debt differ from the one-period case if fiscal policy matters for equilibrium determination. Thus, results under $\phi_\pi > 1$ and $\phi_T > 1 - \beta$ (active monetary, passive fiscal) are omitted here, and we focus on the special case of active fiscal, passive monetary policy $\phi_\pi = \phi_T = 0$.

In such a case, the system of linearized equations for the non-tradable and policy blocks with long-term debt, considering exclusively policy shocks, is

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa}\eta \widehat{q}_t, \tag{D.19}$$

$$\widehat{u}_t^R + \widehat{q}_t = E_t \left\{ \widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N \right\}, \tag{D.20}$$

$$\widehat{u}_t^R + \widehat{Z}_t = \delta\beta E_t \{ \widehat{Z}_{t+1} \}, \tag{D.21}$$

$$\widehat{d}_{t-1} - \widehat{\pi}_t^N + \delta\beta\widehat{Z}_t + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \delta\beta\widehat{Z}_t) = \beta \left[\widehat{d}_t + \widehat{Z}_t + \Omega(-\widehat{Z}_t + \widehat{q}_t) \right] + \widehat{u}_t^\tau. \tag{D.22}$$

Notice that these are akin to those in Appendix C.3, with an extra variable ($\widehat{Z}_t$) and an additional equation (In particular, if $\delta = 0$, we have $\widehat{Z}_t = -\widehat{u}_t^R$, and we recover the one-period debt system).

As we assume $\beta\delta < 1$, iterating forward (D.21) and imposing stationarity yield,

$$\widehat{Z}_t = -\sum_{i=0}^{\infty} (\delta\beta)^i E_t \left\{ \widehat{u}_{t+i}^R \right\}.$$

With i.i.d. shocks, $E_t \left\{ \widehat{u}_{t+i}^R \right\} = 0$ for $i > 0$, which implies

$$\widehat{Z}_t = -\widehat{u}_t^R.$$

Therefore, the system of equations collapses to:

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa}\eta \widehat{q}_t, \tag{D.23}$$

$$\widehat{u}_t^R + \widehat{q}_t = E_t \left\{ \widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N \right\}, \tag{D.24}$$

$$\widehat{d}_{t-1} = \beta\widehat{d}_t + (1-\Omega)\widehat{\pi}_t^N - \Omega(1-\beta)\widehat{q}_t - (1-\Omega)\beta(1-\delta)\widehat{u}_t^R + \widehat{u}_t^\tau. \tag{D.25}$$

This structure is analogous to the one period debt case, with one difference: $\widehat{u}_t^R$ appears in the last equation multiplied by $-(1-\Omega)\beta(1-\delta)$ instead of $-(1-\Omega)\beta$ in the one period case. However, as this is the only place in which δ plays a role, a corollary is that the transmission of the shock $\widehat{u}_t^\tau$ will be the same as in the case with only one-period debt. This is intuitive, as we are analyzing the case with $\phi_T = \phi_\pi = 0$.

To obtain the solution, we proceed as in Appendix C.3. In particular, write (D.25) as

$$\widehat{d}_{t-1} = \beta\widehat{d}_t + aux_t,$$

where here $aux_t \equiv (1-\Omega)\widehat{\pi}_t^N - \Omega(1-\beta)\widehat{q}_t - (1-\Omega)\beta(1-\delta)\widehat{u}_t^R + \widehat{u}_t^\tau$. Iterating the equation forward, taking expectations and imposing the transversality condition, we obtain

$$\widehat{d}_{t-1} = aux_t + \sum_{j=1}^{\infty} \beta^j E_t\{aux_{t+j}\}. \tag{D.26}$$

Leveraging the procedure in Appendix C.3.2, we can express aux_t as

$$aux_t = (1-\Omega)\widehat{\pi}_t^N - \Omega(1-\beta)\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\widehat{\pi}_t^N - \beta\lambda_1\widehat{u}_t^R\right] - \left[(1-\Omega)\beta(1-\delta)\right]\widehat{u}_t^R + \widehat{u}_t^\tau = ...$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N - \beta\left[1-\delta-\Omega\left(1-\delta+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R + \widehat{u}_t^\tau.$$

and, as shocks are i.i.d., also

$$\sum_{j=1}^{\infty} \beta^j E_t\{aux_{t+j}\} = \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right).$$

Substituting these into equation (D.26), we get

$$\begin{aligned}
\widehat{d}_{t-1} &= \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N - \beta\left[1-\delta-\Omega\left(1-\delta+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R + \widehat{u}_t^\tau + \\
&\quad \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right) \\
&= \underbrace{\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{1}{(1-\beta\lambda_1)}}_{\equiv A^{LT}}\widehat{\pi}_t^N + \widehat{u}_t^\tau \\
&\quad - \underbrace{\left\{\beta\left[1-\delta-\Omega\left(1-\delta+(1-\beta)\lambda_1\right)\right] - \widetilde{\kappa}\eta\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\right\}}_{\equiv B^{LT}}\widehat{u}_t^R
\end{aligned}$$

Notice that A^{LT} is the same as A from Appendix C.3.2 (i.e. with one-period debt). In particular, that means that the value $\Omega_{cut,1}$ that makes $A = 0$ (leading to no stationary equilibrium) is the same as in the $\delta = 0$ case.

With this, the solution for $\widehat{\pi}_t^N$ is given by

$$\widehat{\pi}_t^N = \frac{1}{A^{LT}}(\widehat{d}_{t-1} - \widehat{u}_t^\tau) + \frac{B^{LT}}{A^{LT}}\widehat{u}_t^R. \tag{D.27}$$

Also, the solution for $\widehat{q}_t$ is

$$\begin{aligned}
\widehat{q}_t &= \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\left(\frac{1}{A^{LT}}\widehat{d}_{t-1} + \frac{B^{LT}}{A^{LT}}\widehat{u}_t^R - \frac{1}{A^{LT}}\widehat{u}_t^\tau\right) - \beta\lambda_1\widehat{u}_t^R \\
&= \underbrace{\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A^{LT}}}_{\equiv C^{LT}}\left(\widehat{d}_{t-1} - \widehat{u}_t^\tau\right) + \underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1\right]}_{\equiv D^{LT}}\widehat{u}_t^R.
\end{aligned} \tag{D.28}$$

Again, $C^{LT} = C$ from the one-period case. Finally, as $\widehat{\pi}_t^S = \widehat{q}_t + \widehat{\pi}_t^N - \widehat{q}_{t-1}$, we obtain the solution for the nominal depreciation:

$$\widehat{\pi}_t^S = \underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right]\frac{1}{A^{LT}}}_{\equiv F^{LT}}\left(\widehat{d}_{t-1} - \widehat{u}_t^{\tau}\right) + \underbrace{\left[\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B^{LT}}{A^{LT}} - \beta\lambda_1\right]}_{\equiv G^{LT}}\widehat{u}_t^R - \widehat{q}_{t-1}, \tag{D.29}$$

with $F^{LT} = F$. With these results, we now explore how the solution changes as δ varies and how this relationship itself depends on the value of Ω.

Case with $\Omega = 0$

A^{LT} simplifies to $A^{LT} = (1-\beta\lambda_1)^{-1} > 0$, as $\beta, \lambda_1 \in (0,1)$. At the same time, B^{LT} reads

$$B^{LT} = \beta(1-\delta) - \widetilde{\kappa}\eta\frac{\beta\lambda_1}{1-\beta\lambda_1} = \beta\left(1 - \delta - \frac{\widetilde{\kappa}\eta\lambda_1}{1-\beta\lambda_1}\right) = \beta(\lambda_1 - \delta),$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$, from Appendix C.3.2. Thus, as $\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R} = \frac{B^{LT}}{A^{LT}}$, we see that

$$\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R} > 0 \quad \text{if} \quad \delta < \delta_{cut,1},$$
$$\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R} < 0 \quad \text{if} \quad \delta > \delta_{cut,1},$$

where $\delta_{cut,1} \equiv \lambda_1$. Notice also that $\frac{B^{LT}}{A^{LT}}$ is decreasing in δ. Thus, the greater the duration, the smaller (possibly even negative) the reaction of non-traded inflation to an i.i.d. positive monetary shock.

To study the role of δ in shaping $\widehat{q}_t$'s response to $\widehat{u}_t^R$, we first evaluate D^{LT} at $\Omega = 0$,

$$D^{LT} = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}(1-\beta\lambda_1)\beta(\lambda_1-\delta) - \beta\lambda_1 = \ldots$$
$$\frac{\lambda_1}{(1-\lambda_1)}(1-\beta\lambda_1)\beta(\lambda_1-\delta) - \beta\lambda_1,$$

where we used $\widetilde{\kappa}\eta = \frac{(1-\lambda_1)(1-\beta\lambda_1)}{\lambda_1}$, as well as A^{LT} and B^{LT} derived earlier. Simplifying further,

$$D^{LT} = \frac{\beta\lambda_1}{(1-\lambda_1)}\left[(\lambda_1-\delta)(1-\beta\lambda_1) - 1 + \lambda_1\right] = \frac{\beta\lambda_1}{(1-\lambda_1)}\left[-\delta(1-\beta\lambda_1) + \lambda_1(1-\beta-\widetilde{\kappa}\eta)\right],$$

where the last step uses the quadratic formula $\beta(\lambda_1)^2 = (1+\beta+\widetilde{\kappa}\eta)\lambda_1 - 1$. Thus, the sign of D^{LT} depends on how δ compares to $\delta_{cut,2} \equiv \lambda_1(1-\beta-\widetilde{\kappa}\eta)/(1-\beta\lambda_1)$. In turn, $\delta_{cut,2}$'s sign is determined by that of $1-\beta-\widetilde{\kappa}\eta$, given that $\beta\lambda_1 < 1$. **If $\delta_{cut,2} < 0$ (i.e. if $\beta+\widetilde{\kappa}\eta > 1$), as by definition $\delta \geq 0$, we have $D^{LT} < 0$. Otherwise, $D^{LT} < 0$ if $\delta > \delta_{cut,2}$.** Summarizing, $\widehat{q}_t$'s response to a monetary shock is:

$$\frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} < 0 \quad \text{if } \beta+\widetilde{\kappa}\eta > 1 \text{ and } \delta \geq 0, \text{ or if } \beta+\widetilde{\kappa}\eta < 1 \text{ and } \delta > \delta_{cut,2},$$
$$\frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} \geq 0 \quad \text{if } \beta+\widetilde{\kappa}\eta < 1 \text{ and } \delta \leq \delta_{cut,2}.$$

Recall that in our calibration $\beta+\widetilde{\kappa}\eta > 1$: the sign of the response will not depend on δ in quantitative exercises. The magnitude, however, will: the larger the debt's duration δ, the more negative the response of $\widehat{q}_t$.

Finally, we explore the role of δ in shaping the response of nominal-depreciation to $\widehat{u}_t^R$. Recall,

$$G^{LT} = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 = \frac{1}{(1-\lambda_1)}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1.$$

Substituting $\frac{B^{LT}}{A^{LT}}$ yields:

$$G^{LT} = \frac{1}{(1-\lambda_1)}(1-\beta\lambda_1)\beta(\lambda_1 - \delta) - \beta\lambda_1.$$

This can be shown to be positive as long as $\delta < \delta_{cut,3} \equiv \frac{(\lambda_1)^2(1-\beta)}{(1-\beta\lambda_1)}$.[20] Thus, the response of nominal depreciation to policy shocks is given by:

$$\frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^R} > 0 \quad \text{if} \quad \delta < \delta_{cut,3},$$
$$\frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^R} \leq 0 \quad \text{if} \quad \delta \geq \delta_{cut,3}.$$

Moreover, we can see that G^{LT} is decreasing δ; thus, a longer debt duration reduces the magnitude of the nominal depreciation response and may even induce a nominal appreciation, in contrast to the short-term debt case.

What is the intuition behind these results? By increasing the short-term rate, *ceteris paribus*, the net-present value of primary surpluses, as in the one period-debt setup (discounting channel). With long-term debt, the shock **also** reduces the value of outstanding nominal debt $(x + \delta Z_t)D_{t-1}^G$ (by reducing the current price of new debt issuance Z_t, a valuation effect). Thus, both sides of the life-time government constraint are reduced, and the effect on inflation depends on which dominates. If the discounting channel is greater than the valuation effect, qualitatively, results are as in the short-term debt-only case: inflation must increase to make the FTPL equation hold. As the previous derivations show, this happens as long as debt-duration δ is small enough. Otherwise, if the valuation effect outweighs the discounting channel, inflation must fall to compensate; this materializes if δ is large enough.

The relative price of non-tradables $\widehat{q}_t$, according to the UIP condition in real terms iterated forward, depends negatively on the expected path of the domestic policy rate and positively on that of non-traded inflation. By the former, provided that $\phi_\pi = 0$ and $\widehat{u}_t^R$ being i.i.d., the relative price falls after the shock, while by the latter it rises as long as $\widehat{\pi}_t^N$ increases. With $\beta + \widetilde{\kappa}\eta > 1$ and $\delta = 0$, the first dominates and the relative price falls. This generalizes for $\delta > 0$, and as the impact on non-traded inflation weakens with a longer duration, the drop in relative prices is more pronounced for larger values of δ.

Finally, the impact response of the nominal exchange rate is the sum of that of the relative price and non-traded inflation. The former is negative, and the latter is positive for $\delta = 0$, but the latter dominates, leading the nominal exchange rate to jump with the shock. However, a longer duration δ intensifies the fall in $\widehat{q}_t$ and weakens the fall in $\widehat{\pi}_t^N$. For a large enough duration, the nominal exchange rate appreciates following the shock.

[20]For later use, note that $\delta_{cut,3} \equiv \frac{\lambda_1^2(1-\beta)}{1-\beta\lambda_1} = \lambda_1 \cdot \frac{\lambda_1(1-\beta)}{1-\beta\lambda_1}$. Since $\lambda_1 \in (0,1)$ and $\beta \in (0,1)$, it follows that $0 < \frac{\lambda_1(1-\beta)}{1-\beta\lambda_1} < 1$. Therefore, $\delta_{cut,3} < \lambda_1 \equiv \delta_{cut,1}$.

Case with $0 < \Omega \leq 1$

The reaction of non-traded inflation $\widehat{\pi}_t^N$ in the general case is governed by B^{LT}/A^{LT}. We know A^{LT} is the same as with $\delta = 0$; thus, from the results in Appendix C.3.2, $A^{LT} > 0$ if

$$\Omega < \frac{1-\lambda_1}{1-\beta\lambda_1} \equiv \Omega_{\text{cut},1},$$

otherwise $A^{LT} < 0$. With $\Omega = \Omega_{\text{cut},1}$, a stationary solution does not exist, as previously discussed.

Regarding B^{LT},

$$\begin{aligned} B^{LT} &= \beta\left[1-\delta-\Omega\left(1-\delta+(1-\beta)\lambda_1\right)\right] - \widetilde{\kappa}\eta\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)} \\ &= \beta\left[1-\delta-\Omega\left(1-\delta+(1-\beta)\lambda_1\right)\right] - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) + \beta\lambda_1\Omega(1-\beta) \\ &= \beta(1-\delta)(1-\Omega) - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) = (1-\Omega)\beta\left[1-\delta-\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}\right] = (1-\Omega)\beta\left(\lambda_1-\delta\right) \end{aligned}$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$. Hence, for any $\Omega \in (0,1)$ the sign of B^{LT} is pinned down by the ordering between δ and $\delta_{cut,1} \equiv \lambda_1$, while $B^{LT} = 0$ when $\Omega = 1$.

Therefore, the sign of the response to a monetary shock (given by $\frac{B^{LT}}{A^{LT}}$) depends jointly on δ (through B^{LT}) and on Ω (through A^{LT}). We can summarize the alternative combinations as:

Effects on $\widehat{\pi}_t^N$: monetary shock ($\widehat{u}_t^R$)

	Values of Ω		
Values of δ	$0 < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega < 1$	$\Omega = 1$
$0 \leq \delta < \delta_{\text{cut},1}$	$+$	$-$	0
$\delta_{\text{cut},1} < \delta$	$-$	$+$	0

It is also of interest to analyze how $\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R}$ varies with δ and Ω. First, we can calculate

$$\frac{\partial}{\partial\delta}\left(\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R}\right) = -\frac{(1-\Omega)\beta}{A^{LT}},$$

so that $\frac{\partial}{\partial\delta}\left(\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R}\right) < 0$ whenever $\Omega < \Omega_{\text{cut},1}$ (a longer duration dampens the effect on inflation if Ω is small, potentially becoming negative if $\delta > \delta_{cut,1}$), while $\frac{\partial}{\partial\delta}\left(\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R}\right) > 0$ whenever $\Omega_{\text{cut},1} < \Omega < 1$ (a longer duration increases the effect on inflation if Ω is large, potentially becoming positive if $\delta > \delta_{cut,1}$).

In addition, we have

$$\frac{\partial}{\partial\Omega}\left(\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R}\right) = \frac{A^{LT}\frac{\partial B^{LT}}{\partial\Omega} - B^{LT}\frac{\partial A^{LT}}{\partial\Omega}}{(A^{LT})^2}$$

whose sign is determined by the numerator's

$$A^{LT}\frac{\partial B^{LT}}{\partial \Omega} - B^{LT}\frac{\partial A^{LT}}{\partial \Omega} = -\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1}\beta\left(\lambda_1-\delta\right) + ...$$
$$(1-\Omega)\beta\left(\lambda_1-\delta\right)\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)(1-\beta\lambda_1)^{-1} = \frac{\beta\left(\lambda_1-\delta\right)}{(1-\beta\lambda_1)}\frac{(1-\beta)\lambda_1}{(1-\lambda_1)}.$$

This derivative is positive (as in the case with $\delta = 0$) as long as $\delta < \lambda_1$; otherwise, it is negative.

Next, the relative price $\widehat{q}_t$ depends on $\widehat{u}_t^R$ through D^{LT}

$$D^{LT} = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 = \frac{\lambda_1}{(1-\lambda_1)}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1.$$

As B^{LT} depends on debt maturity (recall $B^{LT} = (1-\Omega)\beta(\lambda_1-\delta)$), the sign of D^{LT} depends on whether δ is relative to the threshold $\delta_{\text{cut},1}$. Therefore, we split the analysis into two.

- <u>**Case with** $0 < \delta \leq \delta_{\text{cut},1}$</u>: Here, $B^{LT} \geq 0$. Thus, **if $\Omega > \Omega_{\text{cut},1}$ we have $A^{LT} < 0$, which implies $D^{LT} < 0$.** Alternatively, if $\Omega < \Omega_{\text{cut},1}$ (and hence $A^{LT} > 0$), then $D^{LT} \leq 0$ if and only if:

$$B^{LT} \leq \beta(1-\lambda_1)A^{LT} \iff (1-\Omega)\beta(\lambda_1-\delta) \leq \beta(1-\lambda_1)\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right]\frac{1}{(1-\beta\lambda_1)}$$
$$\iff (1-\Omega)(\lambda_1-\delta)(1-\beta\lambda_1) \leq 1-\lambda_1-\Omega(1-\beta\lambda_1)$$
$$\iff \Omega(1-\lambda_1)(1-\beta\lambda_1) - (1-\Omega)\delta(1-\beta\lambda_1) \leq 1-\lambda_1-\lambda_1+\beta(\lambda_1)^2$$

  From the quadratic formula, $\beta(\lambda_1)^2 - \lambda_1 + 1 = \lambda_1(\beta+\widetilde{\kappa}\eta)$, thus

$$\iff \Omega(1-\lambda_1)(1-\beta\lambda_1) - (1-\Omega)\delta(1-\beta\lambda_1) \leq \lambda_1(\beta+\widetilde{\kappa}\eta-1)$$
$$\iff \Omega(1-\lambda_1+\delta)(1-\beta\lambda_1) \leq \lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1)$$

  Therefore, $D^{LT} \leq 0$ if

$$\Omega \leq \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1)}{(1-\lambda_1+\delta)(1-\beta\lambda_1)} \equiv \Omega^{LT}_{\text{cut},2},$$

  with equality if $\Omega = \Omega^{LT}_{\text{cut},2}$. It can be shown that $\Omega^{LT}_{\text{cut},2} \leq \Omega_{\text{cut},1}$ whenever $\delta \leq \delta_{\text{cut},1} = \lambda_1$.[21]

  Notice that the denominator in $\Omega^{LT}_{\text{cut},2}$ is positive for any $\delta > 0$, as is the numerator if $\beta+\widetilde{\kappa}\eta \geq 1$. Thus, **we conclude that $D^{LT} < 0$ as long as $\beta+\widetilde{\kappa}\eta \geq 1$, $\Omega < \Omega^{LT}_{\text{cut},2}$ and $\delta \leq \delta_{\text{cut},1}$** .

  If instead $\beta+\widetilde{\kappa}\eta < 1$, $\Omega^{LT}_{\text{cut},2}$ can be zero or negative if

$$\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1) \leq 0 \iff \delta \leq \frac{\lambda_1}{(1-\beta\lambda_1)}(1-\beta-\widetilde{\kappa}\eta) = \delta_{cut,2},$$

[21] Proof:

$$\Omega^{LT}_{\text{cut},2} - \Omega_{\text{cut},1} = \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1)}{(1-\beta\lambda_1)(1-\lambda_1+\delta)} - \frac{1-\lambda_1}{1-\beta\lambda_1} = \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1)-(1-\lambda_1)(1-\lambda_1+\delta)}{(1-\beta\lambda_1)(1-\lambda_1+\delta)} = ...$$

$$\frac{\left[\lambda_1(\beta+\widetilde{\kappa}\eta-1)-(1-\lambda_1)^2\right]+\delta\left[(1-\beta\lambda_1)-(1-\lambda_1)\right]}{(1-\beta\lambda_1)(1-\lambda_1+\delta)} = \frac{(\beta-1)(\lambda_1)^2+\delta\lambda_1(1-\beta)}{(1-\beta\lambda_1)(1-\lambda_1+\delta)} = \frac{(\beta-1)\lambda_1(\lambda_1-\delta)}{(1-\beta\lambda_1)(1-\lambda_1+\delta)} \leq 0.$$

with equality if $\delta = \delta_{cut,2}$. Moreover, since $\widetilde{\kappa}\eta > 0$ and $\lambda_1 \in (0,1)$, it follows that $0 < \delta_{cut,2} < \delta_{cut,1} \equiv \lambda_1$ whenever $\beta + \widetilde{\kappa}\eta < 1$. Therefore, within the case analyzed here (i.e. $\delta \in (0, \delta_{cut,1}]$), we must distinguish two sub-cases:

(i) If $0 < \delta \leq \delta_{cut,2}$, then $\Omega^{LT}_{\text{cut},2} \leq 0$. Hence, for any $0 < \Omega < \Omega_{\text{cut},1}$ we necessarily have $D^{LT} > 0$.

(ii) If $\delta_{cut,2} < \delta \leq \delta_{cut,1}$, then $\Omega^{LT}_{\text{cut},2} > 0$ (and $\Omega^{LT}_{\text{cut},2} \leq \Omega_{\text{cut},1}$). In this case, for $0 < \Omega < \Omega_{\text{cut},1}$ we have $D^{LT} \leq 0$ when $0 < \Omega \leq \Omega^{LT}_{\text{cut},2}$, and $D^{LT} > 0$ when $\Omega^{LT}_{\text{cut},2} < \Omega < \Omega_{\text{cut},1}$.

The following table summarizes the findings:

Effects on $\widehat{q}_t$ when $0 < \delta \leq \delta_{\text{cut},1}$

Values of δ	Values of Ω and $\beta + \widetilde{\kappa}\eta$			
	$0 < \Omega < \Omega^{LT}_{\text{cut},2}$			
	$\beta + \widetilde{\kappa}\eta \geq 1$	$\beta + \widetilde{\kappa}\eta < 1$	$\Omega^{LT}_{\text{cut},2} < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq 1$
$0 < \delta \leq \delta_{cut,2}$	$-$	$\varnothing$	$+$	$-$
$\delta_{cut,2} < \delta \leq \delta_{\text{cut},1}$	$-$	$-$	$+$	$-$

– **Case with $\delta > \delta_{\text{cut},1}$:** With this, $B^{LT} \leq 0$ (with equality if $\Omega = 1$). If $0 < \Omega < \Omega_{\text{cut},1}$, then $A^{LT} > 0$, so $B^{LT}/A^{LT} < 0$ and therefore

$$D^{LT} = \frac{\lambda_1}{1-\lambda_1}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 < 0.$$

Thus, $D^{LT} < 0$ if $0 < \Omega < \Omega_{\text{cut},1}$.

Consider instead $\Omega > \Omega_{\text{cut},1}$, so that $A^{LT} < 0$. Since $B^{LT} < 0$ for $\Omega \in (0,1)$, we have $\frac{B^{LT}}{A^{LT}} > 0$, and hence D^{LT} can be of either sign. In particular, $D^{LT} \geq 0$ if and only if

$$\frac{\lambda_1}{1-\lambda_1}\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 \geq 0 \iff \frac{B^{LT}}{A^{LT}} \geq \beta(1-\lambda_1).$$

Since $A^{LT} < 0$ in the present case, multiplying both sides by A^{LT} flips the inequality, so that

$$\frac{B^{LT}}{A^{LT}} \geq \beta(1-\lambda_1) \iff B^{LT} \leq \beta(1-\lambda_1)A^{LT}.$$

Following the same steps as before, we have

$$D^{LT} \geq 0 \iff \Omega(1-\beta\lambda_1)(1-\lambda_1+\delta) \leq \lambda_1(\beta+\widetilde{\kappa}\eta-1) + \delta(1-\beta\lambda_1),$$

or, $D^{LT} \geq 0$ if

$$\Omega \leq \frac{\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1)}{(1-\lambda_1+\delta)(1-\beta\lambda_1)} \equiv \Omega^{LT}_{\text{cut},2}.$$

From the previous proof, $\Omega^{LT}_{\text{cut},2} > \Omega_{\text{cut},1}$ if $\delta > \delta_{cut,1}$.

Previously we noticed that the denominator in $\Omega^{LT}_{\text{cut},2}$ is positive for any $\delta > 0$, as well as it is the numerator if $\beta + \widetilde{\kappa}\eta \geq 1$. Thus, **we conclude that, if $\delta > \delta_{\text{cut},1}$ and $\beta + \widetilde{\kappa}\eta \geq 1$, $D^{LT} \geq 0$ as long as $\Omega_{\text{cut},1} < \Omega < \Omega^{LT}_{\text{cut},2}$, while $D^{LT} < 0$ if $\Omega^{LT}_{\text{cut},2} < \Omega$.**

If instead $\beta + \widetilde{\kappa}\eta < 1$, $\Omega^{LT}_{\text{cut},2}$ can be zero or negative if

$$\lambda_1(\beta+\widetilde{\kappa}\eta-1)+\delta(1-\beta\lambda_1) \leq 0 \iff \delta \leq \frac{\lambda_1}{(1-\beta\lambda_1)}(1-\beta-\widetilde{\kappa}\eta) \equiv \delta_{cut,2},$$

with equality if $\delta = \delta_{cut,2}$, where $\delta_{cut,2}$ was previously defined. However, since $\widetilde{\kappa}\eta > 0$ and $\lambda_1 \in (0,1)$, it follows that $0 < \delta_{cut,2} < \delta_{cut,1} \equiv \lambda_1$ whenever $\beta + \widetilde{\kappa}\eta < 1$. Therefore, in the case analyzed here with $\delta > \delta_{cut,1}$, we necessarily have $\delta > \delta_{cut,2}$, and so $\Omega^{LT}_{\text{cut},2} > 0$ (and, from the previous proof, $\Omega^{LT}_{\text{cut},2} > \Omega_{\text{cut},1}$). Hence, the characterization of D^{LT} for $\Omega > \Omega_{\text{cut},1}$ is the same as in the case $\beta + \widetilde{\kappa}\eta \geq 1$: we have $D^{LT} \geq 0$ if $\Omega_{\text{cut},1} < \Omega < \Omega^{LT}_{\text{cut},2}$ (with equality at $\Omega = \Omega^{LT}_{\text{cut},2}$), while $D^{LT} < 0$ if $\Omega^{LT}_{\text{cut},2} < \Omega \leq 1$. These are summarized in the next table.

Effects on $\widehat{q}_t$ when $\delta > \delta_{\text{cut},1}$			
	Values of Ω		
Effect	$0 < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega < \Omega^{LT}_{\text{cut},2}$	$\Omega^{LT}_{\text{cut},2} < \Omega \leq 1$
$\frac{\partial \widehat{q}_t}{\partial \widehat{u}^R_t}$	$-$	$+$	$-$

We now turn to the sensitivity of D^{LT} with respect to debt maturity:

$$\frac{\partial D^{LT}}{\partial \delta} = \frac{\lambda_1}{1-\lambda_1}\frac{1}{A^{LT}}\frac{\partial B^{LT}}{\partial \delta} = -\frac{\beta\lambda_1(1-\Omega)}{(1-\lambda_1)A^{LT}}.$$

Since A^{LT} does not depend on δ, but its sign depends on Ω relative to $\Omega_{\text{cut},1}$, then $\frac{\partial D^{LT}}{\partial \delta} < 0$ **if** $0 < \Omega < \Omega_{\text{cut},1}$**, while** $\frac{\partial D^{LT}}{\partial \delta} > 0$ **if** $\Omega_{\text{cut},1} < \Omega < 1$ **(with** $\frac{\partial D^{LT}}{\partial \delta} = 0$ **at** $\Omega = 1$). Finally, notice that $\frac{\partial D^{LT}}{\partial \Omega}$ (sensitivity with respect to the share of dollar debt) depends upon that of $\frac{\partial}{\partial \Omega}\left(\frac{\partial \widehat{\pi}^N_t}{\partial \widehat{u}^R_t}\right)$, which we already analyzed.

Finally, for the nominal depreciation $\widehat{\pi}^S_t$, G^{LT} does depend on debt maturity, so again we have two alternatives to consider:

– **Case with** $0 < \delta \leq \delta_{\text{cut},1}$**:** Here $B^{LT} \geq 0$. Also, if $\Omega > \Omega_{\text{cut},1}$ then $A^{LT} < 0$, so

$$G^{LT} = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B^{LT}}{A^{LT}} - \beta\lambda_1 < 0$$

On the other hand, if $\Omega < \Omega_{\text{cut},1}$, then $A^{LT} > 0$, and $G^{LT} > 0$ if:

$$\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B^{LT}}{A^{LT}} > \beta\lambda_1 \iff \frac{1}{1-\lambda_1}\frac{B^{LT}}{A^{LT}} > \beta\lambda_1 \iff B^{LT} > \beta\lambda_1 A^{LT}(1-\lambda_1) \iff$$

$$(1-\Omega)\beta(\lambda_1-\delta) > \beta\lambda_1(1-\lambda_1)\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right]\frac{1}{(1-\beta\lambda_1)} \iff$$

$$(1-\Omega)(\lambda_1-\delta)(1-\beta\lambda_1) > \lambda_1\left[1-\lambda_1-\Omega(1-\beta\lambda_1)\right] \iff$$

$$\Omega\delta(1-\beta\lambda_1) > \lambda_1(1-\lambda_1) - (\lambda_1-\delta)(1-\beta\lambda_1) \iff \Omega\delta(1-\beta\lambda_1) > \delta(1-\beta\lambda_1) - (1-\beta)\lambda_1^2$$

$$\iff \Omega > 1 - \frac{(1-\beta)\lambda_1^2}{\delta(1-\beta\lambda_1)} \equiv \Omega^{LT}_{\text{cut},3}.$$

We next establish that $\Omega^{LT}_{\text{cut},3} \leq \Omega_{\text{cut},1}$ whenever $\delta \leq \delta_{\text{cut},1} \equiv \lambda_1$:

$$\Omega^{LT}_{\text{cut},3} - \Omega_{\text{cut},1} = 1 - \frac{(1-\beta)\lambda_1^2}{\delta(1-\beta\lambda_1)} - \frac{1-\lambda_1}{1-\beta\lambda_1} = \frac{(1-\beta)\lambda_1(\delta-\lambda_1)}{\delta(1-\beta\lambda_1)} \leq 0,$$

and therefore $\Omega^{LT}_{\text{cut},3} \leq \Omega_{\text{cut},1}$, with strict inequality if $\delta < \lambda_1$.

It is relevant to analyze if $\Omega^{LT}_{\text{cut},3}$ can be negative, because, in that case, the condition $\Omega^{LT}_{\text{cut},3} < \Omega$ would hold automatically for any $\Omega > 0$. This can happen if

$$1 - \frac{(1-\beta)\lambda_1^2}{\delta(1-\beta\lambda_1)} < 0 \iff \delta < \frac{(1-\beta)\lambda_1^2}{(1-\beta\lambda_1)} \equiv \delta_{\text{cut},3}.$$

Moreover, $\delta_{\text{cut},3} = \lambda_1 \cdot \frac{\lambda_1(1-\beta)}{1-\beta\lambda_1}$, and since $\frac{\lambda_1(1-\beta)}{1-\beta\lambda_1} < 1$ for $\beta \in (0,1)$ and $\lambda_1 \in (0,1)$, it follows that $\delta_{\text{cut},3} < \delta_{\text{cut},1} \equiv \lambda_1$. Therefore, within the case analyzed here ($0 < \delta \leq \delta_{\text{cut},1}$), we have two sub-cases:

(i) If $0 < \delta \leq \delta_{\text{cut},3}$, then $\Omega^{LT}_{\text{cut},3} \leq 0$; and hence $G^{LT} > 0$ for any $0 < \Omega < \Omega_{\text{cut},1}$ (while $G^{LT} < 0$ for $\Omega_{\text{cut},1} < \Omega \leq 1$)

(ii) If instead $\delta_{\text{cut},3} < \delta \leq \delta_{\text{cut},1}$, then $\Omega^{LT}_{\text{cut},3} > 0$ (and $\Omega^{LT}_{\text{cut},3} \leq \Omega_{\text{cut},1}$), so that for $0 < \Omega < \Omega_{\text{cut},1}$ we have $G^{LT} \leq 0$ when $0 < \Omega \leq \Omega^{LT}_{\text{cut},3}$ and $G^{LT} > 0$ when $\Omega^{LT}_{\text{cut},3} < \Omega < \Omega_{\text{cut},1}$; as before, $G^{LT} < 0$ for $\Omega_{\text{cut},1} < \Omega \leq 1$.

Summarizing:

Effects on $\widehat{\pi}^S_t$: $0 < \delta \leq \delta_{\text{cut},1}$

	Values of Ω		
Values of δ	$0 < \Omega < \Omega^{LT}_{\text{cut},3}$	$\Omega^{LT}_{\text{cut},3} < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq 1$
$0 < \delta \leq \delta_{\text{cut},3}$	$\varnothing$	+	−
$\delta_{\text{cut},3} < \delta \leq \delta_{\text{cut},1}$	−	+	−

– **Case with $\delta > \delta_{cut,1}$:** In this case, $B^{LT} < 0$. If $0 < \Omega < \Omega_{\text{cut},1}$, and thus $A^{LT} > 0$,

$$G^{LT} = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1 \right) \frac{B^{LT}}{A^{LT}} - \beta\lambda_1 < 0.$$

Consider instead $\Omega > \Omega_{\text{cut},1}$, so that $A^{LT} < 0$:

$$G^{LT} = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1 \right) \frac{B^{LT}}{A^{LT}} - \beta\lambda_1 = \frac{1}{1-\lambda_1} \frac{B^{LT}}{A^{LT}} - \beta\lambda_1.$$

We have $G^{LT} \geq 0$ if and only if

$$\frac{1}{1-\lambda_1} \frac{B^{LT}}{A^{LT}} \geq \beta\lambda_1 \iff \frac{B^{LT}}{A^{LT}} \geq \beta\lambda_1(1-\lambda_1).$$

Since $A^{LT} < 0$ in the present case, multiplying both sides by A^{LT} flips the inequality, so that

$$\frac{B^{LT}}{A^{LT}} \geq \beta\lambda_1(1-\lambda_1) \iff B^{LT} \leq \beta\lambda_1(1-\lambda_1)A^{LT}.$$

Thus, $G^{LT} \geq 0$ if

$$\Omega \leq 1 - \frac{(1-\beta)\lambda_1^2}{\delta(1-\beta\lambda_1)} \equiv \Omega^{LT}_{\text{cut},3},$$

and, from the previous derivations, $\Omega^{LT}_{\text{cut},3} > \Omega_{\text{cut},1}$ if $\delta > \lambda_1 = \delta_{cut,1}$.

It is relevant to analyze if $\Omega^{LT}_{\text{cut},3}$ can be negative, because in that case the condition $\Omega \leq \Omega^{LT}_{\text{cut},3}$ could never hold. As before, $\Omega^{LT}_{\text{cut},3} < 0$ would require $\delta < \delta_{\text{cut},3} \equiv \frac{(1-\beta)\lambda_1^2}{(1-\beta\lambda_1)}$. However, since $\delta_{\text{cut},3} < \delta_{\text{cut},1}$ and we are considering $\delta > \delta_{\text{cut},1}$, it follows that $\delta > \delta_{\text{cut},3}$ and therefore $\Omega^{LT}_{\text{cut},3} > 0$. Hence, for $\Omega > \Omega_{\text{cut},1}$ we have $G^{LT} \geq 0$ if and only if $\Omega \leq \Omega^{LT}_{\text{cut},3}$ (with equality at $\Omega = \Omega^{LT}_{\text{cut},3}$), while $G^{LT} < 0$ if $\Omega^{LT}_{\text{cut},3} < \Omega \leq 1$; as before, $G^{LT} < 0$ for $0 < \Omega < \Omega_{\text{cut},1}$. Summarizing:

Effects on $\widehat{\pi}_t^S$ when $\delta > \delta_{\text{cut},1}$			
	Values of Ω		
Effect	$0 < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq \Omega_{\text{cut},3}^{LT}$	$\Omega_{\text{cut},3}^{LT} < \Omega \leq 1$
$\frac{\partial \widehat{\pi}_t^S}{\partial \widehat{u}_t^R}$	$-$	$+$	$-$

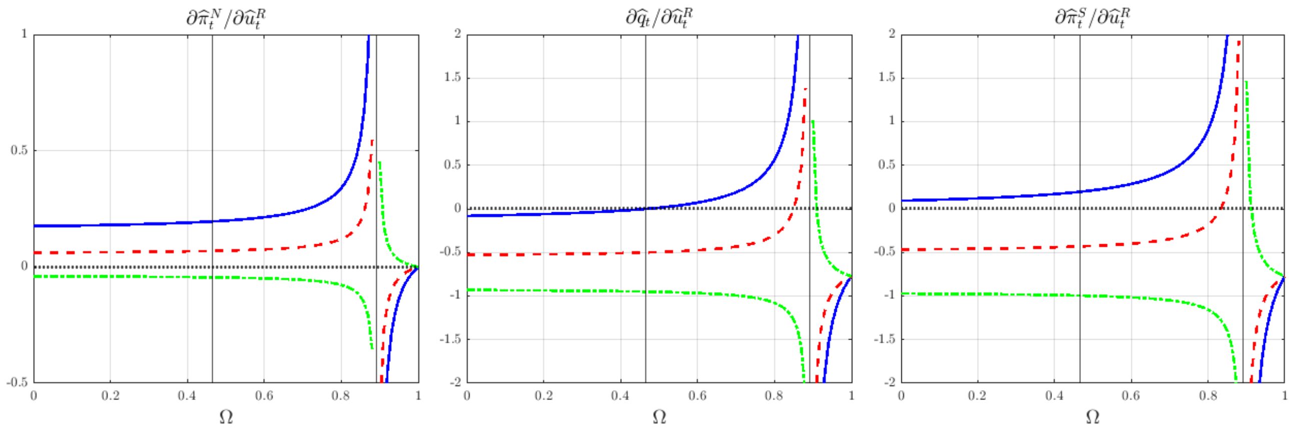
Finally, we examine how the response of nominal depreciation to monetary shocks varies with debt maturity δ:

$$\frac{\partial}{\partial \delta}\left(\frac{\partial \widehat{\pi}_t^S}{\partial \widehat{u}_t^R}\right) = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}+1\right)\frac{1}{A^{LT}}\frac{\partial B^{LT}}{\partial \delta} = -\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}+1\right)\frac{(1-\Omega)\beta}{A^{LT}} = -\frac{(1-\Omega)\beta}{(1-\lambda_1)A^{LT}},$$

where the last equality uses $\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}+1\right) = \frac{1}{1-\lambda_1}$ and $\frac{\partial B^{LT}}{\partial \delta} = -(1-\Omega)\beta$. The sign thus depends on the opposite of A^{LT}'s. Thus, if $0 < \Omega < \Omega_{\text{cut},1}$, we have $\frac{\partial}{\partial \delta}\left(\frac{\partial \widehat{\pi}_t^S}{\partial \widehat{u}_t^R}\right) < 0$, while $\frac{\partial}{\partial \delta}\left(\frac{\partial \widehat{\pi}_t^S}{\partial \widehat{u}_t^R}\right) \geq 0$ for $\Omega_{\text{cut},1} < \Omega \leq 1$ (with equality at $\Omega = 1$).

Overall, to summarize the results, Figure 7 below is analogous to Figure 2 in the main text, adding two cases with duration greater than one: 2 quarters ($\delta = 0.5158$) and 20 quarters ($\delta = 0.98$). Under the baseline calibration $\delta_{\text{cut},1} = 0.7979$, which implies a duration of 4.4136 quarters.

Figure 7: The contemporaneous effect of $\widehat{u}_t^R$ under the N-policy setup depending Ω, for different δ's.

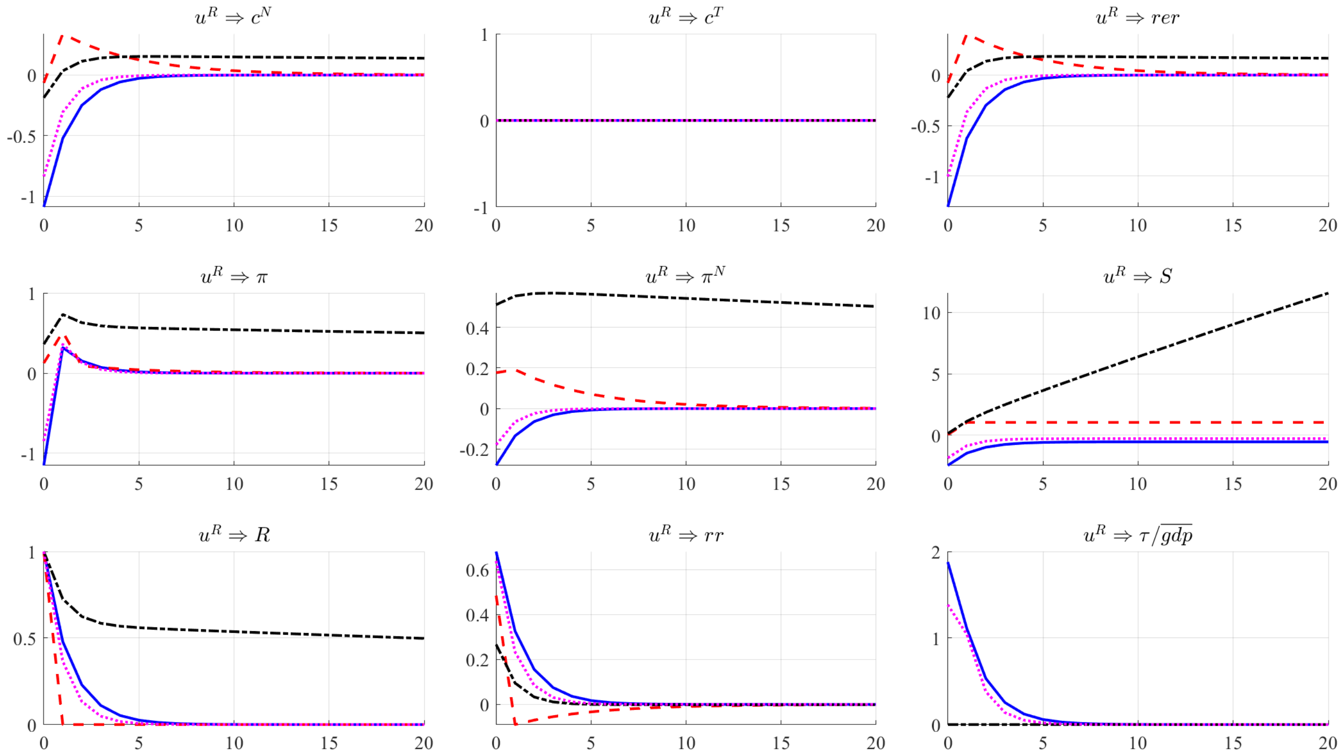


Note: The figure is analogous to Figure 2. Solid-blue lines correspond to the case with Duration=1 ($\delta = 0$), in dashed-red lines Duration=2 ($\delta = 0.5158$), while in dashed-dotted-green lines Duration=20 ($\delta = 0.98$).

F Additional Exercises

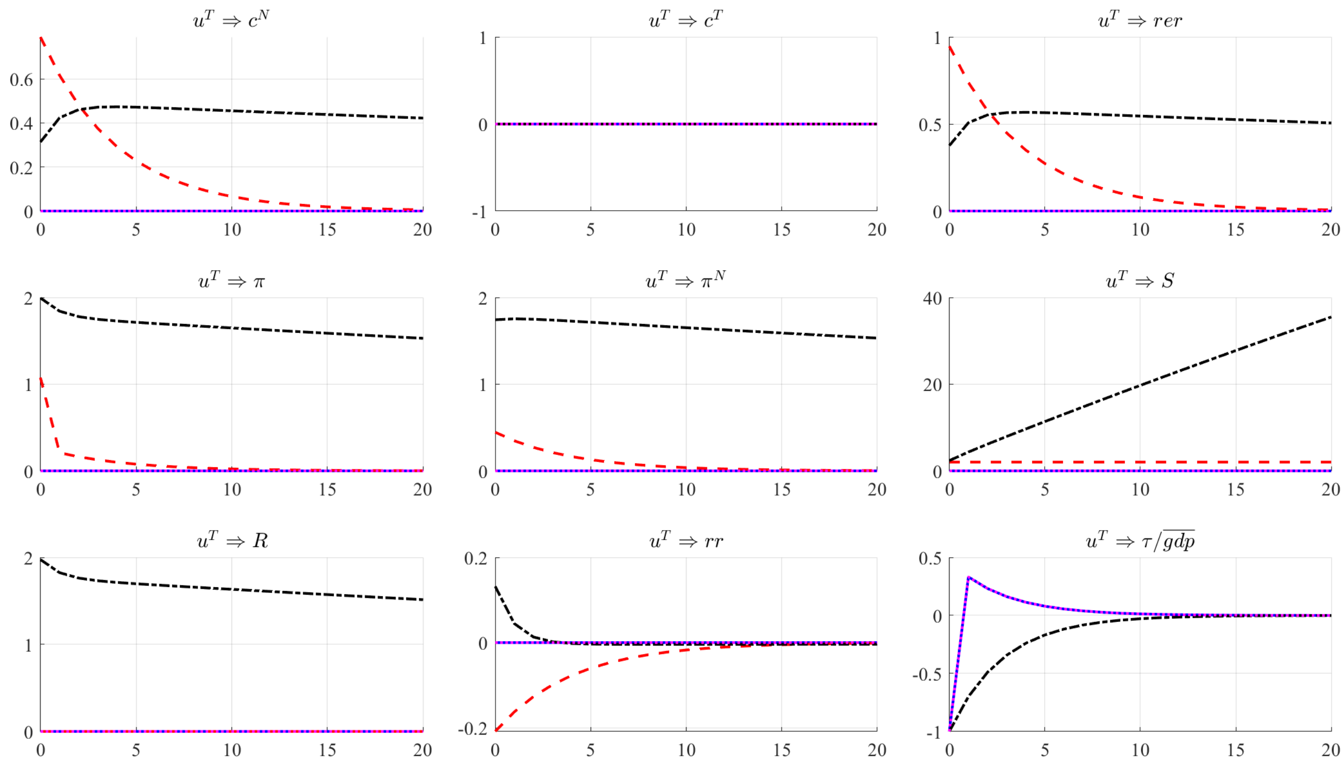
F.1 Baseline model: the role of ϕ_π

Figure E.1: Responses to a monetary shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.



Notes: The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue and dashed red lines are the same as in Figure 1 for comparison (i.e. respectively, $\phi_\pi = 1.5, \phi_T = 1$, and $\phi_\pi = 0, \phi_T = 0$). In addition, the dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99, \phi_T = 0$, while dotted-magenta lines use $\phi_\pi = 1.01, \phi_T = 1$. In all cases, $\Omega = 0$. See Figure 1 for variables' definitions.

Figure E.2: Responses to a lump-sum-tax shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.

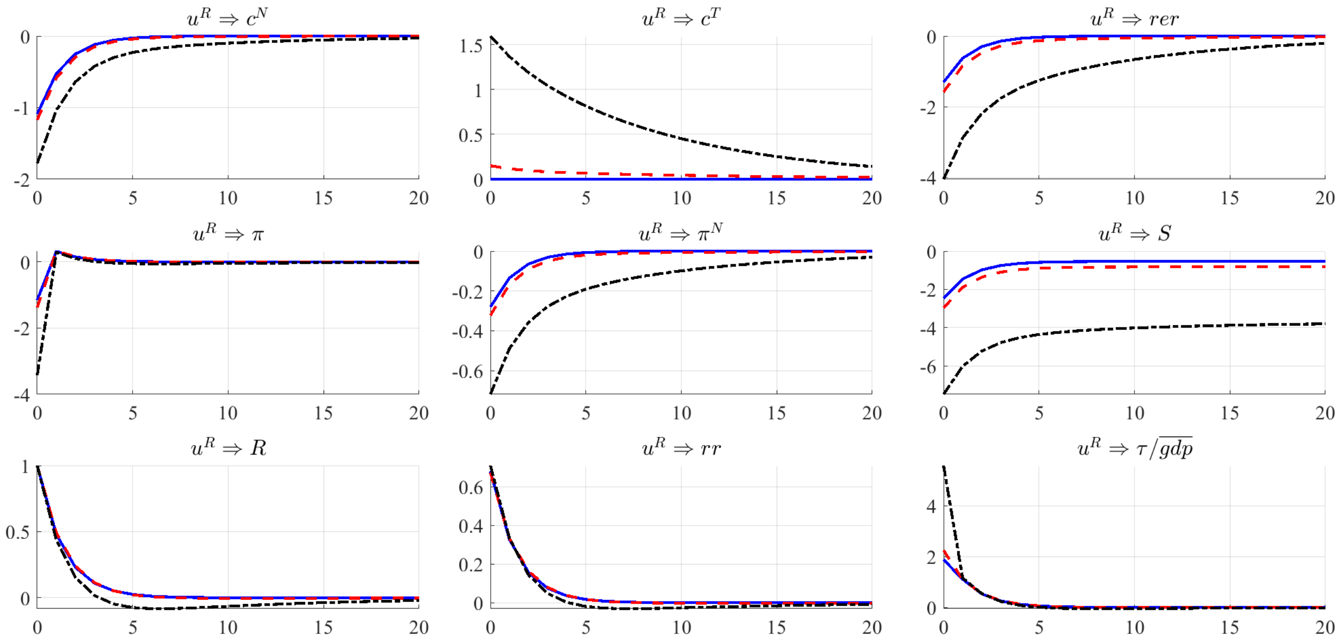


Notes: The shock is a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\Omega = 0$, differing in the fiscal and monetary configuration: solid-blue lines use $\phi_\pi = 1.5$ and $\phi_T = 1$, dashed-red lines $\phi_\pi = 0, \phi_T = 0$, dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99$ and $\phi_T = 0$, and dotted-magenta lines use $\phi_\pi = 1.01$ and $\phi_T = 1$. See Figure 1 for variables' definitions.

F.2 Fisherian deflation

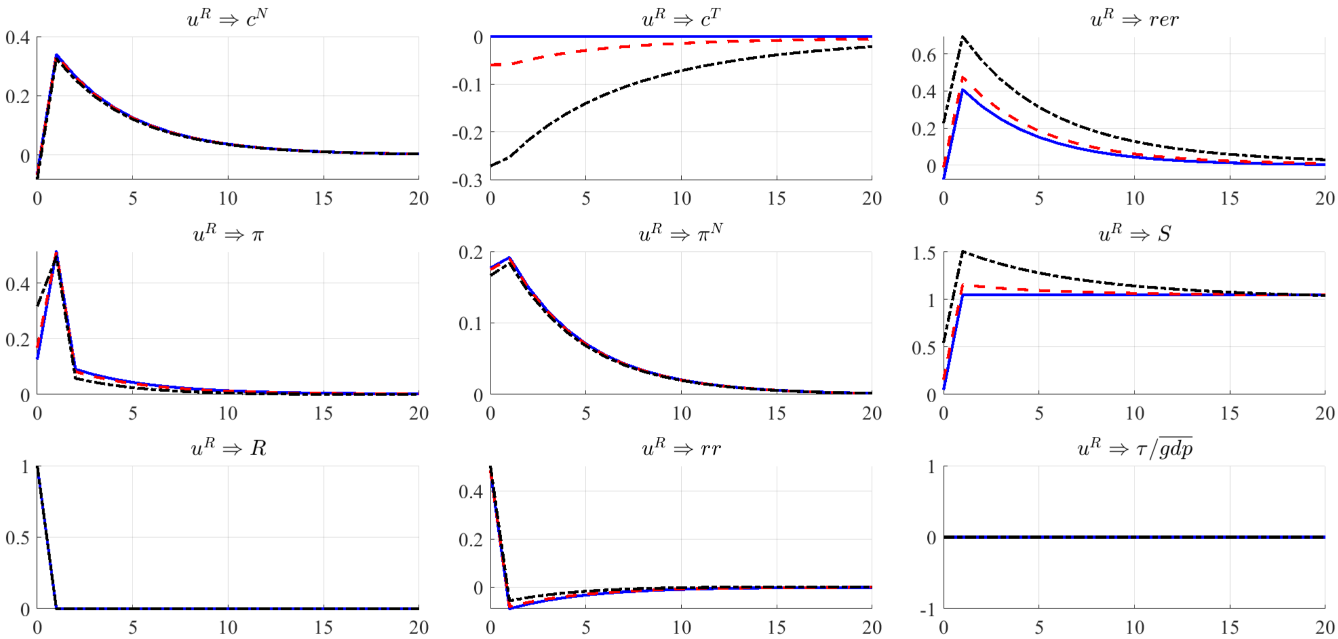
F.2.1 Monetary shocks

Figure E.3: Responses to a monetary shock, Ricardian, different values of ϕ_B.



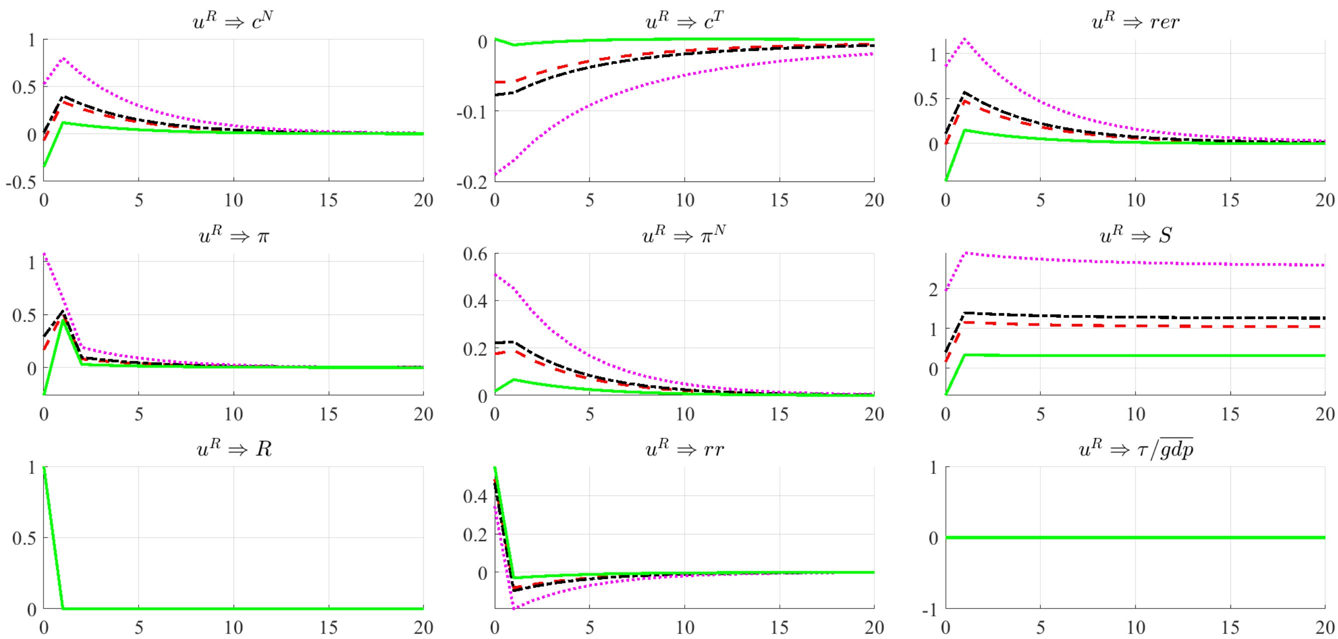
Notes: The figure displays the responses under an active monetary and passive fiscal setup, with $\phi_\pi = 1.5$ and $\phi_T = 1$ and $\Omega = 1$. Each line differs depending on the value for ϕ_B and the premium specification: In dashed-red $\phi_B = 0$, in dashed-dotted black $\phi_B = 0.05$ and in dotted magenta $\phi_B = 0.1$.

Figure E.4: Responses to a monetary shock, Non-Ricardian, different values of ϕ_B, $\Omega = 0$.



Notes: This figure displays the responses under an active fiscal and passive monetary setup, with $\phi_\pi = 0$ and $\phi_T = 0$ and $\Omega = 0$. Each line differs depending on the value for ϕ_B and the premium specification: In dashed-red $\phi_B = 0$, in dashed-dotted black $\phi_B = 0.05$ and in dotted magenta $\phi_B = 0.1$.

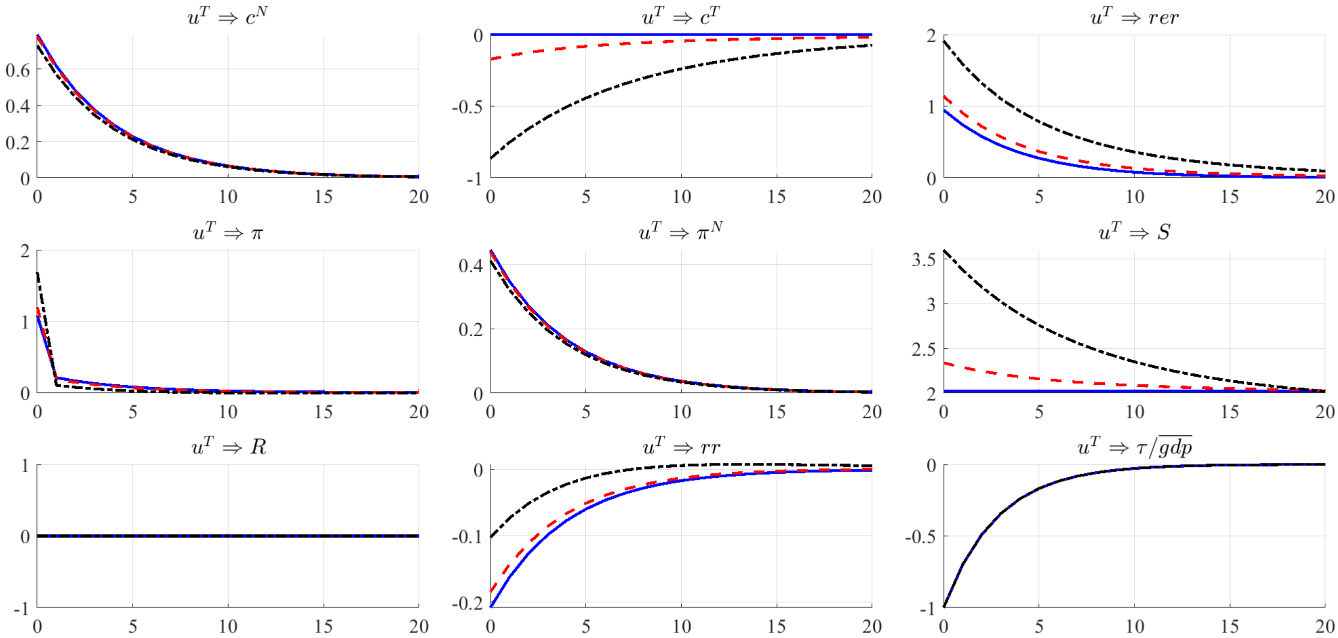
Figure E.5: Responses to a monetary shock, Non-Ricardian, different values of Ω, with $\phi_B = 0.05$.



Notes: The figure is analogous to Figure 3, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, all of them with $\phi_B = 0.05$. They differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.9$, and in solid-green $\Omega = 1$.

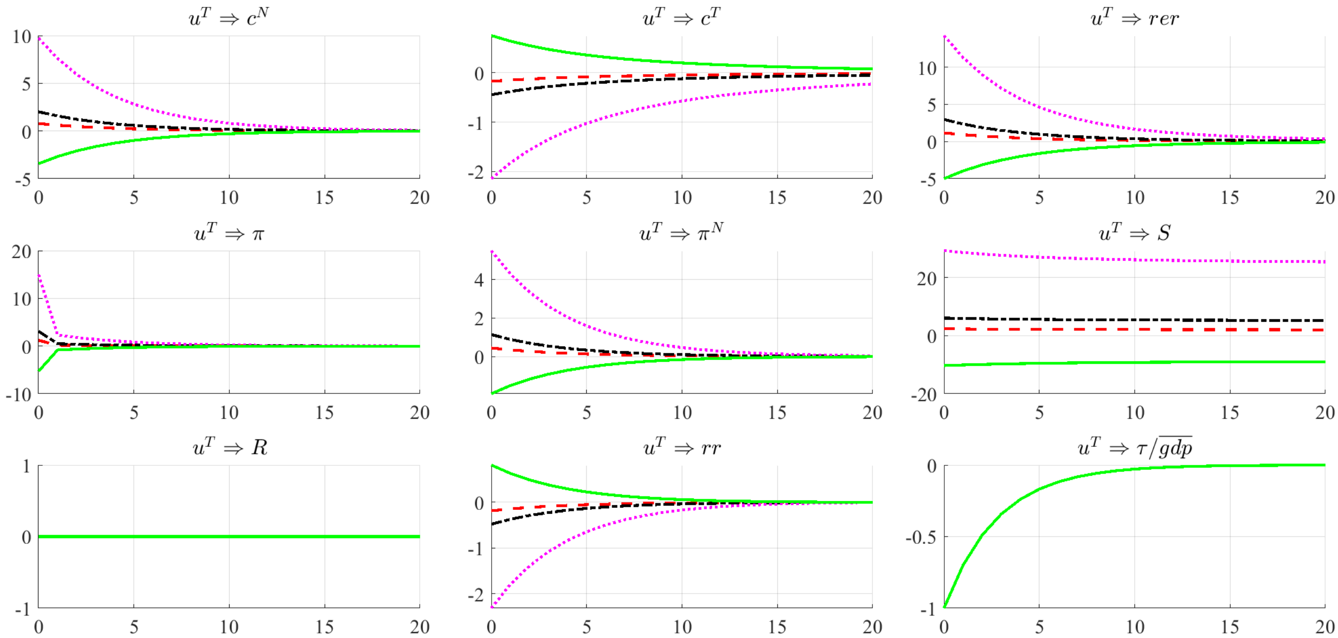
F.2.2 Fiscal shocks

Figure E.6: Responses to a fiscal shock, Non-Ricardian, different values of ϕ_B, $\Omega = 0$.



Notes: This figure displays the responses under an active fiscal and passive monetary setup, with $\phi_\pi = 0$ and $\phi_T = 0$ and $\Omega = 0$. Each line differs depending on the value for ϕ_B and the premium specification: In dashed-red $\phi_B = 0$, in dashed-dotted black $\phi_B = 0.05$ and in dotted magenta $\phi_B = 0.1$.

Figure E.7: Responses to a fiscal shock, Non-Ricardian, different values of Ω, with $\phi_B = 0.05$.



Notes: The figure is analogous to Figure 5, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, all of them with $\phi_B = 0.05$. They differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.9$, and in solid-green $\Omega = 1$.

G Dynamics Induced by Real Shocks

In this section, we study the effects of shocks to traded output y^T (which could also be interpreted as terms-of-trade shocks in this simple setup) and to the world interest rate R^W (shocks to π^* induce effects similar to those generated by R^W). For each of them, we first describe the propagation under Ricardian/passive fiscal policy, then analyze the role played by a Non-Ricardian setup, and finally study the role played by the currency composition of government debt. Unfortunately, the algebraic approach using the NT-based policy setup is not sufficiently clear to derive clean results in the case of real shocks: even if disturbances are i.i.d., as long as they affect tradable consumption and the net-foreign-debt position, endogenous persistence arises, complicating the algebra. For that reason, all results are generated using the CPI-based framework with the baseline calibration.

G.1 Traded output

Figure E.8 displays the dynamics triggered by an increase in y^T, with an autocorrelation of 0.7, normalized to increase traded consumption by 1%, assuming $\Omega = 0$. Before analyzing a specific policy regime, we should note that the shock induces a positive wealth effect, increasing the desired consumption of both types of goods. The increase in c^T is quite persistent, as the interest rate in dollars (R^*) is calibrated to have minor elasticity to the country's net-foreign debt position; thus, convergence back to the steady state is quite slow.[22] Additionally, the higher demand for non-tradables increases their relative price, leading to a real appreciation. If prices were fully flexible, this would materialize instantaneously. However, since non-traded prices are sticky, it takes time to reach their maximum appreciated value. Moreover, the real appreciation arises from a relatively small increase in non-trade inflation and, more significantly, from a nominal appreciation.

The specific dynamics are shaped by the fiscal and monetary regimes. The blue lines display the case with a Ricardian/passive tax response ($\phi_T = 1$) and an active Taylor rule for the interest rate ($\phi_\pi = 1.5$). Under such a configuration, the policy rate falls following the reduction in inflation. This, in turn, induces an even further appreciation: by the UIP condition (9), a fall in the domestic rate requires a further expected appreciation. In addition, as the policy rate falls by more than overall inflation, the real rate also decreases. From the perspective of households, this further increases non-traded consumption (c^N) on impact through inter-temporal substitution.

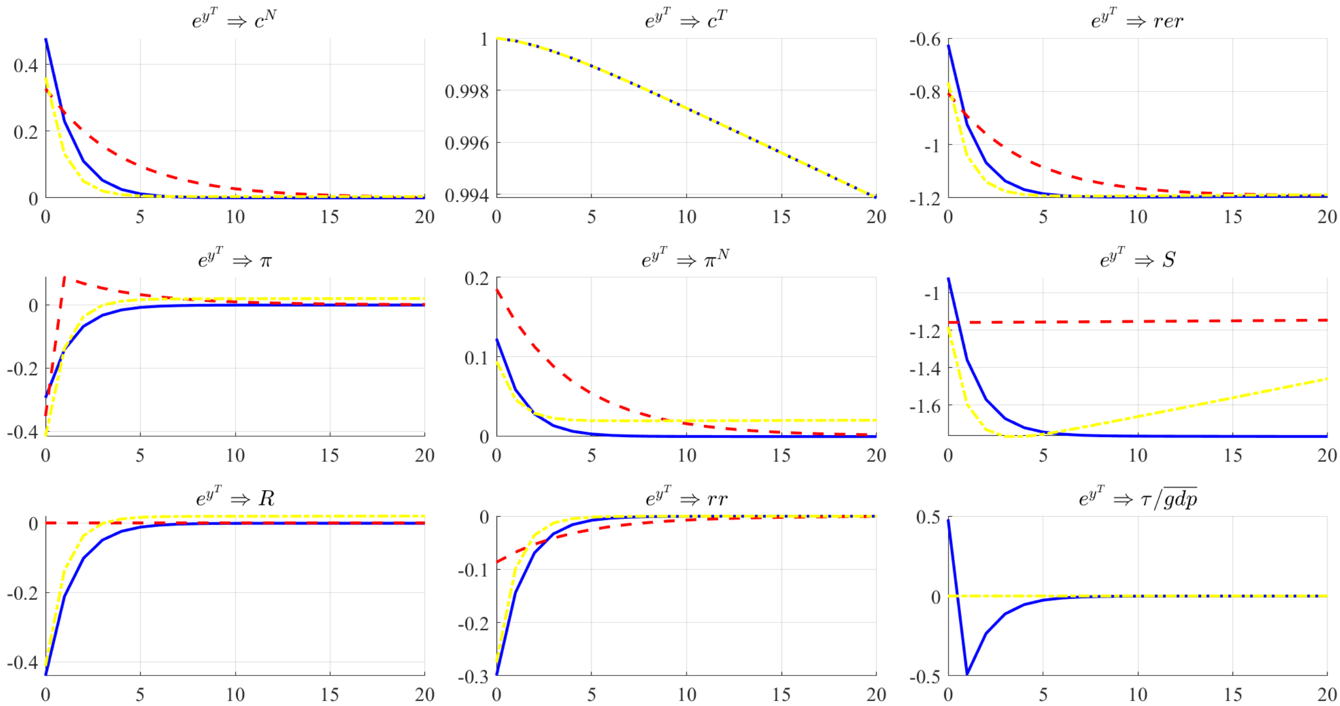
In turn, fiscal policy is also affected by the shock through two different channels. First, the reduction in inflation increases the real value of the outstanding nominal debt. Second, the reduction in the real rate increases the net present value of taxes. The former dominates in equation (8), as can be seen from the fact that lump-sum taxes increase in response to the shock.

The dashed-red lines in Figure E.8 show the responses to the same shock under a Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). While responses are qualitatively similar to those in the previous policy configuration, two quantitative differences emerge from the policy rate remaining fixed. First, the further nominal appreciation in the period following the shock that we described in the blue lines disappears, which in turn eliminates the persistence of the initial fall in overall inflation. Second, as the nominal interest rate does not change but expected inflation is positive, the drop in the real rate is much smaller than in the previous policy configurations.

Moreover, the smaller reduction in the real rate limits the intertemporal substitution that influences non-traded consumption, and therefore c^N increases by less on impact. This also implies that

[22] As this is a transitory shock, R^* persistently falls as an improvement in the current account is generated, causing the elastic premium to decrease.

Figure E.8: Responses to a traded-output shock, Ricardian vs. Non-Ricardian.



Notes: The figure show the responses to a positive shock to y^T, with an autocorrelation of 0.7, normalized to increase c^T by 1% at the moment the shock hits. All cases feature $\Omega = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 1 for variables' definitions.

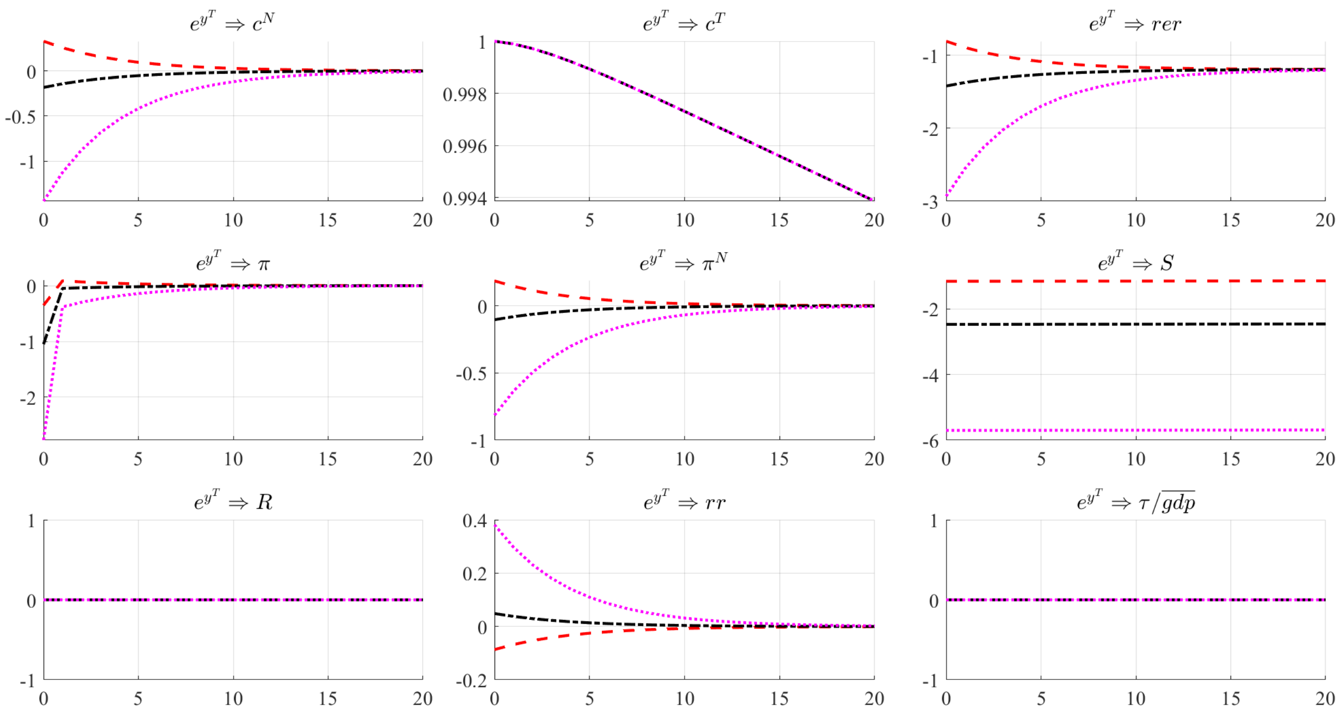
the path of c^N in the periods after the shock hits converges more slowly to the steady state. In turn, as π^N is forward-looking under Calvo prices, the initial increase in non-traded inflation is larger, anticipating the expected higher demand in the future.

In terms of fiscal policy, as lump-sum taxes are fixed, the right-hand side of the FTPL equation (8) increases as the real rate falls. This implies that, relative to the case with Ricardian policy, prices need to fall even more. We have already described the reasons for which this is not going to happen through π^N, which implies that the initial nominal appreciation is larger.

Notice that there are two differences that distinguish the dashed-red lines from the solid-blue ones: both taxes and interest rates are kept constant if $\phi_T = \phi_\pi = 0$. In order to better understand the relative importance of each of them, Figure E.8 includes an additional case (in dashed-dotted yellow lines) where taxes still do not move ($\phi_T = 0$), but the policy rate increases with inflation, using $\phi_\pi = 0.99$ (just below the value required for equilibrium uniqueness under an active fiscal policy). In this third case, the dynamics of both the nominal and real interest rates, non-traded inflation and consumption, as well as the real exchange rate, are similar to those under a Ricardian/passive fiscal policy. Thus, to a large extent, the differences between the solid-blue and dashed-red lines are driven by the different behavior of the policy rate. Still, the FTPL channel continues to play a role. As the real rate drops even further in the dashed-dotted yellow lines, it further increases the right-hand side of the FTPL equation (8). Thus, an initially larger nominal appreciation is still required to compensate if taxes do not change.

Dynamics are significantly altered once we allow for a positive share of debt denominated in foreign currency. Figure E.9 shows four cases, all of which have $\phi_T = 0$ and $\phi_\pi = 0$, but with different values of Ω: 0 (dashed red), 0.5 (dashed-dotted black), and 0.75 (dotted magenta).[23] With the exception of the case with fully dollar denominated debt (which we analyze below), larger values of Ω result in further nominal and real appreciations, while non-traded inflation and consumption now fall. What triggers these differences?

Figure E.9: Responses to a traded-output shock, Non-Ricardian, different values of Ω.



Notes: The figure is analogous to Figure E.8, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.75$.

In the presence of foreign denominated debt, the real appreciation generated by the shock (which has a real origin, present even under flexible prices) tends to reduce the real value (in domestic consumption units) of outstanding dollar-denominated debt if $\Omega > 0$. For a given path of lump-sum taxes (as $\phi_T = 0$), satisfying the FTPL equation (8) requires either an increase in the real value of outstanding peso-denominated debt (which can materialize if the price level drops as the shock hits), a reduction in the net-present value of primary surpluses (which requires the real rate to increase), or a combination of both. These two alternatives help each other in achieving the goal: a higher real rate pushes c^N downwards, which in turn reduces non-traded inflation today, helping to dilute the real value of outstanding debt in pesos. At the same time, the real-exchange rate appreciates even further, which also adds an intra-temporal substitution channel that further reduces c^N and thus π^N. Moreover, these effects are magnified for larger shares $\Omega > 0$.

The figure does not include the case of $\Omega = 1$, which is shown in Appendix G.3, as this case significantly alters the scale of the graphs. However, as hinted at by previous results, if government debt is fully denominated in dollars, the dynamics have the opposite sign. In particular, this means that a real *depreciation* is induced, which might seem counterintuitive. What is behind this result?

As we mentioned, the flexible-price effect triggered by this shock is a drop in both *rer* and R^*. If nothing else were to happen, this is inconsistent with the FTPL valuation equation (8) when $\Omega = 1$:

[23] In Appendix G.3, a similar figure is displayed with $\phi_\pi = 0.99$, where it can be seen that the differences are magnified.

the *rer* falling reduces the real value of outstanding debt (left-hand side), while the drop in R^* (which tends to lower *rr* through UIP) increases the net present value of primary surpluses (right-hand side); thus, the equation does not hold. Therefore, either the real exchange rate should not appreciate, the real interest rate should increase, or a combination of both. However, to obtain a larger real rate (given that the nominal rate is fixed), additional expected non-trade inflation is required, which contradicts a real appreciation unless the nominal exchange rate jumps even further. This explains why not only do responses have the opposite sign, but the responses are much larger in absolute value.[24]

In conclusion, while in the case of government debt fully denominated in pesos, FTPL considerations do not qualitatively alter the propagation of shocks to y^T; this is not the case if a fraction of the debt is denominated in foreign currency. In particular, a positive shock might lead to a contraction in the non-trade sector, even though the real exchange rate is appreciating. This happens because, here, the appreciation is not mainly led by a larger demand in non-tradables, but is produced by the FTPL channel that requires non-traded prices to fall, which materializes via a lower non-traded demand.[25]

G.2 World interest rate

We now turn to the analysis of the consequences generated by a drop in the world interest rate R^W. Figure E.10 displays the dynamics of a shock with a persistence of 0.7, normalized to raise c^T by 1% on impact, under the assumption of $\Omega = 0$. The flexible-price mechanism behind the propagation works as follows. The fall in the international cost of borrowing induces an intertemporal substitution effect and a positive wealth effect (as the country is a net-foreign borrower in our calibration), both of which increase the desired consumption of both types of goods. In particular, the additional demand for c^N increases its relative price, leading to a real appreciation. In a world with sticky prices, the final outcome hinges on the fiscal and monetary configuration.

The solid-blue lines in Figure E.10 correspond to the passive-fiscal, active-monetary setup ($\phi_T = 1$ and $\phi_\pi = 1.5$). We can see that c^N increases on impact, and the real appreciation materializes through the nominal exchange rate. Also, c^N falls below its steady state level in the periods following the realization of the shock, which also explains why non-traded inflation falls further after a few periods. This occurs because there are two effects at play. On one hand, the homotheticity of preferences implies an increase in c^N following the rise in c^T; on the other hand, the fall in *rer* makes non-tradables relatively costlier. Thus, initially, the former effect dominates, while the latter becomes more relevant in the following periods.

Another complementary view of this same effect comes from the dynamics of the domestic real rate *rr*. While it falls following the shock, it returns to the steady state at a faster rate than the world interest rate R^W.[26] In turn, this implies that the real exchange rate (via the UIP condition) also takes longer to converge back to the steady state after the initial drop. As this relatively more appreciated path implies costlier non-traded goods, c^N falls after the initial increase, also lowering π^N as firms anticipate this future lower demand in the presence of Calvo frictions.

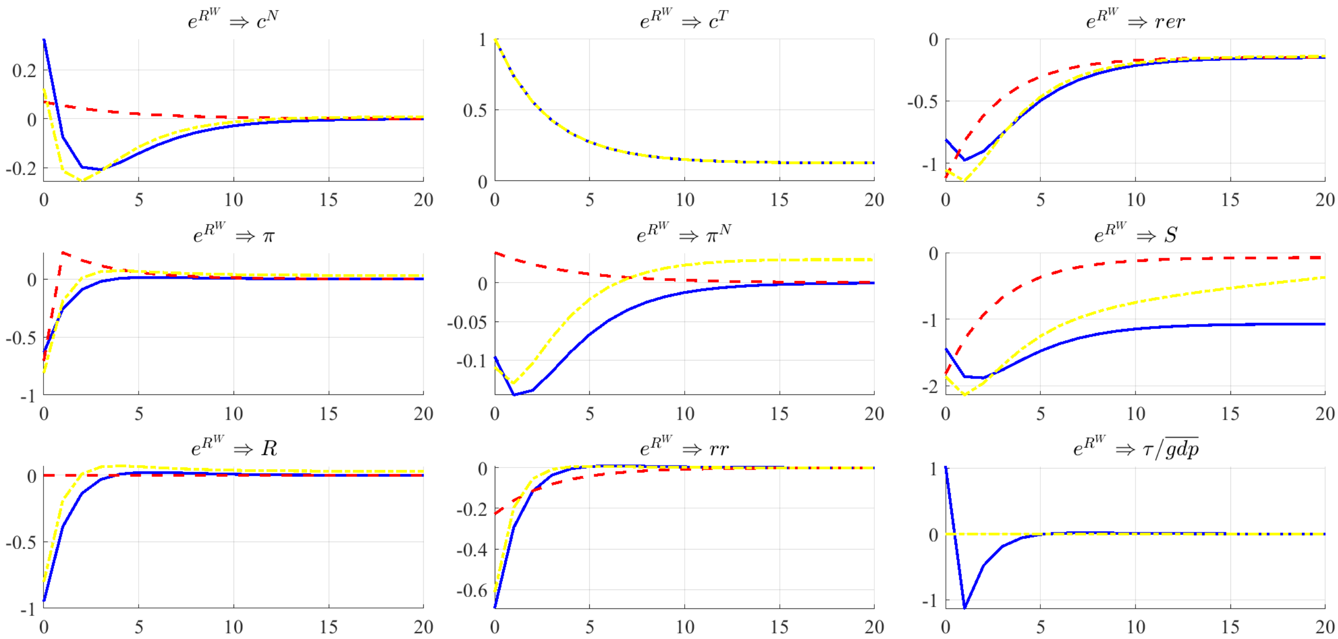
The dashed-red lines in Figure E.10 correspond to the case of Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). In that case, notice that c^N increases and converges from above

[24] One can numerically verify that this happens whenever Ω is larger than a threshold analogous to $\Omega_{\text{cut},1}$, which equals 0.945 in this case, as we previously mentioned.

[25] As shown in Appendix G.3, having a policy rate that reacts to inflation with $\phi_\pi = 0.99$ only magnifies these differences, as the drop in the policy rate in such a case generates a path for the nominal exchange rate that appreciates even further, which requires an even larger and more persistent fall in π^N (and thus c^N).

[26] While that evolution of R^W is not shown in the figure, its autocorrelation of 0.7 implies that, for instance, five periods after the shock hits, it is still at a value of 0.08 below the steady state, whereas *rr* is virtually zero at that point.

Figure E.10: Responses to a world interest-rate shock, Ricardian vs. Non-Ricardian.



Notes: The figure shows the responses to a negative shock to R^W, with an autocorrelation of 0.7, normalized to increase c^T by 1% at the moment the shock hits. All cases feature $\Omega = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 1 for variables' definitions.

to the steady state, which shows how the previously analyzed dynamics (blue lines) in the periods following the shock were heavily influenced by the path of the policy rate. This, in turn, induces a rise in π^N, while the nominal exchange rate converges monotonically after the initial appreciation.

From the FTPL perspective, this initial drop in the nominal exchange rate materializes to compensate for the fact that the fall in R^W puts downward pressure on the relevant discount rate. As lump-sum taxes are fixed, the net present value of primary surpluses increases, requiring a fall in the CPI level to compensate on the left-hand side of (8), which materializes through S (as π^N is increasing). Overall, the dynamics under this alternative policy configuration are quite different from those with passive fiscal policy.

However, as we saw in the analysis of a y^T shock, these differences are mainly generated by the assumption of a constant policy rate. Indeed, the dashed-dotted yellow lines in Figure E.10 illustrate the case of an active fiscal policy ($\phi_T = 0$) but with a monetary-policy rate sensitive to inflation ($\phi_\pi = 0.99$). We can see that the dynamics are much closer to the case of a Ricardian fiscal policy, as represented by the solid-blue lines.

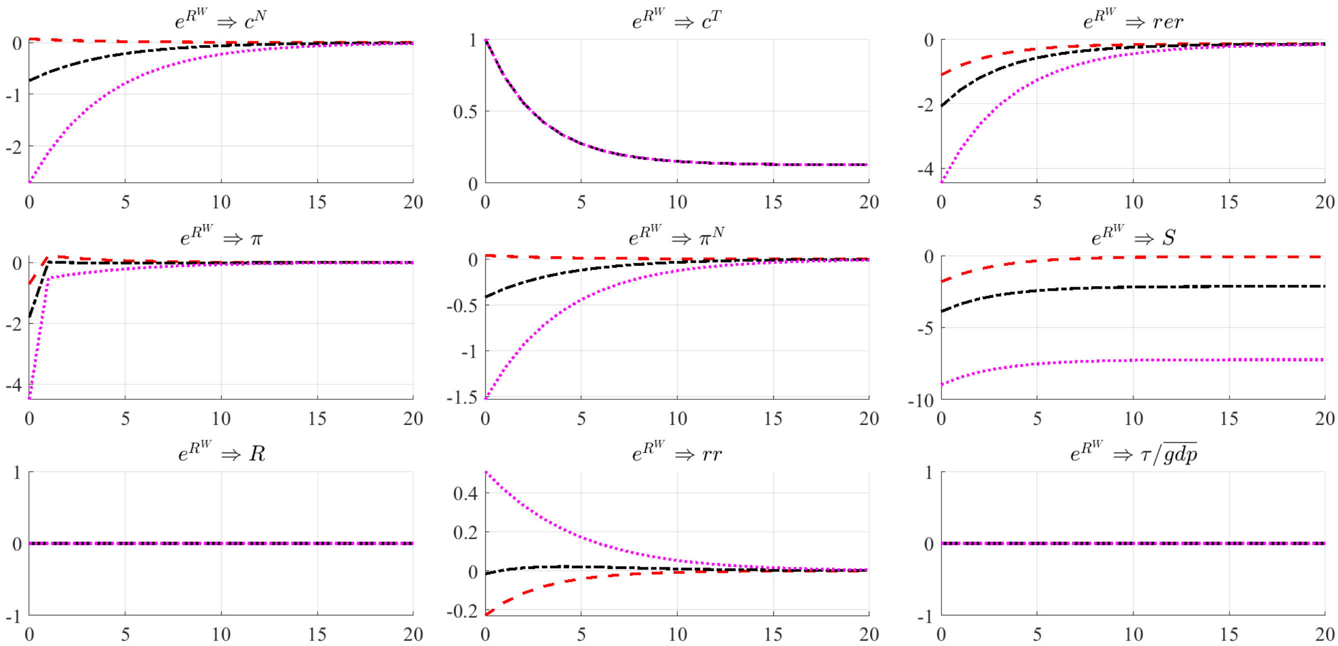
Figure E.11 displays the dynamics under the active-fiscal, passive-monetary setup ($\phi_\pi = 0 = \phi_T = 0$) for different values of the share of government debt denominated in dollars Ω.[27] As the shock induces a real appreciation, it reduces the real value (in domestic units) of outstanding dollar-denominated debt. In order to compensate, either the price level should fall to increase the value of debt obligations in pesos, or the real rate needs to rise to reduce the net-present value of primary surpluses (which is brought about by lower expected inflation), or a combination of both. As a result, the shock further reduces inflation for higher ratios Ω. In turn, non-traded consumption falls for larger shares of dollar-denominated debt: while the increase in c^T pushes to expand c^N, the real

[27] Appendix G.3 also includes a case with $\phi_T = 0$ and $\phi_\pi = 0.99$, showing more exacerbated differences relative to Figure E.11.

appreciation compensates (expenditure switching); and the latter dominates as a higher Ω induces a deeper fall in *rer*.

The case of $\Omega = 1$ is included in Appendix G.3. As expected from our previous analysis, the responses flip signs in this case, particularly inducing a real depreciation and an expansion in non-traded consumption. The effect on *rer* follows from the same arguments presented for the shock to y^T: the fall in the interest rate increases the net present value of primary surpluses, which requires a real depreciation for the valuation equation (8) to hold.

Figure E.11: Responses to a world interest-rate shock, Non-Ricardian, different values of Ω.

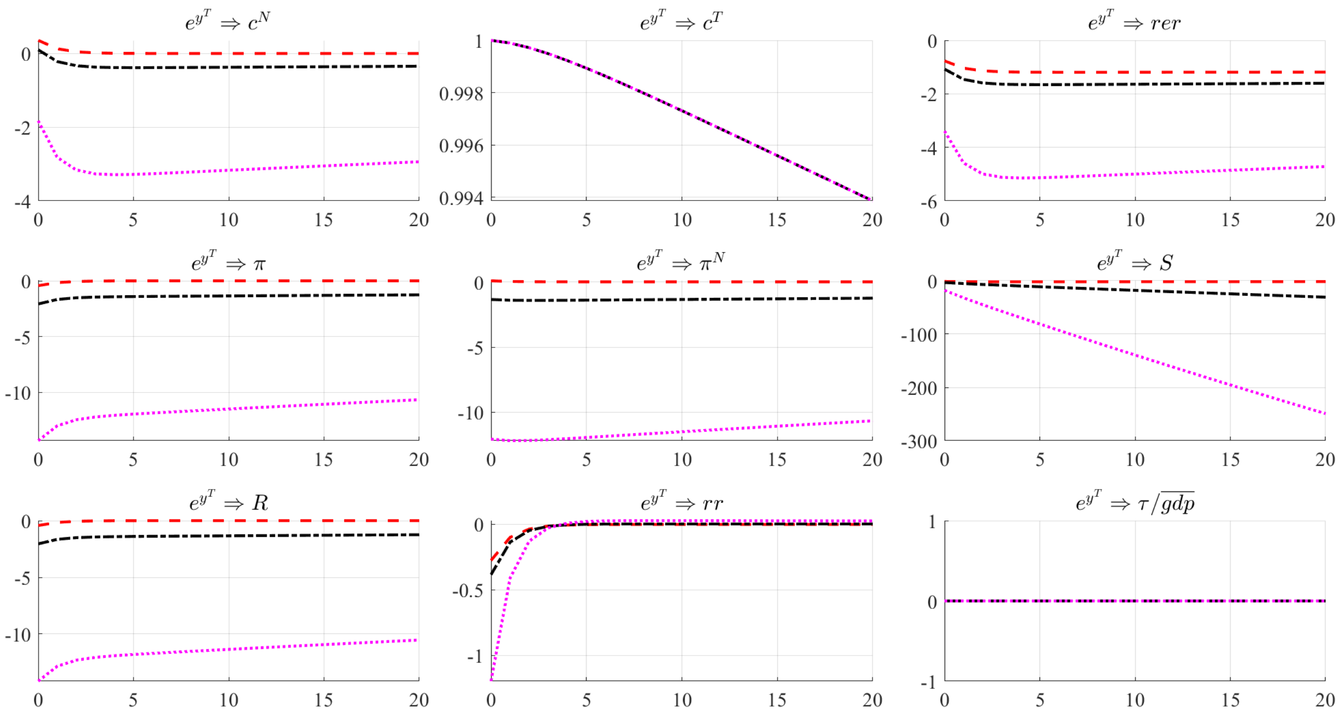


Notes: The figure is analogous to Figure E.10, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for Ω: In dashed-red $\Omega = 0$, in dashed-dotted black $\Omega = 0.5$, in dotted magenta $\Omega = 0.75$.

Overall, we see that the dynamics induced by a shock to the world interest rate are also significantly different when fiscal policy is active. Moreover, the currency composition of the debt is key to determining the results.

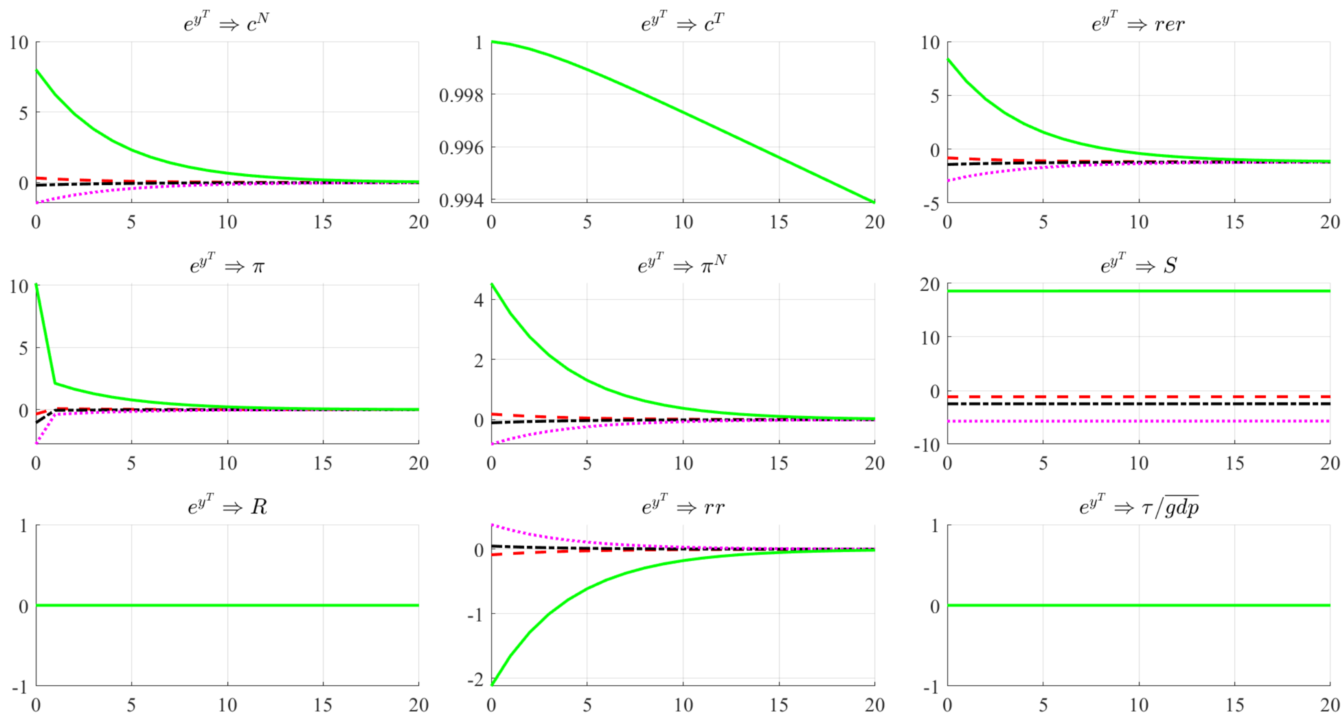
G.3 Additional figures

Figure E.12: Responses to a traded-output shock, Non-Ricardian, different values of Ω, with $\phi_\pi = 0.99$.



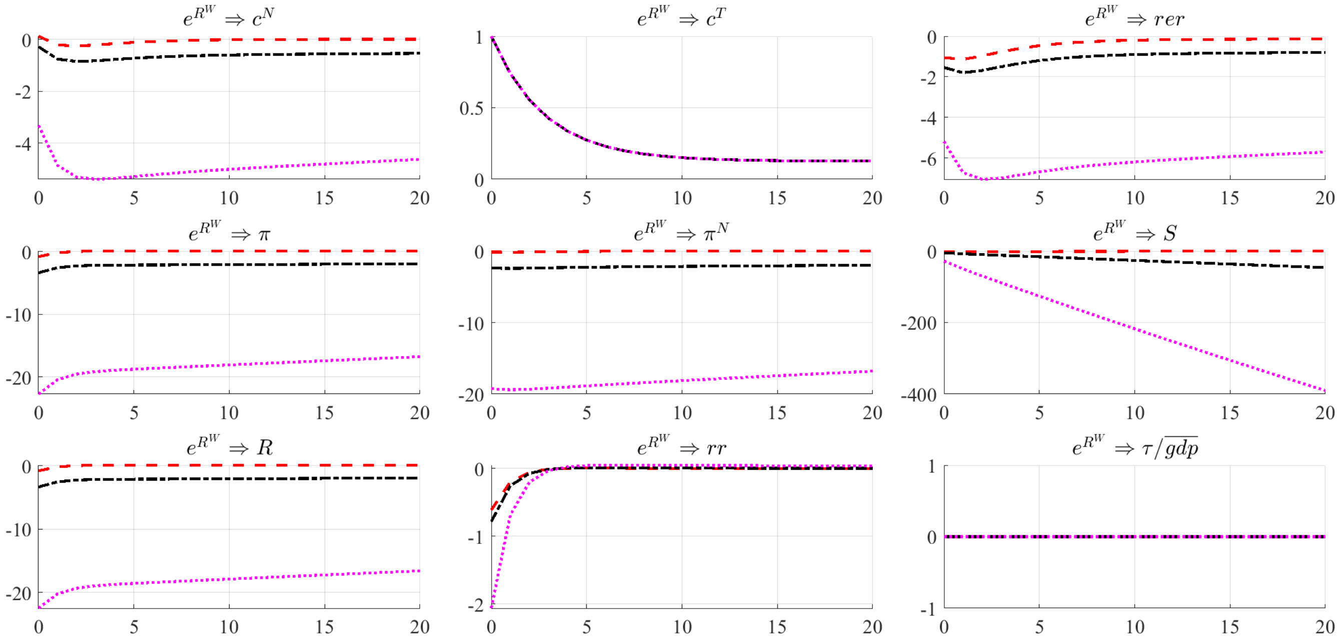
Notes: The figure is analogous to Figure E.9, but using a value for $\phi_\pi = 0.99$.

Figure E.13: Responses to a traded-output shock, Non-Ricardian, different values of Ω.



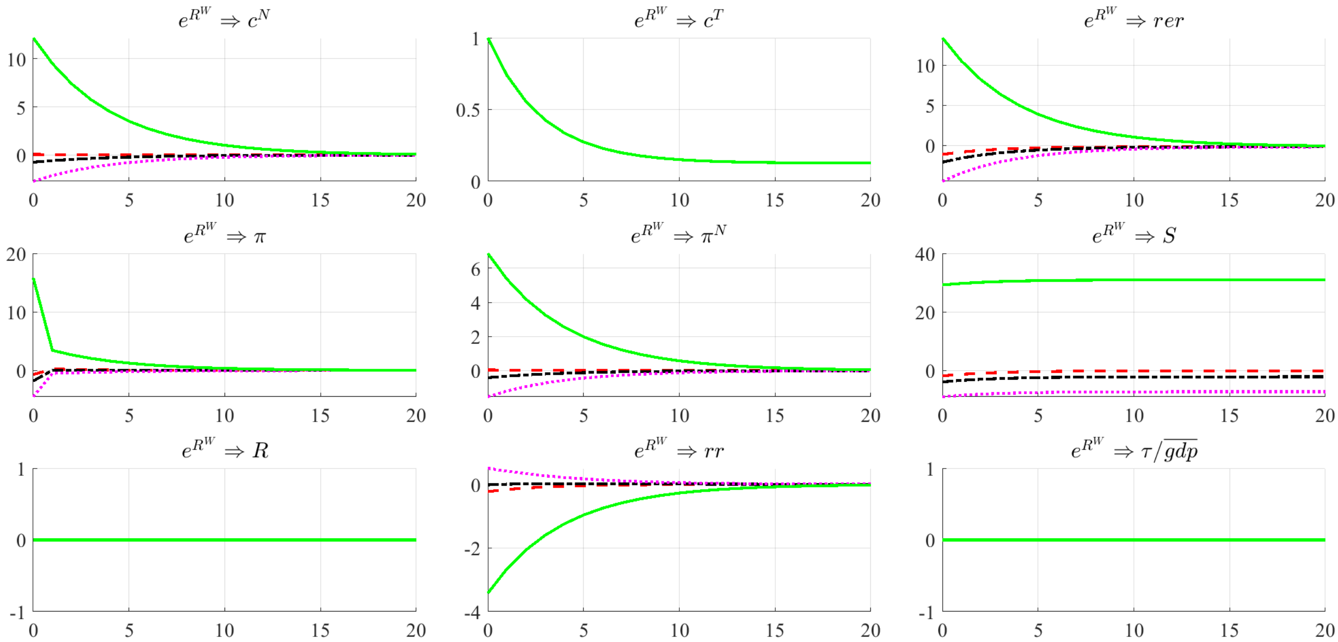
Notes: The figure is analogous to Figure E.9, adding also the case of $\Omega = 1$ in solid-green lines.

Figure E.14: Responses to a world interest-rate shock, Non-Ricardian, different values of Ω, with $\phi_\pi = 0.99$.



Notes: The figure is analogous to Figure E.11, but using a value for $\phi_\pi = 0.99$.

Figure E.15: Responses to a world interest-rate shock, Non-Ricardian, different values of Ω.



Notes: The figure is analogous to Figure E.11, adding also the case of $\Omega = 1$ in solid-green lines.