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The effect of risk aversion and cash flow risk on the equity share distribution in the entrepreneur and venture capital contract

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Abstract

We study the impact of risk aversion and cash flow risk on the allocation of equity shares between entrepreneurs and venture capitalists in a setting characterized by double-sided moral hazard. Cash flows are simultaneously influenced by both price risk and background risk. We evaluate the main results of the model through simulation exercises that highlight the parameters that influence the dynamics of optimal equity share in project cash flows, such as the entrepreneur's risk aversion relative to the VC's and other partner attributes like the productivity and efficiency of their respective efforts. We carried out the analysis under different risk and effort complementarity scenarios. We find that the productivity and efficiency of partners' efforts are dominated by their risk aversion, and that the slope of the effect of these traits on the optimal equity share trajectory is modified by risk parameters and effort complementarity.

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1 Introduction

Entrepreneurial investment is fraught, risky and highly concentrated; nonetheless, empirical data indicates that the elevated risk associated with entrepreneurial activity can be offset by a premium in returns. What motivates a decision-maker to pursue entrepreneurship? A hypothesis suggests that entrepreneurs possess different risk preferences. This idea posits that decision-makers with more risk tolerance are more inclined to opt for entrepreneurship (Kanbur (1979), Kihlstrom and Laffont (1979)). This hypothesis assumes that decision-makers exhibit DARA (decreasing absolute risk aversion) preferences.[1] This indicates a positive association between wealth and the rate of business starts up, since wealth diminishes risk aversion or enhances risk tolerance (Cressy (2000)). Moreover, numerous studies have examined and validated this hypothesis (Cramer, Hartog, Jonker, and Van Praag (2002); Polkovnichenko (2003); Kan and Tsai (2006); Caliendo, Fossen, and Kritikos (2009); Hvide and Panos (2014); Berkhout, Hartog, and van Praag (2016)).

Despite the extensive theoretical and empirical research on the risk preferences of entrepreneurs, this premise is not considered in the analysis of the contractual connection between entrepreneurs and venture capitalists (VCs). In accordance with established research, it is assumed that all decision-makers exhibit risk neutrality, so guaranteeing that the model's outcomes are influenced solely by incentives under asymmetric information, rather than by the players' risk preferences.

VCs are firms that invest in the nascent stages of new enterprises, offering not only financial resources but also management expertise, advice, and a network of connections to the entrepreneur, so enhancing the probability of success for the new venture. There

[1] In a utility function $u(\cdot)$, risk aversion is shown by the negative second derivative of the utility function ($u'' < 0$), and its intensity is quantified by $A = -\frac{u''}{u'}$. Consequently, A represents the concavity of $u(\cdot)$ and quantifies the degree of absolute risk aversion (Arrow (1965); Pratt (1964)). The inverse of absolute risk aversion is termed absolute risk tolerance, represented as $T = \frac{1}{A}$. Since the marginal risk tolerance is represented by $T' = -\frac{A'}{A^2}$, the condition $T' > 0$ is equivalent to $A' < 0$.

exists an intrinsic complementarity between the entrepreneur's work and effort and the soft skills, expertise, and network provided by the VC. A moral hazard problem emerges due to the unobservable effort levels of partners. The issue is double, because both the entrepreneur and the VC provide private efforts that influence the project's probability of success.

This paper challenges the widely held belief in theoretical literature that entrepreneurs and VCs are risk-neutral. To establish an analytical framework that addresses the trade-off between risk and incentives inherent in the contractual connection between entrepreneurs and VCs, we utilize a double-sided moral hazard model. To the best of our knowledge, the connection between entrepreneurs and VCs utilizing changes in risk and risk-aversion as a key characteristic in creating the optimal contract and the rights of the project cash flows has not been covered in the literature.[2]

The risk-neutral contract serves as our benchmark. The equilibrium in this scenario is reached when the entrepreneur's marginal value of effort equals the VC's marginal value of effort. We examine how this equilibrium condition is impacted by the entrepreneur's and VC's risk preferences. This analysis also includes two distinct risk types, a background risk and a price risk, that impact the equilibrium condition. The former is referred to as an additive risk, whereas the latter is defined as a multiplicative risk. We also look at the combined effect of both risks.

To get a clearer understanding of the model's main result, we conduct simulations. Our model posits that players' risk aversion and changes in risk are critical factors in the allocation of equity shares between entrepreneurs and VCs. Through simulation, we assess the model and present the findings in a set of three-dimensional graphs. We look at how a

[2] Amit, Muller, and Cockburn (1995) studied the principal-agent dilemma between averse-risk entrepreneurs and neutral-risk investors. There is one moral hazard issue in this study since entrepreneurs are the sole individuals that realized unobservable efforts. Ewens, Jones, and Rhodes-Kropf (2013) also analyzed the principal-agent problem between VCs and investors. The moral hazard problem alters the negotiation between VCs and entrepreneurs. In this paper, VCs are risk-averse and the only ones who make unobservable efforts.

set of traits of the entrepreneur and the VC impact the dynamics of their individual efforts and equity shares. Productivity, efficiency, and risk aversion make up the parameters. The analysis takes place under various risk scenarios and effort complementarity. These risk scenarios are represented by changes in price risk and background risk variances and their covariance.

We find that the entrepreneur's equity share in the project's cash flows rises in tandem with an increase in the entrepreneur's effort productivity relative to the venture capitalist's effort productivity, and vice versa. When we examine the relative importance of their efforts' efficiency, we get identical results. On the other hand, our findings also suggest that the optimal equity share given to the VC is larger when the entrepreneur is less willing to take risks than the VC. When the VC is more risk averse than the entrepreneur, the opposite happens. A novel finding suggests that risk aversion dominates the trajectory of the optimal equity share when efficiency or productivity interacts with it. Additional interesting results highlight that the slopes of the effects of productivity, efficiency, and risk aversion on optimal shares are modified by risk parameters and the complementarity of efforts.

Our paper is closely connected to other strands of literature. First, the environment in which entrepreneurs and VCs operate is unpredictable due to the rapidly evolving markets. In periods of shocks and crises, uncertainty may intensify to a degree that undermines the contractual connection between entrepreneurs and financiers, such as VCs. Block and Sandner (2009) examined the impact of the global financial crisis on the venture capital market. The financial crisis adversely affected the amount of funds obtained at each funding round, particularly for firms in later financing stages, which got approximately 20% less than they would have before to the crisis. Similar to the global financial crisis, the COVID-19 disruption significantly affected the VC market. Howell, Lerner, Nanda, and Townsend (2020) found that early-stage VC activity fell by 38%. Brown and Rocha (2020) examined VC activities in the United Kingdom during the COVID-19 crisis. The global pandemic adversely impacted the VC market, consistent with other entrepreneurial finance markets like the United States (Howell, Lerner, Nanda, and Townsend (2020)), however to a much lower extent than observed in nations such as China (Brown and Rocha (2020)).

Second, there is a substantial amount of studies on optimal contracts between a VC and an entrepreneur when risk neutrality prevails. In order to examine the dual role of VCs, specifically to finance and advise entrepreneurs, Casamatta (2003) created a double-sided moral hazard model. The key premise of this study is that the efforts are perfect substitutes, which implies that the cash-flow shares are entirely dependent on how efficiently the VC's and entrepreneur's efforts are implemented. De Bettignies and Brander (2007) model requires the entrepreneur to choose between bank finance and VC funding. A VC gives the entrepreneur advice, unlike a bank. The VC's unobserved effort potentially creates a moral hazard problem. The entrepreneur's unobserved effort also creates another potential moral hazard issue. In this case, it is paradoxical to discuss how the entrepreneur's skills are enhanced by the VC's experience and connections, as De Bettignies and Brander (2007), like Casamatta (2003), presume that the players' efforts are perfect substitutes. Therefore, the synergy of efforts is not important. How the project cash flows are allocated in this study depends on how productive the efforts are.

Kanniainen and Keuschnigg (2003, 2004) examined the relationship between an optimal number of portfolio firms and the quality of VC advice, under the assumption of neutral risk. They proved that adding more portfolio firms increases the risk of individual projects and dilutes managerial advice. Both studies made the assumption that VCs' effort is a continuous variable and entrepreneurs' effort is binary. The optimal number of firms that matches the effects of profit creation and profit destruction determines the cash-flow shares. The relationship between the optimal size of VCs' portfolios and the distribution of cash flows between VCs and entrepreneurs was also examined by Bernile, Cumming, and Lyandres (2007). Both theoretically and empirically, this study showed that cash flow shares depend on the size of the portfolio and that partner attributes like productivity and efficiency of effort influence both decision variables. The authors make the assumption that VCs' and entrepreneurs' efforts are additive, or perfect substitutes, and both partners are risk-neutral, as documented in prior research.

As the preceding paragraphs indicate, assuming risk neutrality and perfect substitute efforts is a common practice in the literature that examines the contractual connection between VCs and entrepreneurs.[3] [4] In this study, we remove these assumptions and scrutinize the impact of risk shifts, partners' risk aversion, and the degree of complementarity on the dynamics of shares in the projects' cash flows.

Third, there are connections between our paper and the literature on price risk and background risk. The concept of price risk was first introduced by Sandmo (1971), who showed that a firm's optimal output in a competitive market decreases with a mean-preserving increases in risk (also known as a second-order risk increase) if preferences show decreasing absolute risk aversion (DARA). The impact of a rise in mean-preserving price risk on a competitive firm's output level selection was also examined by Lippman and McCall (1981). They were the first to draw the conclusion that output will either rise or fall based on specific technical factors pertaining to the utility function's curvature and the forms of the average and marginal cost functions.[5] Bonilla, Sabat, and Vergara (2024) proposed a model that allows the simultaneous examination of price risk and background risk's impact on the firm's optimum product.[6] They provide empirical evidence to

[3] Elitzur and Gavious (2003, 2011), and Lukas, Molls, and Welling (2016) also adopt the assumption of neutral risk.

[4] Some studies, such as Repullo and Suarez (2004), De Bettignies (2008), Fairchild (2011), Vergara, Bonilla, and Sepulveda (2016), Narayanan and Levesque (2014), Narayanan and Levesque (2019), and Zeng (2023), use the idea of complementary efforts, but always in the context of risk neutrality.

[5] Leland (1972), Ishii (1977), Wong (2014) and Watt (2020) also examined the connection between price risk and the output's optimal firm.

[6] Wong (1996) also explores this idea.

support their theoretical findings. The Economic Political Uncertainty (EPU) measures background risk, a type of risk that impacts the production environment.[7]

Finally, we connect our work to other articles that quantify risk aversion with a constant absolute risk aversion (CARA) utility function.[8] Polkovnichenko (2003) applied this specific utility function to investigate the correlation between human capital and the choice to pursue entrepreneurship. She found that the presence of human capital significantly reduces the impact of unique and unpredictable risks on the additional compensation that an entrepreneur demands in order to establish a business. Hvide and Panos (2014) used a CARA utility function to establish a link between risk tolerance and entrepreneurship. They used stock market participation as a proxy for the coefficient of absolute risk aversion, along with various supplementary proxy users. The study revealed that individuals who participate in common stock are around 50% more inclined to initiate a business venture. Ewens, Jones, and Rhodes-Kropf (2013) showed how equilibrium asset prices are impacted by the principal-agent conflict between investors and VCs. Their model is defined by entrepreneurs who have ideas, investors who are well-diversified but lack the time and skills to evaluate and manage possible investments, and VCs, who are hired by investors and possess significant experience in evaluating and supervising entrepreneurial ideas. VCs can only manage a small number of investments because it takes a lot of time to oversee them. Because of this, VCs should be compensated for the substantial idiosyncratic risk they bear. The authors represented VCs' risk preferences using the

[7]Other examples of background risk are documented in the literature and include background wealth (Chavas (1985)), fixed costs (Machnes (1993)), health condition (Bleichrodt, Crainich, and Eeckhoudt (2003), Rey and Rochet (2004)), and environmental quality (Kama and Schubert (2004); Xepapadeas (2005)).

[8]The decision-maker's preferences are characterized by a negative exponential utility function known as CARA preferences: The function $u(y)$ is defined as $-\frac{exp(ay)}{a}$, where a represents the coefficient of absolute risk aversion, that is, $A = -\frac{u''}{u'} = a$. It can be established that if y follows a normal distribution with a mean of μ and a variance of σ^2, then the expected value of y is equal to $u(\mu - \frac{1}{2}a\sigma^2)$. The expression $\mu - \frac{1}{2}a\sigma^2$ represents the certainty equivalence. Maximizing expected utility is equivalent to maximizing the certainty equivalent.

CARA utility function. As a result, the equity shares are solely dependent on the VCs' risk aversion coefficient rather than the other players'.

The remainder of the paper is structured as follows: In Section 2, we present the model; in Section 3, we solve the model and present the main result; in Section 4, we simulate the model; and finally, in Section 5, we conclude.

2 Model

We will now analyze the situation of an entrepreneur who starts a project using a new and creative concept. The project necessitates three categories of inputs: an investment level represented by I and two unobservable efforts represented by e and a. The entrepreneur is alone responsible for providing effort level e, while the VC is exclusively responsible for supplying effort level a.

Following Casamatta (2003), an investment I can be provided by both, the entrepreneur and the VC. The opportunity cost of allocating money into the firm is the risk-free rate r, which is standardized to zero for purpose of simplicity. Let us denote the quantity of financing provided by the VC as A_{VC} and the quantity provided by the entrepreneur as $I - A_{VC}$.

Following a structure similar to Bhattacharyya and Lafontaine (1995), the project is risky, and the project's income is modeled using a production function based on two arguments: the entrepreneur's effort and the VC's effort. A general production function is used as follows:

$$\pi = pf(e, a) + \varepsilon \tag{1}$$

where π is the total income earned, p is a stochastic price or productivity shock normally distributed with mean μ_p and variance σ_p^2, and ε is a random variable, earning or cost shock, distributes normally with mean μ_ε and variance σ_ε^2 (Bernhardt (2000)). Therefore,

$\pi \sim N(\mu_p f + \mu_\varepsilon, \sigma_p^2 f^2 + 2f\sigma_{pe} + \sigma_\varepsilon^2)$, where $\sigma_{p\varepsilon}$ represents the covariance between p and ε.

It is assumed that the production function $f(\cdot, \cdot)$ is a neoclassical production function. In other words, the first derivatives f_e and f_a are positive, the second derivatives f_{ee} and f_{aa} are negative, and the cross-derivatives $f_{ea} = f_{ae} > 0$ are positive. It is also assumed that $f(0, a) = f(e, 0) = 0$, $\lim_{e \to 0} f_e = \lim_{a \to 0} f_a = \infty$ and $\lim_{e \to \infty} f_e = \lim_{a \to \infty} f_a = 0$. These assumptions simply underscore how production technology requires both inputs, and thus we can anticipate an interior solution to the problem.

It is assumed that the entrepreneur's effort is not observable by the VC and that the VC's effort is not observable by the entrepreneur, so a double-sided moral hazard issue arises.

Effort levels are costly. If $C(e)$ denotes the disutility of the entrepreneur's effort while $B(a)$ represents the disutility of the VC's effort, it is assumed that the disutility functions are increasing at growing rates, namely $C_e, B_a > 0$, $C_{ee}, B_{aa} > 0$. In addition, $C(0) = B(0) = 0$ and $C_e(0) = B_a(0) = 0$.

The entrepreneur has constant absolute risk-averse (CARA) risk preferences represented by $u(w_E, e) = -exp(-\eta_E[w_E - C(e)])$, where w_E represents net income for the entrepreneur, which is shown by $w_E = (1 - \alpha)(pf(e, a) + \varepsilon) - (I - A_{VC})$, and $\eta_E > 0$ is the entrepreneur's coefficient of absolute risk aversion ($\eta_E = -u''/u'$). The variable α is the fraction of the cash flow share (equity) assigned to the VC in the project flows, and $(I - A_{VC})$ is the investment made by the entrepreneur. It is considered that the VC has also constant absolute risk-averse. The VC's utility function is $v(w_{VC}, a) = -exp(-\eta_{VC}[w_{VC} - B(a)])$, where w_{VC} represents net income for the VC, which is shown by $w_{VC} = \alpha(pf(e, a) + \varepsilon) - A_{VC}$, and $\eta_{VC} > 0$ is the VC's coefficient of absolute risk aversion ($\eta_{VC} = -v''/v'$). The literature on risk aversion and the economics of entrepreneurship typically uses this utility function, as we saw above.

When effort levels are not observable, the way in which cash flows from the project are distributed will affect the way in which efforts are supplied. The level of effort offered by the entrepreneur comes from his incentive-compatibility constraint, denoted by $(IC)_E$:

$$\hat{e} = \underset{e}{\text{argmax}} \quad E(-exp(-\eta_E[(1-\alpha)(pf(e,a)+\varepsilon)-(I-A_{VC})-C(e)]) \tag{2}$$

Equation (2) indicates that the entrepreneur maximizes his expected utility based on the income share stated in the contract, his rational expectation of the other player's effort, his level of investment, and based on the cost of his effort.

The VC chooses his level of effort also on the basis of his incentive-compatibility constraint, denoted by $(IC)_{VC}$:

$$\hat{a} = \underset{a}{\text{argmax}} \quad E(-exp(-\eta_{VC}[\alpha(pf(e,a)+\varepsilon)-A_{VC}-B(a)]) \tag{3}$$

The maximization problem faced by the entrepreneur is then:

$$\max_{\alpha,e,a,A_{VC}} E(-exp(-\eta_E[(1-\alpha)(pf(e,a)+\varepsilon)-(I-A_{VC})-C(e)]) \tag{4a}$$

$s.t$

$$\hat{e} = \underset{e}{\text{argmax}} \quad E(-exp(-\eta_E[(1-\alpha)(pf(e,a)+\varepsilon)-(I-A_{VC})-C(e)]) \tag{4b}$$

$$\hat{a} = \underset{a}{\text{argmax}} \quad E(-exp(-\eta_{VC}[\alpha(pf(e,a)+\varepsilon)-A_{VC}-B(a)]) \tag{4c}$$

$$E(-exp(-\eta_{VC}[\alpha(pf(e,a)+\varepsilon)-A_{VC}-B(a)]) > -exp(-\eta_{VC}\bar{w}) \tag{4d}$$

The last equation serves as the participation constraint for the VC, where $\bar{w}$ denotes the VC's reservation wage.

Given the functional form of the entrepreneur's utility, the entrepreneur's incentive-compatibility equation can be expressed as:

$$\hat{e} = \underset{e}{\text{argmax}} \quad (1-\alpha)(\mu_p f(e,a) + \mu_\varepsilon) - (I - A_{VC}) - C(e) - \frac{\eta_e(1-\alpha)^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} \tag{5}$$

while the VC's incentive-compatibility equation can be expressed as:

$$\hat{a} = \underset{a}{\text{argmax}} \quad \alpha(\mu_p f(e,a) + \mu_\varepsilon) - A_{VC} - B(a) - \frac{\eta_{vc}\alpha^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} \tag{6}$$

Using the first order conditions based on Holmstrom (1979), the problem becomes

$$\max_{\{\alpha,e,a,A_{VC}\}} (1-\alpha)(\mu_p f + \mu_\varepsilon) - (I - A_{VC}) - C(e) - \frac{\eta_e(1-\alpha)^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} \tag{7a}$$

$s.t$

$$(1-\alpha)\mu_p f_e = C_e + \eta_e(1-\alpha)^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e] \tag{7b}$$

$$\alpha\mu_p f_a = B_a + \eta_{vc}\alpha^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a] \tag{7c}$$

$$\alpha(\mu_p f + \mu_\varepsilon) - A_{VC} - B(a) - \frac{\eta_{vc}\alpha^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} \geq \bar{w} \tag{7d}$$

3 The Main Results

In order to start our study, we solve the entrepreneur's maximization problem under the assumption that both the VC and the entrepreneur are risk neutral. This instance will be used as a benchmark for the case where both parties are risk averse.

3.1 Both risk-neutral partners

When both partners are risk neutral, the maximization problem faced by the entrepreneur is reduced to:

$$\max_{\{\alpha,e,a,A_{VC}\}} (1-\alpha)(\mu_p f + \mu_\varepsilon) - (I - A_{VC}) - C(e) \tag{8a}$$

$$s.t$$

$$(1-\alpha)\mu_p f_e = C_e \tag{8b}$$

$$\alpha \mu_p f_a = B_a \tag{8c}$$

$$\alpha(\mu_p f + \mu_\varepsilon) - A_{VC} - B(a) \geq \bar{w} \tag{8d}$$

It should be noted that in the scenario of risk neutrality, the entrepreneur's and the VC's risk aversion coefficients is zero ($\eta_e = \eta_{vc} = 0$). Under these circumstances, neither price risk nor background risk influence the partners' decisions regarding their efforts or the VC's equity share in the project's cash flows.

The project cash flows must be shared, as may be inferred from the incentive-compatibility restrictions. We can prove this easily:

$$\alpha = \frac{f_e B_a}{f_e B_a + f_a C_e} \in (0,1). \tag{9}$$

The optimal equity share is not represented by equation (9). According to this equation, the VC cannot be awarded a share with $\alpha = 0$ or $\alpha = 1$.

Let's assume the following example: $f(e,a) = A[\delta e + \gamma a]$, $C(e) = \beta_E \frac{e^2}{2}$, and $B(a) = \beta_{VC} \frac{a^2}{2}$. Naturally, equation (9) is given by:

$$\alpha = \frac{\delta a/\beta_E}{\delta a/\beta_E + \gamma e/\beta_{VC}} \tag{10}$$

This example illustrates a contract where the disutilities of the parties' efforts are convex and their efforts are perfect substitutes. Such assumptions can be found in articles such as Casamatta (2003) and De Bettignies and Brander (2007).

The entrepreneur must optimize his expected profit while taking into account the VC's participation constraint and compatible incentive constraints in order to get the shares allocated to the VC in the project cash flows. The following proposition describes the equilibrium condition when both partners are risk-neutral.

Proposition 1.

a) When solving this doubled-sided moral hazard problem for the optimal share of the VC, the participation constraint PC_{VC} is binding because his shadow prices λ_3 is positive.

b) The VC's equity participation, which resolves the entrepreneur's optimization problem, meets the following equilibrium condition:

$$\lambda_1 \mu_p f_e = \lambda_2 \mu_p f_a$$

where λ_1 and λ_2 represent the shadow prices of the incentive compatibility constraints of the entrepreneur and the VC, respectively.

c) The equity participation level given to the VC that solves the entrepreneur's problem is non-linear and at a fixed point takes the form of $\alpha^* = h(\alpha^*)$, where:

$$h(\alpha) = \frac{f_a^2((1-\alpha)\mu_p f_{ee} - C_{ee})}{f_a^2((1-\alpha)\mu_p f_{ee} - C_{ee}) + f_e^2(\alpha \mu_p f_{aa} - B_{aa})}$$

Proof. See the Appendix. □

The Lagrange multiplier or shadow price linked to the VC participation restriction is represented by λ_3. This shadow price is positive, which indicates that the VC's participation constraint is binding. Should it not be, raising A_{VC} would raise the entrepreneur's expected income without altering the VC's incentives.

Lagrange multiplier (shadow price) λ_2 is associated with the VC incentive compatibility constraint, while λ_1 is associated with the entrepreneur incentive compatibility constraint. The fact that $\lambda_1, \lambda_2 > 0$ (refer to Appendix) indicates that the principal (i.e. the entrepreneur) expects the agent (i.e. the VC) to exert effort harder than what is actually done in the second-best solution.

Proposition 1 indicates that when the marginal value of the entrepreneur's effort ($\lambda_1 f_e$) equals the marginal value of the VC's effort ($\lambda_2 f_a$), the VC will be given an equity share in the cash flows of the project that resolves the entrepreneur's optimization problem.

To illustrate the optimal solution to the entrepreneur's maximization problem, we will use the example of perfect substitute efforts and convex costs described above. The optimal solution is obtained by solving the system of equations composed of the incentive-compatible constraints and part c) of Proposition 1. This case is expressed in the following corollary.

Corollary 1. *Assuming that the partners' disutilities are convex and that the production function takes the form of perfect substitute efforts, the optimal equity share and optimal efforts are as follows:*

$$\alpha^* = \frac{\gamma^2/\beta_{VC}}{\gamma^2/\beta_{VC} + \delta^2/\beta_E}$$

$$e^* = \frac{(1-\alpha^*)A\mu_p\delta}{\beta_E}$$

$$a^* = \frac{\alpha^* A\mu_p\gamma}{\beta_{VC}}$$

This corollary describes a very simple version of an entrepreneur-VC contract. In this specific case, the entrepreneur's optimization problem has an analytical solution that is determined upon synergy, the expected price, and productivity and efficiency parameters that characterize the partners. Incorporating the concept of complementarity, however, results in a non-linear solution to the problem, requiring the solution of a system of equations incorporating effort and equity participation.

How does the risk aversion of both VCs and entrepreneurs affect the results stated in Proposition 1? That's what we'll see next.

3.2 Both risk-averse partners

The incentive-compatibility equations (7b) and (7c) illustrate the double-sided moral hazard dilemma in the setting of risk-averse entrepreneurs and VCs. These equations indicate that the VC cannot be given a participation share of $\alpha = 0$ or $\alpha = 1$. Consequently, it is necessary to share the project's cash flows. It is simple to show that:

$$\alpha = \frac{f_e(B_a + \eta_{vc}\alpha^2 f_a[\sigma_p^2 f + \sigma_{p\varepsilon}])}{f_e(B_a + \eta_{vc}\alpha^2 f_a[\sigma_p^2 f + \sigma_{p\varepsilon}]) + f_a(C_e + \eta_e(1-\alpha)^2 f_e[\sigma_p^2 f + \sigma_{p\varepsilon}])} \in (0,1) \tag{11}$$

There is no optimal equity share in equation (11). Getting the VC's share of the project cash flows requires the entrepreneur to maximize his expected profit while accounting for the VC's participation constraint and compatible incentive constraints. The equilibrium condition where both partners are risk averse is described by the following proposition.

Proposition 2.

a) When solving this doubled-sided moral hazard problem for the optimal share of the VC, the participation constraint PC_{VC} is binding because his shadow prices λ_3 is positive.

b) The cash-flow share given to the players is found when:

$$\lambda_1 f_e(\mu_p - 2\eta_e(1-\alpha)[\sigma_p^2 f + \sigma_{p\varepsilon}]) + \eta_{vc}\alpha[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] =$$
$$\lambda_2 f_a(\mu_p - 2\eta_{vc}\alpha[\sigma_p^2 f + \sigma_{p\varepsilon}]) + \eta_e(1-\alpha)[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]$$

where λ_1 and λ_2 represent the shadow prices of the incentive compatibility constraints of the entrepreneur and the VC, respectively.

c) The equity participation level given to the VC that solves the entrepreneur's problem is non-linear and at a fixed point takes the form of $\alpha^* = g(\alpha^*)$*, where:*

$$g(\alpha) = \frac{|B_2| f_a(\mu_p - 2\eta_{vc}\alpha[\sigma_p^2 f + \sigma_{p\varepsilon}]) - |B_1| f_e(\mu_p - 2\eta_e(1-\alpha)[\sigma_p^2 f + \sigma_{p\varepsilon}]) + |B|\eta_e[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{(\eta_{vc} + \eta_e)|B|[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]} \tag{12}$$

where $|B|$*,* $|B_1|$ *and* $|B_2|$ *are determinant of the matrices* B*,* B_1 *and* B_2*, respectevely.*

Proof. See the Appendix. □

It is evident from comparing Propositions 1 and 2 that the partners' risk aversion influences both the equity share allocated to the VC in the project's cash flows and the marginal values of their efforts. This equilibrium condition, as well as the dynamics of efforts and equity shares, are also impacted by the price risk and background risk variances and the accompanying covariance between the two risks.

We will utilize the previously mentioned example of perfect substitute efforts and convex costs to show the optimum solution to solve the entrepreneur's maximizing issues. Additionally, we'll suppose that the project's revenue is solely impacted by background risk. The result that follows expresses this case.

Corollary 2. *Assuming that the production function takes the form of perfect substitute efforts and that the partners' disutilities are convex, the optimal equity share and optimal efforts in the context of risk aversion and background risk are as follows:*

$$\alpha^* = \frac{{(\mu_p A\gamma)^2}/{\beta_{VC}} + \eta_e\sigma_\varepsilon^2}{{(\mu_p A\gamma)^2}/{\beta_{VC}} + {(\mu_p A\delta)^2}/{\beta_E} + (\eta_e + \eta_{vc})\sigma_\varepsilon^2}$$

$$e^* = \frac{(1-\alpha^*)A\mu_p\delta}{\beta_E}$$

$$a^* = \frac{\alpha^* A\mu_p\gamma}{\beta_{VC}}$$

The scope of Corollary 1 is expanded by Corollary 2. Although the solution to the entrepreneur's problem is analytical, we can see that background risk and the partners' risk aversion coefficients affect the optimal effort and equity decisions.

Simulations provide an understandable representation of these seemingly complicated expressions, as we shall see in the next section. We will examine how a set of characteristics of the VC and the entrepreneur affect the dynamics of their respective equity shares and efforts. The set of parameters includes the productivity, efficiency, and risk aversion coefficients. We carried out the analysis under effort complementarity and different risk scenarios. Changes in price risk and background risk variances, along with their covariance, represent these risk scenarios.

4 Simulations

In order to provide a clearer explanation of the results, we proceed to simulate them for reasonable parameter values. We assume that the production technology is a CES function of the form of $f(e, a) = A\left[\delta e^{\rho} + \gamma a^{\rho}\right]^{1/\rho}$, where e and a represent the level of efforts supplied by the entrepreneur and the VC, respectively. The parameters δ and γ are elasticities of each the players' efforts, A is a productivity parameter, and ρ is a substitution parameter. If ρ is equal to 1, the efforts are perfect substitutes. If ρ is equal to $-\infty$, the efforts are perfect complements. If $-\infty < \rho < 1$, the efforts are complementary. We also assume that the disutility of partners' efforts are $C(e) = \beta_E \frac{e^{1+h}}{1+h}$ and $B(a) = \beta_{VC} \frac{a^{1+k}}{1+k}$, respectively, where $\beta_E > 0$ and $\beta_{VC} > 0$ measure the efficiency of partners' efforts, and h and k are the degree of cost progression of the entrepreneur and VC, respectively.

We study the behavior for different values of the parameters; the baseline cases differ only in the value for ρ. We have two options: a value of 1 denoting perfect substitute efforts and a value of -1 denoting the case of complementary efforts. The parameters are displayed in the table below. The last row reports the value of α^*, that is, the optimal equity share allocated to VC that solves the entrepreneur's maximization problem. This value is the same in these two cases. Thus, the complementarity of efforts does not alter the equity share in project flows when partners possess the same attributes. Half of the equity share is given to each partner. When the price risk and background risk variances and their covariance are modified, the same behavior happens. Their equity share in the project's cash flows, however, shifts when the partners' traits differ. Online Appendix B reports the effects of changing each of the parameters on α^*. Table 1 in that Appendix presents Cases 1 to 13 in the context of $\rho = 1$, whereas Table 2 presents Cases 14 to 26 in the context of $\rho = -1$. Case 1 is the baseline case when $\rho = 1$, and Case 14 is the baseline case when $\rho = -1$; both cases are shown in the following table.

Parameter	**Baseline cases**
δ	0.5
γ	0.5
β_E	30
β_{VC}	30
h	1
k	1
A	1
ρ	$\{1, -1\}$
μ_p	1
η_E	0.5
η_{VC}	0.5
σ_ε^2	0.06
σ_p^2	0.11
σ_{pe}^2	0.01
α^*	0.50

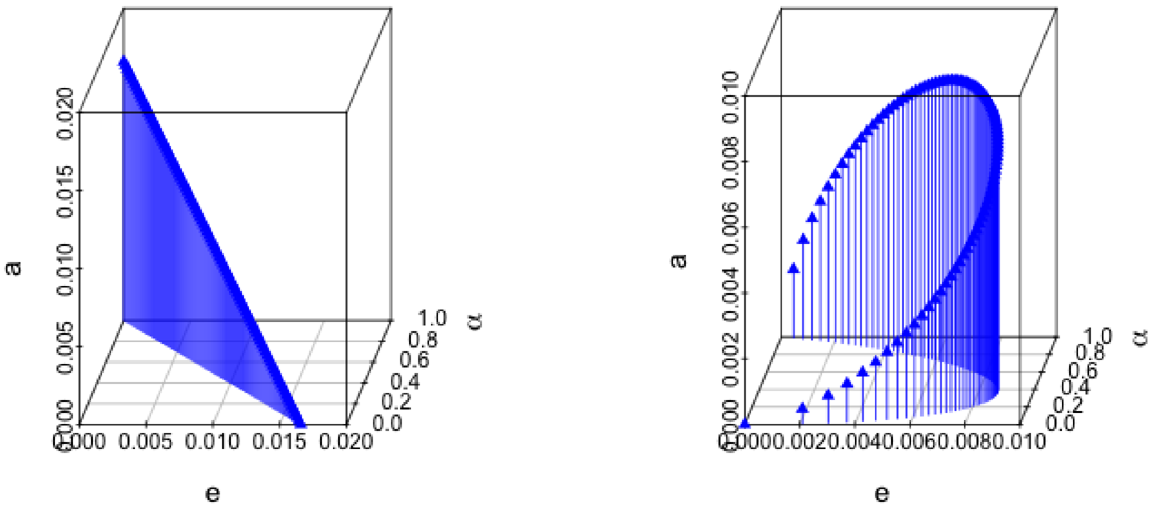
4.1 The dynamics of partners efforts

From the equations (7b) and (7c) (the entrepreneur's and VC's incentive-compatibility constraints), we can obtain the best response functions for the partners to the equity share assigned to the VC in the first stage of the game.

As a first step we compute the values for e y a that satisfy QUE COSA?, using a grid of 100 equidistant values for $\alpha \in (0, 1)$. The following two three-dimensional images in Figure 1 illustrate the baseline cases (Case 1 and Case 14), showing how the corresponding values of the VC's and entrepreneur's efforts modify in response to α changes In the case of $\rho = 1$, the partners' efforts are perfect substitutes, as illustrated in Case 1. In a share α allocated to the VC, the entrepreneur's effort monotonically decreases, whereas the VC's effort monotonically increases by α. Maximum effort is achieved in both cases when α takes extreme levels. In the case of the entrepreneur, $\alpha = 0$ maximizes his efforts, whereas $\alpha = 1$ maximizes the effort of the VC.

When the partners' efforts are complementary, the dynamics of their efforts change. This is evident in Case 14, where $\rho = -1$ is the complementarity parameter. It becomes clear that effort levels for the equity share become concave when complementarity exists. The graphic representations show that a specific level of equity involvement must be given to VC to maximize the entrepreneur's effort. The same is true for the VC's effort; that is, there needs to be a certain level of equity to maximize his effort. In this case, the level of equity share awarded to the VC that maximizes his or the entrepreneur's effort does not occur at the extreme values of α, as it does in Cases 1. The three-dimensional graphic sequence for Cases 1 to 13 in the $\rho = 1$ context and Cases 14 to 26 in the $\rho = -1$ setting are provided in Appendix C1. This graph sequence reveals that the effort dynamics of the partners are similar to those of their respective baseline cases.

Behind the graphs in Figure 1, which show the dynamics of partners' effort levels as a function of the equity share α granted to the VC, are equations (7b) and (7c). The level of α that maximizes the entrepreneur's effort is different from the level of α that



$\rho = 1$ $\rho = -1$

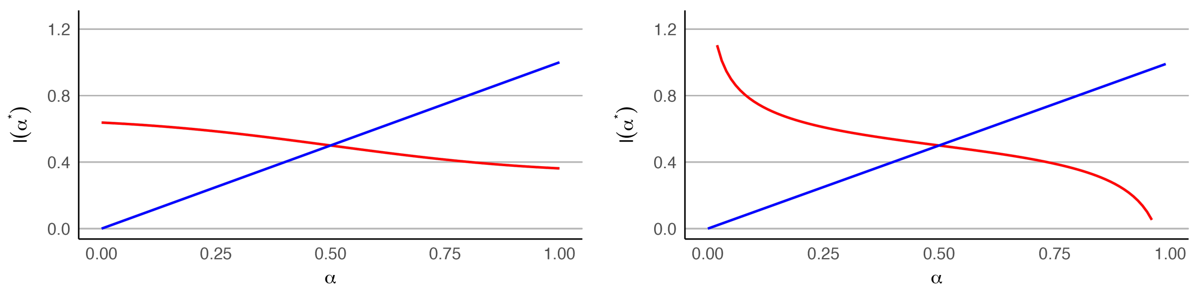
Figure 1: Effort dynamics

maximizes the VC's effort, as we have observed. However, since we only take into account the problem at the level of the partners' incentive-compatible constraints, neither of these equity share levels resolves the entrepreneur's maximization problem. Equations (7b), (7c), and (12) are required to solve the entrepreneur's problem.

4.2 The dynamics of equity share α

Next, we simulate equation (12), which characterizes the behavior of $g(\alpha)$, together with equations (7b) and (7c) for each of the 100 combinations of α, e, and a. The results are shown in Figure 2. We have added the 45 degree line to highlight the optimal value for α^* in each case. The final rows of Table 1 and Table 2 in Online Appendix B report the optimal α^*. We use Case 1 and Case 14 as our benchmark. In these instances, each partner receives 50% of the project cash flows. This occurs when the red curved line, which represents the function $g(\alpha)$, crosses the blue line, which represents the 45 degree line. The equity share awarded to the VC that solves the entrepreneur's problem is therefore non-linear and has the form of $\alpha^* = g(\alpha^*)$, that is, a fixed point. The two-dimensional graphic sequence for Cases 1 to 13 in the $\rho = 1$ context and Cases 14 to 26 in the $\rho = -1$ setting are provided in Online Appendix C2. This graph sequence reveals that the equity share dynamics allocated to VC are similar to those of their respective baseline cases. Nevertheless, the cut-off point changes depending on the parameter that is modified.

Figure 2: Optimal equity share: Baseline cases



The parameter δ represents the elasticity of the entrepreneur's effort, while γ represents the elasticity of the VC's effort. In their study, De Bettignies and Brander (2007) defined δ as the measure of productivity for entrepreneurial effort and γ as the measure of productivity for VC effort. Comparative statics analysis shows that greater entrepreneurial productivity relative to the VC ($\delta/\gamma > 1$) implies greater equity allocation in the project's cash flows, and vice versa ($\delta/\gamma < 1$). The result is similar when analyzing the relative efficiency of the entrepreneur compared to the VC ($\beta_E/\beta_{VC} < (>)1$), where β_E and β_{VC} measure the efficiency of the entrepreneur's effort and VC's effort, respectively, as is defined by Casamatta (2003). The consequence of a change in relative productivity or relative efficiency on the optimal α^* is mitigated when efforts are complementary. With parameters that take account of the degree of cost progression, a different phenomenon takes place. An increase in the entrepreneur's (VC) cost progression degree decreases (increases) the equity share allotted to the VC in the context of perfect substitution efforts. The entrepreneur's (VC) cost progression degree, on the other hand, increases (decreases) the equity share allotted to the VC in the setting of complementary efforts.

Risk aversion coefficients correspond to key partner attributes that impact optimal equity share. The results indicate that when the entrepreneur is more risk-averse than the VC, the optimal α^* allocated to the VC is higher. The opposite occurs when the VC is more risk-averse than the entrepreneur. However, this effect is lessened by the complementarity of efforts.

A set of graphs showing the dynamics of optimal equity share α^* as a function of the entrepreneur's optimization model's parameters is presented in the subsection that follows. To present the results in a concise and clear way, we summarize them in a series of three-dimensional graphics where the title indicates the "new" value for the parameter we affected. The corresponding values for α^* are shown on the vertical axis of each graph. One axis in the first set of graphs shows background risk standard deviation, price risk standard deviation, or the associated covariance between these risks, while the third axis shows additional traits of the partners, including their level of productivity or efficiency.

The entrepreneur's or venture capitalist's risk aversion is included in the analysis in a second set of graphics. These sets of three-dimensional plots are constructed for scenarios in which the complementary parameter $\rho \in \{-1, 1\}$. Finally, we allow this parameter to take different values; that is, we let $\rho \in [-1, 1]$.

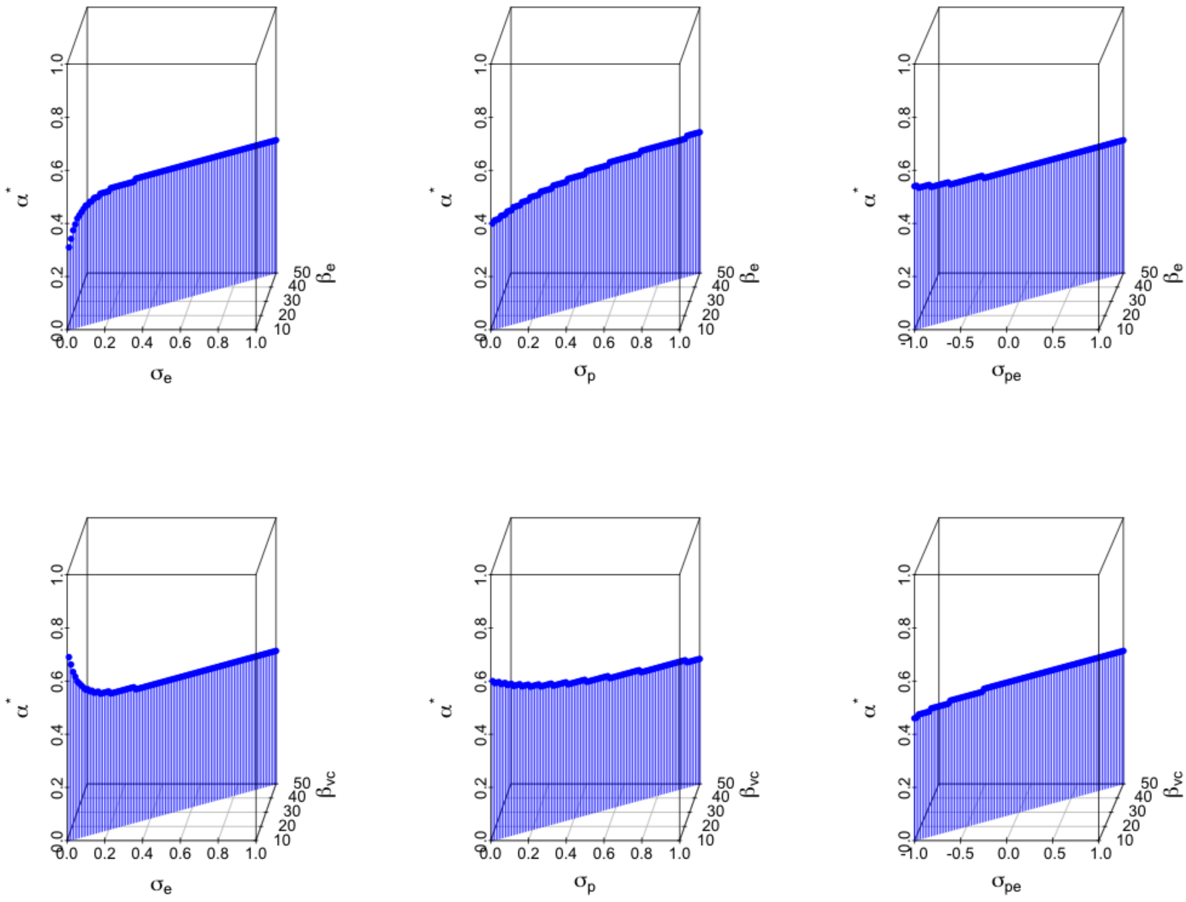
4.3 Dynamics of optimal equity share α^*

4.3.1 Efficiency, productivity and risk parameters under $\rho = 1$

We begin by examining the combined impact of the partner characteristics, as indicated by the productivity and efficiency parameters, and the risk parameters. The above-mentioned comparative static analysis reveals that when partners have the same traits, risk parameters have no effect on the optimal equity share. Conversely, the optimal equity share shifts when the partners' productivity or efficiency varies. We may learn more about the dynamics of the optimal equity share when these parameters vary at the same time from the following three-dimensional graphs.

The graph sequence in the setting of perfect substitutes efforts reported in Figure 3 indicates that the optimal equity share allotted to the VC is increased when the entrepreneur's effort becomes less efficient and when background risk or price risk, measured by the standard deviation, increases. When any of these sources of risk interact with the VC's inefficient parameter, the opposite happens. The dynamics of α^* stay steady since the influence is negligible when the covariance of both risks and the inefficiency of the entrepreneur's or VC's effort are combined.

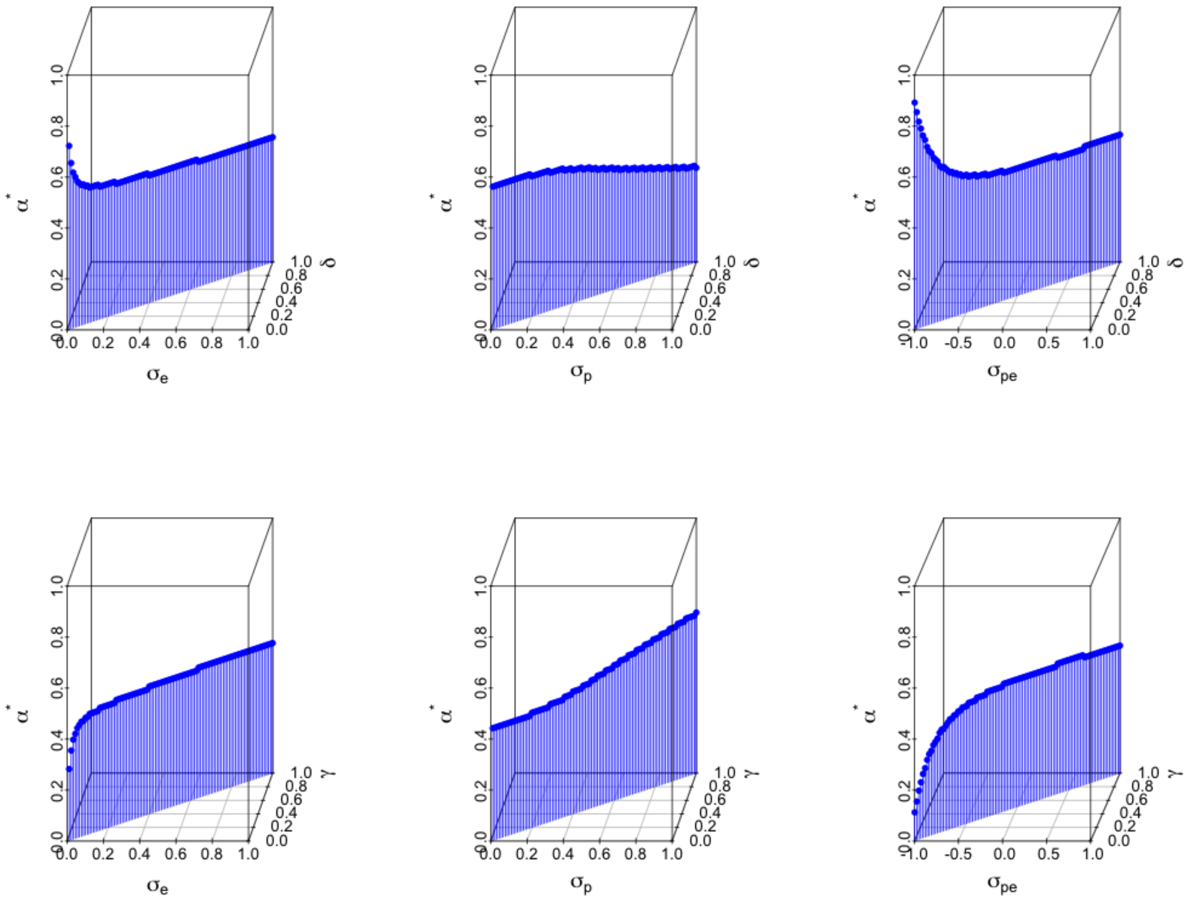
Figure 3: Efficient of efforts and risk parameters under $\rho = 1$



Note: This figure presents 3D surfaces of the optimal equity share α^*, generated by evaluating 100 specific parameter combinations. The methodology involves defining 100 equally spaced values within the range of each parameter. The variances (σ_e, σ_p) range from 0.01 to 1.00, the covariance (σ_{pe}) from -1 to 1, and the efficiency parameter β_e from 10 to 50. These values are paired sequentially (the i-th value of one parameter with the i-th value of the other), and the model computes α^* for each pair. The top row shows the results for σ_e, σ_p, and σ_{pe} against β_e; the bottom row shows the analogous analysis with β_{vc}.

The comparative statistics analysis also indicates that the VC's or entrepreneur's equity share in the project's cash flows increases as their productivity improve. The graph sequence that follows (Figure 4) displays how the optimal equity share assigned to the VC is decreased when the entrepreneur's effort productivity grows and background risk, or price risk, as indicated by the standard deviation, increases. Conversely, when any of these risk factors interact with the VC's productivity measure, the optimal equity share given to the VC increases. When the covariance and productivity of the VC's or entrepreneur's efforts are combined, the optimal equity share exhibits similar patterns. However, the rate at which the optimal equity share granted to the VC increases or falls is what differentiates the dynamics of α^*, which is apparent in the three-dimensional graphs.

Figure 4: Productivity of efforts and risk parameters under $\rho = 1$



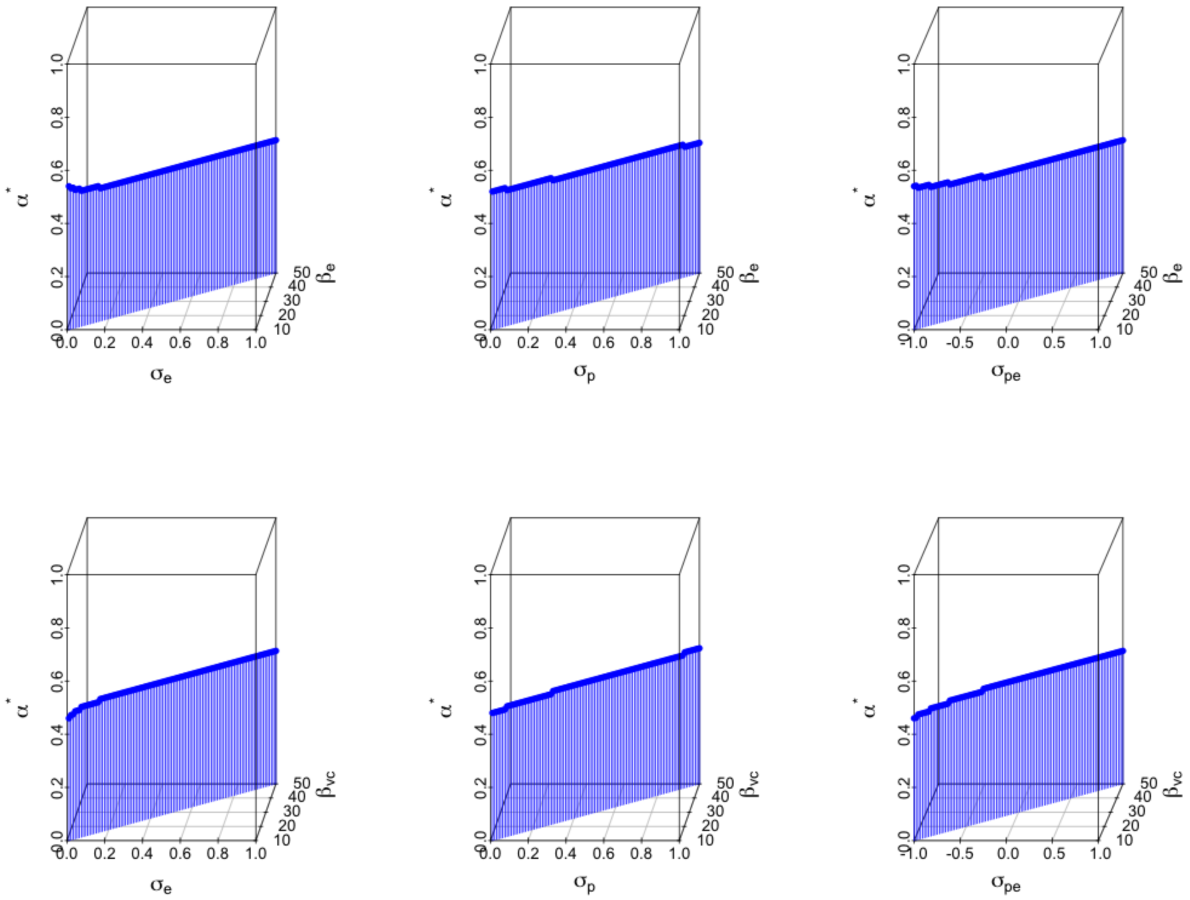
Note: The surfaces in this figure were produced following the same paired-value methodology described for Figure 3. Here, the variance terms (σ_e, σ_p) and the new parameters δ and γ vary between 0.01 and 1.00, while the covariance (σ_{pe}) ranges from -1 to 1. The parameters increase jointly along their 100-point sequences. The top row displays the dynamics of α^* when σ_e, σ_p, and σ_{pe} vary with δ; the bottom row shows the results when they vary with γ.

4.3.2 Efficiency, productivity and risk parameters under $\rho = -1$

In the context of complementary efforts, we examine in this subsection the combined impact of risk factors and the productivity and efficiency of partners' efforts on α^*. With regard to the aforementioned comparative static analysis, changes in the complementary parameter have no effect on the optimal equity share when partners share the same characteristics. Similar results are obtained when varying the risk parameters. The following three-dimensional graphs offer additional insight into the dynamics of the optimal equity share when these parameters are combined with the efficiency or productivity of the partners' efforts.

The graph sequence presented in Figure 5 highlights how the combined influence of risk parameters and VC effort efficiency on the dynamics of α^* is mitigated by the complementarity of efforts. A similar thing happens when the entrepreneur's effort efficiency interacts with risk parameters. Despite different combinations of efficiency and risk parameters, it is shown that the α^* dynamics can be considered stable.

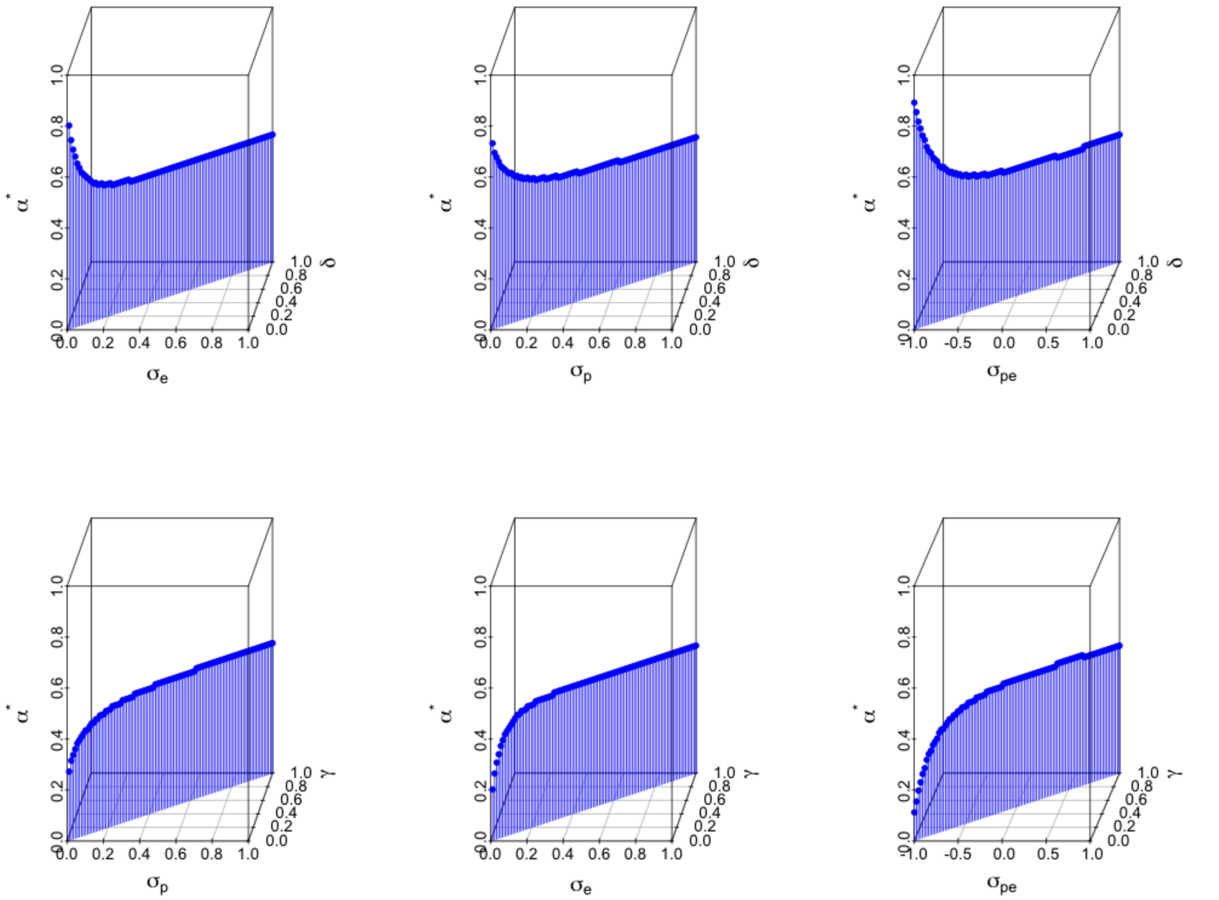
Figure 5: Efficient of efforts and risk parameters under $\rho = -1$



Notes: These graphs were generated using the same methodology and parameter ranges as in Figure 3. The key distinction is that the effort complementarity parameter is fixed at $\rho = -1$ for all calculations. The resulting 3D surfaces depict the optimal α^* for the paired sequences of parameters under this specific condition.

In contrast to the previous series of graphs, the complementarity effect has no discernible effect on the dynamics of α^* when the partners' productivity and risk parameters are combined. Figure 6 reports a graph sequence that is similar to Figure 4's graph sequence.

Figure 6: Productivity of efforts and risk parameters under $\rho = -1$



Notes: This figure follows the methodology of Figure 4, with the parameters δ and γ varying jointly with the variance and covariance terms. The sole difference is that the analysis is conducted under a fixed effort complementarity parameter of $\rho = -1$. The surfaces illustrate the resulting optimal α^* values.

4.3.3 Effect of risk aversion under $\rho = 1$

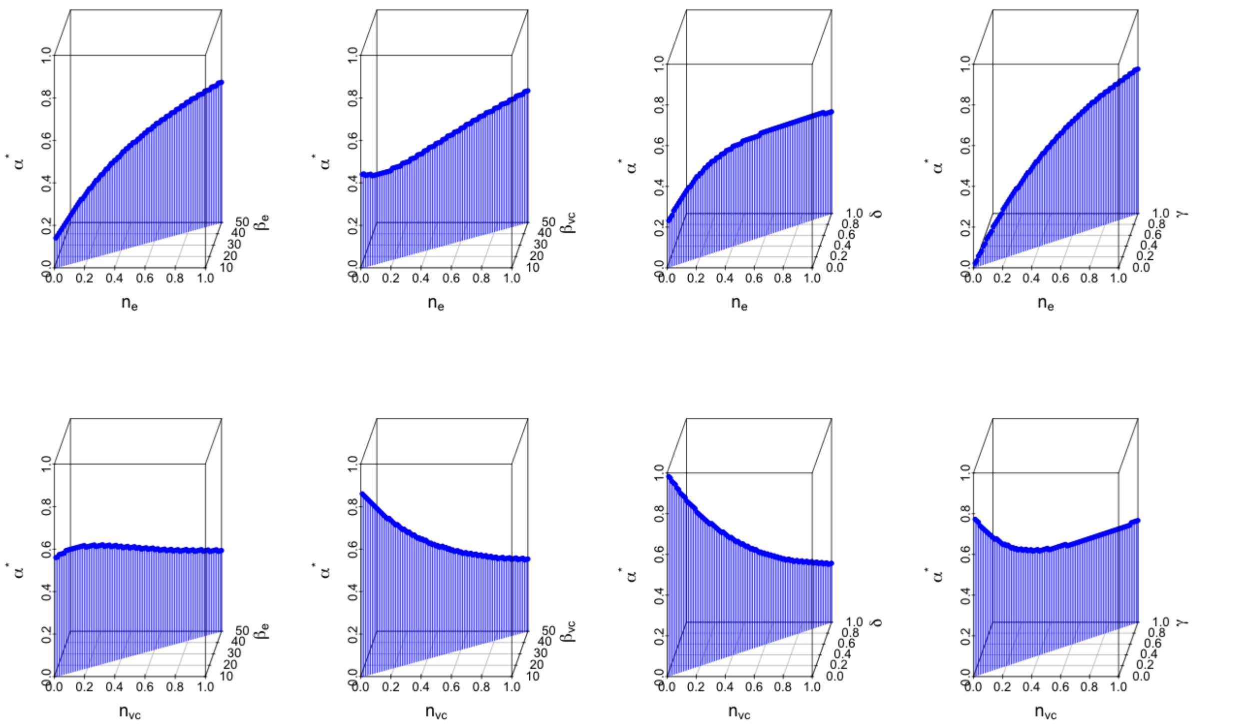
In this subsection, we will examine how the partners' risk aversion, the partners's productivity and efficiency, and risk parameters interact to affect the VC's allocated equity share in the project's cash flows in the context of $\rho = 1$, i.e., perfectly substitutable efforts.

The optimal α^* allotted to the VC is larger when the entrepreneur is more risk-averse than the VC, as indicated by the comparative statistics study. When the VC is more risk averse than the entrepreneur, the opposite happens. As will be seen below, risk aversion's influence on α^* dynamics dominates risk factors and other partner attributes like productivity and efficiency.

The three-dimensional graphic sequence reported in Figure 7 shows that when the entrepreneur's risk aversion increases, α^* rises. However, this effect grows more slowly when the entrepreneur's productivity increases or the VC's effort efficiency decreases. This means that the consequence of an increase in his risk aversion is mitigated; in other words, the slope of the α^* trajectory decreases. On the other hand, the impact of an entrepreneur's risk aversion increase on α^* dynamics is amplified when his inefficiency of effort increases and VC's productivity of effort increases.

The sequence of graphics also illustrates that, as the VC's risk aversion rises, a diminishing dynamic of the equity share assigned to the VC takes place, regardless of the productivity and efficiency characteristics of the partners. Although this decline is mitigated by VC productivity and entrepreneur inefficiency, it is exacerbated by VC inefficiency and entrepreneur productivity.

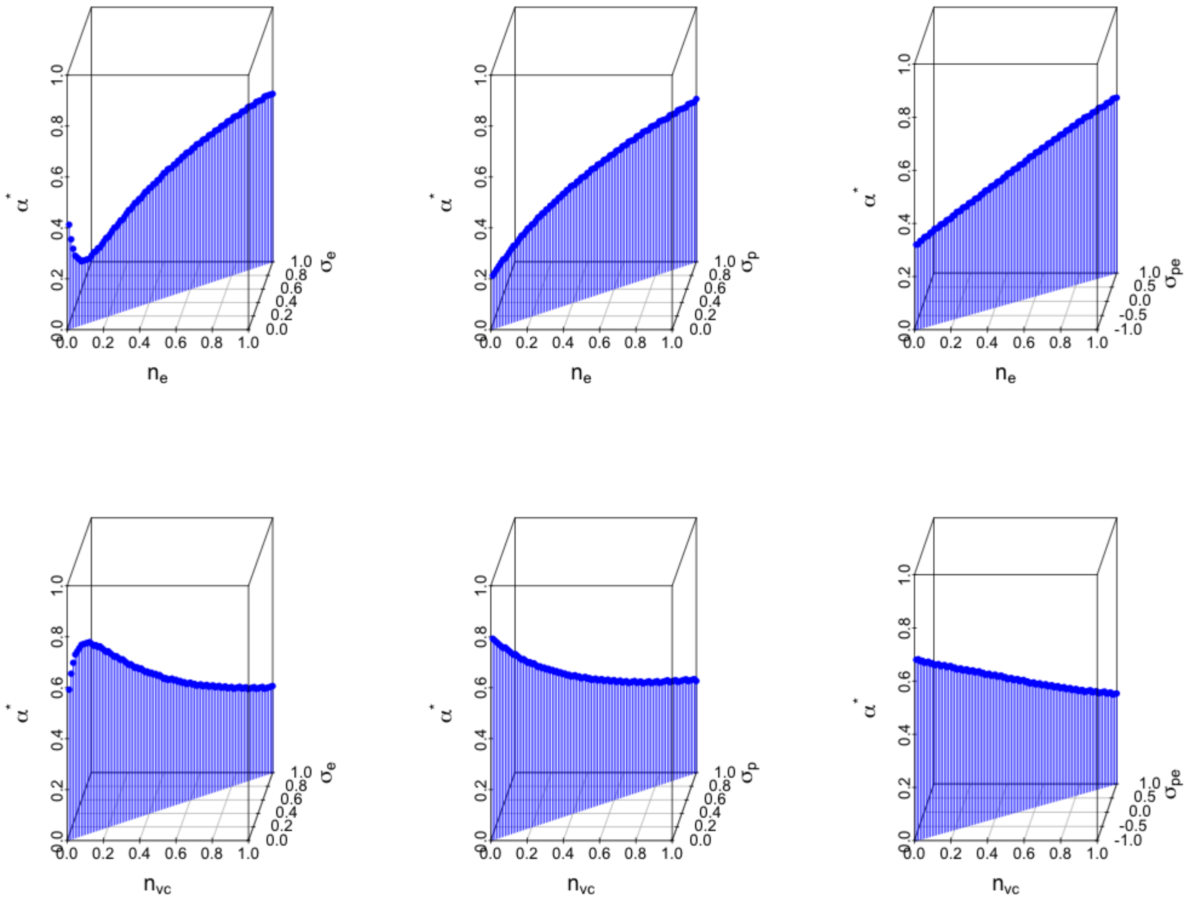
Figure 7: Efficiency, productivity and risk aversion under $\rho = 1$



Notes: This figure introduces the risk aversion coefficients η_e and η_{vc} (varying between 0 and 1), analyzed alongside other parameters using the established paired-value approach. The efficiency coefficients (β_e, β_{vc}) range from 10 to 50, while δ and γ vary from 0.01 to 1.00. The top row shows α^* when η_e varies jointly with β_e, β_{vc}, δ, and γ; the bottom row presents the analogous results for η_{vc}.

The sequence of three-dimensional plots reported in Figure 8 shows the dynamics of α^* when the partners' risk aversion and risk parameters interact. When an entrepreneur's risk aversion rises along with any of the risk indicators, the optimal equity stake given to the VC rises as well. A drop in α^* is only indicated for low values of his risk aversion and background risk, measured by the standard deviation. The dynamics of α^*, on the other hand, are affected in the opposite way when the VC's risk aversion and any of the risk measurements are raised.

Figure 8: Risk aversion and risk parameters when $\rho = 1$



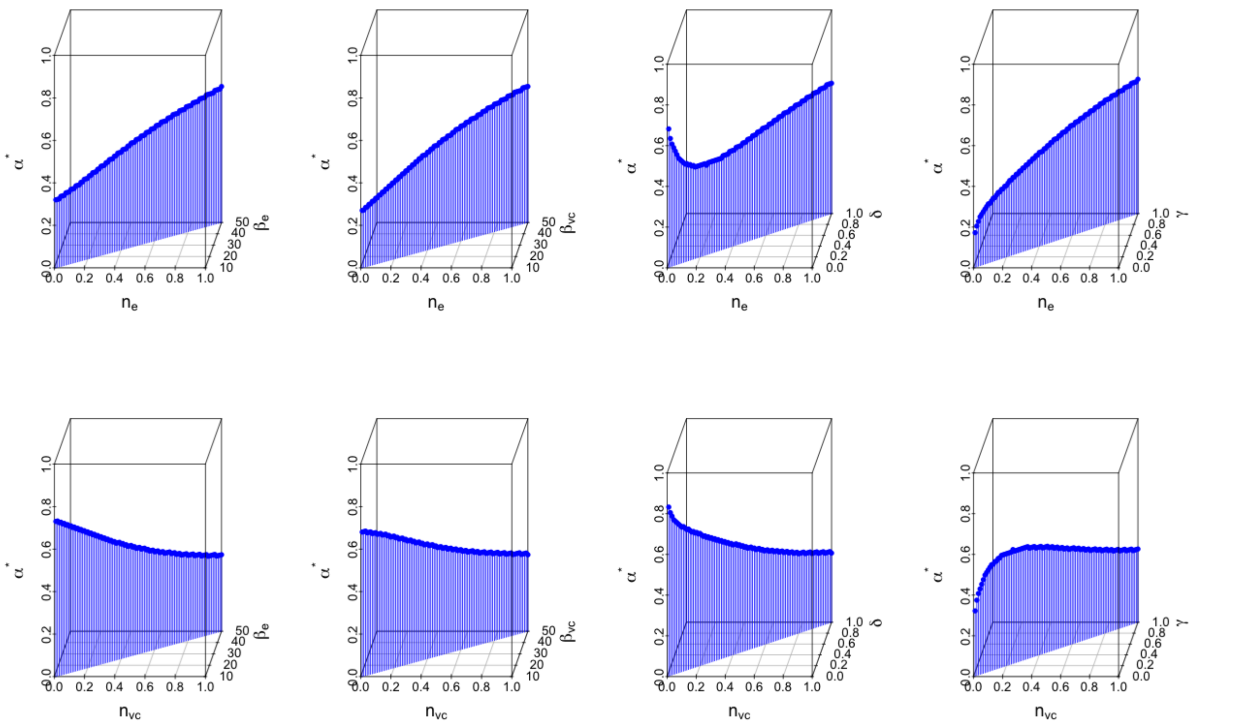
Notes: The surfaces here explore the relationship between the risk aversion parameters and the variance/covariance terms. The paired-value methodology is applied, with η_e and η_{vc} (0 to 1) varying alongside σ_e, σ_p (0.01 to 1.00), and σ_{pe} (-1 to 1). The top row corresponds to η_e, and the bottom row to η_{vc}.

4.3.4 Effect of risk aversion under $\rho = -1$

This subsection will look at how, in the situation of $\rho = -1$, or complementary efforts, the partners' productivity and efficiency, risk parameters, and risk aversion interact to impact the VC's allocated equity share in the project's cash flows. As we observed in the context of perfect substitutes efforts, risk aversion's effect on α^* dynamics in the scenario $\rho = -1$ dominates risk factors as well as other partner traits like productivity and efficiency. We'll observe that while α^*'s trend remains steady, its growth rate shifts.

Figure 9 shows that an increase in the entrepreneur's risk aversion raises α^*, while an increase in the VC's risk aversion reduces α^*. Thus, the strength of the partners' risk aversion dominates the strength of their productivity and efficiency. However, at low levels, the entrepreneur's productivity force outweighs the risk aversion force. The same holds true when comparing these attributes in the case of VC.

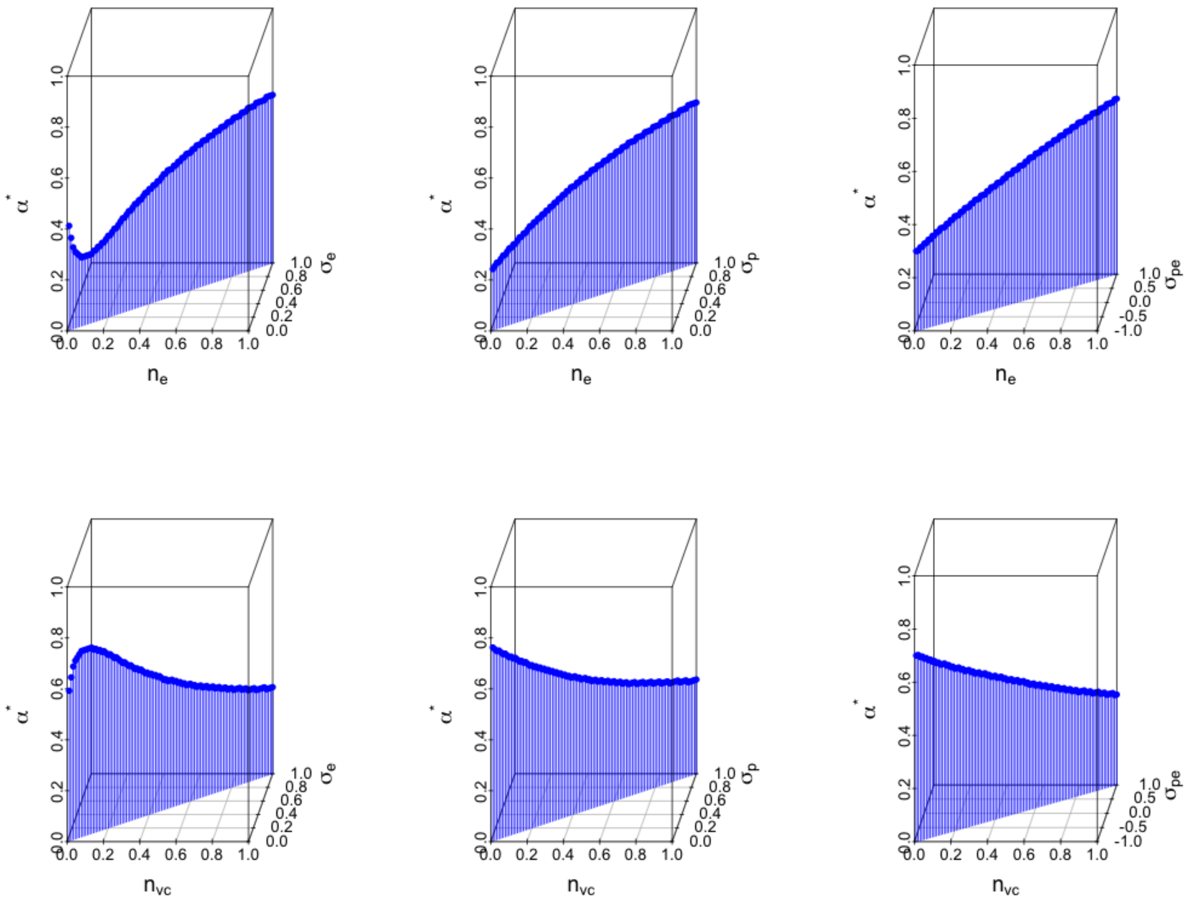
Figure 9: Efficiency, productivity and risk aversion under $\rho = -1$



Notes: The graphs in this figure are generated under the same conditions as Figure 7, with the only difference being that the complementarity parameter is fixed at $\rho = -1$ for all computations.

Figure 10 presents a sequence of three-dimensional plots that illustrate the dynamics of α^* when the partners' risk aversion and risk parameters interact in a setting characterized by complementary efforts. The results are similar to the case where $\rho = 1$, i.e., in the scenario of perfectly substitutable efforts. Thus, we see once again how the force of risk aversion dominates the force of risk indicators.

Figure 10: Risk parameters and risk aversion under $\rho = -1$



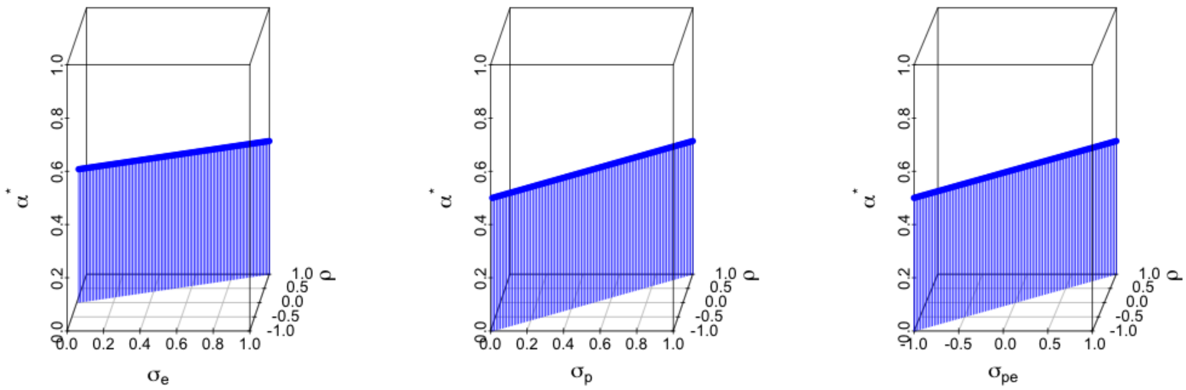
Notes: This figure replicates the analysis of Figure 8, but with the complementarity parameter fixed at $\rho = -1$.

4.3.5 Effect Complementary

This subsection focuses on how complementarity, along with partners' efficiency, productivity, and risk aversion, as well as risk parameters, affects the optimal equity stake granted to the VC. Now we allow that ρ takes values from -1 to 1.

When partners have the same productivity, efficiency, degree of cost progression, and risk aversion, Figure 11 shows that the dynamics of α^* remain unchanged when level of complementarity and risk indicators change. However, when the partners' attributes are modified, the equity share α^* changes.

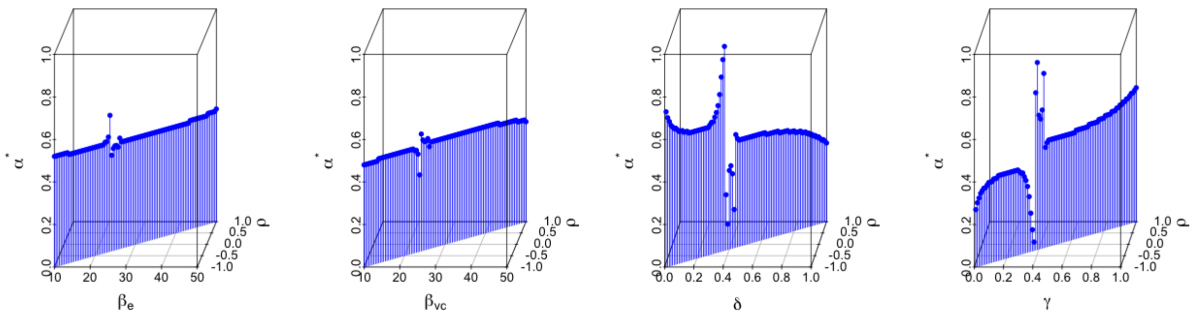
Figure 11: Complementary efforts and Risk parameters



Notes: This figure shows α^* as ρ (-1 to 1) varies jointly with the variance terms (σ_e, σ_p, 0.01 to 1.00) and the covariance (σ_{pe}, -1 to 1).

Figure 12 shows a series of three-dimensional graphs that show how α^* changes when the productivity and efficiency of the partners interact with complementary efforts. The dynamics of α^* are kept relatively steady by the combination of efficiency with the complementary parameter. If the entrepreneur's effort efficiency declines, there will be a slight increase, and if the venture capitalist's effort efficiency declines, there will be a slight drop. The complementarity of efforts is diminished in both situations. In contrast, a decrease in complementarity and a rise in the entrepreneur's productivity reduce α^*. When VC productivity rises and complementarity declines, the opposite happens.

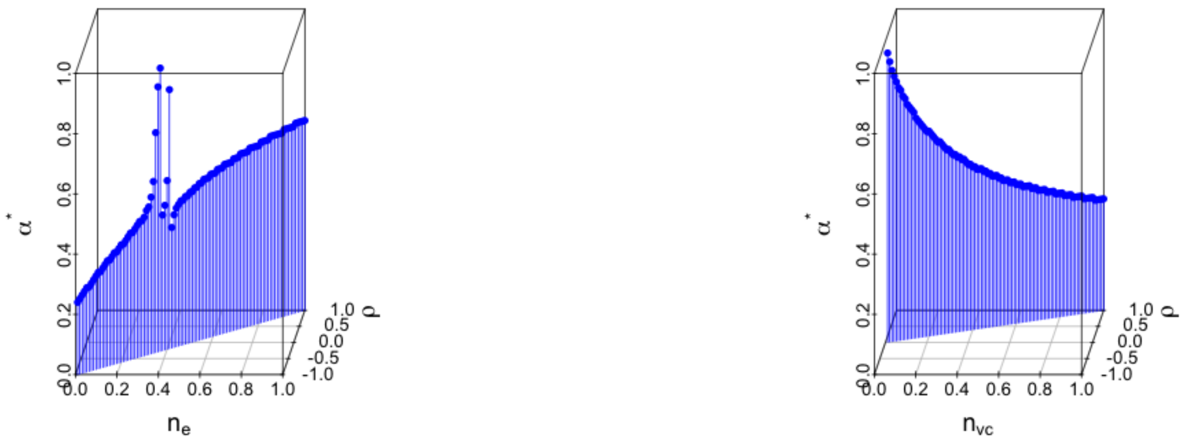
Figure 12: Efficiency, productivity and complementary efforts



Notes: This figure presents the results of varying ρ (-1 to 1) together with δ and γ (0 to 1). For each pair of parameters, the model computes the corresponding optimal α^*, displayed in the 3D surfaces.

The interaction between risk aversion and effort complementarity is depicted in Figure 13. If the entrepreneur's risk aversion increases and the complementarity of efforts decreases, α^* increases; conversely, if the VC's risk aversion increases and the complementarity of efforts decreases, α^* decreases.

Figure 13: Risk aversion and complementary efforts



Notes: This figure presents the results of varying ρ (-1 to 1) together with the risk aversion coefficients n_e and n_{vc} (0 to 1). For each pair of parameters, the model computes the corresponding optimal α^*, displayed in the 3D surfaces.

5 Conclusions

The impact of risk on entrepreneurship and the risk preferences of entrepreneurs are the subject of a large body of theoretical and empirical study. However, the study of the contractual relationship between entrepreneurs and VCs does not take the hypothesis into account. To ensure that the model's results are driven only by incentives under asymmetric information and not by the participants' risk preferences, it is assumed that all decision-makers are risk neutral, in line with recognized studies.

The dynamic nature of markets makes the environment in which VCs and entrepreneurs operate unpredictable. During times of shocks and crises, the increased level of uncertainty may compromise the contractual relationship between entrepreneurs and VCs. In conformity with empirical evidence, we developed a double-sided moral hazard model that takes into account the VC's and entrepreneur's risk preferences in addition to two different types of risk: background risk and price risk.

Our main theoretical finding highlights the way the partners' risk preferences, the efficiency and productivity of their efforts, and the risk and effort complementarity parameters all affect the equilibrium condition. We simulated the model and used three-dimensional graphs to show the relationship between these parameters and the optimal equity share to deepen understanding of the main finding of our paper. We conclude that the productivity and efficiency of each partner's effort is positively associated with their equity share and inversely associated with their risk aversion. Simulations show that risk aversion dominates the trajectory of the optimal equity share when efficiency or productivity interacts with it. Other novel results indicate that risk parameters and the complementarity of efforts alter the slopes of the impacts of productivity, efficiency, and risk aversion on optimal equity shares.

Appendix

When $\eta_e = \eta_{vc} = 0$ is assumed, proposition 2 reduces to proposition 1. Therefore, we shall simply demonstrate Proposition 2.

Proof. The entrepreneur's optimization problem is:

$$
\begin{aligned}
L = {} & (1-\alpha)(\mu_p f + \mu_\varepsilon) - (I - A_{VC}) - C(e) - \frac{\eta_e (1-\alpha)^2 [\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} + \\
& \lambda_1((1-\alpha)\mu_p f_e - C_e - \eta_e(1-\alpha)^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]) \\
& + \lambda_2(\alpha \mu_p f_a - B_a - \eta_{vc}\alpha^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a]) \\
& + \lambda_3(\alpha(\mu_p f + \mu_\varepsilon) - A_{VC} - B(a) - \frac{\eta_{vc}\alpha^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} - \bar{w})
\end{aligned}
$$

and $\{\lambda_1, \lambda_2, \lambda_3, A_{VC}, e, a, \alpha\}$ must be chosen optimally.

The first-order conditions of the entrepreneur's optimization problem are:

1. With respect to the VC's investment, A_{VC}:

$$\frac{\partial L}{\partial A_{VC}} = 1 - \lambda_3 = 0$$

$$\lambda_3 = 1 \tag{A1}$$

2. With respect to the entrepreneur's effort, e:

$$
\begin{aligned}
\frac{\partial L}{\partial e} = {} & (1-\alpha)\mu_p f_e - C_e - \eta_e(1-\alpha)^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e] \\
& + \lambda_1((1-\alpha)\mu_p f_{ee} - C_{ee} - \eta_e(1-\alpha)^2[\sigma_p^2[f_e^2 + f f_{ee}] + \sigma_{p\varepsilon} f_{ee}]) \\
& + \lambda_2(\alpha\mu_p f_{ea} - \eta_{vc}\alpha^2[\sigma^2[f_e f_a + f f_{ae}] + \sigma_{p\varepsilon} f_{ea}]) \\
& + \lambda_3(\alpha\mu_p f_e - \eta_{vc}\alpha^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]) = 0
\end{aligned}
$$

where the first term is zero because of the entrepreneur's incentive-compatibility constraint. Given (A1), the above expression becomes:

$$
\begin{aligned}
&+\lambda_1((1-\alpha)\mu_p f_{ee} - C_{ee} - \eta_e(1-\alpha)^2[\sigma_p^2[f_e^2 + ff_{ee}] + \sigma_{p\varepsilon} f_{ee}]) \\
&+\lambda_2(\alpha\mu_p f_{ea} - \eta_{vc}\alpha^2[\sigma^2[f_e f_a + ff_{ae}] + \sigma_{p\varepsilon} f_{ea}]) = \\
&-\alpha\mu_p f_e + \eta_{vc}\alpha^2[\sigma_p^2 ff_e + \sigma_{p\varepsilon} f_e]
\end{aligned}
\tag{A2}
$$

3. With respect to the VC's effort, a:

$$
\begin{aligned}
\frac{\partial L}{\partial a} &= (1-\alpha)\mu_p f_a - \eta_e(1-\alpha)^2[\sigma_p^2 ff_a + \sigma_{p\varepsilon} f_a] \\
&+\lambda_1((1-\alpha)\mu_p f_{ae} - \eta_e(1-\alpha)^2[\sigma_p^2[f_a f_e + ff_{ea}] + \sigma_{p\varepsilon} f_{ae}]) \\
&+\lambda_2(\alpha\mu_p f_{aa} - B_{aa} - \eta_{vc}\alpha^2[\sigma_p^2[f_a^2 + ff_{aa}] + \sigma_{p\varepsilon} f_{aa}]) \\
&+\lambda_3(\alpha\mu_p f_a - B_a - \eta_{vc}\alpha^2[\sigma_p^2 ff_a + \sigma_{p\varepsilon} f_a]) = 0
\end{aligned}
$$

where the last term is zero because of the VC's incentive-compatibility constraint. So the above expression becomes:

$$
\begin{aligned}
&+\lambda_1((1-\alpha)\mu_p f_{ae} - \eta_e(1-\alpha)^2[\sigma_p^2[f_a f_e + ff_{ea}] + \sigma_{p\varepsilon} f_{ae}]) \\
&+\lambda_2(\alpha\mu_p f_{aa} - B_{aa} - \eta_{vc}\alpha^2[\sigma_p^2[f_a^2 + ff_{aa}] + \sigma_{p\varepsilon} f_{aa}]) \\
&= -(1-\alpha)\mu_p f_a + \eta_e(1-\alpha)^2[\sigma_p^2 ff_a + \sigma_{p\varepsilon} f_a]
\end{aligned}
\tag{A3}
$$

4. With respect to the share assigned to the VC, α:

$$
\begin{aligned}
\frac{\partial L}{\partial \alpha} &= -(\mu_p f + \mu_\varepsilon) + \eta_e(1-\alpha)[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] \\
&+\lambda_1(-\mu_p f_e + 2\eta_e(1-\alpha)[\sigma_p^2 ff_e + \sigma_{p\varepsilon} f_e]) + \lambda_2(\mu_p f_a - 2\eta_{vc}\alpha[\sigma_p^2 ff_a + \sigma_{p\varepsilon} f_a]) \\
&+\lambda_3((\mu_p f + \mu_\varepsilon) - \eta_{vc}\alpha[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]) = 0
\end{aligned}
$$

Given (A1), the above expression becomes:

$$\lambda_1(\mu_p f_e - 2\eta_e(1-\alpha)[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]) + \eta_{vc}\alpha[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] =$$
$$\lambda_2(\mu_p f_a - 2\eta_{vc}\alpha[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a]) + \eta_e(1-\alpha)[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] \tag{A4}$$

5. With respect to λ_1:

$$\frac{\partial L}{\partial \lambda_1} = (1-\alpha)\mu_p f_e - C_e - \eta_e(1-\alpha)^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e] = 0 \tag{A5}$$

6. With respect to λ_2:

$$\frac{\partial L}{\partial \lambda_2} = \alpha\mu_p f_a - B_a - \eta_{vc}\alpha^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a] = 0 \tag{A6}$$

7. With respect to λ_3:

$$\frac{\partial L}{\partial \lambda_3} = \alpha(\mu_p f + \mu_\varepsilon) - A_{VC} - B(a) - \frac{\eta_{vc}\alpha^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} - \bar{w} = 0 \tag{A7}$$

The system of equations is expressed as:

$$\lambda_3 = 1 \tag{A8}$$

$$+\lambda_1((1-\alpha)\mu_p f_{ee} - C_{ee} - \eta_e(1-\alpha)^2[\sigma_p^2[f_e^2 + f f_{ee}] + \sigma_{p\varepsilon} f_{ee}])$$
$$+\lambda_2(\alpha\mu_p f_{ea} - \eta_{vc}\alpha^2[\sigma^2[f_e f_a + f f_{ae}] + \sigma_{p\varepsilon} f_{ea}]) =$$
$$-\alpha\mu_p f_e + \eta_{vc}\alpha^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e] \tag{A9}$$

$$+\lambda_1((1-\alpha)\mu_p f_{ae} - \eta_e(1-\alpha)^2[\sigma_p^2[f_a f_e + f f_{ea}] + \sigma_{p\varepsilon} f_{ae}])$$
$$+\lambda_2(\alpha\mu_p f_{aa} - B_{aa} - \eta_{vc}\alpha^2[\sigma_p^2[f_a^2 + f f_{aa}] + \sigma_{p\varepsilon} f_{aa}])$$
$$= -(1-\alpha)\mu_p f_a + \eta_e(1-\alpha)^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a] \tag{A10}$$

$$\lambda_1(\mu_p f_e - 2\eta_e(1-\alpha)[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]) + \eta_{vc}\alpha[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] = \tag{A11}$$

$$\lambda_2(\mu_p f_a - 2\eta_{vc}\alpha[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a]) + \eta_e(1-\alpha)[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]$$

$$(1-\alpha)\mu_p f_e = C_e + \eta_e(1-\alpha)^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e] \tag{A12}$$

$$\alpha\mu_p f_a = B_a + \eta_{vc}\alpha^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a] \tag{A13}$$

$$\alpha(\mu_p f + \mu_\varepsilon) - A_{VC} - B(a) - \frac{\eta_{vc}\alpha^2[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{2} = \bar{w} \tag{A14}$$

Systems (A9) and (A10) can be expressed as:

$$\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

where

$$b_1 = (1-\alpha)\mu_p f_{ee} - C_{ee} - \eta_e(1-\alpha)^2[\sigma_p^2[f_e^2 + f f_{ee}] + \sigma_{p\varepsilon} f_{ee}]$$

$$b_2 = \alpha\mu_p f_{ea} - \eta_{vc}\alpha^2[\sigma^2[f_e f_a + f f_{ae}] + \sigma_{p\varepsilon} f_{ea}]$$

$$b_3 = (1-\alpha)\mu_p f_{ae} - \eta_e(1-\alpha)^2[\sigma_p^2[f_a f_e + f f_{ea}] + \sigma_{p\varepsilon} f_{ae}]$$

$$b_4 = \alpha\mu_p f_{aa} - B_{aa} - \eta_{vc}\alpha^2[\sigma_p^2[f_a^2 + f f_{aa}] + \sigma_{p\varepsilon} f_{aa}]$$

$$c_1 = -\alpha\mu_p f_e + \eta_{vc}\alpha^2[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]$$

$$c_2 = -(1-\alpha)\mu_p f_a + \eta_e(1-\alpha)^2[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a]$$

Solving the system for λ_1 and λ_2:

$$|B| = \begin{vmatrix} b_1 & b_2 \\ b_3 & b_4 \end{vmatrix} = b_1 \times b_4 - b_2 \times b_3$$

$$|B_1| = \begin{vmatrix} c_1 & b_2 \\ c_2 & b_4 \end{vmatrix} = c_1 \times b_4 - c_2 \times b_2$$

$$|B_2| = \begin{vmatrix} b_1 & c_1 \\ b_3 & c_2 \end{vmatrix} = c_2 \times b_1 - c_1 \times b_3$$

$$\lambda_1 = \frac{|B_1|}{|B|} = \frac{c_1 \times b_4 - c_2 \times b_2}{b_1 \times b_4 - b_2 \times b_3}$$

$$\lambda_2 = \frac{|B_2|}{|B|} = \frac{c_2 \times b_1 - c_1 \times b_3}{b_1 \times b_4 - b_2 \times b_3}$$

If we replace λ_1 and λ_2 in (A11) we get:

$$\frac{|B_1|}{|B|}(\mu_p f_e - 2\eta_e(1-\alpha)[\sigma_p^2 f f_e + \sigma_{p\varepsilon} f_e]) + \eta_{vc}\alpha[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2] =$$
$$\frac{|B_2|}{|B|}(\mu_p f_a - 2\eta_{vc}\alpha[\sigma_p^2 f f_a + \sigma_{p\varepsilon} f_a]) + \eta_e(1-\alpha)[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]$$

Rearranging the previous expression, we obtain:

$$\alpha^* = \frac{|B_2|f_a(\mu_p - 2\eta_{vc}\alpha[\sigma_p^2 f + \sigma_{p\varepsilon}]) - |B_1|f_e(\mu_p - 2\eta_e(1-\alpha)[\sigma_p^2 f + \sigma_{p\varepsilon}]) + |B|\eta_e[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]}{(\eta_{vc} + \eta_e)|B|[\sigma_p^2 f^2 + 2f\sigma_{p\varepsilon} + \sigma_\varepsilon^2]} \tag{A15}$$

□

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