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VERTICAL MERGERS, FORECLOSURE, AND RIVALS' EXIT: THE ROLE OF FRINGE FIRMS

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Abstract

We analyze vertical mergers when the upstream and downstream markets each have a dominant firm and a competitive fringe. Adding a fringe at each stage of the successive monopoly model of [Spengler \(1950\)](#) generates the anticompetitive mechanisms of the post-Chicago literature. The merger eliminates double marginalization but creates incentives for input foreclosure, customer foreclosure, and fringe exit. Our main result links customer foreclosure to input foreclosure. When the merged firm sources internally, independent suppliers with high operating costs can exit. Their exit reduces the supply available to downstream rivals and, relative to retaining that capacity, raises the input price rivals pay and the merged firm's foreclosure wedge. In an equilibrium construction, consumer prices remain below the pre-merger level. With atomless participation, consumer harm is ruled out under stated conditions; with indivisible downstream participation, it can arise. We derive implications for merger review under Section 7 of the Clayton Act.

JEL Codes: L42, L13, L41, K21

Keywords: Vertical mergers, double marginalization, input foreclosure, dominant firm and competitive fringe, raising rivals' costs, non-predatory exit, antitrust policy

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1 Introduction

In 1950, Joseph Spengler cleared away some of the confusion surrounding the economic results of vertical integration. In particular, he demonstrated that vertical integration of an upstream monopolist and an independently owned and operated downstream monopolist would result in lower prices and greater consumption for consumers.¹ Sequential market power generates double marginalization: both firms apply markups, producing output below the level that a vertically integrated monopolist would choose. Vertical integration eliminates one layer of markup. This efficiency gain is referred to as the elimination of double marginalization (EDM). In addition to the positive effect on consumer welfare, the integrated firm's profit would be greater than the sum of the unintegrated profit of the two monopolists. All winners, no losers. Thus, the appropriate enforcement policy should be one of "benign neglect."

Under Spengler's assumptions of constant marginal costs and pure monopolies upstream and downstream, this is sound antitrust policy. There is no rival to foreclose and none to exit. Since pure, i.e., single firm, monopoly is quite rare, this advice is of limited usefulness. In *Standard Oil*, *American Tobacco*, and *Alcoa*,² the antitrust concern involved unlawful monopolization under Section 2 of the Sherman Act. The firms were not monopolists; they were dominant firms.

In this paper, we extend Spengler's analysis to the case of a vertical merger between two dominant firms that each compete with a competitive fringe. This seemingly slight change in the market structure makes the antitrust inquiry far more complicated. No more benign neglect. Adding a competitive fringe at each stage, while preserving the successive monopoly structure, allows us to analyze input foreclosure, customer foreclosure, and fringe participation in the same framework. The consumer welfare comparison then depends on fringe participation. Our main result is that internal sourcing by the merged firm can induce upstream exit, and that exit strengthens input foreclosure.

The post-Chicago literature has demonstrated that vertical integration may have negative

¹Cournot (1838) derived the analogous result for complementary monopolists.

²See *Standard Oil Co. of New Jersey v. United States* (1911), *United States v. American Tobacco Co.* (1911), and *United States v. Aluminum Co. of America* (1945). The government alleged that Standard Oil controlled 90 percent of the business of producing, shipping, refining, and selling petroleum and its products. In its first year of operation, American Tobacco produced about 96 percent of domestic cigarette output. Judge Hand reckoned Alcoa's control of the market for virgin aluminum ingot at over 90 percent.

welfare consequences.³ Thus, the benefits of EDM may be offset to some extent. [Ordover, Saloner and Salop \(1990\)](#) showed that vertical integration creates incentives for input foreclosure. Because the integrated firm internalizes downstream profit, it faces a new trade-off: lower input prices to downstream rivals expand external sales but strengthen competitors. Integration can create a profit-maximizing incentive to worsen rivals’ input terms, raising rivals’ costs ([Salop and Scheffman 1983, 1987](#); [Krattenmaker and Salop 1986](#)) and reducing downstream competition. [Hart and Tirole \(1990\)](#) establish a complementary result: the pre-integration upstream monopolist cannot commit to high (restrictive) supply terms because it faces an incentive to secretly offer lower prices to individual downstream buyers, dissipating monopoly rents. Vertical integration resolves this commitment problem by changing the upstream firm’s incentives, enabling credible supply restriction to downstream rivals and generating equilibrium foreclosure. Crucially, this mechanism requires no short-run profit sacrifice: the merger changes the integrated firm’s objective function, and foreclosure is the equilibrium outcome of profit-maximizing behavior. Both analyses assume away pre-merger double marginalization, so the EDM benefit does not enter the welfare computation. [Chen \(2001\)](#) characterizes conditions under which the EDM benefit dominates the foreclosure harm in a related setting with competing upstream suppliers, a downstream duopoly, and costs of switching suppliers. [De Stefano and Salinger \(2025a,b\)](#) develop this trade-off further, analyzing vertical mergers between an upstream monopolist and a downstream duopoly under successive stages and simultaneous moves timing and proposing the net downstream pricing pressure (NDPP) as a screening statistic for consumer harm. [Harrington \(2021\)](#) shows that merger-related cost efficiencies need not be passed through to consumers when their realization is private information.

We contribute to the literature by studying vertical mergers in the context of the dominant firm and competitive fringe model ([Forchheimer 1908](#); [Worcester 1957](#); [Stigler 1965](#); [Tirole 1988](#); for a graphical treatment, see [Carlton and Perloff 2005](#), pp. 111–119).⁴ The structure

³We use the term “post-Chicago literature” in the sense of [Hovenkamp \(1985\)](#) and [Hovenkamp \(2001\)](#), who introduced it to describe the body of industrial organization theory — exemplified by [Salop and Scheffman \(1983\)](#), [Krattenmaker and Salop \(1986\)](#), [Ordover, Saloner and Salop \(1990\)](#), hereafter OSS), and the literature that followed — identifying strategic conditions under which vertical arrangements may harm competition. Hovenkamp argues that the Chicago School’s modeling assumptions are insufficient to predict firm behavior in markets with strategic interaction and multiple competing firms at each level of the supply chain.

⁴[Ferguson \(1966, ch. 11\)](#) provides one of the earliest analyses of the dominant firm and competitive fringe model under the heading ‘Price Leadership in Oligopoly.’

yields clear predictions: each dominant firm prices against a residual demand net of fringe supply, and the fringe’s response is governed by its supply and participation decisions. A similar market structure has been applied to problems of optimal regulation under incomplete information (Biglaiser and Ma 1995, 1999). Riordan (1998) studies backward integration by a dominant downstream firm into a competitive upstream industry and shows that it raises input and output prices. To our knowledge, the implications for vertical merger analysis have not been studied using this market structure.

We begin with an upstream dominant firm U, an upstream competitive fringe, an independently owned and operated downstream dominant firm D, and a downstream competitive fringe. The pre-merger equilibrium exhibits double marginalization arising from the sequential exercise of market power by U and D. A proposed merger between U and D, forming an integrated firm I, eliminates double marginalization. The paper characterizes the EDM benchmark, joint pricing with input foreclosure, and participation equilibria with fringe exit. Predatory behavior is discussed as an extension.

This paper makes three contributions. First, we identify a chain that links customer foreclosure to input foreclosure through upstream exit. To our knowledge, no prior work derives this mechanism in an equilibrium model.⁵ When the integrated firm sources its input internally, the input price can fall, and upstream fringe firms with high operating costs exit. Because the upstream fringe supplies d as well as D , their exit reduces the input supply available to d . This weakens d ’s outside option and expands I ’s residual demand from d . Proposition 3(i) shows that this loss raises the input price d pays when revenue is strictly concave and upstream supply is convex. The result keeps downstream fringe supply linear and models the loss as a parallel shift in upstream supply. Under a curvature bound and a condition that keeps the standalone benchmark attainable, it raises I ’s foreclosure wedge. Part (ii) shows that integration induces the exit in equilibrium. Relative to the case where that capacity stays active, exit raises the consumer price. Consumer prices remain below the pre-merger level.⁶

⁵Salop (2018, pp. 1975–1976 and n. 82) and Donna and Pereira (2023) describe how customer foreclosure can lead upstream suppliers to exit and raise the input prices that downstream rivals pay. Neither derives the chain from a model of supplier participation. We derive the chain with equilibrium participation and establish its effects on the input price and the foreclosure wedge (Proposition 3).

⁶Ford’s 1961 acquisition of Autolite illustrates why internal sourcing by a major buyer can matter for

Second, the paper studies a non-predatory viability channel. Building on the logic of [Whinston \(1990\)](#), [Donna and Pereira \(2023, 2024\)](#), and [Alderman, Blair and Donna \(2025\)](#), we show that I’s EDM-driven price reduction can push downstream fringe firms below break-even, without any foreclosure conduct by I. At the pre-merger input price, the integrated firm’s lower effective marginal cost reduces its downstream price below p^* . The larger the EDM efficiency gain, the larger the set of fringe firms pushed below break-even. This mechanism is distinct from OSS foreclosure and from predatory exclusion. Whether exit harms consumers depends on how fringe firms participate. Under the conditions of [Proposition 5](#), consumer harm cannot occur with atomless participation. With an indivisible downstream capacity block, exit raises the consumer price, and re-entry is unprofitable even without a re-entry cost ([Proposition 6](#)).

Third, the dominant firm and competitive fringe structure on both sides of the vertical relationship has not been analyzed in the vertical foreclosure literature to the best of our knowledge. [Ordover, Saloner and Salop \(1990\)](#) study a symmetric duopoly at both market levels. [De Stefano and Salinger \(2025a,b\)](#) analyze vertical mergers between an upstream monopolist and one of two competing downstream duopolists under successive stages and simultaneous moves timing assumptions, developing the NDPP as a consumer harm screening statistic. Neither paper includes a competitive fringe at either stage. In the present setting, each dominant firm prices against a residual demand net of fringe supply, so the foreclosure incentive operates on the residual demand margin rather than total market demand. The market structure of the competitive fringe on each side plays a central role in determining both the magnitude of EDM and the consequences of input and customer foreclosure. This structure makes the chain possible: the upstream fringe is both the integrated firm’s former supplier and the rivals’ alternative source.

The paper proceeds as follows. [Section 2](#) presents the model and pre-merger equilibrium. Objective functions and key first-order conditions are displayed in the main text; formal derivations and additional results are collected in the [Appendix](#). [Section 3](#) analyzes vertical

independent suppliers. In [Ford Motor Co. v. United States \(1972\)](#), the Supreme Court described the acquisition as foreclosing independent spark-plug manufacturers from a purchaser of a substantial share of industry output. Chief Justice Burger’s dissent also recounted the district court’s finding that Autolite’s plant could not profitably operate at half its production volume (pp. 585–586). These features motivate examining customer exposure and the scale of capacity decisions. The industry was oligopolistic, so the case motivates these economic questions rather than calibrates the dominant-firm-and-fringe structure or establishes our consumer-harm mechanism.

integration, input foreclosure, and participation, while Section 3.3 establishes the upstream exit chain. Section 4 places the new results in the context of the literature and discusses their scope. Section 5 derives implications for merger evaluation. Section 6 concludes.

2 The Economic Model

In this section, we specify the economic model that we will employ to explore the post-Chicago competitive concerns that arise with what seems to be a minor change to Spengler’s model: the addition of a competitive fringe at each stage.

2.1 Market Structure

We consider the economic effects of a vertical merger between two dominant firms, the upstream dominant firm U and the downstream dominant firm D. At the upstream stage, an input is produced by U and a fringe of small rivals. This input is transformed into a final good by D, which also faces competition from a downstream competitive fringe. Before the merger, none of the firms are vertically integrated.

Each final-good unit uses one unit of input. Let A and C denote the active upstream and downstream fringe sets. For fixed C , let $p^*(w; C)$ denote D’s optimal price. Total pre-merger derived input demand is $X(w; C) = Q(p^*(w; C))$. The dominant upstream firm’s residual demand subtracts $S_A(w)$, the supply of the active upstream fringe. After integration, internal output is supplied at the integrated firm’s upstream marginal cost. External input demand comes only from the downstream fringe.

Fringe firms are price takers with constant marginal costs and unit capacity. Write $c_{ui} \geq c_U$ and $c_{di} \geq c_D$ for their upstream and downstream production costs. An active upstream firm supplies when $w > c_{ui}$; an active downstream firm supplies when $p - w > c_{di}$. We refer to this production decision of an active firm as *dispatch*, to distinguish it from the earlier participation decision, described below. At equality, the firm is indifferent about dispatch. Aggregating these production decisions gives $S_A(w)$ and $S_{D,C}(p, w)$. Operating costs already incurred at the pricing stage do not enter dispatch. Upstream costs bounded below by c_U ensure that internal sourcing at c_U is efficient. The price-taking assumption concerns each fringe firm’s

ability to influence prices, not the fringe's aggregate market share. Many individually small firms can collectively supply a substantial share while each takes prices as given.

Let κ_{ui} and κ_{di} be recurring operating costs that installed fringe firms can avoid by remaining inactive for the operating period. Previously paid installation or entry costs are sunk. The pricing analysis holds active sets fixed. The participation analysis determines those sets beforehand. The baseline allows zero or positive operating costs. Appendix H uses heterogeneous upstream operating costs and zero downstream operating costs. Appendix I adds positive downstream operating costs. Appendix J changes downstream participation from individual atomless decisions to one indivisible capacity block.

Prices are public and linear: w is the actual external input price and p is the final good price. The dominant firms have constant production costs c_U and c_D ; I's internal unit cost is $c_U + c_D$. Flexible contracts, discriminatory offers, and divisional objectives are not part of the baseline game.

For fixed active sets, define

$$q_D(p, w) = Q(p) - S_{D,C}(p, w), \quad X_D(w) = Q(p^*(w; C)) - S_A(w). \quad (1)$$

We suppress A and C in q_D and X_D when those sets are held fixed. On a smooth region, assume $Q' < 0$, $q_{D,p} < 0$, and $q_{D,w} \geq 0$. A strict input-pricing incentive requires $q_{D,w} > 0$.

The timing is as follows. First, the ownership structure is observed. Second, installed fringe firms simultaneously choose whether to incur their avoidable operating costs and remain available. Third, prices and production are determined. Before integration, U chooses w , D then chooses p , and active fringes supply competitively. After integration, I jointly chooses p and external input supply, and independent suppliers clear the input market. Participation anticipates the pricing subgame. The one-period baseline allows reactivation of installed capacity on the same operating-cost terms.

A participation equilibrium consists of active sets and global continuation pricing choices for those sets. Active firms cover their operating costs. Inactive firms cannot profitably activate, using the continuation outcome after their own deviation. Atomless firms take prices as given for an individual deviation. A finite capacity block instead anticipates its effect on

prices.

Remark 1 (Product Differentiation). The final good is homogeneous and production requires one input unit. Differentiated products require a separately specified demand system and strategic pricing responses.

2.2 The Pre-merger Equilibrium

We begin our analysis by spelling out the pre-merger equilibrium for both firms D and U, solving backward from D's pricing problem. For fixed active sets, D solves

$$\max_p (p - w - c_D) q_D(p, w), \quad q_D(p, w) \geq 0. \quad (2)$$

At a positive-output interior optimum, its first-order condition is

$$q_D(p, w) + (p - w - c_D) q_{D,p}(p, w) = 0. \quad (3)$$

We assume a unique global response $p^*(w; C)$ on the relevant input-price interval for pre-merger comparative statics. A sufficient local condition is a negative second derivative of D's objective. Positive pass-through additionally requires $q_{D,w} - q_{D,p} + (p - w - c_D) q_{D,pw} > 0$. Appendix B derives the response derivative.

U anticipates that response and solves

$$\max_w (w - c_U) X_D(w), \quad X_D(w) \geq 0. \quad (4)$$

At a positive-sales interior optimum,

$$X_D(w) + (w - c_U) X'_D(w) = 0, \quad X'_D = Q'(p^*) \frac{dp^*}{dw} - S'_A. \quad (5)$$

Here the derivative includes D's response. Positive pass-through and nondecreasing fringe supply imply $X'_D < 0$.

Definition 1 (Double Marginalization). At an interior pre-merger equilibrium (w^*, p^*) , the markups satisfy $w^* > c_U$ and $p^* > w^* + c_D$. Hence $p^* - (c_U + c_D) = (w^* - c_U) + (p^* - w^* - c_D) >$

0. This decomposition identifies two margins. With competitive fringes, it does not by itself order the jointly optimized post-merger consumer price.

3 Vertical Integration

Integration replaces the two dominant firms' sequential objectives by joint profit maximization. We first isolate the EDM comparison, then solve external input sales and the zero-sales corner. Participation subsequently connects these pricing subgames to equilibrium active sets.

3.1 Elimination of Double Marginalization

Vertical integration produces a single dominant firm, I, that faces competition from fringes at both the upstream and downstream stages. Initially, we hold the downstream fringe's input price at w^* and its active set fixed. In the downstream-only EDM benchmark, I solves

$$\max_p (p - c_U - c_D) q_D(p, w^*), \quad q_D \geq 0. \quad (6)$$

Its interior first-order condition is $q_D + (p - c_U - c_D) q_{D,p} = 0$. Under strict concavity, positive residual output, and interior global optima before and after the cost reduction, the benchmark price p^I is below p^* .

Proposition 1 (EDM Benchmark). *Holding d's input price and the downstream fringe supply schedule fixed at their pre-merger values, and maximizing downstream profit alone, integration reduces the optimal consumer price below p^* . Consumer surplus increases.*

At the old price p^* , the derivative of the benchmark objective equals $(w^* - c_U) q_{D,p} < 0$, using D's pre-merger first-order condition. Strict concavity locates its optimum to the left.

Proposition 1 restates the [Spengler \(1950\)](#) result in the dominant firm and competitive fringe setting. It isolates the EDM effect by abstracting from exit. The EDM efficiency accrues to I. In the benchmark, d continues to buy the input at w^* . The cost asymmetry between I and d follows from integration. Because d's input price is unchanged, the asymmetry does not raise rivals' costs in the sense of [Salop and Scheffman \(1983, 1987\)](#). Being more efficient and pricing aggressively is consistent with competitive conduct (*Pacific Bell Telephone Co.*

v. linkLine Communications, Inc. 2009). Even at w^* , the price reduction $p^I < p^*$ may push the highest-cost downstream fringe firms below break-even. Section 3.4 examines this case (Proposition 4).

3.2 Joint Pricing and Input Foreclosure

In this subsection, we let I choose the input price and expand the analysis to consider the incentive for engaging in input foreclosure. Fix both active sets during pricing and define signed excess input demand $r_A(p, w) = S_{D,C}(p, w) - S_A(w)$. On a smooth region, $r_{A,w} < 0$ and $r_{A,p} \geq 0$. External sales are constrained to be nonnegative and downstream own output is $q_D \geq 0$. The integrated positive-sales objective is

$$\Pi_A(p, w) = (p - c_U - c_D) q_D(p, w) + (w - c_U) r_A(p, w). \quad (7)$$

For a given p , let $\bar{w}_A(p)$ solve $r_A(p, \bar{w}_A(p)) = 0$. This is the input price when the upstream fringe alone supplies d. If I offers an input price w below $\bar{w}_A(p)$, the market price is w and I sells $x = r_A(p, w) > 0$. An offer above $\bar{w}_A(p)$, including refusal to supply, leaves the actual input price at $\bar{w}_A(p)$ and I's external sales at zero. We assume a unique clearing price and restrict attention to feasible prices with nonnegative downstream residual output.

Definition 2 (Input Foreclosure). At a specified downstream price p and active upstream set A , input foreclosure is a higher actual input price, or lower external input supply, than would be chosen by a standalone supplier with the same marginal cost facing $r_A(p, w)$. The benchmark holds p and A fixed to isolate the downstream-profit incentive.

At an interior global optimum with positive external sales and downstream own output, joint pricing requires both first-order conditions

$$\begin{aligned} q_D + (p - c_U - c_D) q_{D,p} + (w - c_U) r_{A,p} &= 0, \\ r_A + (w - c_U) r_{A,w} + (p - c_U - c_D) q_{D,w} &= 0. \end{aligned} \quad (8)$$

The first condition includes the external revenue term $(w - c_U) r_{A,p}$, which the benchmark in (6) omits. To include refusal to supply, choose external quantity $x \geq 0$ and let $W_A(p, x)$ solve $r_A(p, W_A) = x$. The same objective becomes

$$H_A(p, x) = (p - c_U - c_D) q_D(p, W_A(p, x)) + (W_A(p, x) - c_U) x. \quad (9)$$

At an interior downstream price, the sales-corner conditions are $H_{A,p} = 0$, $H_{A,x} \leq 0$, $x \geq 0$, and $xH_{A,x} = 0$. Here

$$H_{A,x} = W_A - c_U + \frac{x + (p - c_U - c_D) q_{D,w}}{r_{A,w}}. \quad (10)$$

Refusal to supply is a form of input foreclosure. With zero external sales, d pays the fringe-only clearing price $\bar{w}_A(p)$. Under the interior positive-sales assumptions of Proposition 2, this price exceeds the standalone supplier's price $w_A^U(p)$. If the standalone optimum is at a boundary, the boundary values must be compared directly.

Proposition 2 (Input Pricing Incentive). *Fix p and the active sets. Suppose the standalone input-profit objective $(w - c_U) r_A(p, w)$ has a unique interior maximizer $w_A^U(p) > c_U$ with positive sales. Suppose $w \mapsto \Pi_A(p, w)$ is continuous and strictly concave on the closed interval of actual input prices defined by $r_A(p, w) \geq 0$ and $q_D(p, w) \geq 0$, including the zero-sales endpoint $\bar{w}_A(p)$. Let $p > c_U + c_D$ and $q_{D,w} > 0$. Then I chooses an actual input price above $w_A^U(p)$, possibly at the zero-sales boundary $\bar{w}_A(p)$.*

Offers above $\bar{w}_A(p)$ leave the actual price at $\bar{w}_A(p)$, so this interval contains every attainable input price. At its endpoint $r_A = 0$, so $\Pi_A(p, \bar{w}_A(p)) = H_A(p, 0)$.

Proof. At $w_A^U(p)$, the standalone first-order condition makes $r_A + (w - c_U) r_{A,w} = 0$. The remaining derivative of joint profit is $(p - c_U - c_D) q_{D,w} > 0$. Strict concavity places the integrated optimum to the right, or at the feasible upper boundary. The same standalone condition implies $r_A = -(w - c_U) r_{A,w} > 0$ when the markup is positive and residual demand slopes downward. Positive standalone sales therefore follow from the existence of an interior standalone optimum. If no interior standalone optimum exists, compare the boundary optima directly. $\square$

3.3 Customer Foreclosure and Upstream Participation

Customer foreclosure is the merging downstream firm's withdrawal as a buyer from independent upstream suppliers. Its production is instead sourced internally at c_U . This reduces their potential demand, but the equilibrium operating decision depends on anticipated prices and avoidable costs.

For any proposed active sets, first solve the continuation pricing problem in Section 3.2. Then check the participation inequalities from Section 2. In the atomless baseline, an active upstream firm requires $\max\{w - c_{ui}, 0\} \geq \kappa_{ui}$ and an active downstream firm requires $\max\{p - w - c_{di}, 0\} \geq \kappa_{di}$. Inactive installed firms require the opposite weak inequalities when activation is available on the same terms.

At any fixed pair (p, w) , $r_{A_1}(p, w) - r_{A_0}(p, w) = S_{A_0}(w) - S_{A_1}(w) \geq 0$. The inequality is strict only if removed capacity would have supplied at w . If the clearing roots are unique, this inward shift also implies $\bar{w}_{A_1}(p) \geq \bar{w}_{A_0}(p)$. Withholding all external supply leaves d facing a weakly higher fringe-only input price.

A coverage comparison can be made at an independently specified benchmark $(\hat{p}, \hat{w})$, such as a fixed-price supply obligation. If $S_{A_0}(\hat{w}) \geq S_D(\hat{p}, \hat{w}) > S_{A_1}(\hat{w})$, d can be supplied entirely by the upstream fringe before exit at an input price no greater than $\hat{w}$, whereas maintaining that same downstream and input price after exit requires positive sales by I . This is a change in coverage at fixed benchmark prices.

The effect on equilibrium input pricing can also be characterized beyond a linear demand specification. For this comparison, normalize $c_U = c_D = 0$, retain $S_D(p, w) = p - w$ with zero downstream operating costs, and write active upstream supply as $S(w) + K$, where $K \in [0, B]$ is the active capacity of a zero-marginal-cost cohort. Let Q and S be twice continuously differentiable on $[0, P]$, with $Q(0) > 0$, $Q(P) = 0$, $Q' < 0$, $S(0) = 0$, $S'' \geq 0$, and $S'(w) > 0$ for $w > 0$. Assume revenue $R(p) = pQ(p)$ is strictly concave, with $R'' < 0$. Let p_m solve $R'(p_m) = 0$, let p_c solve $R'(p_c) = 2p_c$, and assume $0 < B < p_c$. Define $\theta(p) = -R''(p)$, $g(w) = [wS(w)]''$, $\theta_{\max} = \max_{[0, p_m]} \theta$, $g_{\max} = \max_{[0, p_m]} g$, $g_{\min} = \min_{[0, p_m]} g$. Let $(p(K), w(K))$ denote the integrated firm's optimal prices. Let $w_K^U(p)$ maximize standalone input profit $z[p - z - S(z) - K]$ over attainable input prices, those with nonnegative sales and nonnegative

residual output for I, as in Section 3.2. The integrated firm's wedge over this conditional standalone-supplier price is $\Delta(K) = w(K) - w_K^U(p(K))$.

Proposition 3 (Upstream Exit and Input Foreclosure). *(i) Under the preceding conditions, a reduction in K strictly raises the actual input price $w(K)$. If, in addition,*

$$g_{\max} - g_{\min} < \frac{2}{\theta_{\max} + 2}, \quad (11)$$

and $Q(p_m) \geq p_m$, it strictly raises the wedge $\Delta(K)$. These comparisons allow the integrated firm to adjust the consumer price. They hold with positive external input sales and at the zero-sales boundary.

(ii) There is an open set of parameters for which economies with linear demand and linear upstream supply satisfy the conditions in part (i), including (11), and an atomless upstream operating-cost cohort is fully active before integration and inactive afterward. Relative to the post-merger pricing subgame retaining its installed capacity, exit raises the actual input price, the foreclosure wedge, and the consumer price. Both post-merger consumer prices remain below the pre-merger price. The participation outcomes and these strict comparisons persist under sufficiently small quadratic perturbations of demand and upstream supply satisfying part (i). With positive upstream curvature, the perturbed construction has zero external input sales with retained capacity and positive external input sales after exit.

The proof and construction are in Appendix H. Part (i) characterizes the pricing consequences of lost independent capacity. The condition $Q(p_m) \geq p_m$ keeps the standalone benchmark attainable at the integrated consumer price; without it, the benchmark can lie at the boundary where I's residual output is zero, and the wedge need not rise. Part (ii) establishes that integration can induce that loss in an economy satisfying part (i)'s conditions. Suppliers anticipate continuation prices, and I's choices are global optima. Alternative input supply is therefore endogenous to the merger. Preserving independent capacity lowers the consumer price in this family, although integration benefits consumers even after exit. In this family, exit lowers the quantity of input d buys. In the perturbed construction, I sells to d after exit because the cohort's supply is gone, and it charges more than a standalone supplier would. The wedge rises and d's total purchases fall.

The general comparison retains linear downstream fringe supply and models capacity loss as a parallel reduction in upstream supply. Removing capacity with heterogeneous marginal costs requires a separate comparison.

3.4 Downstream Participation and Consumer Prices

An efficiency gain can reduce rivals' operating margins without any profit sacrifice. Whether this leads to exit and consumer harm depends on how rivals participate. With atomless participation, consumer harm cannot occur under the conditions of Proposition 5. A large capacity block whose return lowers prices can remain inactive, and consumers are harmed (Proposition 6).

Proposition 3 isolates the upstream participation channel while holding downstream operating costs at zero. We now allow positive downstream operating costs and determine participation in both fringes.

The downstream-only benchmark holds d 's input price at w^* and computes $p^I < p^*$. At those prices, an installed downstream firm's variable profit is $\max\{p^I - w^* - c_{di}, 0\}$. Comparing this margin with its avoidable operating cost identifies a benchmark viability failure. Equation (A.10) shows the comparison.

A larger upstream markup represents a larger potential cost reduction from EDM. When that reduction lowers the benchmark output price, it can expand the set of firms that fail the viability test.

Proposition 4 (Downstream Viability in the EDM Benchmark). *Hold d 's input price at w^* and its pre-exit active set fixed. The downstream-only EDM price $p^I < p^*$ weakly lowers each downstream firm's variable profit. With positive avoidable operating costs and a positive density of previously active firms whose effective production-plus-operating costs lie in the interval crossed by the price reduction, a positive mass fails the benchmark viability test. A further reduction in p^I weakly expands that set.*

Appendix D shows how removing a fixed amount of downstream supply can offset EDM when the input price and downstream-only optimization are held fixed. To compare ownership regimes with endogenous participation instead, normalize the dominant firms' production

costs to zero and let demand be $Q(p) = a - bp$, with $a, b > 0$. Upstream supply consists of a differentiable zero-operating-cost base $S_0(w)$, with $S_0(0) = 0$ and $S'_0(w) > 0$ at relevant positive prices, and an atomless zero-marginal-cost cohort with capacity B and operating cost $k > 0$. Downstream firms have unit capacity, heterogeneous constant marginal costs with measure dc on $[0, c_{\max}]$, where $c_{\max} > a/b$, and common operating cost $f > 0$. Installed firms can reactivate on the same terms. Subscripts $-$ and $+$ denote pre-merger and post-merger equilibria. Write $T_j = \max\{p_j - w_j - f, 0\}$ for active downstream capacity and $G(w) = S_0(w) + wS'_0(w)$.

Active downstream firms produce their full capacity, because each earns a dispatch margin of at least $f > 0$. Their aggregate supply is locally fixed during each pricing subgame, so $q_{D,w} = 0$. The input pricing incentive of Proposition 2 is absent at the pricing stage.⁷ The input price affects downstream competition only through participation. Appendix I provides the global dispatch function and the continuation pricing conditions.

Proposition 5 (Atomless Participation and Consumer Prices). *Suppose equilibria exist in the extension just specified, under the timing and participation rules in Section 2, the dominant firms have positive quantities, input prices are positive when downstream input demand is positive, and the relevant pricing optima are interior and differentiable locally. Then*

$$p_+ - p_- = \frac{T_- - T_+ - bw_-}{2b}.$$

Thus consumer harm occurs if and only if the loss of active downstream capacity exceeds bw_- . If G is nondecreasing, as for $S_0(w) = w^2$ with $G(w) = 3w^2$, such harm cannot occur. Appendix I gives an example in which all pricing and participation conditions hold globally.

The price decomposition separates the loss of downstream capacity from the removal of the pre-merger input markup. In the unit-density specification, consumer harm would require lost capacity to exceed bw_- . But a smaller active downstream fringe combined with a higher consumer price requires a higher input price. That raises upstream participation and, when G is nondecreasing, conflicts with the integrated pricing conditions. This contradiction produces

⁷The results differ in the downstream fringe's operating costs. In Proposition 3, downstream fringe firms have no operating costs, so the marginal firm responds to the input price and the incentive of Proposition 2 operates. In Propositions 5 and 6, downstream operating costs are positive, so active firms produce at capacity, and the input price affects downstream output only through participation.

the no-harm result.

Proposition 5 relies on atomless participation. An individual installed firm’s return leaves prices unchanged. If consumer prices rise while the input price falls, previously active downstream firms earn higher margins and cannot all remain inactive on unchanged operating terms. We now change the participation margin. One downstream capacity block decides whether to operate and anticipates how its activation changes prices. Its production is competitively dispatched. The block represents a large discrete capacity decision, such as operating an entire facility. Its return changes prices. The following result shows that this difference can sustain consumer harm.

Proposition 6 (Indivisible Participation and Consumer Harm). *Replace individually atomless downstream participation by one installed capacity block that supplies competitively at production but makes a single operating decision. There is an open family of economies with linear demand, quadratic base upstream supply, an atomless upstream operating-cost cohort, and a zero-marginal-cost downstream block in which the unique aggregate pre-merger participation outcome has both the block and cohort active, while the post-merger outcome has both inactive. The post-merger consumer price exceeds the pre-merger price. Activation and reactivation require the same operating costs; no additional re-entry cost is needed.*

Appendix J provides sufficient conditions for this family and proves global continuation optimality and participation. With the block active after integration, EDM lowers the consumer price enough that the block cannot cover its operating cost.⁸ Its withdrawal removes downstream competition, raising the consumer price. Returning restores enough capacity to lower the continuation price and remains unprofitable. This price effect of the block’s own return sustains inactivity. It is absent from an atomless individual deviation.

Propositions 5 and 6 show that the participation margin determines the consumer price comparison. Proposition 6 requires no re-entry cost. Reactivation uses the same operating

⁸Proposition 6 shares features with models in which a merger’s cost reduction deters entry (e.g., [Cabral 2003](#)). The vertical structure adds an element. When the block is active, I supplies its input and sets both the input price and the consumer price, so I’s joint pricing determines the block’s margin $p - w$. The block exits through a margin squeeze: the consumer price falls more than the input price. This is the price squeeze at issue in *United States v. Aluminum Co. of America* (1945) and *Pacific Bell Telephone Co. v. linkLine Communications, Inc.* (2009). Here the squeeze is profit maximizing and involves no predatory pricing. So, absent an antitrust duty to deal, under *linkLine* it is not actionable under Section 2 ([Hovenkamp and Hovenkamp 2009](#)). Merger review may be used to address it.

cost. The block has a common zero marginal cost, so the construction changes downstream cost heterogeneity as well as participation. The sufficient conditions hold on an open set of parameters. Appendix J provides an example where both merging firms supply a majority of pre-merger output and the block supplies 35 percent of final output. The operating cost f must lie in an interval whose width is about 1 percent of f . The interval widens as the block's share rises: at a 48 percent share its width is about 2.7 percent of f . In this family, consumer harm requires a large block or an operating cost in a narrow range.

3.5 Dynamic Profit Sacrifice

Profit-maximizing input foreclosure differs from a strategy that sacrifices current joint operating profit to change future participation. Let $V_A(w) = \max_p \Pi_A(p, w)$ over the feasible pricing problem, including boundary cases. A constrained strategy sacrifices current profit only if its value is below the best feasible joint-profit value.

For a specified strategy, the economic recoupment comparison uses the incremental discounted continuation gain relative to the best feasible baseline, rather than gross future monopoly rents. That gain must exceed the current joint-profit sacrifice. Appendix G states the comparison.

A path with a temporary low price, irreversible exit, and later high prices requires a dynamic model with dated participation decisions and reactivation costs, which is outside the scope of this paper.

4 The New Results in Context

The results connect the familiar vertical-pricing incentives to two equilibrium questions: whether internal sourcing removes independent input capacity, and whether the resulting participation choices overturn EDM's consumer benefit. Proposition 3 establishes the upstream-exit channel. Propositions 5 and 6 characterize how atomless and indivisible participation can produce different consumer-price conclusions.

Salop and Scheffman (1983, 1987), Krattenmaker and Salop (1986), and Ordover, Saloner and Salop (1990) establish RRC logic underlying Proposition 2. The market structure here

is different: Our competitive fringes differ from strategic downstream rivals: output is determined by dispatch at common prices, while participation can be atomless or indivisible. Setting fringe supply to zero produces successive monopoly, so it does not nest the OSS duopoly. In [Riordan \(1998\)](#), a dominant downstream firm integrates into a competitive upstream industry, and integration raises the input price. Here the upstream market has its own dominant firm. Internal sourcing can lower the price independent suppliers receive, and their exit raises the price downstream rivals pay. The distinctive question here is whether withdrawal of an integrated customer’s purchases changes upstream participation and, through the resulting alternative supply, the integrated supplier’s pricing wedge. Customer foreclosure may also arise through agreements that secure a dominant firm’s access to distribution, as in search engine default contracts ([Hovenkamp 2026](#)). The competitive fringe makes alternative supply and the individual participation margin explicit. The indivisible extension isolates a discrete capacity decision. Replacing the fringes with asymmetric oligopolists would add strategic pricing by rivals.

[Salinger \(1988\)](#), [Chen \(2001\)](#), and [De Stefano and Salinger \(2025a,b\)](#) analyze vertical-pricing tradeoffs under different structures and timing. In our model, the gain from external supply must be computed jointly with its effect on downstream profit. A related literature models vertical relations with bargaining over input terms ([Sheu and Taragin 2021](#); [Donna et al. 2024](#)). We study public linear prices, which make refusal to supply and the clearing price supported by the fringe explicit. [Whinston \(1990\)](#) shows why statically optimal conduct can undermine participation without deliberate current-profit sacrifice. The benchmark viability result applies that familiar logic to EDM. Our equilibrium contribution determines what participation implies for consumer prices: [Proposition 5](#) rules out harm under the atomless extension’s restrictions, which remove the input pricing incentive of [Proposition 2](#), while [Proposition 6](#) gives sufficient conditions for harm with indivisible capacity. [Proposition 3](#) characterizes the input-price and foreclosure-wedge effects of capacity loss under the stated demand and supply conditions, and establishes merger-induced exit in an equilibrium construction. The viability comparison also connects to merger-induced exit studied by [Donna and Pereira \(2023\)](#) and [Alderman, Blair and Donna \(2025\)](#).

Contracting and Commitment. [Hart and Tirole \(1990\)](#) allow flexible contracts and endogenous industry structure. We study public linear prices. The clearing price supported by the upstream fringe caps the price d pays, and refusal to supply is one feasible choice for I .

Participation and Dynamics. Exit can persist after prices rise if operating margins remain below avoidable operating costs. If reactivation would otherwise be profitable, continued inactivity requires an additional barrier or a participation decision large enough to change prices, as in [Proposition 6](#).

Profit Maximization and Predation. [Proposition 2](#) describes a change in the profit-maximizing objective after integration. [Proposition 4](#) describes a fixed-price viability comparison. Neither establishes deliberate profit sacrifice, equilibrium consumer harm, or a legal conclusion. Separating these claims avoids treating an incentive to raise rivals' costs as proof that the merger raises consumers' prices.

An improvement in costs or products can displace rivals through ordinary competition ([Kaplow and Shapiro 2007](#)).⁹ In *Utah Pie Co. v. Continental Baking Co.* (1967), the Court upheld liability for price cutting that harmed a rival; *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.* (1993) later required below-cost pricing and a prospect of recoupment. Under that standard, the EDM-driven price reduction behind [Proposition 4](#) is not actionable, even when it pushes fringe firms below break-even. The relevant welfare comparison must account for the gain as well as the participation response. With atomless participation, consumer prices cannot rise under the conditions of [Proposition 5](#). An indivisible return decision can produce a different outcome ([Proposition 6](#)).

Contractual Alternatives. A two-part tariff that charges D the marginal cost c_U and a fixed fee removes the upstream markup on D 's purchases (see [Blair and Kaserman 1980](#)). The fee is fixed when U later sets the price to d , so U 's objective excludes D 's profit. The input pricing incentive of [Proposition 2](#) then requires contingent transfers or shared decision rights. A two-part tariff can therefore eliminate double marginalization without creating that incentive, which bears on whether the EDM efficiency is merger specific. [Angerhofer and Blair](#)

⁹We thank Louis Kaplow for this observation.

(2021) note that several contractual equivalents to vertical integration suggest that the EDM efficiency may not be merger specific, so the agencies may discount it; they also argue that integration can avoid transaction costs these contracts entail. Such a tariff is a contractual arrangement, aside from a vertical merger, that can produce the customer withdrawal and upstream exit of Proposition 3 when the parties can overcome contractual incompleteness (Frydinger and Hart 2024). Appendix K solves this case.

Empirical Interpretation. Applying the model requires demand, production costs, fringe capacities, operating costs, and the alternatives available after inactivity. The relevant alternative supply consists of suppliers able to serve downstream rivals at feasible prices. Following Kaplow (2010), this evidence can be assessed directly, without first defining a market.

Internal Transfer Prices. Joint profit maximization uses c_U as the economic cost of internal input, regardless of an accounting transfer price. If divisional managers instead optimize independent objectives against internal charges, the game changes (Hirshleifer 1956). Because the 2023 Merger Guidelines credit only verifiable efficiencies, the actual internal pricing arrangement is relevant evidence.

Pass-through and Welfare. Cost pass-through depends on demand curvature, fringe dispatch, capacity constraints, and the active sets. Consumer surplus follows from the equilibrium consumer-price comparison for unchanged demand. Total welfare includes operating costs and production efficiency.

5 Implications for Merger Evaluation

When successive monopolists merge, the competitive analysis recommends “benign neglect.” The competitive analysis of vertical integration between successive dominant firms is quite complicated. An accurate assessment of the competitive effects requires a substantial amount of information that is not readily available and may be difficult to obtain.

Merger evaluation must distinguish the internal cost reduction from its consumer benefit. The pre-merger upstream markup $w^* - c_U$ measures the cost reduction per unit. Its pass-

through to consumers depends on I's joint pricing and on how fringe participation responds to the merger (Propositions 5 and 6). The 2023 Merger Guidelines ([U.S. Department of Justice and Federal Trade Commission 2023](#)) credit efficiencies that are verifiable and merger specific. A two-part tariff can remove the upstream markup without integration ([Angerhofer and Blair 2021](#)), which bears on the second requirement.

The input pricing incentive of Proposition 2 corresponds to the ability and incentive analysis of Guideline 5, which asks whether the merged firm may limit rivals' access to an input they use to compete. Proposition 3 adds that rivals' alternative supply responds to the merger. Internal sourcing withdraws the integrated firm's purchases from independent suppliers, and their exit raises the input price d pays and I's wedge relative to retaining that capacity. In the construction of Proposition 3(ii), preserving that capacity would lower the consumer price further below its pre-merger level. The Supreme Court identified the first step of this chain in *Ford Motor Co. v. United States* (1972), where the acquisition of Autolite removed a purchaser of a substantial share of industry output from independent spark plug manufacturers. In *Federal Trade Commission v. Microsoft Corp.* (2023), the court required evidence of both the ability and the incentive to foreclose (see also [Alderman, Blair and Donna 2025](#)). Our results are consistent with that requirement: Proposition 2 establishes an incentive at given prices and active sets, and Propositions 3 and 5 show that a stronger incentive can coexist with lower consumer prices. Section 7 of the Clayton Act reaches mergers whose effect may be substantially to lessen competition, so an agency need not show that foreclosure has occurred or will invariably occur. Unfortunately, this may open the door to the prohibition of sound, pro-competitive mergers. A remedy evaluation needs the same participation and pricing checks under the proposed supply terms.

The harms in Propositions 2, 3, and 6 arise from profit-maximizing conduct after integration. Absent a duty to deal or predatory pricing, Section 2 of the Sherman Act does not reach them (*Pacific Bell Telephone Co. v. linkLine Communications, Inc.* 2009; [Hovenkamp 2022](#)). Merger review under Section 7 is the forum that can address them before they occur.

A downstream viability failure at benchmark prices is a reason to investigate participation. Firms may return when prices change. Propositions 5 and 6 show that the size of an individual participation decision can change the outcome, even when firms face avoidable operating costs

in both ownership regimes.

A supply obligation can constrain the input pricing incentive of Proposition 2. Because a binding obligation holds I below its preferred input price, its effect depends on monitoring and enforcement. It also lowers the price independent suppliers receive, which can change their participation. Its effect on consumers depends on that response. Lower fringe margins from an efficiency gain can coexist with lower consumer prices (Proposition 3). A prohibition or structural remedy may require evidence that participation responses reverse the consumer benefit, as in Proposition 6, measured against the relevant counterfactual.

The model is most relevant where a merging buyer's internal sourcing is large relative to independent suppliers' sales, and downstream rivals depend on those suppliers or on the integrated firm. A merger between a dominant firm and an atomless fringe firm at the other stage generates neither the EDM benefit nor the input pricing incentive: the fringe firm prices at marginal cost and cannot affect prices. For such mergers, Spengler's benign neglect applies.

6 Concluding Remarks

To our knowledge, this paper provides the first analysis of vertical mergers with a dominant firm and a competitive fringe at both stages, a structure in which the merger changes who supplies the input and who participates in each market. The pre-merger equilibrium exhibits double marginalization, which the merger between U and D eliminates. The EDM efficiency is real but coexists with incentives for input foreclosure, customer foreclosure, and competitive fringe exit.

The main result is the customer foreclosure chain. Internal sourcing withdraws the integrated firm's purchases from independent upstream suppliers. Anticipating lower input prices, suppliers with high operating costs exit. Their exit reduces the alternative supply available to downstream rivals. Proposition 3 shows that this exit raises the input price rivals pay and strengthens the foreclosure wedge, relative to retaining independent capacity. The input price comparison allows nonlinear demand and convex upstream supply, with linear downstream fringe supply and a parallel capacity loss; the wedge comparison adds a curvature bound and a condition that keeps the standalone benchmark attainable. In the equilibrium construction,

preserving independent capacity lowers the consumer price, and both post-merger consumer prices remain below the pre-merger price. [Salop \(2018\)](#) and [Donna and Pereira \(2023\)](#) describe this link. To our knowledge, no prior work derives it in an equilibrium model. It implies that merger review should treat rivals' alternative supply as endogenous to the merger.

Building on [Whinston \(1990\)](#) and on the complementary good merger analyses of [Donna and Pereira \(2023\)](#) and [Alderman, Blair and Donna \(2025\)](#), the paper shows that the non-predatory viability channel operates in the vertical merger EDM benchmark. At the pre-merger input price, the integrated firm's lower effective marginal cost reduces its downstream price and can push fringe firms below break-even, without any foreclosure conduct. The larger the upstream markup that EDM eliminates, the larger the set of fringe firms pushed below break-even. Endogenous participation determines whether this channel harms consumers. With atomless participation, consumer harm cannot occur under the conditions of Proposition 5. With an indivisible downstream capacity block, exit raises the consumer price, and returning is unprofitable because the block's return lowers the price (Proposition 6).

The paper examines the EDM benchmark, input foreclosure, customer foreclosure, fringe exit, and possible dynamic profit sacrifice for merger authorities to consider. The distinction among profit-maximizing foreclosure (Proposition 2), foreclosure strengthened by upstream exit (Proposition 3), benchmark viability loss (Proposition 4), and equilibrium consumer harm (Proposition 6) is paramount for both the theory of harm and remedy design.

References

- Alderman, Brianna L., Roger D. Blair, and Javier D. Donna.** 2025. “The Microsoft Acquisition of Activision: Neither Horizontal nor Vertical.” *Managerial and Decision Economics*, 46: 3713–3728.
- Angerhofer, Tirza J., and Roger D. Blair.** 2021. “Successive Monopoly, Bilateral Monopoly and Vertical Mergers.” *Review of Industrial Organization*, 59(2): 343–361.
- Biglaiser, Gary, and Ching-to Albert Ma.** 1995. “Regulating a Dominant Firm: Unknown Demand and Industry Structure.” *RAND Journal of Economics*, 26(1): 1–19.
- Biglaiser, Gary, and Ching-to Albert Ma.** 1999. “Investment Incentives of a Regulated Dominant Firm.” *Journal of Regulatory Economics*, 16(3): 215–235.
- Blair, Roger D., and David L. Kaserman.** 1980. “Vertical Control with Variable Proportions: Ownership Integration and Contractual Equivalents.” *Southern Economic Journal*, 46(4): 1118–1128.
- Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.** 1993. 509 U.S. 209 (1993).
- Cabral, Luís M. B.** 2003. “Horizontal Mergers with Free-Entry: Why Cost Efficiencies May Be a Weak Defense and Asset Sales a Poor Remedy.” *International Journal of Industrial Organization*, 21(5): 607–623.
- Carlton, Dennis W., and Jeffrey M. Perloff.** 2005. *Modern Industrial Organization*. 4th ed. Boston: Pearson Addison Wesley.
- Chen, Yongmin.** 2001. “On Vertical Mergers and Their Competitive Effects.” *RAND Journal of Economics*, 32(4): 667–685.
- Cournot, Antoine Augustin.** 1838. *Recherches sur les Principes Mathématiques de la Théorie des Richesses*. Paris. English translation: *Researches into the Mathematical Principles of the Theory of Wealth*. New York: Macmillan, 1897.
- De Stefano, Martino, and Michael A. Salinger.** 2025a. “The Complicated Simple Economics of Vertical Mergers.” *Journal of Law and Economics*, 68(1): 215–239.

- De Stefano, Martino, and Michael A. Salinger.** 2025*b*. “A Simultaneous Moves Approach to the Complicated Simple Economics of Vertical Mergers.” *Review of Industrial Organization*, 66(2): 115–139.
- Donna, Javier D., and Pedro Pereira.** 2023. “Rivals’ Exit and Vertical Merger Evaluation.” *Journal of Competition Law and Economics*, 19: 220–249.
- Donna, Javier D., and Pedro Pereira.** 2024. “Structural Presumptions for Non-Horizontal Mergers in the 2023 Merger Guidelines: A Primer and a Path Forward.” *Review of Industrial Organization*, 65(1): 303–345.
- Donna, Javier D., Pedro Pereira, Y. Pu, André Trindade, and R. C. Yoshida.** 2024. “Direct Sales and Bargaining.” *RAND Journal of Economics*, 55(4): 749–787.
- Federal Trade Commission v. Microsoft Corp.*** 2023. No. 3:23-cv-02880-JSC (N.D. Cal. July 10, 2023), *aff’d*, No. 23-15992 (9th Cir. May 7, 2025).
- Ferguson, C. E.** 1966. *Microeconomic Theory*. Homewood, IL: Richard D. Irwin.
- Forchheimer, Karl.** 1908. “Zur Theorie des Marktpreises.” *Archiv für Sozialwissenschaft und Sozialpolitik*, 27: 141–162.
- Ford Motor Co. v. United States.*** 1972. 405 U.S. 562 (1972).
- Frydlinger, David, and Oliver Hart.** 2024. “Overcoming Contractual Incompleteness: The Role of Guiding Principles.” *Journal of Law, Economics, and Organization*, 40(3): 625–647.
- Harrington, Joseph E., Jr.** 2021. “There May Be No Pass Through of a Merger-Related Cost Efficiency.” *Economics Letters*, 208: 110050.
- Hart, Oliver, and Jean Tirole.** 1990. “Vertical Integration and Market Foreclosure.” *Brookings Papers on Economic Activity: Microeconomics*, 1990: 205–276.
- Hirshleifer, Jack.** 1956. “On the Economics of Transfer Pricing.” *Journal of Business*, 29(3): 172–184.
- Hovenkamp, Erik.** 2022. “The Antitrust Duty to Deal in the Age of Big Tech.” *Yale Law Journal*, 131(5): 1483–1546.

- Hovenkamp, Erik.** 2026. “The Competitive Effects of Search Engine Defaults.” *Journal of Law and Economics*. Forthcoming.
- Hovenkamp, Erik, and Herbert Hovenkamp.** 2009. “The Viability of Antitrust Price Squeeze Claims.” *Arizona Law Review*, 51: 273–302.
- Hovenkamp, Herbert.** 1985. “Antitrust Policy after Chicago.” *Michigan Law Review*, 84: 213–284.
- Hovenkamp, Herbert.** 2001. “Post-Chicago Antitrust: A Review and Critique.” *Columbia Business Law Review*, 2001(2): 257–337.
- Kaplow, Louis.** 2010. “Why (Ever) Define Markets?” *Harvard Law Review*, 124(2): 437–502.
- Kaplow, Louis, and Carl Shapiro.** 2007. “Antitrust.” In *Handbook of Law and Economics*. Vol. 2, ed. A. Mitchell Polinsky and Steven Shavell, 1073–1225. Amsterdam: North-Holland.
- Krattenmaker, Thomas G., and Steven C. Salop.** 1986. “Anticompetitive Exclusion: Raising Rivals’ Costs to Achieve Power over Price.” *Yale Law Journal*, 96(2): 209–293.
- Ordoover, Janusz A., Garth Saloner, and Steven C. Salop.** 1990. “Equilibrium Vertical Foreclosure.” *American Economic Review*, 80(1): 127–142.
- Pacific Bell Telephone Co. v. linkLine Communications, Inc.*** 2009. 555 U.S. 438 (2009).
- Riordan, Michael H.** 1998. “Anticompetitive Vertical Integration by a Dominant Firm.” *American Economic Review*, 88(5): 1232–1248.
- Salinger, Michael A.** 1988. “Vertical Mergers and Market Foreclosure.” *Quarterly Journal of Economics*, 103(2): 345–356.
- Salop, Steven C.** 2018. “Invigorating Vertical Merger Enforcement.” *Yale Law Journal*, 127: 1962–1994.
- Salop, Steven C., and David T. Scheffman.** 1983. “Raising Rivals’ Costs.” *American Economic Review: Papers and Proceedings*, 73(2): 267–271.

- Salop, Steven C., and David T. Scheffman.** 1987. “Cost-Raising Strategies.” *Journal of Industrial Economics*, 36(1): 19–34.
- Sheu, Gloria, and Charles Taragin.** 2021. “Simulating Mergers in a Vertical Supply Chain with Bargaining.” *RAND Journal of Economics*, 52(3): 596–632.
- Spengler, Joseph J.** 1950. “Vertical Integration and Antitrust Policy.” *Journal of Political Economy*, 58(4): 347–352.
- Standard Oil Co. of New Jersey v. United States.*** 1911. 221 U.S. 1 (1911).
- Stigler, George J.** 1965. “The Dominant Firm and the Inverted Umbrella.” *Journal of Law and Economics*, 8: 167–172.
- Tirole, Jean.** 1988. *The Theory of Industrial Organization*. Cambridge: MIT Press.
- United States v. Aluminum Co. of America.*** 1945. 148 F.2d 416 (2d Cir. 1945).
- United States v. American Tobacco Co.*** 1911. 221 U.S. 106 (1911).
- U.S. Department of Justice and Federal Trade Commission.** 2023. “Merger Guidelines.” December 18, 2023.
- Utah Pie Co. v. Continental Baking Co.*** 1967. 386 U.S. 685 (1967).
- Whinston, Michael D.** 1990. “Tying, Foreclosure, and Exclusion.” *American Economic Review*, 80(4): 837–859.
- Worcester, Dean A.** 1957. “Why ‘Dominant Firms’ Decline.” *Journal of Political Economy*, 65(4): 338–346.

Appendix

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This appendix contains the derivations for the main-text objectives and the constructions and proofs of Propositions 3, 5, and 6. Active sets are fixed during pricing, and participation is checked at continuation outcomes using Section 2’s timing and activation rules. Appendices H–J state the specific primitives and sufficient conditions.

A Upstream Dominant Firm

The profit function of U is:

$$\pi^U(w) = (w - c_U)X_D(w) \quad (\text{A.1})$$

The FONC for an interior maximum with respect to w is:

$$X_D(w) + (w - c_U)X'_D(w) = 0 \quad (\text{A.2})$$

For fixed active sets, the upstream residual-demand derivative includes D’s pricing response:

$$X'_D(w) = Q'(p^*(w)) \frac{dp^*}{dw} - S'_A(w).$$

When the response derivative in Appendix B is positive, $Q' < 0$ and $S'_A \geq 0$ imply $X'_D < 0$. At an interior optimum with $X_D > 0$, the markup is $w^* - c_U = -X_D(w^*)/X'_D(w^*) > 0$. A sufficient condition for a unique global interior optimum on a specified feasible interval is strict concavity, $2X'_D + (w - c_U)X''_D < 0$ throughout that interval, together with derivative signs that place the optimum inside its boundaries. Alternatively, compare global values directly. Appendix H verifies these conditions for its construction.

B Downstream Dominant Firm

The profit function of D, taking w as given, is:

$$\pi^D(p; w) = (p - w - c_D)q_D(p, w) \quad (\text{A.3})$$

The FONC with respect to p is:

$$q_D(p, w) + (p - w - c_D)(\partial q_D / \partial p) = 0 \quad (\text{A.4})$$

Let $m_D = p - w - c_D$ and evaluate derivatives at an interior D optimum. Differentiating (A.4) gives

$$\frac{dp^*}{dw} = -\frac{q_{D,w} - q_{D,p} + m_D q_{D,pw}}{2q_{D,p} + m_D q_{D,pp}}.$$

A negative denominator is the local second-order condition. Positive pass-through also requires a positive numerator. We impose those restrictions when using a smooth increasing response.

C Double Marginalization Decomposition

The total markup over social marginal cost $c_U + c_D$ decomposes as:

$$p^*(w^*) - (c_U + c_D) = [w^* - c_U] + [p^*(w^*) - w^* - c_D] > 0 \quad (\text{A.5})$$

Both bracketed terms are strictly positive under the conditions ensuring interior solutions to (A.2) and (A.4).

D Integrated Firm: EDM Benchmark

The EDM benchmark deliberately excludes external input profit and fixes d 's input price at w^* . I maximizes $(p - c_U - c_D) q_D(p, w^*)$. Its interior first-order condition is:

$$q_D(p, w^*) + (p - c_U - c_D) q_{D,p}(p, w^*) = 0. \quad (\text{A.6})$$

Under strict concavity and positive residual output, the lower effective marginal cost implies $p^I < p^*$.

Corollary A1 (Consumer Harm in a Controlled Exit Comparison). *Hold d 's input price at*

w^* and use downstream-only optimization. Suppose removal adds a constant N to $q_D(p, w^*)$ throughout the relevant price interval because the removed firms would supply their full units at all compared prices. Assume a strictly concave post-removal objective with a unique interior global optimum p^{exit} . Then $p^{exit} > p^*$ if and only if

$$N > (w^* - c_U) |q_{D,p}(p^*, w^*)|.$$

Proof. At p^* , the derivative of the post-removal objective is $q_D + N + (p^* - c_U - c_D) q_{D,p}$. Subtracting D's pre-merger first-order condition leaves $N + (w^* - c_U) q_{D,p}$. Strict concavity makes its positive sign equivalent to a larger optimum. The threshold therefore uses the residual-demand derivative at the pre-merger price; it does not require a globally constant slope. It does require the constant supply removal and fixed-input-price downstream-only objective. It neither determines N from participation nor supplies a general empirical test for joint pricing. $\square$

E Integrated Firm and the External Sales Corner

Fix the active upstream set A . Define signed excess demand $r_A(p, w) = S_D(p, w) - S_A(w)$. On a smooth positive-sales region $r_{A,w} = S_{D,w} - S'_A < 0$, $r_{A,p} = S_{D,p} > 0$, and $q_{D,w} = -S_{D,w} > 0$. Actual sales satisfy $x \geq 0$. Market clearing with I supplying x defines $W_A(p, x)$ by $r_A(p, W_A(p, x)) = x$; at $x = 0$, $W_A(p, 0) = \bar{w}_A(p)$. The implicit derivatives are $W_{A,x} = \frac{1}{r_{A,w}} < 0$ and $W_{A,p} = \frac{-r_{A,p}}{r_{A,w}} > 0$.

$$\Pi_A(p, w) = (p - c_U - c_D) q_D(p, w) + (w - c_U) r_A(p, w). \quad (\text{A.7})$$

At an interior optimum with positive external sales, the joint-price first-order conditions are:

$$\begin{aligned} \Pi_{A,p} &= q_D + (p - c_U - c_D) q_{D,p} + (w - c_U) r_{A,p} = 0, \\ \Pi_{A,w} &= r_A + (w - c_U) r_{A,w} + (p - c_U - c_D) q_{D,w} = 0. \end{aligned} \quad (\text{A.8})$$

For the corner, maximize $H_A(p, x) = (p - c_U - c_D) q_D(p, W_A(p, x)) + (W_A(p, x) - c_U) x$

over feasible p and $x \geq 0$. At an interior downstream price and away from other quantity boundaries, $H_{A,p} = q_D + (p - c_U - c_D)(q_{D,p} + q_{D,w}W_{A,p}) + xW_{A,p} = 0$. The external-sales conditions are $x \geq 0$, $H_{A,x} \leq 0$, and $xH_{A,x} = 0$, where $H_{A,x} = W_A - c_U + \frac{x+(p-c_U-c_D)q_{D,w}}{r_{A,w}}$. These are necessary conditions; joint concavity of H_A on a convex feasible domain makes them sufficient. If concavity fails, compare global optima across the corner and positive-sales branches.

Let p_A^0 maximize $H_A(p, 0)$. The corner derivative is $g_A = \bar{w}_A(p_A^0) - c_U + \frac{(p_A^0 - c_U - c_D)q_{D,w}}{r_{A,w}}$, evaluated at $(p_A^0, \bar{w}_A(p_A^0))$. If $g_A > 0$, zero external sales cannot be optimal: a small increase in x improves joint profit. Under joint concavity, $g_A \leq 0$ together with the p condition supports the zero-sales optimum. A transition from zero to positive sales after exit is therefore established by $g_{A_0} \leq 0 < g_{A_1}$, with the concavity and feasibility conditions maintained.

The standalone benchmark in Proposition 2 is conditional on the same p and A : maximize $(w - c_U)r_A(p, w)$ on the positive-sales branch, including its boundaries. Only when its optimum is interior do we define $w_A^U(p)$ by $r_A + (w_A^U(p) - c_U)r_{A,w} = 0$. With a positive markup and $r_{A,w} < 0$, this equation already implies positive sales. At this price $\Pi_{A,w} = (p - c_U - c_D)q_{D,w} > 0$. Strict concavity of $\Pi_A(p, \cdot)$ on the closed interval of actual input prices, where $\Pi_A(p, \bar{w}_A(p)) = H_A(p, 0)$, proves the conditional price ordering in Proposition 2.

F Exit Conditions

Both fringes choose availability before prices and incur avoidable operating costs κ_{ui} and κ_{di} . Previously paid entry costs are sunk. The baseline uses the same operating-cost terms for staying and activating installed capacity. For atomless firms, write $v_{ui}(w) = \max\{w - c_{ui}, 0\}$ and $v_{di}(p, w) = \max\{p - w - c_{di}, 0\}$. Active firms require $v \geq \kappa$; inactive firms require $v \leq \kappa$. Equality can support partial participation. The active sets and continuation prices must satisfy these inequalities jointly.

$$v_{ui}(w) \geq \kappa_{ui} \quad (i \in A), \quad v_{ui}(w) \leq \kappa_{ui} \quad (i \notin A). \quad (\text{A.9})$$

The downstream equilibrium conditions use $v_{di}(p, w)$ and the active set C in the same way.

These inequalities are evaluated at the anticipated continuation equilibrium, not at a price calculated before other firms' decisions are incorporated.

A finite participation unit instead compares payoffs after its own stay or activation decision, using the resulting continuation prices. Appendix J applies this rule to an indivisible downstream block. If installation or reactivation has an additional cost, replace the inactive firm's cost threshold accordingly and solve that different game; such a barrier is not assumed in Sections H–J.

The downstream viability comparison in Section 3.4 uses the controlled EDM benchmark: input price w^* and output price p^I . A downstream firm's unit-capacity variable profit at those prices is $\max\{p^I - w^* - c_{di}, 0\}$. It fails to cover its recurring fixed operating cost if:

$$\max\{p^I - w^* - c_{di}, 0\} < \kappa_{di}. \quad (\text{A.10})$$

At fixed benchmark prices, $p^I < p^*$ lowers variable profits and can push previously active firms below break-even. An anticipated-participation equilibrium with downstream exit must additionally recompute I's continuation price with the surviving downstream set and check the corresponding stay and re-entry conditions.

G Predatory Foreclosure Rationality Condition

This appendix shows the profitability condition for a specified dynamic strategy. Characterizing a dynamic equilibrium is outside the scope of this paper. For a specified feasible strategy that induces exit, compare its incremental continuation gain with its current joint-profit sacrifice:

$$\begin{aligned} &PV(\delta, \text{continuation profit gain relative} \\ &\quad \text{to the profit-maximizing baseline}) \\ &> \text{current joint-profit sacrifice.} \end{aligned} \quad (\text{A.11})$$

Here δ discounts future profits. The sacrifice is the reduction in joint operating profit relative to the best feasible baseline, allowing p to adjust under both strategies. The continuation gain is incremental to that baseline, rather than gross monopoly rents.

H Upstream Participation and the Foreclosure Wedge

General comparison: proof of Proposition 3(i)

Maintain the conditions of part (i). For active cohort capacity K , external input sales and the integrated firm's downstream output are

$$r_K(p, w) = p - w - S(w) - K, \quad q_D(p, w) = Q(p) - (p - w).$$

Integrated profit is

$$\Pi_K(p, w) = pQ(p) - (p - w)^2 - wS(w) - wK.$$

First omit the constraint $q_D \geq 0$ and maximize over

$$\tilde{\mathcal{F}}_K = \{(p, w) : 0 \leq w \leq p \leq P, r_K(p, w) \geq 0\}.$$

This set is compact and convex. The point $(p_c, 0)$ has $r_K = p_c - K > 0$ and $q_D = Q(p_c) - p_c > 0$, since $Q(p_c) = 2p_c - p_c Q'(p_c) > 2p_c$. It therefore yields strictly positive profit. The Hessian is

$$\nabla^2 \Pi_K = \begin{pmatrix} -\theta(p) - 2 & 2 \\ 2 & -2 - g(w) \end{pmatrix},$$

with determinant $\theta(p)[2 + g(w)] + 2g(w) > 0$. Strict concavity gives a unique global optimum.

The relaxed optimum has $q_D > 0$. If $q_D \leq 0$ and $r_K = 0$, profit is nonpositive. If $q_D \leq 0$ and $r_K > 0$, then

$$\Pi_{K,p} = Q(p) + pQ'(p) - 2(p - w) < 0,$$

so a small reduction in p is feasible and profitable. Thus the relaxed optimum solves the original problem. An optimum with $w = 0$ would have $p = p_c$, but $\Pi_{K,w}(p_c, 0) = 2p_c - K > 0$, and a small increase in w is feasible. Hence $w > 0$. Feasibility then gives $p > w$, and $q_D > 0$ implies $p < P$. Only $r_K \geq 0$ can bind.

Optimal prices are continuous in K : mixing any feasible point with $(p_c, 0)$ gives strictly positive residual sales and a point feasible for nearby capacities; compactness and uniqueness

then give continuity of the maximizer.

The conditional standalone benchmark. Write $\beta(K) = w_K^U(p(K))$. A standalone supplier maximizes $z[p - z - S(z) - K]$ over attainable input prices: sales $r_K(p, z) \geq 0$ and residual output $q_D(p, z) = Q(p) - (p - z) \geq 0$. The derivative of this objective is

$$J(z; p, K) = p - 2z - S(z) - zS'(z) - K, \quad J_z = -[2 + g(z)] < 0.$$

At the integrated optimum, $J(0; p, K) = p - K > 0$, whereas at the fringe-only clearing price $\bar{w}_K(p)$,

$$J(\bar{w}_K(p); p, K) = -\bar{w}_K(p)[1 + S'(\bar{w}_K(p))] < 0.$$

Thus J has a unique root $\beta \in (0, \bar{w}_K(p))$, which satisfies

$$p - 2\beta - S(\beta) - \beta S'(\beta) - K = 0.$$

In the positive-sales regime the integrated input-price condition gives $J(w; p, K) = -p < 0$; at the zero-sales boundary, $J(w; p, K) = -w[1 + S'(w)] < 0$. Hence $0 < \beta < w$ in both regimes. Under the wedge condition $Q(p_m) \geq p_m$, the constraint $q_D \geq 0$ never binds. The integrated optimum has $p < p_m$ in both regimes, as shown below, and $Q(p) - p$ decreases in p , so

$$q_D(p, z) = Q(p) - p + z > Q(p_m) - p_m \geq 0 \quad (z \geq 0).$$

Thus β is the standalone optimum. Without this condition the benchmark can bind at $z = p - Q(p)$, where the wedge equals $q_D(p, w)$. With linear demand and positive external sales, the integrated first-order condition gives $q_D = Q(0)/2$, so a binding benchmark leaves the wedge constant in K . For example, $Q(p) = 1 - p/10$, $S(w) = w/10 + w^2/100$, and $B = 1/100$ satisfy the other conditions of part (i), including (11), but give $\Delta(0) = \Delta(B) = 1/2$.

Positive external sales. The integrated first-order conditions are

$$R'(p) = 2(p - w), \quad 2(p - w) = S(w) + wS'(w) + K.$$

In particular, $R'(p) > 0$, so $p < p_m$. Define

$$\rho = \frac{2}{\theta(p) + 2}, \quad \Psi = g(w) + 2(1 - \rho) > 0.$$

Differentiating the integrated and standalone conditions gives

$$\frac{dw}{dK} = -\frac{1}{\Psi} < 0, \quad \frac{dp}{dK} = -\frac{\rho}{\Psi}, \quad \frac{d\beta}{dK} = \frac{dp/dK - 1}{2 + g(\beta)}.$$

Consequently,

$$\frac{d\Delta}{dK} = \frac{g(w) - g(\beta) - \rho}{[2 + g(\beta)]\Psi} < 0$$

under (11), because

$$g(w) - g(\beta) \leq g_{\max} - g_{\min} < \frac{2}{\theta_{\max} + 2} \leq \rho.$$

The input-price comparison does not require the curvature bound.

Zero external sales. At $r_K = 0$, write

$$p = L(w, K) = w + S(w) + K, \quad h = S'(w), \quad M = 1 + h.$$

Restricted profit is $V(w, K) = R(L) - L[S(w) + K]$. Its first-order condition is

$$V_w = [R'(p) - S(w) - K]M - ph = 0.$$

It implies $R'(p) = S(w) + K + ph/M > 0$, so again $p < p_m$. At a stationary point,

$$-V_{ww} = \Phi = \theta(p)M^2 + 2hM + \frac{pS''(w)}{M} > 0, \quad -V_{wK} = \Gamma = \theta(p)M + 1 + 2h > 0.$$

Therefore

$$\frac{dw}{dK} = -\frac{\Gamma}{\Phi} < 0.$$

Market clearing gives $dp/dK - M dw/dK = 1$. Differentiating the standalone condition then yields

$$\frac{d\Delta}{dK} = \left(1 - \frac{1+h}{2+g(\beta)}\right) \frac{dw}{dK}.$$

Let $\omega = g_{\max} - g_{\min}$. Condition (11) implies $\omega < 1$, and

$$1 + g(\beta) - h \geq 1 + g(w) - \omega - h = 1 + h + wS''(w) - \omega > 0.$$

Thus $d\Delta/dK < 0$. As in the positive-sales regime, $dw/dK < 0$ does not require the curvature bound. Since $0 < \beta < w < p < p_m$ in both regimes, all curvature bounds need only cover $[0, p_m]$.

Comparisons across regimes. There is at most one regime switch as K increases. To see this, solve the positive-sales first-order conditions without imposing $r_K \geq 0$. For $w \in [0, p_m]$, the first condition determines a unique p , ranging from p_c to p_m , with $dp/dw = \rho < 1$. Along this response, the second condition's left-hand side minus its right-hand side is strictly decreasing in w , with derivative $-\Psi$. It is positive at $w = 0$ and negative at $w = p_m$. Thus there is a unique solution for every $K \in [0, B]$, with $0 < w < p < p_m$ and $q_D > 0$.

Along that solution,

$$r_K = \frac{wS'(w) - S(w) - K}{2}, \quad \frac{dr_K}{dK} = \frac{wS''(w) dw/dK - 1}{2} < 0.$$

At $K = 0$, convexity and $S(0) = 0$ imply $r_K \geq 0$. Whenever this unconstrained solution has $r_K \geq 0$, it is the unique global optimum. Once it becomes infeasible, the constrained optimum must have $r_K = 0$. The optimal path is therefore smooth within each regime and continuous at their possible junction. The derivative signs establish $w(K_1) > w(K_0)$ whenever $K_1 < K_0$, and, under (11), $\Delta(K_1) > \Delta(K_0)$. This proves part (i).

Participation and consumer prices. Suppose the cohort has common operating cost κ and is fully active in a pre-merger equilibrium, so $\kappa \leq w^*(B)$. Strict monotonicity of $w(K)$

gives the unique aggregate post-merger participation outcome:

$$K^I = \begin{cases} B, & \kappa \leq w(B), \\ \text{the unique } K \in (0, B) \text{ with } w(K) = \kappa, & w(B) < \kappa < w(0), \\ 0, & \kappa \geq w(0). \end{cases}$$

Hence $w(B) < \kappa \leq w^*(B)$ implies merger-induced capacity loss. Complete exit requires $\kappa \geq w(0)$; strict inequalities give robust participation incentives.

Capacity loss raises the consumer price in the positive-sales regime, since $dp/dK = -\rho/\Psi < 0$. At the zero-sales boundary,

$$\frac{dp}{dK} = 1 - \frac{M\Gamma}{\Phi} = \frac{pS''(w) - [1 + S'(w)]^2}{[1 + S'(w)]\Phi}.$$

Thus the consumer-price effect depends on supply curvature there. It is strictly negative as a derivative with respect to K when upstream supply is linear. The general result retains linear downstream fringe supply and a parallel capacity shift; it does not establish the same comparisons for arbitrary changes in either supply schedule.

Equilibrium construction: proof of Proposition 3(ii)

Normalize costs as in part (i). Let $Q(p) = a - p$. The upstream base is an atomless unit-capacity population with marginal-cost density $\mu > 0$ and zero operating cost. A second atomless cohort has installed capacity B , zero marginal cost, and avoidable operating cost κ per unit. The downstream fringe has unit marginal-cost density, unit capacity, and zero operating cost. Choose cost supports longer than a , so

$$S_K(w) = \mu w + K, \quad S_D(p, w) = p - w, \quad q_D = a - 2p + w.$$

The base populations remain available and can idle without recurring costs. The cohort's participation is determined before prices as in Section 2; each atomless firm takes aggregate capacity and continuation prices as given. Installation costs are sunk, and reactivation is

available on the same operating-cost terms.

Choose parameters satisfying

$$0 < B < \frac{a\mu}{2(1+2\mu)}, \quad \frac{a}{2(1+2\mu)} < \kappa < \frac{3a-4B}{6+8\mu}. \quad (\text{A.12})$$

The capacity bound makes the interval for κ nonempty and implies $B < a/4 = p_c$. Here $p_m = a/2$, $\theta(p) = 2$, and $g(w) = 2\mu$, so all conditions of part (i), including (11), hold. The benchmark condition holds with equality, $Q(p_m) = a/2 = p_m$.

Pre-merger equilibrium. For fixed K , D maximizes $(p-w)(a-2p+w)$ and chooses $p^*(w) = a/4 + 3w/4$. U's residual input demand is $3a/4 - (3/4 + \mu)w - K$. Its profit is strictly concave in w , giving

$$w^*(K) = \frac{3a-4K}{6+8\mu}, \quad p^*(K) = \frac{a}{4} + \frac{3}{4}w^*(K). \quad (\text{A.13})$$

Both dominant firms have positive residual quantities at these prices. Since $w^*(B) > \kappa$ and $w^*(K)$ decreases in K , every cohort member strictly prefers to operate at every candidate aggregate capacity. Full participation is the unique aggregate pre-merger outcome.

Post-merger continuation prices. The unique global pricing optimum for each $K \in [0, B]$ is

$$\begin{aligned} w(K) &= \frac{a(1+\mu) - (3+4\mu)K}{2(1+\mu)(1+2\mu)}, \\ p(K) &= \frac{a(1+\mu) - K}{2(1+2\mu)}, \\ r_K &= 0, \quad \Delta(K) = \frac{w(K)}{2}. \end{aligned} \quad (\text{A.14})$$

To verify this, substitute $p = (1+\mu)w + K$ into integrated profit and maximize along the zero-sales boundary. The displayed prices have $w > 0$ and $q_D > 0$ for $K \leq B < a/4$. The multiplier on $r_K \geq 0$ is $\lambda = K/(1+\mu) \geq 0$, so the first-order and complementary-slackness conditions hold. Strict concavity makes them sufficient for the unique global optimum.

At fixed p and K , the conditional standalone optimum is $w_K^U(p) = (p-K)/[2(1+\mu)]$. Since market clearing gives $p-K = (1+\mu)w(K)$, this benchmark equals $w(K)/2$, establishing the wedge formula. It has positive residual sales and leaves I positive residual output,

$$q_D(p(K), w(K)/2) > 0.$$

Participation, price comparisons, and robustness. The continuation input price decreases strictly in K . Under (A.12),

$$w(B) < w(0) = \frac{a}{2(1+2\mu)} < \kappa < w^*(B). \quad (\text{A.15})$$

No positive active capacity can cover the cohort's operating cost after integration. At $K = 0$, an individual atomless firm's reactivation is also unprofitable. Complete exit is therefore the unique aggregate post-merger participation outcome.

The formulas show $w(0) > w(B)$, $\Delta(0) > \Delta(B)$, and $p(0) > p(B)$. The input d buys, $p(K) - w(K) = \mu w(K) + K$, falls with exit:

$$[p(0) - w(0)] - [p(B) - w(B)] = -\frac{(2+3\mu)B}{2(1+\mu)(1+2\mu)} < 0.$$

Both post-merger consumer prices are below the pre-merger price because

$$p^*(B) - p(0) = \frac{a(3+10\mu) - 12B(1+2\mu)}{8(3+4\mu)(1+2\mu)} > 0, \quad (\text{A.16})$$

where $B < a/4$ makes the numerator exceed $4a\mu > 0$.

For example, $a = 12$, $\mu = 1$, $B = 1$, and $\kappa = 2.1$ give

$$(w^*(B), p^*(B)) = \left(\frac{16}{7}, \frac{33}{7}\right),$$

$$(w(B), p(B), \Delta(B)) = \left(\frac{17}{12}, \frac{23}{6}, \frac{17}{24}\right), \quad (w(0), p(0), \Delta(0)) = (2, 4, 1).$$

The parameter inequalities and price comparisons are strict and depend continuously on (a, μ, B, κ) . They therefore persist under sufficiently small changes within this family.

Small curvature perturbations. The construction also extends to

$$Q(p) = a - p - \eta p^2, \quad S(w) = \mu w + \varepsilon w^2,$$

for sufficiently small $\eta \geq 0$ and $\varepsilon > 0$. Here p_m solves $a - 2p_m - 3\eta p_m^2 = 0$, and condition (11) becomes

$$6\varepsilon p_m < \frac{1}{2 + 3\eta p_m}.$$

This inequality and $B < p_c$ hold for sufficiently small perturbations. The benchmark condition holds because $Q(p_m) = p_m + 2\eta p_m^2 \geq p_m$. The pre-merger pricing objectives remain strictly concave near the linear benchmark, and the post-merger problem remains strictly concave under part (i). Their optimal prices are continuous in these parameters. Thus the strict participation inequalities $w(0) < \kappa < w^*(B)$ and all endpoint price, wedge, and purchase comparisons persist. Pre-merger $w^*(K)$ remains decreasing in K , since capacity enters the strictly concave upstream objective as $-wK$. Together with part (i), these inequalities imply full pre-merger participation and complete post-merger exit.

At retained capacity, the benchmark multiplier $B/(1 + \mu) > 0$ preserves the zero-sales optimum under sufficiently small perturbations. At $K = 0$, the unconstrained integrated first-order conditions imply

$$r_0 = \frac{wS'(w) - S(w)}{2} = \frac{\varepsilon w^2}{2} > 0.$$

That solution is therefore feasible and is the unique global optimum. Exit now induces positive external sales as well as a higher input price and foreclosure wedge. The input d buys falls, so the sales by I replace only part of the lost cohort supply. Taking $\eta > 0$ and $\varepsilon > 0$ sufficiently small gives an open set of parameters with nonlinear demand and upstream supply satisfying part (i). This proves the robustness claim and completes the proof of part (ii).

I Consumer Prices with Atomless Participation

Normalize the integrated firm's upstream and downstream marginal costs to zero. Let demand be $Q(p) = a - bp$, with $a > 0$ and $b > 0$. Independent upstream supply contains a differentiable zero-operating-cost base $S_0(w)$ with $S_0(0) = 0$ and $S'_0(w) > 0$ at positive relevant prices and an atomless zero-marginal-cost cohort with installed capacity B and common avoidable operating cost $k > 0$. Let K_j be that cohort's active capacity in ownership regime j . The construction

in Appendix H uses the same upstream side with $S_0(w) = \mu w$, $b = 1$, and $k = \kappa$, and sets downstream operating costs to zero.

Installed downstream firms have unit capacity, heterogeneous constant marginal costs with measure dc on $[0, c_{\max}]$, where $c_{\max} > a/b$, and a common strictly positive avoidable operating cost f . The measure places one unit of installed capacity per unit of marginal cost; it is not a probability density. Every price with positive demand satisfies $p \leq a/b$, so the support covers every relevant margin. Both fringes choose participation before prices. Pre-merger, U then chooses w and D chooses p ; post-merger, I jointly chooses p and external input supply. An atomless firm cannot change continuation prices by an individual participation deviation. An inactive installed firm may operate on the same cost terms; no additional reactivation penalty is imposed.

At a participation equilibrium, downstream active capacity is

$$T_j = \max\{p_j - w_j - f, 0\}.$$

Every active downstream firm produces its full capacity. The highest active marginal cost is T_j , while its dispatch margin is $f > 0$. Consequently, the supply of the fixed active set is locally constant during pricing, even though aggregate participation is endogenous. For the uniform-cost specification, its global dispatch function is

$$s_T(p, w) = \min\{\max\{p - w, 0\}, T\}.$$

Using $s = p - w - f$ directly as the pricing-stage supply function would incorrectly let participation adjust after prices. Global optimization must also compare prices that cross the dispatch kink.

For an interior pre-merger equilibrium with positive dominant-firm quantities, D maximizes $(p - w)(a - bp - T_-)$, giving

$$p_- = \frac{a - T_- + bw_-}{2b}, \quad \frac{dp_-}{dw} = \frac{1}{2}.$$

Define $G(w) = S_0(w) + wS'_0(w)$. U's first-order condition is

$$G(w_-) + K_- = a - bp_- - \frac{bw_-}{2}.$$

At a post-merger equilibrium with positive downstream active capacity, I locally maximizes

$$p(a - bp - T_+) + w(T_+ - S_0(w) - K_+).$$

The equilibrium has positive external sales when $w_+ > 0$ and $S'_0(w_+) > 0$. At a zero-sales corner, lowering w slightly leaves downstream output unchanged locally and yields positive input revenue, contradicting optimality. The first-order conditions are therefore

$$p_+ = \frac{a - T_+}{2b}, \quad G(w_+) + K_+ = T_+ = a - 2bp_+.$$

These equations are necessary conditions for global equilibria, not sufficient conditions for existence. A candidate must maximize against all dispatch branches and satisfy both fringes' participation conditions.

Proof of Proposition 5. The price identity follows by subtracting the two downstream pricing equations. Suppose, for contradiction, that $p_+ > p_-$. The identity implies $T_+ < T_-$. Because downstream participation is monotone in the common margin $p - w$, the decline in active capacity implies

$$w_+ - w_- > p_+ - p_- > 0.$$

Upstream cohort participation is monotone in w : its active capacity is zero below k , B above k , and possibly intermediate at k . Hence $w_+ > w_-$ implies $K_+ \geq K_-$. If G is nondecreasing,

$$G(w_+) + K_+ \geq G(w_-) + K_-.$$

But the pricing conditions give

$$G(w_+) + K_+ = a - 2bp_+ < a - 2bp_- < a - bp_- - \frac{bw_-}{2} = G(w_-) + K_-.$$

The second strict inequality follows from $p_- > w_-$, which is implied by a positive pre-merger downstream markup and nonnegative costs. This is a contradiction. If post-merger downstream capacity is zero, the competitive input-clearing price is zero; higher consumer prices would then make some previously active atomless downstream firms profitable again, also contradicting participation. $\square$

Harm can arise when a participation decision changes prices, as in Proposition 6.

The result extends to heterogeneous positive downstream operating costs bounded away from zero, provided active supply remains locally constant and both regimes use identical firm-specific costs and participation opportunities. For an arbitrary base upstream supply, the proof requires nondecreasing G , rather than quadratic supply specifically.

There is also a broader participation restriction. In a single anticipated-pricing period with unchanged operating costs, exit of any previously active atomless upstream firm requires $w_+ \leq w_-$. The inequality is strict when the firm's participation comparisons are strict. If the consumer price rose at the same time, each previously active downstream firm's margin would increase, preventing downstream exit. Therefore upstream exit, downstream exit, and consumer harm cannot all be supported by this one-period atomless participation logic. A temporary price reduction followed by irreversible exit and later higher prices is a different game.

Example. Let $a = 100$, $b = 1$, $S_0(w) = w^2$, $B = 1$, $k = 1$, $f = 28.5$, and $c_{\max} > 100$. Before the merger, $w_- = (\sqrt{24313} - 7)/36 \approx 4.137$, $p_- = 1153/27 + \sqrt{24313}/54 \approx 45.591$, and $T_- \approx 12.954$. After the merger, $w_+ = 2$, $p_+ = 43.5$, and $T_+ = 13$, with external sales $T_+ - S_0(w_+) - B = 8$. The cohort is active in both regimes because $w_j > k$, and downstream participation satisfies $T_j = p_j - w_j - f$. D's full-dispatch price is a global optimum when partial dispatch yields no higher profit. Evaluating at participation-consistent prices, where $p - w = T + f$, this condition reduces to

$$f \geq \frac{T_-}{2} \left[\sqrt{1 + 1/b} - 1 \right] \approx 2.68.$$

Before integration, feasible input prices satisfy $w \leq \sqrt{99}$. On that interval D's full-dispatch price is a global optimum, and U's resulting objective is strictly concave. After integration, partial dispatch with nonnegative external sales implies $p \leq 13 + \sqrt{12}$. Profit on that branch is at most $p(100 - p) \leq 1375.35$, below the full-dispatch profit of 1908.25. Thus U's input price and I's prices are global optima. Consumer prices fall, $p_+ < p_-$, as Proposition 5 implies. In this example the downstream margin rises from about 41.45 to 41.5, and active downstream capacity increases. Proposition 5 concerns consumer prices; margin compression and exit require separate conditions.

J Indivisible Downstream Participation

Replace the atomless downstream participation margin by a single installed block of capacity N with zero marginal cost. The block supplies competitively at the production stage but makes one strategic decision to incur operating cost Nf or deactivate all capacity. Returning restores the whole block and changes continuation prices. The block is therefore not an atomless fringe firm; this is the additional assumption that permits the result.

Let the base upstream supply be w^2 , and keep the atomless operating-cost cohort of capacity B . Both participation decisions occur before pricing. The downstream block and every upstream cohort member anticipate continuation prices after their own participation deviations. Reactivation requires the same operating cost, without an added sunk-entry barrier.

Sufficient conditions for Proposition 6. Let $d = a - N$ and assume

$$0 < B < N < a, \quad \frac{d}{2} > \sqrt{\frac{N}{3}}.$$

When the downstream block is active, define

$$u(K) = \frac{\sqrt{1 + 6(a + N - 2K)} - 1}{6}, \quad v(K) = \sqrt{\frac{N - K}{3}}.$$

Suppose

$$u(0) < d, \quad 0 < k < u(B), \quad N > u(B),$$

and

$$\frac{d}{2} - v(B) < f < \frac{d}{2} - \frac{u(0)}{2}.$$

Then the unique aggregate participation outcome before the merger has the downstream block active and $K = B$. After the merger the downstream block and the upstream operating-cost cohort are inactive. The consumer prices satisfy

$$p_- = \frac{d + u(B)}{2} < \frac{a}{2} = p_+.$$

All inequalities are strict, so the result persists in an open neighborhood of the parameters.

Proof of the pricing subgames. With the block active and input price $w < d$, D's unique optimum is $p = (d + w)/2$. U maximizes

$$w \left(\frac{a + N - w}{2} - w^2 - K \right),$$

whose derivative is $(a + N)/2 - w - 3w^2 - K$ and whose second derivative is $-1 - 6w < 0$. Its optimum is $u(K)$. The condition $u(0) < d$ keeps these optima in the full-block dispatch region. Prices $w \geq d$ cannot support positive upstream dominant-firm sales: $d^2 > N$ follows from $d/2 > \sqrt{N/3}$, so base fringe supply already exceeds downstream demand at such input prices.

With the block active after the merger, full dispatch gives the integrated objective

$$p(d - p) + w(N - K - w^2).$$

Its unique unconstrained optimum is $p = d/2$ and $w = v(K)$. The stated conditions ensure $p > w$, positive own downstream quantity, and positive external sales for every $K \in [0, B]$.

A partially dispatched block requires $p = w$, because its marginal production cost is zero. At that boundary integrated profit is

$$p(a - p - p^2 - K),$$

independently of the block's dispatched quantity. Any feasible partial-dispatch outcome also has $p^2 + K \leq N$, hence $p \leq \sqrt{N} < d$. Its profit can therefore be reproduced by full dispatch at the same $p = w$, with nonnegative own output and external sales. It cannot exceed the strictly concave full-dispatch optimum. This proves global optimality, including the dispatch boundary. $\square$

Proof of participation. Pre-merger, $u(K) \geq u(B) > k$ for every $K \in [0, B]$ when the block is active, so the upstream cohort strictly prefers full participation. The downstream block's margin is $d/2 - u(K)/2 \geq d/2 - u(0)/2 > f$. It strictly prefers to operate. If it is inactive, entering restores the same profitable active-block continuation for any fixed K ; hence an inactive-block pre-merger outcome cannot be a participation equilibrium.

Post-merger, an active block earns the per-unit margin $d/2 - v(K) \leq d/2 - v(B) < f$ for every $K \in [0, B]$. It strictly prefers deactivation. With no downstream block, there is no external input demand. The operating-cost upstream cohort cannot cover positive k and is inactive. A unilateral downstream reactivation at $K = 0$ restores the active-block continuation and remains unprofitable. Thus reactivation does not defeat the exit equilibrium. I then chooses the monopoly price $a/2$. The inequality $N > u(B)$ establishes consumer harm. $\square$

Numerical illustration of Proposition 6

Let $a = 100$, $N = 20$, $B = 1$, $k = 1$, and $f = 37.7$. Thus $d = 80$. All sufficient conditions above hold. In particular, the operating cost interval is

$$40 - \sqrt{\frac{19}{3}} \simeq 37.4834 < f < 40 - \frac{\sqrt{721} - 1}{12} \simeq 37.8457.$$

The pre-merger equilibrium is

$$w_- = \frac{\sqrt{709} - 1}{6} \simeq 4.2712, \quad p_- = 40 + \frac{w_-}{2} \simeq 42.1356.$$

The downstream block earns $p_- - w_- \simeq 37.8644$ per unit, above its operating cost of 37.7. The upstream cohort covers its operating cost of 1. Total pre-merger output is $Q_- = 100 - p_- \simeq 57.8644$. D produces $Q_- - N \simeq 37.8644$, about 65.4 percent of final output. U

produces $Q_- - (w_-^2 + B) \simeq 38.62$, about 66.7 percent of total input output. The downstream block supplies about 34.6 percent of final output.

After integration, if the block stayed active, the consumer price would be 40 and the input price would range from $\sqrt{\frac{20}{3}} \simeq 2.5820$ at $K = 0$ to $\sqrt{\frac{19}{3}} \simeq 2.5166$ at $K = 1$. The block's margin is at most $40 - \sqrt{\frac{19}{3}} \simeq 37.4834 < 37.7$. Exit is optimal for every upstream active capacity. Returning after exit yields $40 - \sqrt{\frac{20}{3}} \simeq 37.4180 < 37.7$ per unit. No additional re-entry cost is needed.

The post-merger consumer price is 50. Under demand $100 - p$, consumer surplus falls from about 1674.15 to 1250. This is consumer harm, not a calculation of the total welfare effect. Total surplus rises in the example, from about 3305 to 3750. Exit saves the operating costs $Nf = 754$ and $Bk = 1$ and the base suppliers' production costs, and these savings exceed the fall in gross consumer benefit.

The interval for f has width about 0.36, or 0.96 percent of f . The width rises with the block's share of output. With $N = 30$ and holding a, B, k fixed and choosing f at the midpoint of the corresponding interval, the width is about 2.7 percent of f , and the block supplies about 48 percent of final output. With $N = 15$, the width is about 0.13 percent of f , and the block supplies about 27 percent. Consumer harm thus requires a large block or an operating cost in a narrow range.

K Two-Part Tariff

This appendix solves the contractual alternative of Section 4 in the linear family of Appendix H. There is no merger. Let $Q(p) = a - p$, $S_K(w) = \mu w + K$, $S_D(p, w) = p - w$, and $c_U = c_D = 0$. U offers D a two-part tariff with a unit price equal to U's marginal cost, zero, and a fixed fee F . The fee is set before U chooses the price w to the downstream fringe, so U's objective excludes D's profit. Inputs supplied to D under the tariff cannot be resold. The cohort decides whether to operate after the contract is signed and before prices are set.

Pricing. D buys all its input from U at zero, so its purchases are withdrawn from independent suppliers. Facing input price w for d, D maximizes $p(a - 2p + w)$ and chooses

$p^T(w) = (a + w)/4$. U's external sales are $r = p^T(w) - w - \mu w - K = a/4 - (3/4 + \mu)w - K$, and U maximizes $wr + F$. The objective is strictly concave in w , so

$$w^T(K) = \frac{a - 4K}{6 + 8\mu}, \quad p^T(K) = \frac{a + w^T(K)}{4}. \quad (\text{A.17})$$

External sales are $r = (a - 4K)/8 > 0$ and D's own output is $[a(7 + 8\mu) - 4K]/[4(3 + 4\mu)] > 0$ for $K \leq B < a/4$.

Participation. The cohort operates only if $w^T(K) \geq \kappa$. The price $w^T(K)$ decreases in K , and $w^T(0) = a/(6 + 8\mu) < a/[2(1 + 2\mu)] < \kappa$ under (A.12). No positive active capacity covers the operating cost, and reactivation at $K = 0$ is unprofitable. Complete exit is the unique aggregate participation outcome under the tariff. Exit raises the input price d pays and the consumer price:

$$w^T(0) - w^T(B) = \frac{2B}{3 + 4\mu} > 0, \quad p^T(0) - p^T(B) = \frac{B}{2(3 + 4\mu)} > 0.$$

Acceptable fees. If D rejects the tariff, the pre-merger equilibrium of Appendix H applies, with the cohort active. Joint profit of U and D under the tariff, with complete exit, minus joint pre-merger profit equals

$$\frac{(a - B) [(14 + 16\mu)B - a]}{4(3 + 4\mu)^2}.$$

A fee acceptable to both parties exists when $B > a/(14 + 16\mu)$. This interval intersects (A.12) when $8\mu^2 + 5\mu > 1$.

Comparison with integration. With complete exit in both arrangements,

$$w(0) - w^T(0) = \frac{a(1 + \mu)}{(1 + 2\mu)(3 + 4\mu)} > 0, \quad p(0) - p^T(0) = \frac{a(5 + 6\mu)}{8(1 + 2\mu)(3 + 4\mu)} > 0.$$

The tariff produces the customer withdrawal and the upstream exit. If this tariff is the no-merger counterfactual, integration raises the consumer price. Integration adds the input-pricing incentive of Proposition 2, which this fixed-fee tariff does not create, and exit amplifies

it, since $\Delta(0) > \Delta(B)$. The tariff price still exceeds the conditional standalone benchmark, because U anticipates D's price response:

$$w^T(K) - w_K^U(p^T(K)) = \frac{w^T(K)}{8(1 + \mu)} > 0.$$

For the example in Appendix H, with $a = 12$, $\mu = 1$, $B = 1$, and $\kappa = 2.1$, the tariff gives $(w^T, p^T) = (4/7, 22/7)$ with the cohort active and $(6/7, 45/14)$ after exit. The cohort exits. Joint profit rises from about 20.94 to about 21.95. Both parties strictly prefer the tariff for $55/7 < F < 869/98$.