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Export Survival with Uncertainty and Experimentation*

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Abstract

Two central facts characterize the dynamics of firm exports. One is the known fact that export survival rates are strikingly low one year after entering a foreign market. The other is the novel fact that re-entrants in export markets are more likely to survive than first-time entrants. Traditional models of exporter dynamics cannot explain these two facts. In this paper, we develop a tractable model of exporter dynamics that can explain them by introducing uncertainty and experimentation. The model delivers analytical predictions on survival probabilities upon entry in a foreign market. We test the main mechanism of the model by exploiting variation in the degree of uncertainty across products and markets. The results support the relevance of uncertainty and experimentation as a central feature that characterize exporter dynamics.

JEL codes: F10, F12, F14

Keywords: Exporter dynamics, uncertainty, experimentation, foreign demand, geometric brownian motion.

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1 Introduction

Both developed and developing countries display a variety of policies and dedicated agencies aimed at helping firms establish a sustained presence in foreign markets. Underlying these policies is the view that increasing the set of domestic firms capable of achieving sustained exports is key to foster aggregate export growth and, potentially, economic development. Recent evidence by Eaton et al. (2008), Freund and Pierola (2010), and Lederman, Rodríguez-Clare and Xu (2011) for Colombia, Peru, and Costa Rica, respectively, supports this view by showing that a considerable fraction of aggregate exports in a given year is accounted for by firms that were not exporting a few years earlier. However, while new entries in foreign markets can potentially have a relevant long run impact, this potential is usually unrealized as most export incursions do not become established export businesses. In fact, about two thirds of firms that make an incursion in a specific foreign market do not continue to export to that market in the subsequent year (Eaton et al. (2014); this paper). The reasons for such short spells are not yet well understood. In particular, little is known about what determines export survival upon entry in a foreign market.

Only recently has a growing literature started to uncover empirical regularities about the more general dynamic process of firm exports, of which export survival is one of its salient manifestations. The regularities tend to mimic analogous patterns long identified in the (domestic) firm dynamics literature. For example, new exporters, like new firms, are smaller, tend to grow faster conditional on survival, and are less likely to survive (Eaton et al. (2008), Arkolakis (2016)), while the size distribution of export sales, as the distribution of firm sales, resembles a Pareto distribution (Eaton, Kortum and Kramarz (2011)). Notwithstanding the similarities, two facts distinctly characterize the dynamics of firm exports. One of these facts, emphasized in recent work, is that the survival rate of export incursions is strikingly low in the first year after entry – particularly lower than the survival rates of domestic firms – and drops further only modestly in subsequent years (e.g., Eaton et al. (2008), Ruhl and Willis (2017)). The second fact, which is novel, is that re-entrants in export markets are more likely to survive than first time entrants to those markets. These two facts describe central features of exporter survival. As such, they are also central to characterize, more generally, the dynamics of firm exports. The distinguishing nature of these facts suggests that standard models of firm dynamics might not be appropriate to explain exporter dynamics, which might be characterized by distinct ingredients. We show that these facts also set tight constraints on the class of models that can explain them and thus are critical in guiding the construction of a relevant theory of exporter dynamics.

We build a theoretical model of exporter dynamics guided by these two facts. The estimated model can explain these facts as well as other relevant facts that have been the focus of previous work. The main feature of the model is the existence of uncertainty about foreign market profitability that can only be resolved by actively exporting (Segura-Cayuela and Vilarrubia (2008), Freund and Pierola (2010), Albornoz et al. (2012), Nguyen (2012), Eaton et al. (2014)). As a result, firms experiment under losses to resolve this uncertainty. The model is flexible yet it is parsimonious, and exhibits a number of tractable features that we exploit to obtain analytical results on survival probabilities. Those results help us estimate the model and derive predictions that we contrast with the data. In order to establish the key role that uncertainty and experimentation play in the dynamics of exports, we follow a two pronged approach. First, we show that other models often used in the literature which do not include uncertainty and experimentation are unable to generate both facts as a joint prediction. Second, we test the central mechanism of the model by exploiting hypothesized variation in the degree of uncertainty by product and distance to the destination. The implied predictions of uncertainty variation on survival probabilities are confirmed by the data in most cases.

We model a simple uncertainty and experimentation mechanism embedded in a theoretical framework with otherwise standard elements. A firm's operating profit in an export market is initially determined by an idiosyncratic time-varying component that follows a Geometric Brownian Motion (GBM) and a constant and idiosyncratic market-specific component. Operation in a foreign market requires that firms pay a continuous, constant, and idiosyncratic fixed cost while firms are allowed to enter and exit the market freely, particularly since there are no sunk costs.

The uncertainty and experimentation mechanism operates as follows. Before entering a foreign market, firms are uncertain about their potential profitability. This uncertainty can only be resolved by actively exporting. Artopoulos, Friel and Hallak (2013) argue that adapting products and marketing practices to match foreign market tastes and ways of doing business is critical for long run export success. Our model postulates that firms are uncertain about the extent to which they will be able to match those foreign tastes and business practices. Hence, they are willing to experiment to find this out by initially exporting at a loss. Specifically, the model includes a multiplicative shock to operating profits with Poisson arrival rate, which increases profits in expected value. The firm knows the parameters of the distribution where the shock comes from and the Poisson arrival rate but is uncertain about the particular realization of both random processes. In other words, it is uncertain about how much and how fast profits will jump. In this environment, the firm enters the foreign market even when operating profits are lower than fixed costs in the expectation of eventually improving performance and justifying the initial investment. Once it has received the shock, however, the firm only stays active if operating profits are higher than fixed costs as there is no further uncertainty to resolve. In the empirical section, we parametrize this shock with a Pareto distribution with scale parameter 1.

A key analytical result is that the probability of survival upon entry at any given horizon is independent of the firm-specific profitability shifters and fixed costs. Firms time their entry and exit decisions as a function of these heterogeneous parameters precisely in a way that cancels out their potential impact on survival probabilities once we condition on entry. Hence, those probabilities are identical across firms and can be obtained without information on the firm-specific parameters or the probability distribution that generates them as they only depend on common parameters. This is one of the main advantages of focusing on survival upon entry. Since observed survival rates average the realization of a common probability across firms, we can use them as empirical counterparts of those theoretical predictions to estimate the common parameters of the model. Although these are only a small subset of all the parameters, they alone determine some of the most important features of the dynamics of firm exports. These features include not only those related to export survival but also those related to export growth.

The fact that the survival rate of export incursions is strikingly low cannot be easily accounted for by standard models. This notion is illustrated by a special case of our model where we shut down the uncertainty and experimentation mechanism. In order to fit the survival rate in the first year, this special case needs to set such a negative trend on the GBM process (relative to its volatility) that it severely underpredicts survival rates at later horizons. In contrast, uncertainty and experimentation arise as a natural explanation for the observed survival patterns. The low initial survival rate of export incursions is an expected outcome of the experimentation process. In turn, deaths can occur disproportionately during the first year as long as the resolution of uncertainty occurs sufficiently fast. This uncertainty and experimentation mechanism also explains why re-entrants exhibit higher survival rates than entrants. Since a large fraction of re-entrants have already resolved their uncertainty during their initial export spell, their re-entry decision is not driven by an intention to experiment. Thus, they are more conservative to enter and as a result survive more.

We estimate the model using firm-level customs data of exports from Peru for the period 1993-2009. We calculate survival rates one to five years after entry both for entrants and for re-entrants. These ten moments are used to estimate the parameters of the model with the Simulated Method of Moments (SMM). Before performing the estimation we develop a correction in the theoretical survival probabilities that accounts for the mismatch between a model set in continuous time and data recorded over discrete time periods. One correction is the "partial-year" effect emphasized by Berthou and Vicard (2015) and Bernard et al. (2017), which deals with the fact that firms may enter the export market at different points along the year. The second correction is the "re-entry" effect, which deals with the possibility that a firm may be out of the export market at the time of computing the instantaneous probability but re-enter it during the relevant discrete (calendar) year. Although the latter effect has been neglected so far both in the firm dynamics and in the exporter dynamics literatures, it is the one with the largest impact on predicted survival probabilities.

The estimated model predicts quite closely the survival rates of export incursions. The predictions are slightly below the survival rates in the data by an average of three percentage points over the first five horizons. The parameter estimates also indicate that uncertainty is resolved notably fast. A firm that continuously exports has a 56.5% probability of receiving the multiplicative shock in less than a month. The shock also has a considerable dispersion (0.54), which justifies the willingness of firms to experiment in foreign markets in the hope of benefiting from a good realization of this random variable. The estimated model also predicts the qualitative fact that survival rates are higher for re-entrants than for entrants. However, it overpredicts the gap. While in the data the average survival rate of re-entrants over the first five horizons is 0.12 percentage points higher than for entrants, the model predicts a 0.19 percentage point difference between the two average rates.

We use hypothesized variation in uncertainty across products and markets to test our uncertainty and experimentation mechanism. Specifically, the model predicts that survival probabilities should be lower the higher is the variance of the shock, which is a measure of the degree of uncertainty that can be resolved by exporting. First, we postulate that the degree of uncertainty should be higher in the case of differentiated products, where firms need to adapt to idiosyncratic tastes and find distributors that help propel sales (Artopoulos, Friel and Hallak (2013)). Consistent with the predictions of the model, survival rates are lower for entrants in differentiated products than for entrants in homogenous products. Second, since a fraction of re-entrants have resolved their uncertainty, this prediction should hold attenuated in the case of re-entrants. We find that the gap between survival rates of re-entrants of differentiated and homogenous products is indeed lower but not as much lower as predicted. Third, we postulate that in the case of differentiated products, the degree of uncertainty over export market profitability should increase with the distance to the destination. Consistent with the model predictions, we find that survival rates in differentiated products are lower the farther away is the destination market. Finally, though this prediction should also be attenuated in the case of re-entrants, we find that survival rates decrease with distance as much for re-entrants as for entrants. Examined together, the evidence of this section is consistent with the main predictions of the uncertainty and experimentation mechanism of our model though not with all of its subtler implications. As a whole, we find it strongly supportive of the notion that uncertainty and experimentation are central components in the dynamics of firm exports.

Our choice of a GBM to model the evolution of the profitability process is made primarily for analytical simplicity. This choice is nevertheless not unjustified since the evolution of export sales has been found to be a highly persistent process (Roberts and Tybout (1997), Das, Roberts and Tybout (2007)). In any event, we also consider a broader class of models and show that, in the absence of uncertainty and experimentation, they are unable to match the two distinguishing facts that we highlight. In particular, as in Arkolakis (2016) we postulate a more general model that embeds as special cases a GBM and a Geometric Ornstein-Uhlenbeck process expanded with a trend in its long-run value. While there exist parameter configurations in this encompassing model that can explain the survival rates of export incursions at various horizons, they are unable to account for the higher survival rates of export re-entrants. First, to fit the decreasing decay in survival rates it is necessary that the process has a negative long-run trend. Second, since re-entrants tend to be older firms, they reach the re-entry point (when they do) farther away from the long run value and thus mean-revert more strongly. As a result, they survive less, not more, than entrants.

This paper is connected to several strands of literature. The oldest and most influential literature on exporter dynamics has focused on the hysteresis implications of exporting sunk costs (Baldwin and Krugman (1989), Dixit (1989), Alessandria and Choi (2007), Impullitti, Irarrazabal and Opromolla (2013)). In this paper, we show that despite their starring role in the literature, sunk costs do not appear to be necessary to deliver predictions on export survival that can fit the data. On the contrary, they only exacerbate the predictive shortcomings of models that do not include uncertainty and experimentation. A more recent literature develops methods to structurally estimate sunk costs (Das, Roberts and Tybout (2007), Morales, Sheu and Zahler (2017)). Our results suggest that such methods may yield sunk-cost estimates that are largely determined by assumptions about the relationship between domestic and foreign profitability. We generate predictions that do not require making assumptions on this relationship.

Except for our inclusion of uncertainty and experimentation, this paper follows closely the work of Arkolakis (2016). In particular, we also model a GBM process and allow for free exit and re-entry in foreign markets. However, we do not include market penetration costs (Arkolakis (2010)), which are critical in that work and in Eaton, Kortum and Kramarz (2011) to generate the observed size distribution of exporters. We show that, in the absence of sunk costs, market penetration costs do not change the survival predictions of standard models and hence their shortcomings in matching the facts that we document. Furthermore, our uncertainty and experimentation mechanism could be an alternative explanation for the observed deviations from Pareto in the lower tail of the exporter distribution since it reduces the size of new, usually small, exporters.

The essence of our uncertainty and experimentation mechanism has already been postulated in various forms in previous studies of exporter dynamics (Segura-Cayuela and Vilarrubia (2008), Freund and Pierola (2010), Albornoz et al. (2012), Nguyen (2012)). We build on this literature by embedding this mechanism in a more general framework that can deliver a wider set of quantitative and qualitative predictions. In this goal, this paper complements Eaton et al. (2014) by sacrificing a relevant dimension in newer foreign transaction databases – relationships with distributors on the importer side – in the sake of parsimony and tractability. Finally, one additional contribution to this literature is that we test for the relevance of uncertainty and experimentation in exporter dynamics and show the limitations of models do not account for these features.

This paper is also related to work specifically oriented to explain exporter survival (Békés and Muraközy (2012); Albornoz, Fanelli and Hallak (2016)). While our model and theoretical results emphasize exporter survival, we hope to make a contribution to a broader literature on exporter dynamics.[1]

The rest of the paper is organized as follows. Section 2 describes the two distinguishing facts about exporter survival that we emphasize in this paper. Section 3 sets up the model and derives predictions on survival probabilities. Section 4 estimates the model, compares its predictions with the data, and discusses why standard models in the literature cannot explain the two facts. Section 5 tests for the uncertainty and experimentation mechanism of the model by looking at its implications across products and markets. Section 6 provides concluding remarks.

2 Two central facts about exporter survival

A vast amount of literature has established a number of facts about patterns of firm dynamics related to their survival (e.g., Mansfield (1962), Evans (1987), Dunne, Roberts and Samuelson (1988, 1989)),

[1] Preliminary results suggest our model is also able to explain most of the relevant facts on the dynamics of firm exports that the literature has highlighted and that motivated work on alternate models. We hope to include these results in the next version of this paper.

growth rates (e.g., Hart and Prais (1956), Mansfield (1962), Evans (1987), Hall (1987), Dunne, Roberts and Samuelson (1989), Davis and Haltiwanger (1992)), and size distribution (Simon and Bonini (1958), Cabral and Mata (2003), Luttmer (2007)). A more incipient strand of literature has recently uncovered analogous patterns in the dynamics of firm exports. For example, smaller and younger exporters, like smaller and younger domestic firms, are less likely to survive and display higher growth rates conditional on survival (Eaton et al. (2008), Berthou and Vicard (2015), Arkolakis (2016)). Also, the upper tail of the size distribution of export sales resembles a Pareto (Eaton, Kortum and Kramarz (2011), Arkolakis (2016)). In spite of the notable similarities, two facts uniquely distinguish exporter dynamics. The first is that the survival profile (i.e. the line connecting survival rates at different horizons) of export entrants is low and flat. The second is that the survival profile of export re-entrants is higher than the survival profile of entrants. This section describes these two facts and discusses how they guide our search for a parsimonious model of exporter dynamics that can explain them.

First, we briefly discuss some definitions and basic data issues. We employ firm-level customs data from Peru for the period 1993-2009 graciously provided to us by the Trade and Integration Unit of the World Bank Research Department.[2] Our dataset covers all export transactions from Peru between 1993 and 2009 by firm, destination country (i.e. export market), and year. We define an export "incursion" as the first entry of a firm in an export market. The "survival rate" S_T is the proportion of incursions that are active in the corresponding export market T years after entry. We follow an incursion up to five years. Hence, the "survival profile" includes the set of survival rates $\{S_T\}_{T=1,..,5}$. Since we do not observe data before 1993, we only consider incursions starting in 1997 to minimize the chances of falsely identifying as incursions export instances with an antecedent before 1993.[3] Also, since we track survival up to five years after entry, we restrict the sample to incursions starting no later than 2004. Our definition of survival does not impose consecutive activity as an exporter up to T. Thus, an incursion that exited at $T = 2$ but is active at $T = 3$ after re-entering the market is considered a survivor in the latter horizon.

If the firm does not maintain a continuous presence in the market during all consecutive years after the incursion, subsequent entries are defined as "re-incursions" or "re-entries". We define an export

[2]The dataset was collected by this unit as part of their efforts to build the Exporter Dynamics Database. Details of its construction are described in the Annex of Cebeci et al. (2012).

[3]For example, incursions in 1997 would be false if the firm exported in the past but not in the last four years. Using the latest years in our database, we find the proportion of incursions that have exported in the past but not in the last four years to be 8.4%. As we consider incursions in later years, false incursions will arise only after a longer period of inactivity. For example, the proportion of false incursions is 3%(1%) when we firms are inactive for 7(10) years. Averaging across incursions in all years, we estimate the proportion of false incursions to be 3.3%.

re-entry as the start of a new spell of exports to a destination by a firm that has exported to that destination in the past but has not done so in the previous year. Re-incursions may also be instances of survival for the original incursion. In the example above, the survival status of the firm at $T = 4$ and $T = 5$ is taken into account both for the final years of the survival profile of entrants and for the first two years of the survival profile of re-entrants.

Fact 1: The exporter survival profile is low and flat

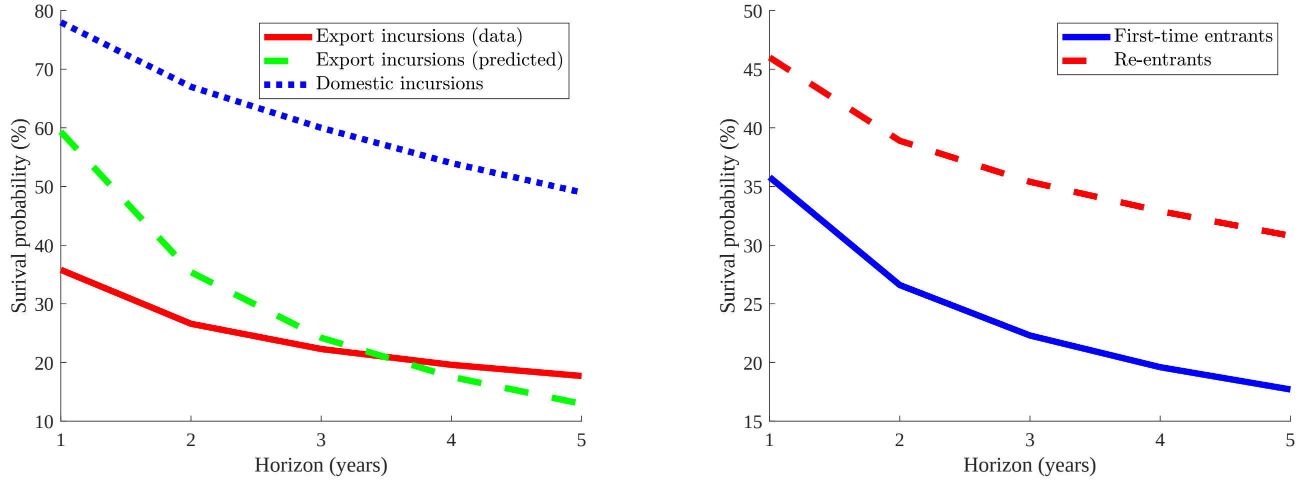
Figure 1a shows the survival profile of export incursions in our dataset (red solid line). A striking feature of this profile is the low survival rates it displays. Only 35.8% of Peruvian export incursions are still active one year after entry. Five years after entry, the survival rate is 17.7%. Low survival rates are not specific to Peru. Using data from the Exporter Dynamic Database, Cebeci et al. (2012) report that the average and median one-year survival rates across 38 countries are both 43%.[4] Another salient feature of the survival profile is the flat slope after $T = 1$. In contrast to the vast fraction of firms that exits just after entering the export market, further increases in the fraction of non-survivors at longer horizons are considerably more gradual. As a reference, Figure 1a displays the domestic survival profile, which is the survival profile of firms as production units (blue dotted line). We denote it "domestic" since all producing firms, except for a negligible fraction of them, sell in the domestic market.[5] Compared to exporter survival rates, domestic survival rates are substantially higher. The first year after entry, 77.9% of U.S. firms in an entry cohort are still in operation. Five years after entry, the survival rate is 49.1%.[6]

The features of the exporter survival profile depicted in Figure 1a are not driven by composition. To control for other covariates, we can obtain the survival profile from a regression framework. First, we regress the survival status of incursions in each of the first five horizons on horizon dummies. This exercise is equivalent to simply calculating the survival rate per horizon as we did in the figure. The results of this regression are displayed in column 1 of Table 7. Then, we add a set of fixed effects by product (2-digit Harmonized System), destination country, and year (i.e. the year corresponding to

[4]Splitting the sample into developed and developing countries, the average survival rate is 43% for each of the two groups. For Peru, they find a survival rate of 44%. Their reported rates are higher because they are calculated by previously merging all destinations into one aggregate export market.

[5]Domestic survival rates are computed using the number of firms by entry cohorts reported in the Business Dynamics Statistics (BDS) constructed by the Bureau of the Census. For comparison with export survival rates, we only consider tradable-firm producers (agriculture, mining, and manufacturing) in entry cohorts 1997-2004. The survival profile is almost unaffected if we include only manufacturing firms or firms in all remaining sectors.

[6]Domestic and exporter survival rates are not strictly comparable. While domestic survival rates capture persistence as an employer, exporter survival rates capture persistence as a seller in a specific market.



(a) Survival profile: Domestic vs. exporter

(b) Survival profile: first entry vs. re-entry

Figure 1: Two key facts of exporter dynamics

the survival status). We can see in column 2 that adding these flexible controls in all three dimensions has a negligible impact on the estimated survival rates.

The fact that exporter survival rates are notoriously low has already been emphasized in the literature.[7] Freund and Pierola (2010), Albornoz et al. (2012), Nguyen (2012), and Eaton et al. (2014) provide a plausible explanation for this fact. If export profitability has an uncertain component that can only be resolved by being actively exporting, firms have incentives to export as an experiment to resolve their uncertainty. Thus, export entry is consistent with low survival rates since firms are betting on a relatively unlikely outcome. This is also the core mechanism operating in our model. As long as firms resolve their uncertainty sufficiently fast, this mechanism can explain both features of the exporter survival profile. It is low at early horizons because firms soon find that exporting is not a profitable activity. It is flat because firms that resolve their uncertainty favorably are less likely to exit afterwards.

As an additional reference, Figure 1a also displays the best prediction of a special case of our model – *the benchmark model* – where this source of uncertainty is removed (green dashed line).[8] As we can see in the figure, the benchmark model is unable to predict the exporter survival profile observed in the data as it predicts too high survival rates early upon entry together with too low survival rates at longer horizons. Despite the specificity of this special case, its inability to fit the exporter survival profile captures a broader implication of standard firm and exporter dynamics models whenever profitability

[7]See, among others, Eaton et al. (2008), Volpe Martincus and Carballo (2009), Nguyen (2012), and Ruhl and Willis (2017).

[8]Section 4.6 discusses the estimation of the benchmark model.

follows a persistent process. These models have difficulty explaining low survival rates at early horizons without also predicting a steep survival profile.

Fact 1 has been key to motivate recent work on uncertainty and experimentation in models of exporter dynamics. Nevertheless, it is the novel fact we present next that, combined with fact 1, makes a substantially stronger case for the relevance of such models.

Fact 2: The survival profile is higher for re-entrants than for (first-time) entrants

It is frequent that firms temporarily cease to export only to re-enter the same market later on. In our dataset, the total instances of re-entry represent 26.4% of total incursions. Figure 1b compares the survival profile of re-entrants with the profile for (first-time) entrants displayed in Figure 1a. Re-entrants have uniformly higher survival rates. Most of the difference already takes place in the first year after entry, when the survival rate is 46.0% for re-entrants versus 35.8% for entrants. Over longer horizons, this gap is preserved with only slight changes. Like fact 1, fact 2 is not driven by composition either. Columns 3 and 4 of Table 7 display analogous results including re-incursions.[9] In column 3, we simply include horizon dummies for re-entrants, which delivers the survival rates depicted in Figure 1b. In column 4, we include a full set of dummies by product, destination, and year. Again, we find that these controls for composition do not substantially affect the survival profiles depicted in the figure.[10]

Fact 2 has no corresponding analog in the firm dynamics literature. As a matter of fact, we are not aware of any study that has computed re-entrant domestic survival rates. A likely reason is that instances of domestic re-entry are much more infrequent than in the case of exports and are typically either dismissed as nuisance or tinkered with assuming they are due to measurement error.[11]

An appealing explanation for the higher survival rates of re-entrants arises naturally from the experimentation mechanism described above. Firms that exit and re-enter have already resolved their uncertainty and hence do not enter to experiment. As their (re-)entry decisions are made with more accurate information about their potential profitability, they tend to survive longer. This is indeed

[9] Standard errors are computed by clustering by firm-destination. This allows for arbitrary correlation between the survival status of incursions and re-incursions of a firm in a given market at any horizon.

[10] We note that since the horizon dummies sum up to a constant, like the different sets of fixed effects, there is a degree of freedom to set the level of the survival profile at any arbitrary level by choosing an appropriate normalization of the fixed effects. To ease readability, we choose normalizations that leave the coefficient on the horizon dummies at similar levels as the observed survival rates. In any event, those normalizations do not affect the decay of the survival profile.

[11] Due to how "entry" is defined in standard firm dynamic databases (Baldwin, Beckstead and Girard (2002)), recorded re-entry instances might be spurious. For example, the BDS reports that re-entry instances represent 7% of incursions. However, since the database only includes firms with at least one employee in its payroll, a large fraction of this percentage probably comes from transitions in and out of employer status (Jarmin and Miranda (2002)).

how our model explains fact 2. Furthermore, in Section 4.6 we show that a broad class of models of exporter dynamics is unable to explain facts 1 and 2 jointly in the absence of uncertainty and experimentation.

3 The model

3.1 Set up

Firms go through two stages in their lifetime as exporters in a given market. At first, they are *inexperienced* and earn flow profits

$$\pi_i(\theta_t) = \left\{ \begin{array}{c} \kappa\theta_t - F \text{ if export at } t \\ 0 \text{ otherwise} \end{array} \right\}$$

(subindex i is for inexperienced) where θ_t is a time-varying index of profitability, κ is a profitability shifter and F is a fixed cost. We allow firms to be heterogeneous in κ and F, as well as in their particular trajectory $\{\theta_t\}$. However, we assume all firms have the same law of motion for θ_t, which for analytical tractability we assume to be a geometric brownian motion (GBM),[12]

$$d \log \theta_t = \mu dt + \sigma dZ_t. \tag{1}$$

In other words, we assume that μ, σ, and the initial level of the process $\bar{\theta}$ are common across firms. We assume that the firm's discount factor satisfies $r > \mu + \frac{1}{2}\sigma^2$ so that expected profits are finite. Furthermore, to guarantee the existence of a stationary distribution, we follow Arkolakis (2016) and assume that the mass of firms that are born each instant grows at rate $g_B > 0$.[13]

Since all firms are born with $\bar{\theta}$, κ is an index of initial profitability in the market. For example, a high value of κ may capture a prior understanding of demand characteristics in the export market that allows the firm to make product adaptations that match their idiosyncratic characteristics (Artopoulos, Friel and Hallak (2013)). This parameter may also capture an advantage in communicating or conducting transactions with foreign agents at lower variable trade costs, e.g. due to family ties. The fixed expenses F represent the costs incurred in activities such as sustaining a distribution network

[12]The profitability parameter θ_t can be microfounded as the combination of random processes for demand and productivity jointly determined by a multivariate GBM in a stationary competition environment with CES preferences. See Luttmer (2007).

[13]We could also assume an exogenous death rate $\delta > 0$. The only difference is that this parameter would directly affect the prediction of the probability of survival of first-time entrants while g_B does not.

and conducting marketing efforts in the foreign market, which are paid on a continual basis while exporting.

For inexperienced firms, exporting yields additional benefits beyond receiving flow profits. Inexperienced firms know that their current profitability level in the export market is only transient and they will eventually become *experienced* if they keep exporting. More specifically, while exporting, inexperienced firms become experienced with intensity λ.[14] An experienced firm earns flow profits

$$\pi_e\left(\theta_t;\psi\right)=\left\{\begin{array}{c}\psi\kappa\theta_t-F\text{ if export at }t\\0\text{ otherwise}\end{array}\right\}$$

where ψ is the new profitability component that separates an experienced firm from an inexperienced firm.[15] The new component ψ intends to capture the fact that by engaging in the exporting activity the firm might acquire fine-grained knowledge about the tastes and needs of consumers or reconfigure its distribution network by finding more suitable partners (Eaton et al. (2014)).

A key feature of our model is that ψ is unknown ex-ante by inexperienced firms. Those firms only know the distribution of ψ, which we assume is common across firms and satisfies $E\left(\psi\right)\geq 1$, implying that being experienced is desirable in expectation. The possible sources of uncertainty are various. One of them stems from the need to adapt products to satisfy demand idiosyncrasies in foreign markets (Artopoulos, Friel and Hallak (2013), Eaton et al. (2014)). Firms may be uncertain about the extent to which their product adaptations match those idiosyncrasies and experiment in the market to figure this out. Another source of uncertainty stems from the need to match with distributors that will exert effort to push their products in the destination market (Artopoulos, Friel and Hallak (2013), Eaton et al. (2014)). Firms may also be uncertain about their ability to find such distributors.

Note that we do not include entry sunk costs in the model, so firms may exit and re-enter markets freely. While this is an assumption made for simplicity, we argue later that sunk costs are neither necessary to obtain the qualitative predictions of the model nor do they help improve its quantitative predictions.

Finally, note that we presented the setup for a generic market. In doing so, we implicitly assumed that while the exogenous part of profitability may be correlated across markets (θ_t), its endogenous part (ψ) is independent across markets.[16] In other words, there are no complementarities in entry

[14] One could expect λ to increase with the length of the exporting experience or the sales volume. This, however, would imply a substantial loss of tractability.

[15] Alternatively, we could have modelled ψ affecting fixed costs rather than operating profits. This decision is inconsequential for the purpose of explaining facts related to exporter survival.

[16] ψ is endogenous in the sense that the timing of its realization depends on the export behavior of the firm. We also

decisions across markets. While we think exploring these complementarities is interesting, they are outside the scope of this paper and are left for future research.

3.2 Entry and exit decisions

It will prove convenient to work with normalized profits defined as $\tilde{\theta}_t \equiv \frac{\kappa\theta_t}{F}$. By Ito's Lemma, $\tilde{\theta}_t$ is a GBM with the same parameters as θ_t. Let $y_x \in \{0,1\}$ be an indicator function that takes the value of one if the firm exports when its status is $x = i, e$. The firm's problem is to maximize its discounted expected profits by choosing an exporting policy $\left\{y_e\left(\tilde{\theta}_t\right), y_i\left(\tilde{\theta}_t\right)\right\}_{t=0}^{\infty}$. We will solve this problem in two steps. Since $x = e$ is an absorbing state, we first solve for the optimal policy of an experienced firm $\left\{y_e^*\left(\tilde{\theta}_t;\psi\right)\right\}$. Then, we solve for the optimal policy $\left\{y_i^*\left(\tilde{\theta}_t\right)\right\}$ of an inexperienced firm taking into account that once it becomes experienced it will follow policy $\left\{y_e^*\left(\tilde{\theta}_t;\psi\right)\right\}$.

The experienced firm

An experienced firm receives profits given by $\pi_e\left(\theta_t;\psi\right) = \psi\kappa\theta_t - F = F\left(\psi\tilde{\theta}_t - 1\right)$ if it exports and 0 otherwise. The value of an experienced firm (V_e) at $t = 0$ is the solution to the following problem:

$$V_e\left(\tilde{\theta}_0;\psi\right) = \sup_{\left\{y_e\left(\tilde{\theta}_t\right)\right\}} E\left(\int_0^{\infty} e^{-rt} F\left(\psi\tilde{\theta}_t - 1\right) y_e\left(\tilde{\theta}_t\right) dt\right)$$

subject to (1) with $\tilde{\theta}_0$ given, where r is the discount rate.

Suppose a firm follows any constant policy $y_e \in \{0,1\}$ during an interval of time $[t, t+\tau]$. Exploiting the stationarity of the problem, we can write the problem recursively as

$$V_e\left(\tilde{\theta}_t;\psi\right) = \max_{y_e\in\{0,1\}} E\left(\int_0^{\tau} e^{-rs} F\left(\psi\tilde{\theta}_{t+s} - 1\right) y_e ds + e^{-r\tau} V_e\left(\tilde{\theta}_{t+\tau};\psi\right)\right).$$

Taking the limit $\tau \to 0$ and rearranging we obtain

$$rV_e\left(\tilde{\theta};\psi\right) dt = \max_{y_e\in\{0,1\}} \left\{F\left(\psi\tilde{\theta} - 1\right) y_e\right\} dt + E\left(dV_e\left(\tilde{\theta};\psi\right)\right) \tag{2}$$

where due to the stationarity of the problem we drop subscript t. This equation says that the return of the firm is the sum of the instantaneous profit flow plus the expected appreciation. Since future profitability is independent from the firm's actions and there are no exit or re-entry costs, the exporting

rule out cases in which fixed costs across markets have a common component that needs to be paid only once, as in Albornoz, Fanelli and Hallak (2016).

decision only depends on whether current profits are non-negative. Thus, the firm's optimal policy is simply $y_e^*\left(\tilde{\theta};\psi\right)=1$ if $\tilde{\theta}\geq\frac{1}{\psi}$ and $y_e^*\left(\tilde{\theta};\psi\right)=0$ if $\tilde{\theta}<\frac{1}{\psi}$.

The inexperienced firm

First, we make a technical assumption so that the inexperienced firms' problem is well-defined: we assume that the distribution of ψ is such that $E_\psi\left(V_e\left(\tilde{\theta};\psi\right)\right)$ satisfies a polynomial growth condition.[17] Let t denote the (random) time at which a firm becomes experienced. Given that this event occurs with intensity λ only if the firm exports, the probability density function (p.d.f) of t depends on the export policy. At time $t=0$, this density is given by $\lambda y_i\left(\tilde{\theta}_t\right)e^{-\int_0^t\lambda y_i\left(\tilde{\theta}_s\right)ds}$, where the exponent term captures the probability that the shock did not take place until t and $\lambda y_i\left(\tilde{\theta}_t\right)$ is the instantaneous arrival rate. Then, the inexperienced firm's problem can be written as

$$V_i\left(\tilde{\theta}_0\right)=\sup_{\left\{y_i\left(\tilde{\theta}_t\right)\right\}}\left(E\int_0^\infty\left[\int_0^t e^{-ru}F\left(\tilde{\theta}_u-1\right)y_i\left(\tilde{\theta}_u\right)du+e^{-rt}E\left(V_e\left(\tilde{\theta}_t;\psi\right)\right)\right]\lambda y_i\left(\tilde{\theta}_t\right)e^{-\int_0^t\lambda y_i\left(\tilde{\theta}_s\right)ds}dt\right) \tag{3}$$

subject to (1) with $\tilde{\theta}_0$ given. Fixing a time t at which the firm receives the shock, the term in square brackets in (3) captures the expected discounted profits, which consist of the discounted stream of net profit flows $F\left(\tilde{\theta}_u-1\right)du$ accumulated during export periods up to t and the discounted expected value of being an experienced firm. Note that by exporting the firm may become experienced sooner, which is always desirable because it implies a higher profit flow on average.

Manipulating (3), we can rewrite the inexperienced firm's problem as[18]

$$V_i\left(\tilde{\theta}_0\right)=\sup_{\left\{y_i\left(\tilde{\theta}_t\right)\right\}_{t=0}^\infty}E\left(\int_0^\infty e^{-rt-\lambda\int_0^t y_i\left(\tilde{\theta}_s\right)ds}\left\{F\left(\tilde{\theta}_t-1\right)+\lambda E_\psi V_e\left(\tilde{\theta}_t\right)\right\}y_i\left(\tilde{\theta}_t\right)dt\right) \tag{4}$$

subject to (1) and $\tilde{\theta}_0$ given. Consider a firm that follows any constant policy $y_i\in\{0,1\}$ during an interval of time $[t,t+\tau]$. Exploiting the stationarity of problem (4) we can write it recursively as

$$V_i\left(\tilde{\theta}_t\right)=\max_{y_i\in\{0,1\}}E\left(\int_0^\tau e^{-(r+\lambda y_i)s}\left\{F\left(\tilde{\theta}_{t+s}-1\right)+\lambda E_\psi V_e\left(\tilde{\theta}_{t+s};\psi\right)\right\}y_i ds+e^{-(r+\lambda y_i)\tau}V_i\left(\tilde{\theta}_{t+\tau}\right)\right). \tag{5}$$

[17] We say that $f:[0,\infty)\to\mathbb{R}$ satisfies a polynomial growth condition if there exist $M>0$ and $\nu>0$ such that $|f(\theta)|\leq M\left(1+\theta^\nu\right)$

[18] Distribute the term $\lambda y_i\left(\tilde{\theta}_t\right)e^{-\lambda\int_0^t y_i\left(\tilde{\theta}_s\right)ds}$ inside the parenthesis and note that $\int_0^\infty\left(\int_0^t e^{-ru-\lambda\int_0^t y_i\left(\tilde{\theta}_s\right)ds}\lambda y_i\left(\tilde{\theta}_t\right)F\left(\tilde{\theta}_u-1\right)y_i\left(\tilde{\theta}_u\right)du\right)dt=\int_0^\infty\left(\int_s^\infty e^{-ru-\lambda\int_0^t y_i\left(\tilde{\theta}_s\right)ds}\lambda y_i\left(\tilde{\theta}_t\right)dt\right)F\left(\tilde{\theta}_u-1\right)y_i\left(\tilde{\theta}_u\right)du=\int_0^\infty e^{-ru-\lambda\int_0^u y_i\left(\tilde{\theta}_s\right)ds}F\left(\tilde{\theta}_u-1\right)y_i\left(\tilde{\theta}_u\right)du$.

Taking the limit $\tau \to 0$ and rearranging, we obtain

$$rV_i\left(\tilde{\theta}\right)dt = \max_{y_i \in \{0,1\}} \left\{F\left(\tilde{\theta}-1\right) + \lambda\left(E_\psi V_e\left(\tilde{\theta};\psi\right) - V_i\left(\tilde{\theta}\right)\right)\right\} y_i dt + E\left(dV_i\left(\tilde{\theta};\psi\right)\right). \tag{6}$$

The term in brackets in equation (6) clarifies the potential trade-off involved in the firm's exporting decision. On the one hand, by exporting there is a chance that the firm will become experienced. Accordingly, the term $\lambda\left(E_\psi V_e\left(\tilde{\theta};\psi\right) - V_i\left(\tilde{\theta}\right)\right)$ captures the benefits of experimentation, which are always non-negative since profits are on average higher for an experienced firm. In fact, when $\tilde{\theta}^* \geq 1$ and experimentation is meaningful (ie. ψ is not identical to 1), these benefits are strictly positive. Thus, inexperienced firms unambiguously prefer to export when $\tilde{\theta}^* \geq 1$. On the other hand, when $\tilde{\theta} < 1$ the first term becomes negative, i.e. $F\left(\tilde{\theta}-1\right) < 0$. Thus, when $\tilde{\theta} < 1$ the firm faces a trade-off: by exporting it earns the possibility of becoming experienced at the cost of incurring a loss. The following proposition shows that there exists a region $\left(\tilde{\theta}^*, 1\right)$ where firms choose to experiment.

Proposition 1. *(a) There exists an optimal policy characterized by a threshold $\tilde{\theta}^* \in [0,1]$ such that if $\tilde{\theta} < \tilde{\theta}^*$, the firm does not export while if $\tilde{\theta} \geq \tilde{\theta}^*$, the firm exports. This policy is the unique piecewise continuous optimal policy. Furthermore, if the distribution of ψ is not degenerate at 1, then $\tilde{\theta}^* < 1$.*
(b) $\tilde{\theta}^*$ solves $\pi_i\left(\tilde{\theta}^*\right) + \lambda\left(E_\psi V_e\left(\tilde{\theta}^*;\psi\right) - V_i\left(\tilde{\theta}^*\right)\right) = 0$.

Proof. See Appendix A.1.

Proposition 1 states that there exists a threshold $\tilde{\theta}^* \leq 1$ such that the firm exports iff $\tilde{\theta} \geq \tilde{\theta}^*$. In fact, this result holds in a more general set up than assuming a GBM and a multiplicative shock. In Appendix A.1, we specify a set of sufficient conditions such that the firm follows a threshold strategy. The key condition, which is satisfied in our setup, is:

$$\frac{d\lambda E_\psi\left(\max\left\{\pi_e\left(\theta_t;\psi\right),0\right\}\right)}{d\theta_t} > \frac{d\lim_{dt\to 0}\left\{E\left(e^{-rdt}\pi_i\left(\theta_{t+dt}\right)\right) - \pi_i\left(\theta_t\right)\right\}}{d\theta_t}.$$

This condition says that the expected flow benefits of becoming experienced should increase faster than the costs of experimenting today rather than tomorrow (recall $\pi_i < 0$ in the relevant region).[19]

[19] This rules out cases in which there is a region for θ where experienced-firm profits are relatively high but inexperienced-firm losses from exporting are strongly decreasing in θ, inducing firms to wait, and another region in which inexperienced-firm losses from exporting are flat in θ and experienced-firm profits are low but high enough so that firms want to export.

Proposition 1 also states that optimality at $\tilde{\theta}^*$ requires that:

$$F\left(\tilde{\theta}^* - 1\right) + \lambda\left(E_\psi V_e\left(\tilde{\theta}^*; \psi\right) - V_i\left(\tilde{\theta}^*\right)\right) = 0. \tag{7}$$

Appendix A.2 shows that (7) can be solved to obtain

$$\tilde{\theta}^* - 1 + \lambda\left(\frac{2}{J + \tilde{J}}\right)\left[\begin{array}{c} \int_{\tilde{\theta}^*}^{\infty}\left(\frac{\tilde{\theta}^*}{z}\right)^{\tilde{\beta}_1}\left(E_\psi\left(\max\left(\psi z - 1, 0\right)\right) - (z-1)\right)\frac{dz}{z} \\ + \int_0^{\tilde{\theta}^*}\left(\frac{\tilde{\theta}^*}{z}\right)^{\beta_2} E_\psi\left(\max\left(\psi z - 1, 0\right)\right)\frac{dz}{z} \end{array}\right] = 0 \tag{8}$$

where $J = \sqrt{\mu^2 + 2r\sigma^2}$, $\tilde{J} = \sqrt{\mu^2 + 2\left(r + \lambda\right)\sigma^2}$, $\tilde{\beta}_1 = \frac{-\mu + \tilde{J}}{\sigma^2} > 1$ and $\beta_2 = \frac{-\mu - J}{\sigma^2} < 0$.

The intuition for (8) is as follows. First, note that for any GBM we can write the solution as an integral of the flow over states z multiplied by a "weight" for that state.[20] The weight represents the length of time the process spends in each state, taking into account the proper discounting. For states with $z > \tilde{\theta}^*$, the correct discount – which is reflected in $\tilde{\beta}_1$ – is $r + \lambda$ since the inexperienced firm becomes experienced at rate λ. Since in that region the inexperienced firm exports, the integrand is the difference between the (expected) flow profits of an experienced firm and that of an inexperienced firm. Note $\tilde{\beta}_1 > 0$ since larger z are less likely and therefore more heavily discounted. For states $z < \tilde{\theta}^*$, only some experienced firms export. Hence, we only have the (expected) flow profits of an experienced firm. The proper discount, which is reflected in β_2, is now r since an inexperienced firm remains inexperienced in this region. Note that $\beta_2 < 0$, reflecting that when $z < \tilde{\theta}^*$ lower states are less likely and, thus, more heavily discounted.

Equation (8) shows that our uncertainty and experimentation model preserves a tractable property of simpler continuous-time models as we only need to solve one equation in one unknown to characterize the whole strategy of the firm.[21,22] While this is special to the GBM framework, we can still allow an arbitrary distribution for ψ, F and κ. Furthermore, note that F and κ do not appear in (8). Hence, $\tilde{\theta}^*$ does not depend on these parameters. In other words, θ^* is proportional to κ and $\frac{1}{F}$. Intuitively, the firm "undoes" the effect of κ and F by timing its entry decision: a low-κ firm will wait longer until θ is large enough to perfectly offset the effect of κ. This property of the model is going to be

[20]The weight here is $\left(\frac{2}{J+\tilde{J}}\right)\left(\frac{\tilde{\theta}^*}{z}\right)^{\tilde{\beta}_1}\frac{1}{z}$ for $\tilde{\theta} > \tilde{\theta}^*$ and $\left(\frac{2}{J+\tilde{J}}\right)\left(\frac{\theta^*}{z}\right)^{\beta_2}\frac{1}{z}$ for $\tilde{\theta} < \tilde{\theta}^*$. This is a property of GBM processes (see Stokey (2009)).

[21]By contrast, note that solving for the optimal strategy in discrete-time dynamic models requires the computationally-intensive procedure of iterating on the Bellman equation.

[22]π_i and π_e being both linear in θ (or, equivalently, ψ being multiplicative) is not important for this result. In Appendix A.2 we show that with general profit functions $\pi_i\left(\theta\right)$ and $\pi_e\left(\theta; \psi\right)$ the problem can still be reduced to one equation in one unknown, as long as the conditions in Proposition 1, specified in Appendix A.1, hold.

very important in the next section and in the empirical exercise.

Let us recap the alternative export trajectories a firm can exhibit. The firm is originally inexperienced and stays away from the market as long as normalized profits are below $\tilde{\theta}^*$. As soon as $\tilde{\theta}$ crosses this entry threshold it starts to export. The purpose of entering the market is initially to experiment, albeit incurring losses, in the expectation of resolving the uncertainty with respect to their profitability shifter ψ. Eventually, one of three events might occur: (a) $\tilde{\theta}$ might decrease and cross $\tilde{\theta}^*$ from above, in which case the firm will stop exporting; (b) $\tilde{\theta}$ might increase above 1, in which case it will turn losses into profits; (c) the firm might receive the ψ shock. In the last case, the firm will keep exporting only if the shock has been sufficiently large to generate a net profit flow (i.e. if $\psi \geq \frac{1}{\tilde{\theta}}$).

3.3 The probability of survival

Henceforth, we assume that all firms are born inactive in the export market, ie. $\frac{\kappa\bar{\theta}}{F} < \tilde{\theta}^*$. Normalizing the entry time to $t = 0$, the firm enters the foreign market with $\tilde{\theta}_0 = \tilde{\theta}^*$. Since $\tilde{\theta}_t$ is a GBM,

$$\ln \tilde{\theta}_T = \ln \tilde{\theta}^* + \mu T + \sigma Z_T$$

is a SBM where Z_T is a standard normal random variable. Defining $x_T \equiv \frac{\ln \tilde{\theta}_T - \ln \tilde{\theta}^*}{\sigma}$,

$$x_T = \frac{\mu}{\sigma} T + Z_T.$$

First, note that, while a firm is inexperienced, it is active iff

$$\ln \tilde{\theta}_T > \ln \tilde{\theta}^* \Leftrightarrow x_T > 0 \Leftrightarrow \frac{\mu}{\sigma} T + Z_T > 0. \tag{9}$$

Thus, the likelihood of this event depends only on $\frac{\mu}{\sigma}$. Since an exporter becomes experienced at an intensity governed by λ, the likelihood of being experienced at any point in time depends only on $\frac{\mu}{\sigma}$ and λ.

Second, define $\tilde{\psi} \equiv \left(\frac{\psi}{\psi_m}\right)^{\frac{1}{\sigma}}$ for some $\psi_m > 0$. $\tilde{\psi}$ will later be useful to compare different distributions of ψ that differ only in a scale parameter ψ_m. Note that, while a firm is experienced, it is active iff

$$\ln \psi + \ln \tilde{\theta}_T > 0 \Leftrightarrow \ln \tilde{\psi} + \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma} + \ln x_T > 0 \Leftrightarrow \ln \tilde{\psi} + \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma} + \frac{\mu}{\sigma} T + Z_T > 0 \tag{10}$$

Let $\tilde{\Psi}$ denote the set of parameters that characterize the distribution of $\tilde{\psi}$. Thus, the likelihood of this event depends only on $\frac{\mu}{\sigma}$, $\frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}$ and $\tilde{\Psi}$.

Let $y(t)$ be an indicator function that takes the value of 1 if the firm is an exporter at t. Putting both parts together, we see that knowing $\Upsilon = \left\{\frac{\mu}{\sigma}, \lambda, \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}, \tilde{\Psi}\right\}$ is sufficient to determine the likelihood of any given trajectory of $\{y(t)\}$. Thus, the expectation of any function of those trajectories also depends on Υ. The following proposition formally states this result:

Proposition 2. *Take any function $f : A \to \mathbb{R}$ with $A \subset \mathcal{P}(\{y(t)\}_{t=0}^{\infty})$. Then, $E(f)$ depends solely on Υ. In particular, the probability of survival of an entrant, at horizon T (p_T), depends only on Υ.*

Proof. Since A is a set of subsets of export trajectories $\{y(t)\}_{t=0}^{\infty}$, and the likelihood of each trajectory $\{y(t)\}_{t=0}^{\infty}$ only depends on Υ, the likelihood of an event $a \in A$ only depends on Υ. Since f takes different values depending only on which event $a \in A$ occurs, it follows that $E(f)$ depends only on Υ. For example, in the case of the probability of survival, take $f = y(T)$ and apply the result. □

As we will discuss later, Proposition 2 shows that using only information on entrant survival allows us to identify only a subset of the parameters of the model. Furthermore, the following corollary shows that the probability of survival of incumbents also depends only on this combination of parameters,

Corollary 1. *The probability of survival of incumbents depends only on Υ.*

Proof. Since incumbents are just a weighted sum of entrants of different ages and the probability of survival for entrants of a given age falls within the scope of Proposition 2, it immediately follows that the probability of survival of incumbents is also a function of Υ. □

Another consequence of Proposition 2 is the following,

Corollary 2. *p_T is independent of κ and F.*

Proof. By Proposition 2, p_T only depends on Υ. From equation (8) it follows that $\tilde{\theta}^*$ is independent of κ and F. Thus, p_T is also independent of κ and F.

Corollary 2 is a key result. It establishes that the probability of survival of an export incursion is independent of κ and F and hence only depends on parameters that are common across firms.[23] The main implication of this result is that all firms entering a given market have the same probability

[23] An analogous result with respect to heterogeneous market-specific profitability shifters is obtained in Albornoz, Fanelli and Hallak (2016) in a framework without experimentation and with homogenous fixed costs.

of survival T periods after entry. The strength of this prediction is achieved despite a substantial amount of heterogeneity in the model allowed for by heterogeneous profit shifters (κ) and fixed costs (F) across firms and markets. Heterogeneity in κ allows the model to affect the likelihood of any entry sequence into foreign markets. This parameter, however, does not provide any information about the probability of survival in the market once it has entered it.

Since entry profits are given by $\pi_o \sim F$ (the common factor of proportionality is $\tilde{\theta}^*$), heterogeneity in F also implies heterogeneous sales at the time of entry. For example, if sales are a constant proportion of profits, entry sales will also be proportional to fixed costs. Thus, the model has a degree of freedom left to rationalize the shape of the size distribution of entrants – and potentially the size distribution of incumbents – by adjusting accordingly the distribution of fixed costs. Most results in this paper do not depend on specific assumptions about this distribution, which we do not need to impose. The two implications of Corollary 2 highlight an advantage of focusing on entrant survival since we can obtain sharp predictions on observables without sacrificing flexibility over firm-specific parameters we have little information about.

Next, we will compute p_T. Since the firm can only receive shock ψ while it exports, it will be useful to define the occupation time s as the total length of time the stochastic process x_t spends above 0 between $t = 0$ and $t = T$:

$$s = \int_0^T \mathbf{1}_{x_t \geq 0} dt$$

where $\mathbf{1}_{x_t \geq 0}$ is an indicator function for the event $\{x_t \geq 0\}$. Given an occupation time s, the probability of not receiving the shock between 0 and T is $P(no\ jump\,|_s) = e^{-\lambda s}$.

Denote by $\omega_T(s, x)$ the joint density of an occupation time of s (between 0 and T) and $x_T = x$. Then, we can express the joint probability of not receiving the shock until T and $x_T = x$ as

$$P(no\ jump, x_T = x) = \int_0^T e^{-\lambda s} \omega_T(s, x) ds$$

while the analogous joint probability for the case in which the firm receives the shock is

$$P(jump, x_T = x) = \int_0^T (1 - e^{-\lambda s}) \omega_T(s, x) ds.$$

Conditional on x_T, the probability of survival of an experienced firm at T is $P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)$.

Now we have all the required elements to compute p_T. Taking into account that the ψ shock and the process x_t are mutually independent, we can decompose this probability into two terms. If $x_T \leq 0$

then the firm will only survive if it has received the shock *and* the shock was sufficiently large. If $x_T \geq 0$ then two things may happen: (a) if the firm has received the shock, then survival depends on the magnitude of the shock; (b) if the firm has not received the shock, then it will be exporting at T since it finds it profitable (in expected terms) to keep waiting for the shock. Thus, p_T can be written as:

$$p_T = \begin{cases} \int_{-\infty}^{0} \int_{s=0}^{T} \left(1 - e^{-\lambda s}\right) P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) \omega_T\left(s, x\right) dsdx \\ \int_{0}^{\infty} \int_{s=0}^{T} \left\{\left(1 - e^{-\lambda s}\right) P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) + e^{-\lambda s}\right\} \omega_T\left(s, x\right) dsdx. \end{cases} \tag{11}$$

Pechtl (1999) shows that $\omega_T\left(s, x\right)$ has the following closed form solution:

$$\omega_T(s,x) = \begin{cases} \exp\left\{-\frac{\left(\frac{\mu}{\sigma}\right)^2 T}{2} + \frac{\mu}{\sigma}x\right\} \frac{|x|}{2\pi} \int_{T-s}^{T} \frac{\exp\left\{-\frac{x^2}{2(T-t)}\right\}}{[t(T-t)]^{3/2}} dt & \text{if } x \geq 0 \\ \exp\left\{-\frac{\left(\frac{\mu}{\sigma}\right)^2 T}{2} + \frac{\mu}{\sigma}x\right\} \frac{|x|}{2\pi} \int_{s}^{T} \frac{\exp\left\{-\frac{x^2}{2(T-t)}\right\}}{[t(T-t)]^{3/2}} dt & \text{if } x < 0. \end{cases}$$

Therefore, given a parametrization of the distribution of ψ (we do this in the next section) equation (11) is easy to compute numerically.

The introduction of uncertainty and experimentation allows the model to explain fact 1. Given μ and σ, a model with uncertainty about ψ can consistently predict lower survival rates over a finite horizon than a "benchmark model" without experimentation (in which case $P(\psi = 1) = 1$) and hence help explain the low survival profile exhibited by fact 1. This result is established in the following proposition:

Proposition 3. *Define p_{BT} as the probability of survival at horizon T in the benchmark model. Then, given (μ, σ, r), any distribution of ψ with unbounded support, and any interval $\left[\underline{T}, \overline{T}\right]$ with $\underline{T} > 0$ and $\bar{T} < \infty$, we can pick $\bar{\lambda}$ such that for any $\lambda > \bar{\lambda}$, $p_T < p_{BT}$ for $T \in \left[\underline{T}, \overline{T}\right]$.*

Proof. See Appendix A.3. □

The intuition of this result is the following. Firms are willing to enter the foreign market with a low value $\tilde{\theta}^*$ to experiment and resolve their uncertainty regarding ψ. Since there are no sunk costs, the only cost of this strategy is the accumulation of losses until the firm becomes experienced. Lured by potential future profits, firms are willing to bet on this uncertain event even if only a good shock will justify their permanence as exporters. Furthermore, for a sufficiently high probability that the jump occurs fast (a high λ) the firm will be willing to accept a very low survival probability in exchange of the chance of getting a good ψ draw.[24] This is the main insight of the proposition. In addition to explaining a low export survival profile for any set of dynamic parameters (μ, σ), this mechanism can also explain the flatness of the survival profile observed in the data. While in the short run the uncertainty regarding ψ implies many firms die because of the shock, firms that do receive the ψ shock are placed far away from the threshold. Hence, they take longer to exit than firms in the benchmark model. In contrast, as we discuss in the next section, the benchmark model can only match a low survival rate at a specific horizon by also predicting a steep survival profile.

The introduction of uncertainty can also help explain fact 2. This is established by a simple corollary to Proposition 3:

Corollary 3. *Define p_{RT} as the probability of survival over horizon T for a re-entrant to the market. Then, given (μ, σ, r), any distribution of ψ with unbounded support, and any interval $\left[\underline{T}, \overline{T}\right]$ with $\underline{T} > 0$ and $\bar{T} < \infty$, we can pick $\bar{\lambda}$ such that for any $\lambda > \bar{\lambda}$, $p_{RT} > p_T$ for $T \in \left[\underline{T}, \overline{T}\right]$.*

Proof. There are two classes of re-entrants. Re-entrants that have not received the shock re-enter and exit at the same thresholds as first-time entrants. Hence, they have their same probability of survival (p_T). Re-entrants that have received the shock enter and exit at $\theta^* = \frac{1}{\psi}$. Although this threshold is different from the entry and exit thresholds of firms in the benchmark model, the fact that entry and exit thresholds coincide in both cases implies that they have identical survival probabilities (p_{BT}). Denoting by $\rho_s > 0$ the probability that a re-entrant has received the shock, the survival probability for a re-entrant is $p_{RT} = (1 - \rho_s)p_T + \rho_s p_{BT}$. Using Proposition 3, this directly implies that $p_{RT} > p_T$. $\square$

This result is driven by the fact that a fraction of re-entrants already knows ψ and hence is only willing to re-enter when they can make positive profits. As a result, they enter with a higher value of θ_t and thus are more likely to survive. The role of uncertainty to explain fact 2 is much more transparent here since in this case the predictive failure of the benchmark model is qualitative. In the absence of uncertainty, entrants and re-entrants are predicted to have the same probability of survival.

While the firm is uncertain about the value of ψ, it also knows that $E(\psi) \geq 1$. However, the results of Proposition 3 and Corollary 3 are driven by the fact that ψ is ex-ante unknown. To see why, consider a family of distributions Ψ that satisfy $\psi = \psi_m \tilde{\psi}$, i.e. they are identical up to a scale parameter ψ_m. Hence, $E(\psi) \geq 1$ can be decomposed into $\psi_m \geq 1$ and $E(\tilde{\psi}) = 1$. The following

[24] The role of the unbounded support for the distribution of ψ is to make the threshold $\tilde{\theta}^*$ arbitrarily small when $\lambda \to \infty$. This is a sufficient condition to make the entrant probability of survival arbitrarily small in that limiting case.

proposition states that the probability of survival goes up with ψ_m. Thus, the fact that $E(\psi) \geq 1$ could not alone generate Proposition 3 and Corollary 3 if the distribution of $\tilde{\psi}$ were degenerate.

Proposition 4. *Let $\psi = \psi_m \tilde{\psi}$. Then, the probability of survival at any horizon increases with ψ_m.*

Proof. See Appendix A.4. □

Proposition 2 establishes that survival only depends on $\psi_m \tilde{\theta}^*$. In fact, it follows immediately from (10) that the probability of survival increases with this product. The key step in the result is proving that $\psi_m \tilde{\theta}^*$ increases with ψ_m (note that $\tilde{\theta}^*$ is a decreasing function of ψ_m). Recall that $\tilde{\theta}^*$ is independent of κ, i.e. if κ increases then θ^* decreases exactly such that $\kappa\theta^*$ stays constant. An increase in ψ, realization by realization, is similar to an increase in κ except during the experimentation period. Thus, the firm does not fully offset the larger ψ with a smaller $\tilde{\theta}^*$. If the distribution of ψ were degenerate (i.e. deterministic), then $E(\psi) \geq 1$ would imply that $\psi_m > 1$. Thus, in this case the previous result implies that firms in the full model would survive at least as much as firms in the benchmark model, which would contradict fact 2. It would also imply that in an uncertain world making the shock more attractive realization by realization (ie. more learning by exporting) would worsen the ability of the model to explain the data.

4 Estimation

In this section, we estimate the model parametrizing the shock with a Pareto distribution. We describe the data and the estimation strategy, and we discuss the ability of the model to explain facts 1 and 2. This section also discusses the extent to which alternate models often used in the literature that do not account for uncertainty and experimentation can explain these two facts.

4.1 Survival probabilities under time aggregation

Our model is set in continuous time and hence assumes that firms make export decisions at every instant in time. The data, however, record transactions over discrete time periods. This mismatch introduces a time aggregation problem. We describe here how we correct for it.

For expositional transparency, let us focus on the "benchmark model" (i.e. the special case without uncertainty where either $\alpha \to \infty$ or $\lambda \to 0$). In the benchmark model, the decision to export does not bear dynamic consequences. Hence, the firm exports whenever it makes positive profits ($\tilde{\theta}^* = 1$). For the normalized process x_t, this implies that entry an exit thresholds coincide at $x_t = 0$. Given these thresholds and the fact that x_t follows a SBM with drift $\widetilde{\mu} = \frac{\mu}{\sigma}$ and diffusion parameter 1, the probability of survival at instant T after entry is simply given by:

$$p_{BT} = \Phi\left(\widetilde{\mu}\sqrt{T}\right). \tag{12}$$

This probability depends only on $\widetilde{\mu}$, not on the individual values of μ and σ.

Equation (12) predicts an "instantaneous" probability, i.e. the probability that a firm entering at $t = 0$ is still active at $t = T$. For example, for a firm that entered on January 1st of calendar year 0 this formula provides the probability that it exports on January 1st of calendar year T. However, this is not how we observe the data. First, calendar year T will report positive exports even if the firm was not exporting at $t = T$ (January 1st) as long as it re-entered anytime between that instant and $t = T + 1$. We call this the "re-entry effect". Second, the firm could have first entered the market at any instant $t \in [0, 1)$ (e.g. August 14th) rather than at $t = 0$ (January 1st). In that case, the relevant horizon to compute survival in calendar year T is shorter. This is the "partial year effect" emphasized by Berthou and Vicard (2015) and Bernard et al. (2017). Thus, time aggregation requires two adjustments to make p_{BT} a proper theoretical counterpart of survival rates as we observe them in the data.

Denote by $P_{BT}(\tau)$ the probability of survival adjusted for the re-entry effect for an incursion made at $\tau \in [0, 1)$. Assuming a uniform density for the time of entry throughout the year, we can account for the partial year effect by computing $P_{BT} = \int_0^1 P_{BT}(\tau)d\tau$.[25] In turn, $P_{BT}(\tau)$ consists of two parts. The first is the instantaneous component. It captures the probability that a firm that entered the market at $t = \tau$ is actively exporting in the instant $t = T$. This event will happen if $x_T \geq 0$. The second part is the re-entry component. It captures the probability that the firm is not exporting at $t = T$ but does it at some point during calendar year T. This event will occur if $x_T < 0$ but $x_t \geq 0$ for some $t \in [T, T + 1)$. Denote by a the first instant in time, starting from $t = T$, that $x_t \geq 0$. This is known as the "first passage time" (FPT) and can be defined as $a = \inf\{t : t \geq T \;\; s.t. \; x_t \geq 0\}$. Then, the two parts of $P_{BT}(\tau)$ can be written as:

$$P_{BT}(\tau) = P(a = T) + P(T < a \leq T + 1). \tag{13}$$

Computing the instantaneous part just requires modifying the formula in equation (12) to account

[25] Using Peruvian export data at a monthly frequency, Bernard et al. (2017) show that actual export entry throughout the year is close to uniform.

for the fact that the relevant time horizon is $T-\tau$ rather than T. Thus, $P(a=T)=\Phi\left(\widetilde{\mu}\sqrt{T-\tau}\right)$. Computing the re-entry part requires that we appeal to known formulas for the FPT of a SBM. We omit the resulting formula but note that it can be derived as a mathematical expression that can be solved up to integrals that need to be numerically computed. Suffice it to say here that the re-entry component is also only a function of $\widetilde{\mu}=\frac{\mu}{\sigma}$. Thus, so are $P_{BT}(\tau)$ and P_{BT}.

Adjusting for the re-entry effect and the partial-year effect has a considerable impact on survival probabilities. To assess the quantitative importance of these two adjustments, in Table 1 we report, for values of $\widetilde{\mu}$ ranging from -0.1 to -0.9: (a) the instantaneous probability of survival without adjustment (i.e. calculated according to (12)); (b) the instantaneous probability of survival adjusted for the partial year effect assuming uniform entry; (c) the probability of survival adjusted only for the re-entry effect; and (d) the probability of survival adjusted for both the partial year effect and the re-entry effect. Panel A reports these probabilities at horizon $T=1$ while panel B reports them at horizon $T=5$. As we can see in the table, the combined impact of the two effects is substantial. For example, in the case of $\widetilde{\mu}=-0.5$, accounting for both effects more than doubles the survival prediction when $T=1$ while it increases it by 68% when $T=5$. The table also shows that the re-entry effect is more important than the partial-year effect. In the case of $\widetilde{\mu}=-0.5$, the re-entry effect alone accounts for 60% of the total adjustment when $T=1$ while it accounts for 77% when $T=5$.

While the exposition in this section focused on the predictions for survival of the benchmark model, analogous adjustment for re-entry and partial-year effects can be made on the predictions for survival of the full model derived in Section 3.3. Although they can also be derived as mathematical expressions that can be solved up to integrals that need to be numerally computed, the estimation of the full model, described next, simulates those adjusted probabilities.

4.2 Parameter identification

Proposition 2 states that survival probabilities only identify particular combinations of the model's parameters, $\Upsilon=\left\{\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma},\frac{\mu}{\sigma},\tilde{\Psi},\lambda\right\}$. For parsimony, we specify ψ to follow a Pareto distribution with location parameter ψ_m and scale parameter α. This, in turn, implies that $\tilde{\psi}=\left(\frac{\psi}{\psi_m}\right)^{\frac{1}{\sigma}}$ follows a Pareto distribution with scale parameter 1 and shape parameter $\sigma\alpha$. In other words, $\tilde{\Psi}=\{\sigma\alpha\}$.

Although the model has six unknowns, $\Phi=\{\mu,\sigma,\alpha,\psi_m,\lambda,r\}$, Υ has only four elements. In fact, using the threshold equation (8) we can vary r or ψ_m, and change σ while keeping $\alpha\sigma$, λ and $\frac{\mu}{\sigma}$ constant in order to keep $\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}$ constant. In other words, without more information we cannot separately identify r, ψ_m and the level of the parameters of the profitability process α,μ and σ. Put differently,

Table 1: Adjustments of survival probabilities for time aggregation effects

	Fixed parameters				
μ/σ	-0.1	-0.3	-0.5	-0.7	-0.9
	Survival probabilities				
	Panel A: One year horizon				
Instantaneous	0.46	0.38	0.31	0.24	0.18
Partial-year correction	0.47	0.42	0.37	0.32	0.28
Re-entry correction	0.71	0.62	0.52	0.43	0.33
Fully-adjusted	0.79	0.73	0.66	0.59	0.52
	Panel B: Five year horizon				
Instantaneous	0.41	0.25	0.13	0.06	0.02
Partial-year correction	0.42	0.26	0.14	0.07	0.03
Re-entry correction	0.54	0.36	0.20	0.10	0.04
Fully-adjusted	0.55	0.38	0.22	0.11	0.05

Note: Reported figures survival probabilities computed as described in Section 4.3.

we do not need to identify these parameters to obtain predictions on survival probabilities.

Now focus on $\widetilde{\mu} = \frac{\mu}{\sigma}$. In the benchmark model, equation (12) shows that survival probabilities depend only on this ratio. In the full model, although this ratio is only one of four arguments in Υ, the levels of μ and σ are still only weakly identified. The identification problem is illustrated in Table 2. First, fixing $\mu = -0.1$, $\psi_m = 1$ and $r = 0.1$, we estimate σ, α, and λ by the Simulated Method of Moments (SMM). This is in fact our baseline estimation, discussed below, which we perform using survival rates of entrants and re-entrants in the first five horizons. Alternatively, we fix $\mu = -0.05$ and $\mu = -0.15$, set σ to maintain the same ratio $\frac{\mu}{\sigma}$, and estimate α and λ by the SMM with the same moments. The identification problem is manifest in the comparison across columns. Despite the wide variation of μ and σ across the three parameter configurations, they all deliver similar survival predictions.

Given the identification scope of survival probabilities, we set $\mu = -0.10$, $r = 0.10$ and $\psi_m = 1$ and estimate $\{\sigma, \alpha, \lambda\}$. Note that the assumption that the scale parameter of the Pareto distribution is 1 implies that the firm will always improve profitability when the shock arrives. In this sense, we can interpret it as a learning shock. We think this is a reasonable assumption as firms are likely to go through a learning period while operating in a foreign market.

Table 2: Identification

	Fixed parameters		
μ	-0.05	-0.1	-0.15
σ	0.14	0.28	0.42
λ	10.0	10.0	10.2
α	6.9	3.7	2.8
	Predicted survival probabilities		
	Panel A: Entrants		
Year 1	0.372	0.357	0.357
Year 2	0.226	0.213	0.213
Year 3	0.197	0.182	0.185
Year 4	0.177	0.166	0.167
Year 5	0.161	0.150	0.152
	Panel B: Re-entrants		
Year 1	0.628	0.617	0.627
Year 2	0.463	0.459	0.462
Year 3	0.388	0.386	0.385
Year 4	0.334	0.332	0.332
Year 5	0.294	0.292	0.292

Note: Reported survival probabilities are computed using the parameters specified in the corresponding column.

4.3 Estimation strategy

We simulate the model by generating 10000 artificial export profit trajectories. An export profit trajectory consists of three independent random components: a GBM trajectory for $\tilde{\theta}_t$, a realization of the learning shock ψ, and a Poisson process governing its arrival. In the computer, we can only simulate an approximation to a continuous process – both for the GBM and the Poisson processes – by discretizing the time space. We artificially create calendar years and divide each in $N = 1000$ intervals (each represented by its middle point) to make this approximation as precise as possible subject to computing constraints. Since all predictions of the model can be expressed in terms of normalized profits and since all firms first enter at $\tilde{\theta}_t = \tilde{\theta}^*$, we generate 10000 trajectories for $\tilde{\theta}_t$ that start at this threshold value. We assume that firms' entrance is uniformly distributed along the unit interval during year 0. To gain simulation precision minimizing loss in computer efficiency, each simulation is used 1000 times by *sliding* the entry time during the first year along each of its 1000 intervals.

For each simulation, we track whether $\psi\tilde{\theta}_t \geq 1$ for an experienced firm or $\tilde{\theta}_t \geq \tilde{\theta}^*$ for an inexperienced firm in each of the 1000 intervals of each calendar year. Whenever a firm satisfies this condition in at least one interval of a calendar year, we consider it to have survived in that year. For entrants, we only need to keep track of survival status in the next five years after the start of the simulation process. The survival probabilities $\{P_T\}_{T=1,\ldots,5}$ are computed as the proportion of surviving firms in each of the first five years. Computing the survival probabilities of re-entrants (P_{RT}) is more involved. As discussed in Section 3.3, this probability is a weighted average of the survival probability of entrants, P_T, which applies to inexperienced re-entrants, and the survival probability in the benchmark model, P_{BT}, which applies to experienced re-entrants. While both of these probabilities are easier to compute, we still need the probability that a re-entrant is experienced to apply appropriate weights (the proportion of experienced re-entrants is 82% when we simulate the model with the parameters of the baseline estimation). Furthermore, as this probability varies with age, we also need the age distribution of re-entrants in the steady state. Hence, we simulate survival probabilities of re-entrants by running each simulation up to $T = 31$ and tracking survival status for five periods after each instance of re-entry.

The estimated parameters minimize the following objective function:

$$\widehat{H}(\sigma, \alpha, \lambda) = \left[S - \widehat{P}(\sigma, \alpha, \lambda)\right]' W \left[S - \widehat{P}(\sigma, \alpha, \lambda)\right] \tag{14}$$

where $\widehat{P}(\sigma, \alpha, \lambda)$ is the simulated vector of survival probabilities, S is the vector of observed survival rates, and W is a block diagonal weighting matrix. The matrix W has two blocks, $\widehat{W}_E$ and $\widehat{W}_R$, where $\widehat{W}_{j=E,R}$ are the inverse matrices of the sample analogs of the variance-covariance matrices $E\left[(S_j - E(S_j))(S_j - E(S_j))'\right]$. The standard errors we report for the estimated parameters are preliminary and incorrectly-estimated statistics that need to be taken with caution. First, we have yet not accounted for the various sources of simulation error. Second, we have computed the variance-covariance matrix as if W were the efficient weighting matrix, which is not the case. For this reason, we report them only as indicative but refrain at this point from making comments on their values. We are currently working on these issues.

4.4 Descriptive statistics

Table 3 provides descriptive statistics about exporters and incursions in our dataset (left panel) and about the macroeconomic environment in Peru (right panel) during the sample period. The first column details the number of exporters each year. The second column details the number of instances, which are any active firm-market-year combination. The third and fourth columns display the number of incursions and re-entries, respectively. Note that an exporting firm might not have made an incursion

Table 3: Descriptive Statistics

	Firms and Entries					Macro Variables		
	Number of:							
	Firms	Instancies	Incursions	Re-entries	Survival rate of incursions (%): 2 years	Total Exports (US$ mill.)	Real GDP Growth (annual variation, %)	Real Exchange Rate (annual variation, %)
1997	3,775	9,859	4,081	700	25.7	6,825	6.477	2.335
1998	3,563	9,116	3,522	729	29.0	5,757	-0.392	4.110
1999	3,895	10,425	4,251	1,102	27.5	6,088	1.495	14.11
2000	4,016	11,347	4,535	1,106	24.2	6,955	2.694	2.725
2001	4,347	11,536	4,244	1,175	25.9	7,026	0.618	1.351
2002	4,685	12,042	4,222	1,338	27.3	7,714	5.454	1.677
2003	5,094	13,171	4,836	1,458	27.4	9,091	4.165	-1.057
2004	5,467	14,318	5,139	1,597	26.3	12,809	4.958	-2.820
2005						17,368	6.285	-1.770
2006						23,830	7.529	0.518
2007						28,094	8.518	-3.422
2008						31,018	9.143	-8.233
2009						26,962	1.049	-0.314
Total	34,842	91,814	34,830	9,205	26.6			

Note: Left panel based on Peruvian customs dataset (World Bank). First two columns of right panel based on INEI. Real exchange rate multiplies nominal exchange rate by US PPI (BLS) and divides it by Peruvian CPI (INEI).

during the sample period. In total, during the sample period (1997-2004) we identify $34,830$ incursions by $13,664$ unique firms and 9205 re-incursions by 3090 unique firms. The first four columns in the left panel display evidence of growing exporting activity in Peru during the sample period. The last column shows the survival rate for each cohort of incursions in a two-year horizon. The survival rate hovers around an average of 26.6%.

The right panel of the table displays summary indicators of the macroeconomic performance of Peru. The information is provided for an expanded period that includes both the sample years used to identify incursions (1997-2004) and the years used to compute survival (2005-2009). The first column of this panel shows a strong positive trend for aggregate exports in Peru. Similarly, the second column shows a strong positive trend in the evolution of GDP, particularly in the later years of the sample. The last column displays the evolution of the real exchange rate, which exhibits an accumulated depreciation of 29% during the period 1996-2002 followed by an accumulated appreciation of 16% during the period 2002-2009. While our model does not account for changes over time in potential export profitability due to changes in the real exchange rate, by focusing on averages over a time period that includes both appreciation and depreciation of the domestic currency we hope to capture patterns in the data that approximate those that would arise in a fully stable macroeconomic environment.

4.5 Results

The top part of Table 4 displays the estimation results. The estimate of σ is 0.279. Given $\mu = -0.10$, this estimate implies that $\widehat{\widetilde{\mu}} = -.359$. This value is not very different from the method-of-moments estimates of $-.0279$ and -0.270 obtained by Luttmer (2007) and Arkolakis (2016), respectively, for this ratio. The similarity might seem striking given that those two papers estimate $\widetilde{\mu}$ based on *domestic* survival rates. However, our model generates lower *exporter* survival rates primarily due to the uncertainty and experimentation mechanism rather by imposing a more negative trend in the profitability process.[26] The estimate for the parameter of the Poisson process is $\widehat{\lambda} = 10$. This is a very high value. It implies that a firm that continuously exports has a 56.5% probability of receiving the learning shock in less than a month. Finally, $\widehat{\alpha} = 3.736$, which implies a standard deviation of 0.54 for the multiplicative shock ψ.

The second part of Table 4 compares the data with the model predictions. A visual representation of the same information is provided in Figure 2. The model does a good job predicting the survival rates

[26]This result also suggest that our model could potentially provide a unifying framework for understanding both domestic and export survival.

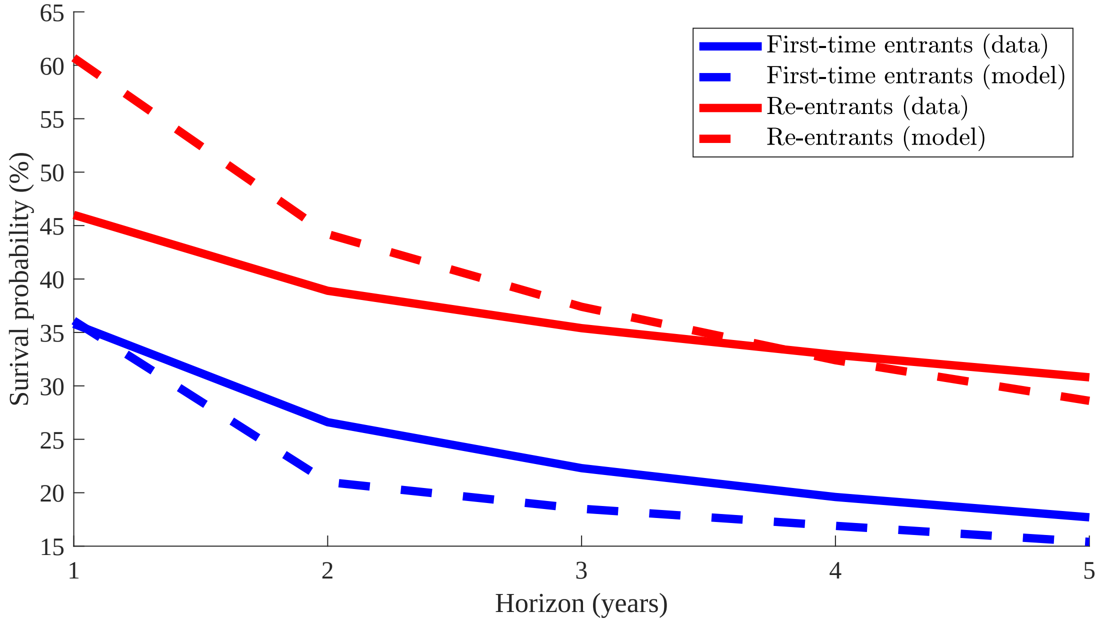


Figure 2: Survival profiles predicted by the model

Table 4: SMM Estimation results

	Fixed parameters	
μ	-0.1	
r	0.1	
	Estimated parameters	
	Coefficient	Std. Dev.
σ	0.279	0.0005
λ	9.993	0.1830
α	3.736	0.0352
	Survival probabilities	
	Panel A: Entrants	
	Model	Data
Year 1	0.36	0.36
Year 2	0.21	0.27
Year 3	0.19	0.22
Year 4	0.17	0.2
Year 5	0.15	0.18
	Panel B: Re-entrants	
Year 1	0.61	0.46
Year 2	0.44	0.39
Year 3	0.37	0.35
Year 4	0.32	0.33
Year 5	0.29	0.31

of entrants. In particular, it predicts a survival profile that is both low and flat. The average absolute discrepancy between data and predictions is three percentage points, with the largest discrepancy in the second year (27% in the model versus 21% in the data). The model predictions are less accurate in the case of re-entrant survival rates. In this case, the average discrepancy between data and model predictions is five percentage point with a particularly large underprediction in the first year (46% in the model versus 61% in the data). Nevertheless, the model is still able to deliver the qualitative fact that re-entrants survive more than entrants. It is interesting to notice that the difficulty of the model to explain the low survival rate of re-entrants also explains why there is a relatively large discrepancy for entrants in period 2. The following trade-off arises. Setting a higher λ, the model can achieve a better fit for the survival profile of entrants by flattening its slope between periods 1 and 2. However, a higher λ also implies a larger proportion of experienced re-entrants and hence even higher predictions for their survival in the earliest periods. A potential hypothesis that might explain the quantitative discrepancy between predictions and data in the case of re-entrants is that the resolution of uncertainty takes places in "steps" rather than in one event. This alternative structure for the resolution of uncertainty might generate survival predictions for re-entrants that are closer to those for entrants. In the sake of parsimony, we leave such extension of our model for future research.

4.6 Alternate models without uncertainty and experimentation

In this section, we discuss why other models that do not include uncertainty and experimentation are unable to explain facts 1 and 2. First, we estimate the benchmark model and extensions that include sunk costs and an exogenous death rate. Then, we estimate another version of the benchmark model where we substitute a mean-reverting Geometric Ornstein-Uhlenbeck (GOU) process for the GBM profitability process we have been assuming so far. Finally, we nest the GBM and GOU processes by expanding the latter allowing long-run profitability to (potentially) drift downwards over time.

Brownian motion The benchmark model is the starting point in the class of models that assumes a GBM profitability process but does not assume uncertainty and experimentation. It is easy to see why this model is not a useful alternative. Since entrants and re-entrants enter and exit at $\tilde{\theta}^* = 1$, in both cases survival probabilities are identical. Therefore, this model is unable to explain fact 2. In fact, the benchmark model is also unable to explain fact 1. The green-colored dashed line in Figure 1a (discussed in Section 2) corresponds to the best prediction of the benchmark model. This prediction is obtained by estimating the model with the SMM using only the survival profile of entrants $\{S_t\}_{t=1,\ldots,5}$.

Table 5: SMM estimation results
(Benchmark model)

	Estimated parameter	
	Coefficient	Std. Dev.
μ/σ	-0.67	0.002
	Survival probabilities: Entrants	
	Model	Data
Year 1	0.594	0.358
Year 2	0.354	0.266
Year 3	0.242	0.223
Year 4	0.176	0.196
Year 5	0.130	0.177

In this case, the only parameter to estimate is the ratio $\widetilde{\mu} = \frac{\mu}{\sigma}$. Table 5 shows that the estimate of this parameter is $\widehat{\widetilde{\mu}} = -0.67$, which is much more negative than the estimate for this ratio that we obtain in the full model. The table also presents the predicted survival rates using this estimate. These are the predictions depicted in Figure 1a. We can see that the model overpredicts survival rates at short horizons while it underpredicts them at longer horizons. In unreported results, we have also included a constant exogenous death rate in the benchmark model and included it as an additional parameter to estimate. The SMM estimation delivers a value of 0 for this parameter. Thus, this exogenous proportional death rate cannot help explain the disproportionate amount of exit during the first year.

Next, we extend the benchmark model to include sunk costs (S). Sunk costs have been the focus of most theoretical and empirical work on exporter dynamics (Baldwin and Krugman (1989), Dixit (1989), Roberts and Tybout (1997), Alessandria and Choi (2007), Das, Roberts and Tybout (2007), Impullitti, Irarrazabal and Opromolla (2013), Morales, Sheu and Zahler (2017)). More specifically, we assume that when firms switch from non-exporter to exporter in a given market for the first time, they need to pay a sunk cost S. Given that incurring the sunk cost is an irreversible decision, firms only enter with profitability above the exit threshold, which raises the probability of survival at short horizons relative to longer ones. This is exactly the opposite of what is needed to fit the observed survival profile. Hence, sunk costs do not help explain fact 1.

Finally, although we have assumed that sunk costs only need to be paid the first time a firm exports, a straightforward extension is to assume that a firm needs to pay a fraction $\phi \in [0, 1]$ of the original sunk cost in subsequent export experiences. This would generate an inaction region $\left[\underline{\theta}_{FT}, \bar{\theta}_{FT}\right]$ such that firms start exporting when $\theta \geq \bar{\theta}_{FT}$ and stop exporting when $\theta \leq \underline{\theta}_{FT}$ during their first export experience while it would similarly generate an inaction region for re-entrants $\left[\underline{\theta}_{RE}, \bar{\theta}_{RE}\right]$ with $\underline{\theta}_{RE} = \underline{\theta}_{FT}$ and $\bar{\theta}_{RE} \leq \bar{\theta}_{FT}$ (with equality iff $\phi = 1$). If $\phi = 1$, then (first-time) entrants and re-entrants have equal survival probabilities. In the more general case of $\phi < 1$, re-entrants survive less than entrants as their inaction region is smaller. This prediction goes against explaining fact 2.[27] Hence, this extension of the benchmark model is also qualitatively unable to explain this fact.[28]

More general diffusions Most empirical work on export dynamics has modelled the logarithm of operating profits as a mean-reverting process. To accommodate this possibility, we change the specification of the profitability process to the continuous-time analog of an AR1 in logarithms, a Geometric Ornstein-Uhlenbeck process (GOU):

$$d\theta_t = \eta \left(\log\left(\bar{\theta}\right) - \log\left(\theta_t\right)\right) \theta_t dt + \sigma \theta_t dW_t.$$

As is usual in the exporter dynamics literature, we assume that the parameters that govern the law of motion of profitability (i.e. η, $\log\left(\bar{\theta}\right)$ and σ) are common across firms after controlling for observable characteristics. We also allow for an exogenous death rate $\delta > 0$, which in the context of a stationary process becomes necessary to generate the downward sloping profile of survival probabilities, particularly at long horizons.

Although this specification of the profitability process might improve the ability of the model to fit fact 1, since all entries and exits take place at the same threshold level θ^*_{GOU}, the model still predicts the same survival probability for entrants and re-entrants. Thus, it is also unable to explain fact 2. This result is in fact more general. Assume that θ follows any general diffusion,

$$d\theta_t = \mu\left(\theta_t\right) dt + \sigma\left(\theta_t\right) dZ_t, \tag{15}$$

that satisfies that if $\theta' > \theta''$, then $F\left(\theta_{t+s}|\theta'\right) \leq F\left(\theta_{t+s}|\theta''\right)$ (a first-order-stochastic-dominance condition) and regularity conditions such that there is a unique solution to (15) and the value functions are bounded.[29] Then, the optimal policy will still be characterized by a unique entry and exit threshold

[27] In fact, if some entrants were allowed to be born above the entry threshold then, since re-entrants necessarily enter the market at the threshold, the counterfactual prediction would arise even with $\phi = 1$.

[28] Note that fact 2 may explain why Das, Roberts and Tybout (2007) find that sunk-costs fully depreciate ($\phi = 1$). By having entrants and re-entrants pay similar sunk costs, the estimated model might want to minimize the failure of the sunk-cost model to explain fact 2.

[29] The first-order stochastic dominance condition is a natural and common assumption in the literature that prevents strange cases in which the firm is very profitable today but does not pay the sunk cost because it knows this high profitability will be the cause of low profitability tomorrow.

both for entrants and for re-entrants and hence produce the identical survival probabilities. As a result, the inability to explain fact 2 generalizes to this broader class of models. Furthermore, introducing entry sunk costs $S \geq 0$ and re-entry sunk costs $\phi S \geq 0$, as analyzed in the case of a GBM, just worsens the problem by inducing lower survival rates for re-entrants.

Estimated and calibrated models of exporter dynamics (Alessandria and Choi (2007), Das, Roberts and Tybout (2007), Ruhl and Willis (2017)) also have two additional features that are worth considering. A first feature is that while we assume that firms are born below the threshold θ^*, these models assume that firms are born with a profitability taken from the stationary distribution (as implied for example by an AR1). As discussed earlier, having entrants be born above the exit threshold only worsens the inability of this model to explain fact 2. A second feature of those models is allowing for idiosyncratic random processes – i.i.d. across firms and over time – for fixed and sunk costs. However, given that firms in these models are born with the stationary distribution, the composition of the pool of first-time entrants is exactly the same as the pool of re-entrants (when $\phi = 1$; if $\phi < 1$ as usual the pool of re-entrants has lower sunk-costs), so entrants and re-entrants have the same probability of survival.

Finally, following Arkolakis (2016), we extend the GOU model to include a deterministic trend in long run profitability. More specifically, θ now follows

$$d \log(\theta_{a,t}) = \eta \left(\log\left(\overline{\theta}\right) + \mu a - \log(\theta_{a,t}) \right) da + \mu da + \sigma dW_a$$

where a is the age of the firm and t is calendar time.[30] As in Luttmer (2007) and Arkolakis (2016), the negative trend captures the fact that older technologies become obsolete, implying that we need to keep track of the age of the firm. This process nests the GBM and GOU processes we have considered so far. If $\eta = 0$, then we obtain a GBM. If $\eta > 0$ but $\mu = 0$, then we obtain a GOU. Note also that if $\sigma = 0$, so that the process is deterministic, firm profits drift downwards at log-rate μ. This flexible specification is able to fit the survival profile of entrants (fact 1). In addition, the deterministic trend generates heterogeneity in survival probabilities across entrants since older firms have lower long run profitability and, thus, survive less as they mean-revert more rapidly. Thus, it also generates a composition effects that may explain fact 2. However, setting parameters that allow the model to match fact 1, the composition effect goes in the opposite direction of explaining fact 2. Since re-entrants tend to be older firms, they re-enter further away from their long-term profitability and

[30] We set $\delta = 0$ for this exercise.

hence tend to survive less than entrants.

5 Mechanisms

We have presented two facts about exporter survival and developed a model with uncertainty and experimentation in export markets that naturally explains them. We have also shown that the most representative exporter dynamics models that do not include these elements are unable to explain the two facts simultaneously. In this section, we provide further evidence of the relevance of uncertainty and experimentation in the dynamics of firm exports by associating variation in the parameter α to observed characteristics of products and markets.

When α is lower, the distribution of ψ has fatter tails. Furthermore, both the mean and the variance of the distribution of ψ increase. While the variance may be associated with uncertainty, the fact that the mean increases is associated with learning-by-exporting. Proposition 4 helps us separate both effects. Suppose we lowered α while varying ψ_m in order to keep the mean of the distribution constant, i.e. $\psi_m = \frac{\alpha-1}{\alpha}$. It can be checked that the variance of the shock decreases with α even after the compensating ψ_m effect (provided the variance exists, i.e. $\alpha > 2$). Next, note that by Proposition 4, decreasing ψ_m lowers the probability of survival. In other words, if we lowered α alone and the probability of survival decreased, then correcting for the effect of the mean with ψ_m can only exacerbate the effect on the probability of survival. Henceforth, we ignore the effect on the mean and talk about decreasing α as intensifying uncertainty but it should be clear that the effect of this change on survival probabilities would be stronger if we corrected for the mean.

Since profits are increasing and convex in ψ, it is unsurprising that the threshold $\tilde{\theta}^*$ goes up with α. Consider a decrease in α such that the variance increases. Conditional on a realization of ψ the firm will survive less. On the other hand, a larger variance of ψ implies that there are more firms that receive a realization of the shock from the upper tail of the distribution, who are less likely to exit later on. Since the estimated GBM process has a negative drift, this effect is relatively more relevant in the long run, where only firms that have benefited from a very good draw of ψ survive. Although the net effect is ambiguous, we show with simulations that the "threshold effect" dominates. That is, a lower (higher) α implies a lower (higher) probability of survival. This result is displayed in Table 6. Across columns from left to right, we maintain all parameters in their estimated values but set α so that the dispersion of the learning shock decreases from four times to a quarter of its estimated value. Accordingly, the predicted survival rates increase uniformly as α increases.

Table 6: Effect of α on survival probabilities

	Fixed parameters				
μ	-0.1	-0.1	-0.1	-0.1	-0.1
σ	0.279	0.279	0.279	0.279	0.279
λ	10.0	10.0	10.0	10.0	10.0
α	2.3	2.8	3.7	5.6	9.4
Pareto std. dev.	2.144	1.072	0.536	0.268	0.134
Normalized Pareto std. dev.	4	2	1	1/2	1/4
	Survival probabilities				
	Panel A: Entrants				
Year 1	0.262	0.286	0.357	0.493	0.620
Year 2	0.090	0.123	0.213	0.348	0.455
Year 3	0.069	0.104	0.182	0.303	0.386
Year 4	0.061	0.095	0.166	0.263	0.332
Year 5	0.056	0.088	0.150	0.233	0.290
	Panel B: Re-entrants				
Year 1	0.510	0.570	0.617	0.658	0.677
Year 2	0.353	0.403	0.459	0.487	0.508
Year 3	0.286	0.331	0.386	0.401	0.423
Year 4	0.250	0.282	0.332	0.345	0.356
Year 5	0.220	0.245	0.292	0.302	0.308

We do not observe α. However, the magnitude of this parameter can be linked to observable characteristics of products and export destinations. Since α governs the variance of the shock, its magnitude captures the degree of uncertainty about the component of export market profitability that can be resolved by experimenting.[31] As discussed in Section 3, possible sources are the need to adapt products to satisfy foreign demand idiosyncrasies and the need to match with distributors that will make efforts to propel sales of the firm's products. It is reasonable to assume that these sources of uncertainty are more relevant for differentiated products than for homogeneous products. Thus, we expect a lower α, and hence a lower P_T, for the former type of products.

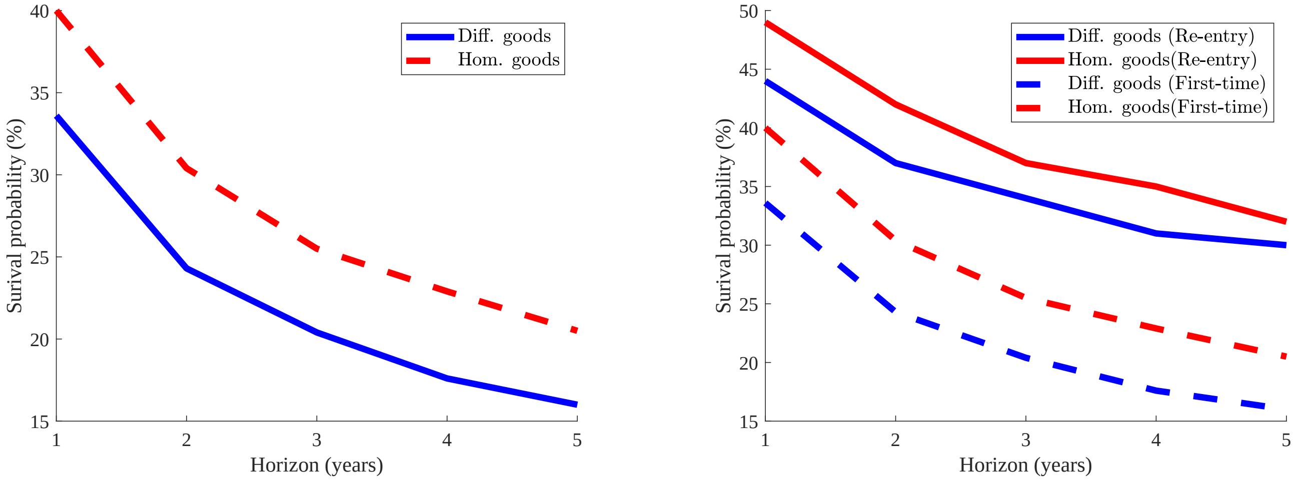
To test this implication, we classify all incursions in our database in either of two categories, differentiated or homogeneous, following Rauch (1999).[32] We first map export data classified at the Harmonized System 10-digit level into Rev.2 SITC 4-digit categories using the Conversion Tables from the United Nations Statistics Division. Then, we map the latter categories into one of our two categories.[33] Finally, we identify the category with the largest value of exports in the year of entry and assign the incursion to that category. There are $20,907$ differentiated incursions and $13,120$ homogenous incursions in our database. For each of these categories, the survival profile is displayed in Figure 3a. Consistent with the hypothesis of a lower α for differentiated products, these products display uniformly lower survival rates. For example, the survival rate is more than six percentage points lower in the first year after entry and more than four percentage points lower in the fifth year. Similar results are obtained in a regression framework, where we can perform inference and control for other covariates. We regress the survival status of each incursion-horizon combination on dummies for horizon and a dichotomous variable for differentiated products. The results, displayed in column 1 of Table 8, show that the survival rate of differentiated products is significantly lower than for homogeneous products. We also interact the differentiated dummy with horizon dummies. Column 2 shows that the survival rate of incursions in differentiated products is lower at all horizons. Columns 3 and 4 show that these results are not an artifact of composition effects. The results are very similar when we replicate the regressions in the first two columns by adding a full set of product (2-digit HS), destination, and year fixed effects.

Since a fraction of re-entrants in differentiated products has already resolved their uncertainty, we

[31]The remaining component, i.e. uncertainty about the future trajectory of the GBM process, remains unresolved after the shock.

[32]We merge homogeneous and referenced-priced categories in Rauch (1999) into only one "homogeneous" category.

[33]The mapping from SITC to Rauch leaves 5.74% of the instances unclassified. We reduce this proportion to 2.33% by arbitrarily assigning unclassified SITC 4-digit categories the classification of similar SITC 4-digit categories. Of the remaining unclassified instances, 60% are transactions without reported HS-code.



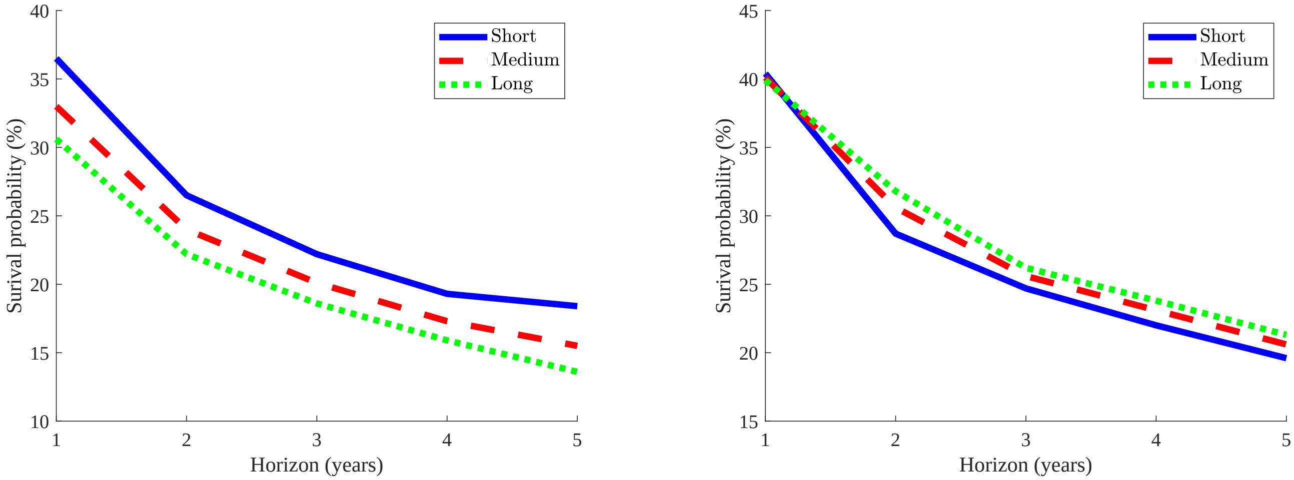
(a) First-time entrants

(b) First entry vs. re-entry

Figure 3: Survival profile by type of product

should expect a smaller gap between re-entrant survival rates in differentiated versus homogeneous products than between entrant survival rates in the two types of products. Figure 3b displays re-entrant survival profiles in these two cases (for reference we also include survival profiles for entrants). As predicted, the gap is smaller for re-entrants than for entrants. The prediction is formally tested by performing a diff-in-diff estimation. We regress the exporting status of each incursion-horizon and re-incursion-horizon combination on entrant horizon dummies, re-entrant horizon dummies, a dummy for differentiated products, and an interaction dummy for re-entrants in differentiated products. We cluster standard errors by firm-destination allowing for correlation between incursions and re-incursions at any horizon. Consistent with the prediction of a smaller gap for re-entrants in differentiated versus homogenous products, we find a positive and significant (at the 10% level) estimated coefficient on the interaction term (column 5). A similar result is obtained by interacting the differentiated product dummy with horizon dummies (column 6) and by adding a full set of product, destination, and year fixed effects (columns 7 and 8). We note, however, that given the high proportion of experienced re-entrants in the baseline estimation, the predicted magnitude of the gap is substantially larger than is observed in the data. This result is in line with the quantitative difficulty of the model to match the survival rates of re-entrants discussed in Section 4.5. Re-entrants exhibit survival patterns that are considerably more similar to those for entrants than predicted by the model.

The uncertainty surrounding export market profitability could also be hypothesized to vary according to distance to the destination. In the first place, neighboring countries are more likely to have similar income levels and thus share similar consumption patterns. In the second place, even control-



(a) Differentiated goods **(b)** Homogeneous goods

Figure 4: Survival profile by distance

ling for income levels, demand idiosyncrasies are more likely to coincide the closer are the exporter and the importer. In the third place, less distant countries are more likely to have a more similar business culture that facilitates communication with distributors and anticipation of their actions. As a result, we could expect a higher degree of uncertainty about export market profitability in more distant destinations. Setting a smaller α for those destinations, the model predicts lower survival probabilities in those cases. To assess this prediction, we divide export destinations into three groups according to their distance from Peru. Short distance destinations are those with a distance smaller than 3440 km. Medium distance destinations are those with a distance between 3440 km. and 10100 km. Long distance destinations are those with a distance above 10100 km. The cut-offs are chosen so that each distance group has an equal number of incursions. Figure 4a displays the survival profile for each distance group in the case of differentiated goods. We can see that the profile is uniformly lower the farther away is the destination. For example, one year after entry the survival rate for the long distance group is six percentage points lower than for the short distance group while five years after entry the survival rate for the former group is five percentage points lower. Figure 4b displays analogous information in the case of homogeneous products, where it is unclear whether distance should matter. In this case, we do not see lower survival rates in more distant destinations. If anything, the opposite seems to be the case.

These results also arise in a regression framework where in addition to controlling for other covariates we can also control for distance as a continuous variable. The results are displayed in Table 9. In column 1 we only control for horizon fixed effects, a fixed effect for differentiated products, and the interaction of distance with fixed effects for differentiated and homogenous products, respectively. In accordance with the graphical results, we see that distance decreases survival rates in the case of differentiated products but increases them in the case of homogenous products. Column 2 presents the results of analogous regression interacting the variables above with horizon dummies. While the coefficient on the interactions for differentiated products are uniformly negative and significant at every horizon, the magnitude and statistical significance of the positive coefficients for analogous interactions in the case of homogenous products are more disparate. Similar results are obtained when we reproduce these regressions including the usual set of product, destination, and year fixed effects.

In the case of differentiated products, where distance matters, we should also expect a smaller gap for re-entrants across distance groups. Figure 5 displays the relevant survival profiles (survival profiles for entrants are also included for reference). In this case, the prediction is not borne by the data. The gap in survival rates across distance groups does not dwindle for re-entrants. Regression analysis confirms this result. Column 1 of Table 10 reports the result of regressing survival status of entrants and re-entrants on horizon dummies, a dummy for re-entrants, a control for distance, and this control interacted with the re-entrant dummy. The interest is in the coefficient on the latter interaction. The estimated sign is opposite to the prediction albeit not significantly different from zero. Similar results are obtained when all three terms are interacted with horizon dummies (column 2) and when we include the full set of fixed effects (columns 3 and 4). Once again, re-entrants display survival patterns that are more similar to those of entrants than predicted by the model.

In sum, the results of this section show that reasonable assumptions about how α varies across products and destinations yield predictions that are consistent with the data in most cases. We regard those results as supportive of the notion that uncertainty and experimentation are crucial features in the dynamics of firm exports.

6 Concluding remarks

This paper develops a model of exporter dynamics with uncertainty and experimentation. The model is parsimonious and has tractable features that allow us to obtain analytical results on survival probabilities. Those results explain two central facts about export survival in foreign markets that existing models that neglect uncertainty and experimentation are unable to account for. The first fact is that the survival profile of export entrants is low and flat. The second is that re-entrants to foreign markets display higher survival rates than first-time entrants. Based on the analytical results we derive, we

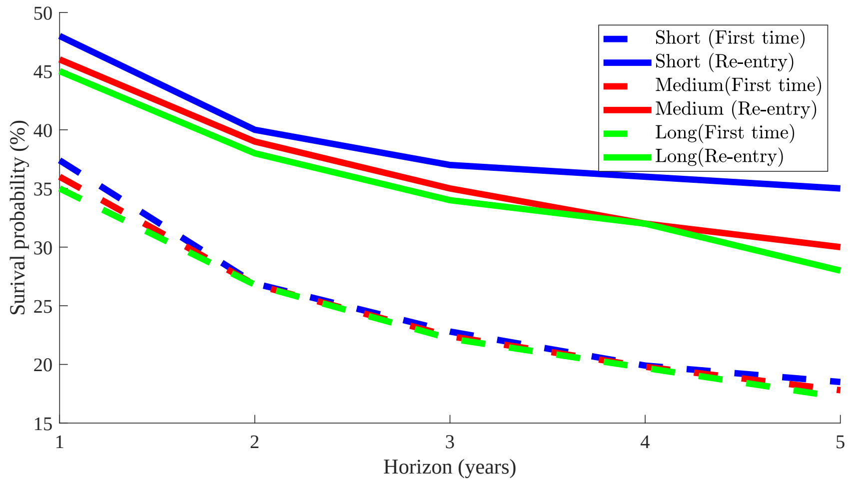


Figure 5: Survival profile by distance and type of incursion

can estimate the parameters of the model and derive quantitative predictions on these two facts. The importance of uncertainty and experimentation in exporter dynamics is further supported by evidence that exploits hypothesized variation in the degree of uncertainty about foreign market profitability across products and distance to the destination.

The paper also makes a methodological contribution to the literature on firm and exporter dynamics by proposing a correction for the mismatch between a model set in continuous time and data recorded in discrete-time periods. This correction has a substantial impact on the model predictions. Conceptually, the correction is more general than our specific application here since the source of mismatch arises even in discrete time models. In particular, it should be applied whenever there is a discrepancy between the frequency at which firms make decisions and the frequency at which the data are recorded. In addition to correcting survival predictions, analogous adjustment could be made to correct for other key variables in dynamic models such as the amount of sales and their growth. We are currently working on developing such corrections.

While focusing on export survival, we hope to contribute to a broader literature that attempts to characterize the main features in the dynamics of firm exports. A next natural step would be to study the implications of our theoretical framework for other moments analyzed in the exporter dynamics literature. We are currently working in this direction. Preliminary results are promising and will hopefully be included in future versions of this paper.

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A Proofs and Derivations

A.1 Proof of Proposition 1

We will prove the result under the following (more general) conditions,

Assumption 1. *$E_\psi \pi_e(\psi, \theta) \geq \pi_i(\theta)\ \forall \psi, \theta$. π_e is continuous, π_i belongs to C^2 and both are weakly increasing in $\theta\ \forall \psi$. ψ and θ are independent.*

Assumption 2. *Let $h \equiv \lambda E_\psi(\max\{\pi_e(\theta;\psi), 0\}) - \lim_{dt\to 0}\left\{E\left(e^{-rdt}\pi_i(\theta_{t+dt})\right) - \pi_i(\theta_t)\right\}$. If $\lambda > 0$, $h(\theta)$ is weakly increasing in θ. Furthermore, $E\left[\int_0^\infty e^{-rt} h(\theta_t)\, d\theta_t | \theta_0\right]$ satisfies a polynomial growth condition.*[34]

Assumption 3. *There exists $\bar{\theta}$ such that $\forall \theta > \bar{\theta}$, flow profits are positive even for inexperienced firms, $\pi_i(\theta) \geq 0$.*

Assumption 4. *The profitability process $\{\theta_t\}$ is assumed to follow a diffusion,*

$$d\theta_t = \mu_\theta dt + \sigma_\theta dZ_t \tag{16}$$

where Z_t is a standard brownian motion. We assume $\mu(\theta)$ and $\sigma(\theta)$ are continuous functions of θ that satisfy Lipschitz and growth conditions on μ and σ.[35] *Furthermore, if $\theta'' > \theta'$, then $F(\theta|\theta'') \succsim_{FOSD} F(\theta|\theta')$.*

Assumption 5. *$E_\psi V_e$ satisfies a polynomial growth condition $\forall \theta$*

[34] We say that $f : [0, \infty) \to \mathbb{R}$ satisfies a polynomial growth condition if there exist $M > 0$ and $\nu > 0$ such that $|f(\theta)| \leq M(1 + \theta^\nu)$

[35] We say that μ satisfies a Lipschitz condition if there exists $k > 0$ such that

$$|\mu(\theta) - \mu(\theta')| \leq k|\theta - \theta'|.$$

This ensures the existence of a strong solution to (15)

Assumption 1 is satisfied in the model in the text because $E(\psi) \geq 1$. Applying Ito's Lemma to Assumption 2 we get

$$h \equiv \lambda E_{\psi} \pi_e(\theta; \psi) + r\pi_i(\theta) - \mu_{\theta} \frac{d\pi_i(\theta)}{d\theta} - \frac{1}{2}\sigma_{\theta}^2 \frac{d^2\pi_i}{d\theta^2}$$

In the model in the text,

$$h \equiv E_{\psi}\left[\max\left\{\psi\tilde{\theta} - 1, 0\right\}\right] + \frac{r}{\lambda}\left(\tilde{\theta} - 1\right) - \frac{\mu\tilde{\theta}}{\lambda}$$

which is clearly increasing in $\tilde{\theta}$ (recall $r - \mu > 0$). Furthermore Assumption 3 is satisfied taking $\bar{\theta} = \frac{F}{\kappa}$ and Assumption 4 is satisfied by the GBM assumption ($\mu_{\theta} = \mu\theta$ and $\sigma_{\theta} = \sigma\theta$). Finally Assumption 5 is satisfied by assumption. In the Pareto case, for example, it can be shown that

$$E_{\psi} V_e\left(\tilde{\theta}\right) = F \begin{cases} \frac{2}{(\alpha-1)(\beta_1-\alpha)(\alpha-\beta_2)\sigma^2}\tilde{\theta}^{\alpha} - \frac{A_{e1}\alpha}{\beta_1-\alpha}\tilde{\theta}^{\beta_1} & \text{if } \tilde{\theta} < 1 \\ \frac{\alpha A_{e2}}{\alpha-\beta_2}\tilde{\theta}^{\beta_2} + \frac{\alpha}{(r-\mu')(\alpha-1)}\tilde{\theta} - \frac{1}{r} & \text{if } \tilde{\theta} \geq 1 \end{cases},$$

for some constants A_{e1} and A_{e2}, which clearly satisfies a polynomial growth condition.

First, we prove the following result,

Lemma 1. *Exporting is optimal for an inexperienced firm when $\theta > \bar{\theta}$.*

Proof. Exporting while $\theta > \bar{\theta}$ yields additional flow profits $\pi_i(\theta) \geq 0$ in $[\bar{\theta}, +\infty)$ if the firm remains inexperienced and increases the odds of becoming experienced, which increases profits in expectation by $E_{\psi}(\max\{\pi_e(\theta;\psi), 0\}) - \max\{\pi_i(\theta), 0\} \geq 0\ \forall\theta$. Hence, exporting is optimal in this region.

□

Define $\pi^{EE}(\theta) \equiv E_{\psi}(\max\{\pi_e(\theta,\psi), 0\})$. Note that the flow benefits of exporting (W) are given by

$$W = \pi_i + \lambda(V_e - V_i).$$

Since y_i is piecewise continuous, V_i is continuous. Given that π_i and V_e are continuous, this implies W is continuous. Assuming an indifferent firm exports, a firm will export iff $W \geq 0$. By Assumption 1 and the possibility of inaction we know that $0 \leq V_i(\theta) \leq V_e(\theta) < \infty\ \forall\theta$. Moreover, since W is continuous and π_e and π_i are continuous, by the Feynman-Kac Theorem we know that $V_i, V_e \in C^2$ and, thus, $W \in C^2$. Hence, V_e and V_i satisfy the following Hamilton-Jacobi-Bellman equations,

$$rV^E = \pi^{EE} + \mu_{\theta}\frac{dV_e}{d\theta} + \frac{1}{2}\sigma_{\theta}^2\frac{d^2V_e}{d\theta^2}\ \forall\theta \tag{17}$$

$$
\begin{aligned}
(r+\lambda)\,V_i &= \pi_i + \lambda V_e + \mu_\theta \frac{dV_i}{d\theta} + \frac{1}{2}\sigma_\theta^2 \frac{d^2 V_i}{d\theta^2} \text{ when } W(\theta) \geq 0 && (18)\\
rV^I &= \mu_\theta \frac{dV^I}{d\theta} + \frac{1}{2}\sigma_\theta^2 \frac{dV_i}{d\theta} \text{ when } W(\theta) < 0 && (19)
\end{aligned}
$$

Next, substract (27) and (28) from (17) to obtain,

$$
\begin{aligned}
(r+\lambda)\,(V_e - V_i) &= \pi^{EE} - \pi_i + \mu_\theta \left(\frac{dV_e}{d\theta} - \frac{dV_i}{d\theta}\right) + \frac{\sigma_\theta^2}{2}\left(\frac{d^2 V_e}{d\theta^2} - \frac{d^2 V_i}{d\theta^2}\right) \text{ when } W(\theta) \geq 0 && (20)\\
r\,(V_e - V_i) &= \pi^{EE} + \mu_\theta \left(\frac{dV_e}{d\theta} - \frac{dV_i}{d\theta}\right) + \frac{\sigma_\theta^2}{2}\left(\frac{d^2 V_e}{d\theta^2} - \frac{d^2 V_i}{d\theta^2}\right) \text{ when } W(\theta) < 0 && (21)
\end{aligned}
$$

Rewrite (20) and (21) in terms of W to obtain

$$
\begin{aligned}
\left(\frac{r+\lambda}{\lambda}\right)(W - \pi_i) &= \pi^{EE} - \pi_i + \frac{\mu_\theta}{\lambda}\left(\frac{dW}{d\theta} - \frac{d\pi_i}{d\theta}\right) + \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\left(\frac{d^2 W}{d\theta^2} - \frac{d^2 \pi_i}{d\theta^2}\right) \text{ when } W(\theta) \geq 0\\
\left(\frac{r}{\lambda}\right)(W - \pi_i) &= \pi^{EE} + \frac{\mu_\theta}{\lambda}\left(\frac{dW}{d\theta} - \frac{d\pi_i}{d\theta}\right) + \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\left(\frac{d^2 W}{d\theta^2} - \frac{d^2 \pi_i}{d\theta^2}\right) \text{ when } W(\theta) < 0
\end{aligned}
$$

where we used the fact that $\pi_i \in C^2$. Rearranging,

$$
\begin{aligned}
\left(1 + \frac{r}{\lambda}\right) W &= \pi^{EE} + \frac{r}{\lambda}\pi_i - \frac{\mu_\theta}{\lambda}\frac{d\pi_i}{d\theta} - \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2\pi_i}{d\theta^2} + \frac{\mu_\theta}{\lambda}\frac{dW}{d\theta} + \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2 W}{d\theta^2} \text{ when } W(\theta) \geq 0\\
\left(1 + \frac{r}{\lambda}\right) W &= W + \pi^{EE} + \frac{r}{\lambda}\pi_i - \frac{\mu_\theta}{\lambda}\frac{d\pi_i}{d\theta} - \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2\pi_i}{d\theta^2} + \frac{\mu_\theta}{\lambda}\frac{dW}{d\theta} + \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2 W}{d\theta^2} \text{ when } W(\theta) < 0.
\end{aligned}
$$

Define $h \equiv \pi^{EE} + \frac{r}{\lambda}\pi_i - \frac{\mu_\theta}{\lambda}\frac{d\pi^I}{d\theta} - \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2\pi_i}{d\theta^2}$, which is exactly Assumption 1 after applying Ito's Lemma. We can rewrite this as

$$
\left(1 + \frac{r}{\lambda}\right) W = W\mathbf{1}_{W<0} + \pi^{EE} + \frac{r}{\lambda}\pi_i - \frac{\mu_\theta}{\lambda}\frac{d\pi^I}{d\theta} - \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2\pi_i}{d\theta^2} + \frac{\mu_\theta}{\lambda}\frac{dW}{d\theta} + \frac{1}{2}\frac{\sigma_\theta^2}{\lambda}\frac{d^2 W}{d\theta^2}. \tag{22}
$$

By assumptions 2 and 5, we know that W and h satisfy a polynomial growth condition. Furthermore, we know that W is continuous. Hence, by Feynman-Kac theorem (Duffie, Appendix E, p.344), the unique solution that satisfies a polynomial growth condition to (22) is given by

$$
W(\theta_0) = E\left(\int_0^\infty e^{-\left(1+\frac{r}{\lambda}\right)t}\left\{W(\theta_t)\,\mathbf{1}_{W(\theta_t)<0} + h(\theta_t)\right\} d\theta_t | \theta_0\right).
$$

The solution requires $W \geq 0$ for $\theta \geq \bar{\theta}$. Thus, W solves

$$W(\theta) = E\left(\int_0^\infty e^{-\left(1+\frac{r}{\lambda}\right)t}\left\{W(\theta_t)\,\mathbf{1}_{W(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\right\}d\theta_t|\theta_0\right) \tag{23}$$

$$W(\theta) \geq 0 \text{ for } \theta_t \geq \bar{\theta} \tag{24}$$

First, we solve (23) disregarding (24),

$$\tilde{W}(\theta) = E\left(\int_0^\infty e^{-\left(1+\frac{r}{\lambda}\right)t}\left\{\tilde{W}(\theta_t)\,\mathbf{1}_{\tilde{W}(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\right\}d\theta_t|\theta_0\right) \tag{25}$$

Lemma 2. *There is a unique continuous solution $\tilde{W}$ to (25).*

Proof. Define the operator $T : C(X) \to C(X)$ as the RHS on (25) restricted to $\left[0,\bar{\theta}\right]$, where C is the space of continuous and bounded functions. Note that T is well-defined in the sense that if $f \in C$, $Tf \in C$. Next, we show that T satisfies monotonicity and discounting:

(i) Monotonicity. Take $f \geq g$. Then,

$$\begin{aligned}
Tf(\theta_0) &= E[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\{f(\theta_t)\mathbf{1}_{f(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\}d\theta_t|\theta_0] \\
&\geq E[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\{g(\theta_t)\mathbf{1}_{f(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\}d\theta_t|\theta_0] \\
&\geq E[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\{g(\theta_t)\mathbf{1}_{g(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\}d\theta_t|\theta_0] = Tg(\theta_0)
\end{aligned}$$

The first step uses $f \geq g$ while the second step uses the fact that if $f(z) < 0 \Rightarrow g(z) < 0$ so $g(z)\,1_{g(z)<0} = g(z)\,1_{f(z)<0} + g(z)\,1_{f(z)\geq 0\cap g(z)<0} \leq g(z)\,1_{f(z)<0}$.

(ii) Discounting. Take $a > 0$. Then,

$$\begin{aligned}
T(f(\theta_0)+a) &= E[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\{(f(\theta_t)+a)\mathbf{1}_{f(\theta_t)+a<0\cap\theta_t<\bar{\theta}} + h(\theta_t)\}d\theta_t|\theta_0] \\
&= E[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\{(f(\theta_t)+a)\mathbf{1}_{f(\theta_t)<0\cap\theta_t<\bar{\theta}} + h(\theta_t) - (f(\theta_t)+a)\mathbf{1}_{-a\leq f(\theta_t)<0\cap\theta_t<\bar{\theta}}\}d\theta_t|\theta_0] \\
&\leq Tf(\theta_0) + aE[\int_0^\infty e^{-(1+\frac{r}{\lambda})}\mathbf{1}_{f(\theta_t)<0\cap\theta_t<\bar{\theta}}d\theta_t|\theta_0] \\
&\leq Tf(\theta_0) + \frac{a}{1+\frac{r}{\lambda}}
\end{aligned}$$

Since $r > 0$ by Assumption 5, the result follows. Thus, by Blackwell's theorem $T : C(X) \to C(X)$ is a contraction. Since $\tilde{W}_{[0,\bar{\theta}]} \in C(X)$, by the contraction mapping theorem there exists a unique continuous $\tilde{W} : [0,\bar{\theta}] \to \mathbb{R}$ that solves (25). Given this, $\tilde{W}(\theta)$ for $\theta > \bar{\theta}$ is uniquely defined from (25).

$\square$

Next, we show that $W = \tilde{W}$,

Lemma 3. $W = \tilde{W}$

Proof. Let V_i be the value function associated with the export strategy "export iff $\tilde{W} \geq 0$ or $\theta \geq \bar{\theta}$". Note that $\tilde{W} = \pi_i + \lambda (V_e - V_i)$. Since $V_i \leq V_i \leq V_e$, it follows that $\tilde{W}(\theta) \geq \pi_i(\theta) \geq 0 \; \forall \theta \geq \bar{\theta}$. Hence, $\tilde{W}$ is a solution of (23). Note that since $\tilde{W}$ is unique, there can be no other continuous solution.

□

Lemma 4. W *is weakly increasing*

Proof. Take some weakly increasing function f and apply T for $\theta \in [0, \bar{\theta}]$,

$$Tf(\theta) = E\left(\int_0^\infty e^{-\left(1+\frac{r}{\lambda}\right)} \left\{ f(\theta_t) \mathbf{1}_{f(\theta_t)<0 \cap \theta_t < \bar{\theta}} + h(\theta_t) \right\} d\theta_t | \theta_0 \right)$$

Since $f(z) 1_{f(z)<0 \cap \theta < \bar{\theta}} + h(z)$ is weakly increasing and θ has the FOSD property, Tf is also weakly increasing. Since the space of bounded, continuous and weakly increasing functions is complete, $\tilde{W}$ is also weakly increasing in $\left[0, \bar{\theta}\right]$. By Lemma 3, $W(\theta)$ is weakly increasing in $\left[0, \bar{\theta}\right]$. Since $W \geq 0$ for $\theta \geq \bar{\theta}$, (23) immediately implies W is weakly increasing also for $\theta \geq \bar{\theta}$.

□

Now we are ready to prove the main result,

Proposition. *The unique piecewise continuous optimal strategy features a threshold θ^* for $\theta < \theta^*$ not exporting is optimal while for $\theta > \theta^*$ exporting is optimal.*

Proof. Since W is continuous, $V_i \in C^2$ everywhere and satisfies the following HJB,

$$rV^I = \max\{W, 0\} + \mu' \theta \frac{dV^I}{d\theta} + \frac{1}{2}\sigma^2 \theta^2 \frac{d^2 V_i}{d\theta^2}.$$

If $\lambda = 0$, then $W = \pi_i$ and by A1 the result follows. If $\lambda > 0$, then by Lemma 3 W is weakly increasing, so it follows that there exists a unique $\theta^* \in \left[0, \bar{\theta}\right]$ such that $W \geq 0$ for $\theta > \theta^*$ and $W < 0$ for $\theta < \theta^*$.

□

Note that since $\theta^* \leq \bar{\theta} = \frac{F}{\kappa}$, $\tilde{\theta}^* \leq 1$. Furthermore, since $W\left(\tilde{\theta}^*\right) = 0$ and $E_\psi V_e - V_i > 0$ when $\tilde{\theta} = 1$ and $\psi \neq 1$, it follows that $\tilde{\theta}^* < 1$ in this case.

A.2 Derivation of the threshold equation (8)

In the GBM case, the HJB equations become

$$rE_{\psi}\left(V_e\right) = \pi^{EE}\left(\theta_t\right) + \mu \frac{dE_{\psi}V_e}{d\theta} + \frac{1}{2}\sigma^2 \frac{d^2 E_{\psi} V_e}{d\theta^2} \tag{26}$$

$$\begin{aligned} (r+\lambda) V_i &= \pi_i + \lambda E_{\psi} V_e + \mu \frac{dV_i}{d\theta} + \frac{1}{2}\sigma^2 \frac{d^2 V_i}{d\theta^2} \text{ when } \theta > \theta^* & (27) \\ rV^I &= \mu \frac{dV_i}{d\theta} + \frac{1}{2}\sigma^2 \frac{dV_i}{d\theta} \text{ when } \theta < \theta^* & (28) \end{aligned}$$

Define $\Delta V \equiv E_{\psi}\left(V_e\right) - V_i$. Substracting (27) and (28) from (26) yields

$$\begin{aligned} (r+\lambda) \Delta V &= \pi^{EE}(\theta) - \pi_i + \mu \frac{d\Delta V}{d\theta} + \frac{1}{2}\sigma^2 \frac{d^2 \Delta V}{d\theta} \text{ when } \theta > \theta^* & (29) \\ r\Delta V &= \pi^{EE}(\theta) + \mu \frac{d\Delta V}{d\theta} + \frac{1}{2}\sigma^2 \frac{d^2 \Delta V}{d\theta} \text{ when } \theta < \theta^* & (30) \end{aligned}$$

When $\theta > \theta^*$, the solution to (29) is given by[36]

$$\begin{aligned} \Delta V(\theta) = \frac{1}{\tilde{J}} & \left[\int_{\theta}^{\infty} \left(\frac{\theta}{z}\right)^{\tilde{\beta}_1} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z} + \int_{\theta^*}^{\theta} \left(\frac{\theta}{z}\right)^{\tilde{\beta}_2} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z} \right] \\ & + C_{1U}\theta^{\tilde{\beta}_1} + C_{2U}\theta^{\tilde{\beta}_2} \end{aligned}$$

where

$$\begin{aligned} \tilde{J} &= \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2(r+\lambda)\sigma^2} \geq \left|\mu - \frac{1}{2}\sigma^2\right| \\ \tilde{\beta}_1 &= \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) + \tilde{J}}{\sigma^2} > 1 \\ \tilde{\beta}_2 &= \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) - \tilde{J}}{\sigma^2} < 0 \end{aligned}$$

and C_{1U} and C_{2U} are unknown constants. Using the transversality condition, $C_{1U} = 0$.

[36] See formula 5.24. in Stokey (2008).

Note the derivative wrt θ is

$$\frac{d\Delta V}{d\theta} = \frac{1}{\theta}\left[\begin{array}{c} \tilde{\beta}_1 \frac{1}{\tilde{J}} \int_\theta^\infty \left(\frac{\theta}{z}\right)^{\tilde{\beta}_1} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z} \\ +\tilde{\beta}_2 \frac{1}{\tilde{J}} \int_{\theta^*}^\theta \left(\frac{\theta}{z}\right)^{\tilde{\beta}_2} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z} \\ +\tilde{\beta}_2 C_{2U} \theta^{\tilde{\beta}_2} \end{array}\right]$$

When $\theta < \theta^*$, the solution to (30) is given by

$$\Delta V(\theta) = \frac{1}{J}\left[\int_\theta^{\theta^*} \left(\frac{\theta}{z}\right)^{\beta_1} \pi^{EE}(z) \frac{dz}{z} + \int_0^\theta \left(\frac{\theta}{z}\right)^{\beta_2} \pi^{EE}(z) \frac{dz}{z}\right] + C_{1D}\theta^{\beta_1} + C_{2D}\theta^{\beta_2}$$

where

$$\begin{aligned} J &= \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2} \geq \left|\mu - \frac{1}{2}\sigma^2\right| \\ \beta_1 &= \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) + J}{\sigma^2} > 1 \\ \beta_2 &= \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) - J}{\sigma^2} < 0 \end{aligned}$$

and C_{1D} and C_{2D} are unknown constants. Using the initial condition $\Delta V(0) = 0$, $C_{2D} = 0$.

Note the derivative wrt θ is

$$\frac{d\Delta V}{d\theta} = \frac{1}{J}\frac{1}{\theta}\left[\begin{array}{c} \beta_1 \int_\theta^{\theta^*} \left(\frac{\theta}{z}\right)^{\beta_1} \pi^{EE}(z) \frac{dz}{z} \\ +\beta_2 \int_0^\theta \left(\frac{\theta}{z}\right)^{\beta_2} \pi^{EE}(z) \frac{dz}{z} \\ +\beta_1 C_{1D} \theta^{\beta_1} \end{array}\right]$$

We have three unknowns, C_{1D}, C_{2U} and θ^*. Using the fact that ΔV is C1 at θ^*,

$$\begin{aligned} \frac{1}{\tilde{J}}\left[\int_{\theta^*}^\infty \left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z}\right] + C_{2U}\theta^{*\tilde{\beta}_2} &= \frac{1}{J}\left[\int_0^{\theta^*} \left(\frac{\theta^*}{z}\right)^{\beta_2} \pi^{EE}(z) \frac{dz}{z}\right] \\ &\quad + C_{1D}\theta^{*\beta_1} \\ \frac{1}{\tilde{J}}\left[\tilde{\beta}_1 \int_{\theta^*}^\infty \left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1} \left(\pi^{EE}(z) - \pi_i(z)\right) \frac{dz}{z}\right] + \tilde{\beta}_2 C_{2U}\theta^{*\tilde{\beta}_2} &= \frac{1}{J}\left[\beta_2 \int_0^{\theta^*} \left(\frac{\theta^*}{z}\right)^{\beta_2} \pi^{EE}(z) \frac{dz}{z}\right] \\ &\quad + \beta_1 C_{1D}\theta^{*\beta_1}. \end{aligned}$$

Next, multiply the first equation by β_1 and substract the second equation to obtain,

$$\left(\frac{\beta_1-\tilde{\beta}_1}{\tilde{J}}\right)\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}$$
$$+\left(\beta_1-\tilde{\beta}_2\right)C_{2U}\theta^{*\tilde{\beta}_2}=\left(\frac{\beta_1-\beta_2}{J}\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}(z)\frac{dz}{z}$$

Rearranging,

$$C_{2U}=\frac{\theta^{*-\tilde{\beta}_2}}{\beta_1-\tilde{\beta}_2}\{\left(\frac{\beta_1-\beta_2}{J}\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}(z)\frac{dz}{z}+ \left(\frac{\tilde{\beta}_1-\beta_1}{\tilde{J}}\right)\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}\} \tag{31}$$

Since $\pi^{EE}-\pi_i\geq 0$ and $\tilde{\beta}_1\geq\beta_1$, it follows that $C_{2U}\geq 0$.

Next, multiply the first equation by $\tilde{\beta}_2$ and substract the second equation to obtain,

$$\left(\frac{\tilde{\beta}_2-\tilde{\beta}_1}{\tilde{J}}\right)\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}=\left(\frac{\tilde{\beta}_2-\beta_2}{J}\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}(z)\frac{dz}{z}$$
$$+\left(\tilde{\beta}_2-\beta_1\right)C_{1D}\theta^{\beta_1}$$

Rearranging,

$$C_{1D}=\frac{\theta^{*-\beta_1}}{\beta_1-\tilde{\beta}_2}\{\left(\frac{\tilde{\beta}_1-\tilde{\beta}_2}{\tilde{J}}\right)\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z} +\left(\frac{\tilde{\beta}_2-\beta_2}{J}\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}(z)\frac{dz}{z}\} \tag{32}$$

The remaining equation is the fact that by continuity the conjecture can only be true if at the threshold the firm is indifferent between exporting and not exporting, ie. $\pi_i(\theta^*)+\lambda\Delta V(\theta^*)=0$,

$$\pi_i(\theta^*)+\frac{1}{\tilde{J}}\lambda\left[\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}+C_{2U}\theta^{*\tilde{\beta}_2}\right]=0$$

Substituting in (31),

$$\pi_i(\theta^*)+\lambda\left[\begin{array}{c}\frac{1}{\tilde{J}}\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}+\left(\frac{1}{J}\right)\left(\frac{\beta_1-\beta_2}{\beta_1-\tilde{\beta}_2}\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}(z)\frac{dz}{z}\\ +\left(\frac{1}{\tilde{J}}\right)\left(\frac{\tilde{\beta}_1-\beta_1}{\beta_1-\tilde{\beta}_2}\right)\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}(z)-\pi_i(z)\right)\frac{dz}{z}\end{array}\right]=0$$

Simplifying,

$$\pi_i\left(\theta^*\right)+\frac{\lambda}{\beta_1-\tilde{\beta}_2}\left[\begin{array}{c}\left(\tilde{\beta}_1-\tilde{\beta}_2\right)\frac{1}{\tilde{J}}\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}\left(z\right)-\pi_i\left(z\right)\right)\frac{dz}{z}\\+\frac{1}{J}\left(\beta_1-\beta_2\right)\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}\left(z\right)\frac{dz}{z}\end{array}\right]=0. \tag{33}$$

Next, note

$$\begin{aligned}\beta_1-\beta_2 &= \frac{2J}{\sigma^2}\\ \tilde{\beta}_1-\tilde{\beta}_2 &= \frac{2\tilde{J}}{\sigma^2}\\ \beta_1-\tilde{\beta}_2 &= \frac{J+\tilde{J}}{\sigma^2}\end{aligned}$$

Thus,

$$\pi_i\left(\theta^*\right)+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(\pi^{EE}\left(z\right)-\pi_i\left(z\right)\right)\frac{dz}{z}+\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}\pi^{EE}\left(z\right)\frac{dz}{z}\right]=0.$$

As suggested in the text, this equation shows that the model boils down to one equation in one unknown even if ψ is not multiplicative. For the case in the text, note $\pi^{EE}=E_\psi\left[\max\left\{\psi\frac{\kappa\theta}{F}-1,0\right\}\right]$ and $\pi_i=\kappa\theta-F$. Replacing,

$$\kappa\theta-F+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c}\int_{\theta^*}^{\infty}\left(\frac{\theta^*}{z}\right)^{\tilde{\beta}_1}\left(E_\psi\left[\max\left\{\psi\kappa z-F,0\right\}\right]-\left(\kappa z-1\right)\right)\frac{dz}{z}\\+\int_0^{\theta^*}\left(\frac{\theta^*}{z}\right)^{\beta_2}E_\psi\left[\max\left\{\psi\kappa z-F,0\right\}\right]\frac{dz}{z}\end{array}\right]=0$$

In terms of $\tilde{\theta}$ and redefining $z=\frac{\kappa z}{F}$.

$$\tilde{\theta}-1+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c}\int_{\tilde{\theta}^*}^{\infty}\left(\frac{\tilde{\theta}^*}{z}\right)^{\tilde{\beta}_1}\left(E_\psi\left[\max\left\{\psi z-1,0\right\}\right]-\left(z-1\right)\right)\frac{dz}{z}\\+\int_0^{\tilde{\theta}^*}\left(\frac{\tilde{\theta}^*}{z}\right)^{\beta_2}E_\psi\left[\max\left\{\psi z-1,0\right\}\right]\frac{dz}{z}\end{array}\right]=0.$$

A.3 Proof of Proposition 3

First, we show that when $\lambda\to\infty$, $\tilde{\theta}^*\left(\lambda\right)\to 0$. Recall the threshold equation,

$$\tilde{\theta}^*\left(\lambda\right)-1+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c}\int_{\tilde{\theta}^*(\lambda)}^{\infty}\left(\frac{\tilde{\theta}^*(\lambda)}{z}\right)^{\tilde{\beta}_1}\left(E_\psi\left[\max\left(\psi z-1,0\right)\right]-\left(z-1\right)\right)\frac{dz}{z}\\+\int_0^{\tilde{\theta}^*(\lambda)}\left(\frac{\tilde{\theta}^*(\lambda)}{z}\right)^{\beta_2}E_\psi\left[\max\left(\psi z-1,0\right)\right]\frac{dz}{z}\end{array}\right]=0.$$

Since $\frac{\lambda}{J+\tilde{J}} \to \infty$, it must be that

$$\lim_{\lambda\to\infty}\left[\begin{array}{c} \int_{\theta^*(\lambda)}^{\infty}\left(\frac{\theta^*(\lambda)}{z}\right)^{\tilde{\beta}_1}\left(E_\psi\left[\max\left(\psi z-1,0\right)\right]-\left(z-1\right)\right)\frac{dz}{z} \\ +\int_0^{\theta^*(\lambda)}\left(\frac{\theta^*(\lambda)}{z}\right)^{\beta_2}E_\psi\left[\max\left(\psi z-1,0\right)\right]\frac{dz}{z} \end{array}\right]=0$$

Note that for any θ^*, the first term goes to 0. Then,

$$\lim_{\lambda\to\infty}\left[\int_0^{\theta^*(\lambda)}\left(\frac{\theta^*\left(\lambda\right)}{z}\right)^{\beta_2}E_\psi\left[\max\left(\psi z-1,0\right)\right]\frac{dz}{z}\right]=0.$$

Since $h>0\ \forall\psi>M$, $E_\psi\left[\max\left(\psi z-1,0\right)\right]>0\ \forall z\neq 0$. Then,

$$\lim_{\lambda\to\infty}\theta^*\left(\lambda\right)=0.$$

Next, recall the formula for p_T,

$$p_T=\left\{\begin{array}{l} \int_{-\infty}^{0}\int_{s=0}^{T}\left(1-e^{-\lambda s}\right)P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)\omega_T\left(s,x\right)dsdx \\ \int_0^{\infty}\int_{s=0}^{T}\left\{\left(1-e^{-\lambda s}\right)P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)+e^{-\lambda s}\right\}\omega_T\left(s,x\right)dsdx \end{array}\right.$$

Note that p_{BT} is given by

$$p_{BT}=\left\{\int_0^{\infty}\int_{s=0}^{T}\omega_T\left(s,x\right)dsdx\right.$$

Substracting p_{BT} from p_T,

$$\begin{aligned} p_T-p_{BT} &= \int_{-\infty}^{0}\int_{s=0}^{T}\left(1-e^{-\lambda s}\right)P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)\omega_T\left(s,x\right)dsdx \\ &\quad -\int_0^{\infty}\int_{s=0}^{T}\left\{\left(1-e^{-\lambda s}\right)\left(1-P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)\right)\right\}\omega_T\left(s,x\right)dsdx \end{aligned}$$

Next, pick some $\tilde{z}>0$ and rewrite the previous expression as

$$\begin{aligned} p_T-p_{BT} &= \int_{-\infty}^{\tilde{z}}\int_{s=0}^{T}\left(1-e^{-\lambda s}\right)P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)\omega_T\left(s,x\right)dsdx \\ &\quad -\int_0^{\tilde{z}}\int_{s=0}^{T}\left(1-e^{-\lambda s}\right)\omega_T\left(s,x\right)dsdx \\ &\quad -\int_{\tilde{z}}^{\infty}\int_{s=0}^{T}\left\{\left(1-e^{-\lambda s}\right)\left(1-P\left(\ln\tilde{\psi}>-x_T-\frac{\ln(\psi_m\tilde{\theta}^*)}{\sigma}\right)\right)\right\}\omega_T\left(s,x\right)dsdx \end{aligned}$$

Next, pick some $\tilde{s} < \underline{T}$ and note

$$\begin{aligned} p_T - p_{BT} \quad < \quad & \int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \left(1 - e^{-\lambda s}\right) P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) \omega_T(s,x)\, ds dx \\ & - \int_{0}^{\tilde{z}} \int_{s=\tilde{s}}^{T} \left(1 - e^{-\lambda s}\right) \omega_T(s,x)\, ds dx \\ & - \int_{\tilde{z}}^{\infty} \int_{s=0}^{T} \left\{\left(1 - e^{-\lambda s}\right)\left(1 - P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)\right)\right\} \omega_T(s,x)\, ds dx \end{aligned}$$

Next, note that since $1-e^{-\lambda s} \to 1$ when $\lambda \to \infty$, then $\forall \epsilon_1 \in (0,1)$, $\exists \bar{\lambda}_1$ such that for $\lambda > \bar{\lambda}_1$, $1-e^{-\lambda s} > 1-\epsilon_1$ $\forall s > \tilde{s}$. Hence, for $\lambda > \bar{\lambda}_1$,

$$\begin{aligned} p_T - p_{BT} \quad < \quad & \int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \left(1 - e^{-\lambda s}\right) P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) \omega_T(s,x)\, ds dx \\ & - \int_{0}^{\tilde{z}} \int_{s=\tilde{s}}^{T} (1 - \epsilon_1)\, \omega_T(s,x)\, ds dx \\ & - \int_{\tilde{z}}^{\infty} \int_{s=0}^{T} \left\{\left(1 - e^{-\lambda s}\right)\left(1 - P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)\right)\right\} \omega_T(s,x)\, ds dx \end{aligned}$$

Since $1 - e^{-\lambda s} < 1$,

$$\begin{aligned} p_T - p_{BT} \quad < \quad & \int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} P\left(\ln \tilde{\psi} > -x - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) \omega_T(s,x)\, ds dx - \int_{0}^{\tilde{z}} \int_{s=\tilde{s}}^{T} (1 - \epsilon_1)\, \omega_T(s,x)\, ds dx \\ & - \int_{\tilde{z}}^{\infty} \int_{s=0}^{T} \left\{\left(1 - e^{-\lambda s}\right)\left(1 - P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)\right)\right\} \omega_T(s,x)\, ds dx \end{aligned}$$

Next, note $\forall \epsilon > 0, \tilde{z} > 0$ $\exists \bar{\lambda}_2$ such that $\forall \lambda > \bar{\lambda}_2$, $P\left(\ln \tilde{\psi} > -\tilde{z} - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) < \epsilon_2$ (this uses the fact that for all M there exists $\bar{\lambda}$ such that $\forall\ \lambda > \bar{\lambda}$, $\theta^* < M$). Next, pick $\epsilon_2 > 0$ and corresponding $\bar{\lambda}_2$ such that

$$\epsilon_2 < (1 - \epsilon_1) \inf_{T \in \left[\underline{T}, \overline{T}\right]} \left(\frac{\int_0^{\tilde{z}} \int_{s=\tilde{s}}^{T} \omega_T(s,x)\, ds dx}{\int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \omega_T(s,x)\, ds dx}\right). \tag{34}$$

Since $\tilde{s} < \underline{T}$, the numerator is strictly positive $\forall T$. Since $T > 0$, the denominator is also strictly positive. Since ω is continuous in T, the integrals are continuous functions of T and, since the denominator is always strictly positive, the quotient of the integrals is continuous in T. Hence, given that $\left[\underline{T}, \overline{T}\right]$ is compact, the infimum is attained and, thus, $\inf_{t \in \left[\underline{T}, \overline{T}\right]} \left(\frac{\int_0^{\tilde{z}} \int_{s=\tilde{s}}^{T} \omega(s,z;t) ds dz}{\int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \omega(s,z;t) ds dz}\right) > 0$. Hence, ϵ_2 is well-defined.

Picking $\bar{\lambda}_3 > \max\left\{\bar{\lambda}_1, \bar{\lambda}_2\right\}$ and noting $P\left(\ln \tilde{\psi} > -z - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) \leq P\left(\ln \tilde{\psi} > -\tilde{z} - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right) < \epsilon_2$

for $z \leq \tilde{z}$, implies

$$\begin{aligned} p_T - p_{BT} &< \epsilon_2 \int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \omega_T(s,x)\, dsdx - (1-\epsilon_1) \int_0^{\tilde{z}} \int_{s=\tilde{s}}^{T} \omega_T(s,x)\, dsdx \\ &- \int_{\tilde{z}}^{\infty} \int_{s=0}^{T} \left\{ \left(1 - e^{-\lambda s}\right) \left(1 - P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)\right)\right\} \omega_T(s,x)\, dsdx \end{aligned}$$

From the definition of ϵ_2 it follows that

$$\epsilon_2 \int_{-\infty}^{\tilde{z}} \int_{s=0}^{T} \omega_T(s,z)\, dsdz < (1-\epsilon_1) \int_0^{\tilde{z}} \int_{s=\tilde{s}}^{T} \omega_T(s,z)\, dsdz$$

$\forall T \in \left[\underline{T}, \overline{T}\right]$. Hence, picking $\bar{\lambda}_3 = \max\left\{\bar{\lambda}_1, \bar{\lambda}_2\right\}$,

$$p_T - p_{BT} < -\int_{\tilde{z}}^{\infty} \int_{s=0}^{T} \left\{ \left(1 - e^{-\lambda s}\right) \left(1 - P\left(\ln \tilde{\psi} > -x_T - \frac{\ln(\psi_m \tilde{\theta}^*)}{\sigma}\right)\right)\right\} \omega_T(s,x)\, dsdx,$$

which immediately implies $p_T - p_{BT} < 0$.

A.4 Proof of Proposition 4

Define $\hat{\theta} = \psi_m \tilde{\theta}$ and $\tilde{\psi} = \frac{\psi}{\psi_m}$ and rewrite equation (8),

$$\frac{1}{\psi_m}\hat{\theta} - 1 + \lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c} \int_{\frac{1}{\psi_m}\hat{\theta}^*}^{\infty} \left(\frac{\hat{\theta}^*}{\psi_m z}\right)^{\tilde{\beta}_1} \left(E\max\left\{\psi_m\tilde{\psi}z - 1\right\} - (z-1)\right)\frac{dz}{z} \\ + \int_0^{\frac{1}{\psi_m}\hat{\theta}^*} \left(\frac{\hat{\theta}^*}{\psi_m z}\right)^{\beta_2} E\max\left\{\psi_m\tilde{\psi}z - 1\right\}\frac{dz}{z} \end{array}\right] = 0$$

Let $\hat{z} \equiv \psi_m z$. Then,

$$
\begin{aligned}
&\frac{1}{\psi_m}\hat{\theta} - 1 \\
+\lambda\left(\frac{2}{J+\tilde{J}}\right)&\left[\begin{array}{c} \int_{\hat{\theta}^*}^{:\infty}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\tilde{\beta}_1}\left(E\max\left\{\tilde{\psi}\hat{z}-1,0\right\} - \frac{1}{\psi_m}\hat{z}+1\right)\frac{d\hat{z}}{\hat{z}} \\ +\int_0^{\hat{\theta}^*}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\beta_2}E\max\left\{\tilde{\psi}\hat{z}-1\right\}\frac{d\hat{z}}{\hat{z}} \end{array}\right] = 0 \\
&\frac{1}{\psi_m}\left(\hat{\theta} - \lambda\left(\frac{2}{J+\tilde{J}}\right)\int_{\hat{\theta}^*}^{:\infty}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\tilde{\beta}_1}d\hat{z}\right) \\
+\lambda\left(\frac{2}{J+\tilde{J}}\right)&\left[\begin{array}{c} \int_{\hat{\theta}^*}^{:\infty}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\tilde{\beta}_1}\left(E\max\left\{\tilde{\psi}\hat{z}-1,0\right\}+1\right)\frac{d\hat{z}}{\hat{z}} \\ +\int_0^{\hat{\theta}^*}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\beta_2}E\max\left\{\tilde{\psi}\hat{z}-1\right\}\frac{d\hat{z}}{\hat{z}} \end{array}\right] = 0 \\
&\frac{1}{\psi_m}\hat{\theta}\left(1-\lambda\frac{2}{J+\tilde{J}}\frac{1}{\tilde{\beta}_1-1}\right) \\
+\lambda\left(\frac{2}{J+\tilde{J}}\right)&\left[\begin{array}{c} \int_{\hat{\theta}^*}^{:\infty}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\tilde{\beta}_1}\left(E\max\left\{\tilde{\psi}\hat{z}-1,0\right\}+1\right)\frac{d\hat{z}}{\hat{z}} \\ +\int_0^{\hat{\theta}^*}\left(\frac{\hat{\theta}^*}{\hat{z}}\right)^{\beta_2}E\max\left\{\tilde{\psi}\hat{z}-1\right\}\frac{d\hat{z}}{\hat{z}} \end{array}\right] = 0
\end{aligned}
$$

Since $1-\lambda\frac{2}{J+\tilde{J}}\frac{1}{\tilde{\beta}-1} \geq 0$, the LHS decreases with ψ_m.

Changing the dummy of integration to $m = \frac{z}{\hat{\theta}^*}$,

$$
\frac{1}{\psi_m}\hat{\theta}\left(1-\lambda\frac{2}{J+\tilde{J}}\frac{1}{\tilde{\beta}_1-1}\right)+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c} \int_1^{:\infty} m^{-\tilde{\beta}_1}\left(E\max\left\{\tilde{\psi}m\hat{\theta}^*-1,0\right\}+1\right)\frac{d\hat{z}}{\hat{z}} \\ +\int_0^1 m^{-\beta_2}E\max\left\{\tilde{\psi}m\hat{\theta}^*-1\right\}\frac{dm}{m} \end{array}\right] = 0
$$

The first derivative wrt $\hat{\theta}^*$ yields

$$
\frac{1}{\psi_m}\left(1-\lambda\frac{2}{J+\tilde{J}}\frac{1}{\tilde{\beta}_1-1}\right)+\lambda\left(\frac{2}{J+\tilde{J}}\right)\left[\begin{array}{c} \int_1^{\infty} m^{-\tilde{\beta}_1+1}\frac{dE\left[\max\left\{\psi m\tilde{\theta}^*-1\right\}\right]}{dm\tilde{\theta}^*}\frac{dm}{m} \\ +\int_0^1 m^{-\beta_2+1}\frac{dE\left[\max\left\{\psi m\tilde{\theta}^*-1\right\}\right]}{dm\tilde{\theta}^*}\frac{dm}{m} \end{array}\right] > 0.
$$

Hence, the LHS increases with $\hat{\theta}^*$. Thus, by the implicit function theorem, $\frac{d\hat{\theta}^*}{d\psi_m} > 0$.

B Facts in Regression Framework

Table 7: Facts 1 and 2 controlling for composition

	(1)	(2)	(3)	(4)
	Dependent variable: Survival status			
	Entrants		Re-entrants	
Year 1	0.358***	0.361***	0.358***	0.360***
	(0.00257)	(0.0886)	(0.00257)	(0.0744)
Year 2	0.266***	0.263***	0.266***	0.262***
	(0.00237)	(0.0885)	(0.00237)	(0.0744)
Year 3	0.223***	0.222**	0.223***	0.218***
	(0.00223)	(0.0885)	(0.00223)	(0.0745)
Year 4	0.195***	0.195**	0.195***	0.188**
	(0.00212)	(0.0885)	(0.00212)	(0.0745)
Year 5	0.177***	0.178**	0.177***	0.169**
	(0.00204)	(0.0886)	(0.00204)	(0.0745)
Year 1*Re-ent.			0.101***	0.0899***
			(0.00578)	(0.00579)
Year 2*Re-ent.			0.123***	0.113***
			(0.00564)	(0.00565)
Year 3*Re-ent.			0.131***	0.121***
			(0.00554)	(0.00554)
Year 4*Re-ent.			0.133***	0.122***
			(0.00550)	(0.00551)
Year 5*Re-ent.			0.131***	0.121***
			(0.00542)	(0.00544)
Sector FE	no	yes	no	yes
Destination FE	no	yes	no	yes
Year FE	no	yes	no	yes
Observations	174,150	174,150	220,175	220,175
R-squared	0.261	0.287	0.294	0.315

Clustered errors by firm-destination in parenthesis.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 8: Effect of type of product on survival

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: Survival status							
	Entrants				Re-entrants			
Differentiated	-0.0547***		-0.0634***		-0.0547***		-0.0644***	
	(0.00375)		(0.00388)		(0.00375)		(0.00387)	
Year 1*Diff.		-0.0635***		-0.0730***		-0.0645***		-0.0746***
		(0.00538)		(0.00547)		(0.00513)		(0.00521)
Year 2*Diff.		-0.0610***		-0.0695***		-0.0621***		-0.0717***
		(0.00499)		(0.00509)		(0.00482)		(0.00491)
Year 3*Diff.		-0.0508***		-0.0593***		-0.0507***		-0.0602***
		(0.00472)		(0.00484)		(0.00457)		(0.00468)
Year 4*Diff.		-0.0526***		-0.0615***		-0.0531***		-0.0630***
		(0.00452)		(0.00465)		(0.00437)		(0.00449)
Year 5*Diff.		-0.0454***		-0.0538***		-0.0429***		-0.0523***
		(0.00434)		(0.00447)		(0.00420)		(0.00432)
Re-entr.*Diff					0.0157*	0.0157*	0.0175**	0.0175**
					(0.00869)	(0.00869)	(0.00866)	(0.00866)
Destination FE	no	no	yes	yes	no	no	yes	yes
Year FE	no	no	yes	yes	no	no	yes	yes
Observations	170,135	170,135	170,135	170,135	215,045	215,045	215,045	215,045
R-squared	0.265	0.265	0.272	0.272	0.297	0.297	0.303	0.303

All regressions include horizon dummies. Regressions in columns (5)-(8) also include the interaction of horizon and re-entrant dummies. They are omitted to save space, since the previous results are roughly unchanged. Clustered errors by firm-destination in parenthesis.

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

Table 9: Effect of distance on survival

	(1)	(2)	(3)	(4)
	Dependent variable: Survival status			
log(dist)*Diff.	-0.0254***		-0.0253***	
	(0.00289)		(0.00289)	
log(dist)*Homog.	0.00836**		0.00828**	
	(0.00377)		(0.00377)	
Year 1*Diff.*log(dist)		-0.0343***		-0.0340***
		(0.00424)		(0.00424)
Year 2*Diff.*log(dist)		-0.0246***		-0.0247***
		(0.00387)		(0.00387)
Year 3*Diff.*log(dist)		-0.0206***		-0.0206***
		(0.00366)		(0.00367)
Year 4*Diff.*log(dist)		-0.0197***		-0.0197***
		(0.00347)		(0.00347)
Year 5*Diff.*log(dist)		-0.0277***		-0.0275***
		(0.00336)		(0.00336)
Year 1*Homog.*log(dist)		-0.00244		-0.00284
		(0.00536)		(0.00536)
Year 2*Homog.*log(dist)		0.0168***		0.0170***
		(0.00496)		(0.00497)
Year 3*Homog.*log(dist)		0.00824*		0.00825*
		(0.00472)		(0.00472)
Year 4*Homog.*log(dist)		0.00984**		0.00974**
		(0.00454)		(0.00454)
Year 5*Homog.*log(dist)		0.00937**		0.00920**
		(0.00438)		(0.00439)
Year FE	no	no	yes	yes
Observations	168,315	168,315	168,315	168,315
R-squared	0.267	0.268	0.268	0.268

All regressions include horizon dummies, a differentiated good dummy, and the interaction between a differentiated good dummy and horizon dummies (omitted). Clustered errors by firm-destination in parenthesis.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 10: Effect of distance on survival for differentiated goods only

	(1)	(2)	(3)	(4)
	Dependent variable: Survival status			
log(dist)*Re-ent.	-0.00917		-0.00902	
	(0.00728)		(0.00729)	
Year 1*log(dist)*Re-ent.		-0.00578		-0.00560
		(0.00986)		(0.00987)
Year 2*log(dist)*Re-ent.		0.00107		0.00121
		(0.00972)		(0.00972)
Year 3*log(dist)*Re-ent.		-0.0102		-0.0101
		(0.00948)		(0.00948)
Year 4*log(dist)*Re-ent.		-0.0130		-0.0126
		(0.00944)		(0.00944)
Year 5*log(dist)*Re-ent.		-0.0179*		-0.0180*
		(0.00933)		(0.00933)
Year FE	no	no	yes	yes
Observations	128,885	128,885	128,885	128,885
R-squared	0.277	0.277	0.277	0.277

All regressions include horizon dummies, a re-entrant dummy, and log(distance), which are omitted. In addition, columns (2) and (4) include the interaction of horizon dummies with log(distance) , which are also omitted.Clustered errors by firm-destination in parenthesis.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.