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# Markov Chains, Eigenvalues and the Stability of Economic Growth Processes

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**Abstract**

In this paper we explore the data on economic growth processes in the last decades, assuming they follow Markov processes. We look for the regimes guiding them and define Markov chains according to which the time series switch from one regime to another. Our findings show that most of the growth processes are quite stable in the sense of remaining most of the time in a dominant regime. Furthermore, we do not find support for the hypothesis of convergence of economies. The main conclusion of our analysis is that growth processes can be better understood in terms of their idiosyncratic dominant regimes.

Keywords: Markov Process, Regime Switching, Economic Growth.

## 1 Introduction

Markov processes and, in particular, Markov chains have been applied to represent phenomena in many different fields. Memoryless stochastic processes can be seen in the light of these models, in which the future is conditioned only on the current state of the system (Privault (2018), Gagniuc (2017)).

In Economics, Hamilton (1989) developed a tool for analyzing the switches of observable variables between different regimes. For instance, between "high" and "low" levels of

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a time series. This *Markov switching* model allows to characterize how a non-stationary series transitions between different regimes, drawing the probability distribution of the switches between those regimes.

This approach allowed to study the properties of business cycles in different economies. So Lam (1990) and Boldin (1996) showed that Markov switching models with two and three regimes provided good forecasts of the cycles of the American economy. Filardo and Gordon (1998) found that Markov switching provided good predictions of the duration of cycles in the U.S. economy. Clements and Krolzig (2003) found similar results for Australia, Canada, France, Germany, Japan and the U.K., while Buckle et al. (2004) did the same for New Zealand. The approach has also been used to study (combined with EGARCH) the impact of oil shocks on stock markets in Aloui and Jammazi (2009) and to the detection of currency manipulation in Park and Kim (2019). While the accuracy of the method has been criticized by Harding and Pagan (2003), in Artis et al. (2004) a Markov switching vector autoregressive model was used to identify an European (i.e. multi-country) business cycle.

In this paper we will apply this perspective to analyze the distribution and the dynamics between different regimes in the context of the growth processes of a set of countries. We are also interested in the way in which the growth rate is affected by the Markov chain over regimes. While the regime switches are determined by the first eigenvalue, the components of the corresponding eigenvector do not bode well as explanatory variables of the actual values of per-capita growth rates. But other indexes derived from the transitions matrix of the process show significant relations with growth rates. So, for instance, the second eigenvalue, which is closely related to the *mixing time* in an ergodic process, has a significant impact on the long-term behavior of that variable.

The aforementioned property of economic growth, jointly with the fact that few of the real-world series of growth rates are non-stationary have relevant implications for the understanding of this phenomenon. We focus on the spectral properties of the transition matrix of Markov processes and their impact on the long-term behavior of growth series and their *stability*, understood as the dominance of one its *regimes*. We also check whether in the long run the economies tend to converge to similar average growth rates. This is the so-called *convergence hypothesis*.[1]

The conclusion is that in most cases the processes guiding the growth of the economies are quite stable, with one of the regimes largely prevailing over the other. Thus, the cases of countries like Argentina, with frequent regime changes seem quite unusual. Our results lead also to the rejection of the convergence hypothesis, since most countries remain under their dominant regime, high for some and low for others.

We find that the stability of the growth processes is quite robust to the introduction of

[1] A weaker version assumes the existence of *clubs* of convergence Quah (1997).

traditional explanatory variables, even those representing cross-country influences. This indicates that stability seems to be an inherent property of an economy, as well as its dominant regime.

The plan of this paper is as follows. We present the basics of the Markov switching method in section 2. In section 3 we present the data of growth processes around the world, the different indexes that can be derived from the matrix of transitions of the corresponding Markov processes. We see these indexes as candidates to be explanatory variables for average per-head growth. In Section 4 we presents the results of the corresponding OLS analysis. In subsection 4.1 we check the robustness of those results by adding another determinants of growth. In section 5 we present the conclusions of this study.

# 2 Markov Switching

Let us briefly recall what a Markov chain is. Consider a system with a finite number of states $1, \ldots, N$, such that any period $t \in \mathbb{N}$ the distribution of possible instations of the state variable $s_t$ satisfies the following condition:

$$P\left\{s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots\right\} = P\left\{s_t = j | s_{t-1} = i\right\} = p_{ij}$$

with $p_{i1} + p_{i2} + \cdots + p_{iN} = 1$.

Thus, each $p_{ij}$ represents the probability of the transition from state $i$ to state $j$. It is useful to express $P$ as a left stochastic matrix:[2]

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}$$

In the case of a reduced number of states, a graphical representation comes handy to understand the behavior of the process. Figures 1 and 2 represent Markov chains with two and three states, respectively.

The transition matrix $P$ is a useful tool to analyze the dynamics of process. So, for instance, in the case of that $P$ is *irreducible*, the *steady state* of the system is understood as an $N$-components vector $\pi = (\pi_1, \ldots, \pi_N)$ such that each $\pi_i$ is the long-term probability

[2]The literature usually represents $P$ as *right* stochastic matrix. The results in this paper are independent of this choice.

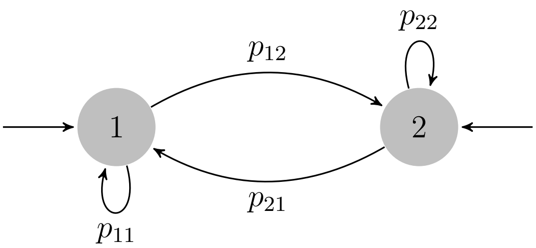



Figure 1: A two states Markov chain

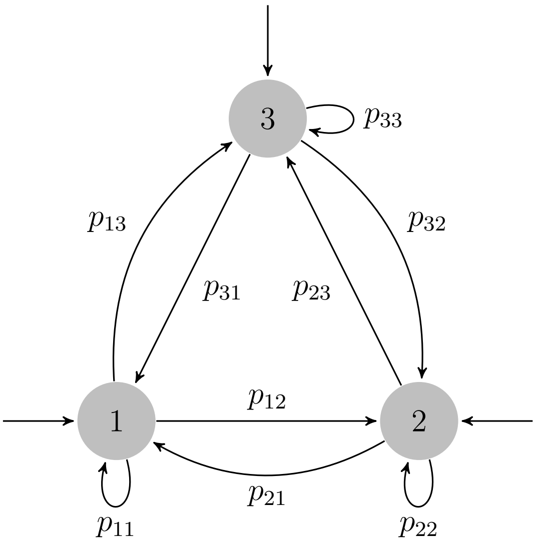



Figure 2: A three state Markov chain

of finding the system at state $i$. Then $\sum_i^N \pi_i = 1$. $\pi$ satisfies the following condition:

$$P\pi = \pi$$

which can also be seen, if $\lambda_1 = 1$ is the first eigenvalue of $P$, as indicating that $\pi$ is its associated eigenvector.

Now consider a time series $\{y_t\}_{t\geq 0}$. This means, in particular, that if the values are drawn from a compact set $Y$, the distributions over $Y$ at $t$, $F_t$ and at any period $t+k$, $F_{t+k}$ do not necessarily verify that $F_t = F_{t+k}$.[3]

This behavior can be interpreted as indicating that the values of $y_t$ go through different *regimes*. Enumerating the regimes as $1, \ldots, N$, the behavior of the series can be described as follows (Hamilton (1989), Hamilton (1994)):

$$y_t - \mu_{s_t^*} = \phi \left( y_{t-1} - \mu_{s_{t-1}^*} \right) + \varepsilon_s \tag{1}$$

where $\mu_{s_t^*} \in Y$ corresponds to the state $s_t^* \in \{1, \ldots, N\}$. If $s_t^* = j$ and $s_{t-1}^* = i$, at $t-1$,

[3]Note that this refers to the non-stationarity of the series of *observable* variables, not to the *non-observable* states of the system.

$\mu_i$ is followed in $t$ by $\mu_j$, with $\mu_i \neq \mu_j$. The transition from $\mu_{s^*_{t-1}}$ to $\mu_{s^*_t}$, corresponding to transition from state $j$ to state $i$ has probability $p_{ij}$. Thus $\phi$ is a function that embodies the combined action of $P$ and, for each state $i$ and period $t$, the conditional distribution $F_t(y|i)$.

Another possible specification is to switch between variances. The process can be still expressed as in (1) but in this case if $s^*_t = i$ the process has $\sigma_i$ and the same follows for any state in $\{1, \ldots, N\}$ with $\sigma_{s^*_t}$. Finally, we can include changes in mean and variance for each regime.[4]

The one with the highest Akaike Information Criterion (AIC) is usually chosen. This procedure embodies the idea that the model that captures better the behavior of the system should be the one with the least loss of information.

Then, the behavior of the variable $y_t$ follows a random walk on the graph of the Markov chain, specified by $P$ and other significant variables, which we are interested to disclose in the case of economic growth processes.

In the case of an *ergodic* Markov chain, there exists a positive probability of being at any of the states. Furthermore, and according to the Perron-Frobenius theorem (Levin and Peres (2017)), the corresponding transition $P$ will have a single eigenvalue equal to 1, leading to a unique steady state distribution over states. The rest of the eigenvalues $\lambda$ are such that $|\lambda| < 1$.

Of the eigenvalues of ergodic Markov chains other than 1, the second largest, $\lambda_2$, plays a relevant role in defining the *spectral gap*. If $\lambda_1 = 1$, the spectral gap is $S_\lambda = \lambda_1 - \lambda_2 = 1 - \lambda_2$.[5]

The *Cheeger constant* measures the connection among states in the Markov chain. It is defined as follows Cheeger (1969):

$$\chi \;=\; \min_{E \subset \{1,\ldots,n\}, \pi(E) \leq \frac{1}{2}} \frac{\sum_{j \in E, i \in E^c} P_{ij}\pi(j)}{\pi(E)} \tag{2}$$

where $E \cup E^c = \{1, \ldots, N\}$, $E \cap E^c = \emptyset$ and $\pi(E) = \sum_{i \in E} \pi(i)$.

We can interpret $\chi$ as a measure of the (inverse) length of the *mixing time*. Letting $P^t$ be the $t$-times product of $P$, the mixing time can be defined as the time $\hat{t}$ such that $|P^{\hat{t}}\pi_0 - \pi| < \epsilon$ for a given small constant $\epsilon$, starting from any initial distribution $\pi_0$. That is, the lower $\chi$, the longer time the series will take to converge, and, thus, the longer the series will remain at a given state without transitioning to another state. Hence, $\chi$ provides a notion of stability by measuring the amount of time at which the series remains in the same state. In our case, it indicates whether an economy which is at a high growth regime will remain growing at high rates for a long period afterwards.

[4] We allow 2 and 3 states with each state having it's own mean and variance. More states can be used for longer series if needed.

[5] An alternative definition of the spectral gap is as the smallest non-zero eigenvalue.

Although useful, the computation of $\chi$ according to its definition is involved. Thus, here it is where the spectral gap defined above becomes useful. Indeed, according to Alon (1986), the Cheeger constant of a Markov chain can be related with its spectral gap by means of the following equation

$$\frac{\chi^2}{2} \leq S_\lambda \leq 2\chi. \tag{3}$$

The condition in (3) implies that the smaller the spectral gap, the smaller must be $\chi$, and, thus, the lower the chance that the system switches from a set of states to its complement set in a single iteration. Furthermore, a smaller $S_\lambda$ makes the event that the system gets out of its current state less probable. As a consequence, the time required for the system to stabilize on a state increases.

For the problem we have at hand, all this means that the eigenvalues of $P$, inferred from the analysis of the time series of growth rates of an economy, will provide us with the information not only to say at which regime of growth is at a given time, but also to determine the stability of its *growth regimes*. From this analysis, we can deduce whether growth processes are stable, and if so, will allow us to predict which economies can be expected to be in a high growth regime in the future.

# 3 Data and regimes

Our dataset consists of 87 time series of rates of growth of per capita GDP for all the years of the 1960 to 2018 period, each one corresponding to a country with data reported in World Bank (2019). These countries and their codes are enumerated in tables 1 and 2, in which they are classified according to their levels of income, informed by the WB.

The first step in our analysis is to check whether these series are non-stationary. For this, we run Augmented Dickey-Fuller tests for the null hypothesis of a unit root of a univariate time series with one lag on three specifications of a linear model. One with a trend but without drift, another with drift without trend and finally one with both drift and trend. We find that the corresponding p-values under the three specifications of the results allow to discard the hypothesis of non-stationarity of the growth series in 80 or more series from among the 87 analyzed (see 8 in the Appendix).[6]

For each series we test six specifications of $s_t^*$, described in table 3. They differ in the number of states of the Markov process and a variable (or variables) that are associated to each state. We consider the mean, the variance or both. Checked in the table are the cases for which a change of state can be detected. We assign to each country the transition matrix $P$ corresponding to the most informative specification, i.e. the one yielding the

[6] Similar results are obtained with 0, 2 and 3 lags.

Table 1: Countries, country code and income group

| High Income | | Middle-High Income | |
|---|---|---|---|
| Australia | AUS | Argentina | ARG |
| Austria | AUT | Belize | BLZ |
| Belgium | BEL | Brazil | BRA |
| Chile | CHL | Botswana | BWA |
| Denmark | DNK | China | CHN |
| Spain | ESP | Colombia | COL |
| Finland | FIN | Costa Rica | CRI |
| France | FRA | Dominican Republic | DOM |
| Great Britain | GBR | Algeria | DZA |
| Greece | GRC | Ecuador | ECU |
| Iceland | ISL | Egypt | EGY |
| Israel | ISR | Fiji | FJI |
| Italy | ITA | Gabon | GAB |
| Japan | JPN | Guatemala | GTM |
| Republic of Korea | KOR | Guyana | GUY |
| Luxembourg | LUX | Mexico | MEX |
| Netherlands | NLD | Malaysia | MYS |
| Norway | NOR | Peru | PER |
| Panama | PAN | Paraguay | PRY |
| Puerto Rico | PRI | Suriname | SUR |
| Portugal | PRT | Thailand | THA |
| Singapore | SGP | Turkey | TUR |
| Sweden | SWE | Saint Vincent and the Grenadines | VCT |
| Seychelles | SYC | South Africa | ZAF |
| Trinidad and Tobago | TTO | | |
| Uruguay | URY | | |
| United States | USA | | |

Note: Country names are shown with their corresponding World Bank acronyms.

Table 2: Countries, country code and income group

| Middle-Low Income | | Low Income | |
|---|---|---|---|
| Bangladesh | BGD | Burundi | BDI |
| Bolivia | BOL | Benin | BEN |
| Ivory Coast | CIV | Burkina Faso | BFA |
| Cameroon | CMR | Central African Republic | CAF |
| Republic of Congo | COG | Democratic Republic of the Congo | COD |
| Ghana | GHA | Haiti | HTI |
| Honduras | HND | Madagascar | MDG |
| Indonesia | IDN | Malawi | MWI |
| India | IND | Nigeria | NER |
| Kenya | KEN | Nepal | NPL |
| Lesotho | LSO | Rwanda | RWA |
| Myanmar | MMR | Sierra Leone | SLE |
| Mauritania | MRT | Chad | TCD |
| Nigeria | NGA | Togo | TGO |
| Nicaragua | NIC | | |
| Pakistan | PAK | | |
| Philippines | PHL | | |
| Papua New Guinea | PNG | | |
| Sudan | SDN | | |
| Senegal | SEN | | |
| Zambia | ZMB | | |
| Zimbabwe | ZWE | | |

Note: Country names are shown with their corresponding World Bank acronyms.

lowest AIC index.

We derive from the corresponding matrix $P$ nine different indexes, defined in table **??** It should be noted that, given that we have chosen the "optimal" Markov process for each

Table 3: Specifications of $s_t^*$

| Specification | States | Mean | Variance |
|---|---|---|---|
| (1) | 2 | ✓ | |
| (2) | 2 | | ✓ |
| (3) | 2 | ✓ | ✓ |
| (4) | 3 | ✓ | |
| (5) | 3 | | ✓ |
| (6) | 3 | ✓ | ✓ |

country (as indicated by the AIC index), the computation of these indexes must take into account whether $P$ is a $2 \times 2$ or a $3 \times 3$ matrix.

Table 4: Indexes

| | | |
|---|---|---|
| Index 1 (Trace) | 2 states | $2 - p_{11} - p_{22}$ |
| | 3 states | $3 - p_{11} - p_{22} - p_{33}$ |
| Index 2 (Power) | 2 states | $(1 - p_{11})^2 + (1 - p_{22})^2$ |
| | 3 states | $(1 - p_{11})^2 + (1 - p_{22})^2 + (1 - p_{33})^2$ |
| Index 3 (Square Root) | 2 states | $[(1 - p_{11})^2 + (1 - p_{22})^2]^{\frac{1}{2}}$ |
| | 3 states | $[(1 - p_{11})^2 + (1 - p_{22})^2 + (1 - p_{33})^2]^{\frac{1}{2}}$ |
| Index 4 (Polynomial) | 2 states | $(1 - p_{11})^2 + (1 - p_{22})^2 + 2(1 - p_{11})(1 - p_{22})$ |
| | 3 states | $(1 - p_{11})^2 + (1 - p_{22})^2 + (1 - p_{33})^2 +$ $2(1 - p_{11})(1 - p_{22}) + 2(1 - p_{11})(1 - p_{33}) + 2(1 - p_{22})(1 - p_{33})$ |
| Index 5 (Eigenvalue) | 2 states | $\lambda_2$ |
| | 3 states | $\lambda_2$ |
| Index 6 (Eigenvalue*) | 3 states | $\lambda_3$ |
| Index 7 (1st Eigenvector) | 2 states | $2 \mid (\frac{1}{2} - \pi_1) \mid$ |
| | 3 states | $\mid (\frac{1}{3} - (\max(\pi))) \mid + \mid (\frac{1}{3} - (\min(\pi))) \mid$ |
| Index 8 (State Probability) | 2 states | $p_{11}$ |
| | 3 states | $p_{11}$ |
| Index 9 (Cross Probability) | 2 states | $p_{12}$ |
| | 3 states | $p_{12}$ |

The indexes obtained from $P$ capture different aspects of the dynamic process. Indexes

1 to 4 share a common pattern: a value of 0 means that every state is absorbing. The farther the index is from 0 the less stable the series is, since switches between states become more frequent. Indexes 5 and 6 use the eigenvalues of the transition matrix. Index 5 is the second eigenvalue ($\lambda_2$) of $P$ and thus conveys the same information as the *spectral gap* of the matrix. Index 6, the third eigenvalue ($\lambda_3$), is only defined for 3-state processes and also carries the information of the spectral gap, this time in its alternative characterization as the smallest positive eigenvalue. In the case of these last two indexes, as indicated in expression (3), values closer to 0 indicate that the process exhibits a fast convergence to a steady distribution of probabilities. Index 7 is based on the eigenvector $\pi$ associated with $\lambda_1 = 1$. It is defined in terms of the distance of the first component of $\pi$ ($\pi_1$) to $\frac{1}{2}$ when the process has 2 states and the sum of the distances of the maximum and minimum components of $\pi$ to $\frac{1}{3}$ in the case of 3 states. In both cases a large value of the index indicate that the process is more stable, understood as being more frequently in a given state. Finally, the last two indexes capture aspects of the Cheeger constant, representing either the probability of remaining in the first state or the probability of transitioning from state 1 to another state.

Figures 7 depicts the correlogram between the different indexes[7]in the case of $2 \times 2$ transition matrixes. It can be seen that indexes 1 to 4 are highly and positively correlated among them and highly but negatively correlated with indexes 5 and 8. That is, frequent regime changes are associated to a slow convergence towards a steady distribution and to a low probability of staying in the first state.

Table 7 in the Appendix presents the results for the sample of 87 countries plus the World economy (the growth of the aggregate of the 87 countries in the sample). Each one of the series has either two or three states, indicating their corresponding mean, standard deviation (in parentheses) and the percentage of time that economy spends at each one. The indexes defined in the previous section can be defined since all these Markov processes are *ergodic* since any state in them can be reached with positive probability form any other state in a finite number of steps. The states can be identified as *regimes of growth*.[8]

A first piece of evidence shows that the different regimes are not coordinated. The evidence in table 7 shows that the *dominant* state (i.e. the one at which the system is more frequently) is not uniformly the high or low one. Furthermore, the mean values corresponding to the regimes in different economies are quite different. These two facts seem to contradict the hypothesis of convergence among economies. As a support for this claim, see Figure 10 in the Appendix, which depicts the distribution for mean growth rates identified with the dominant regimes. Three features of this distribution are: i) the normality test is rejected, ii) many countries are below zero and iii) the median is greater

[7]The other variables will become relevant in Section 4.

[8]From now on we will use indistinctly *states* or *regimes* to denote them.

than the mean for this distribution (i.e. it is left skewed).

As an illustration, let us see the processes corresponding to Argentina, Australia and the World economy. Their evolution in time are depicted in Figures 3, 4 and 5, respectively. We can see that, Argentina and Australia are best modeled as being under two regimes, each one depicted with a different color, while the World economy can be described with three regimes. The gap between the lowest and highest regime is larger for Argentina, which spent 76% of the time in the low regime, unlike Australia that not only has a strictly positive lowest state but also stayed 86% of the period in the high regime.

Table 5 presents the values of the indexes computed for Argentina, Australia and the World economy. According to the interpretation given above, we can see that under indexes 1 to 4, Australia tends to switch less tan Argentina between states, while the World seems to be subject to more frequent switches than the two countries. This is not surprising considering that it results from aggregating a heterogeneous class of economies, with very different dynamics. This is confirmed further by the values of index 7. In turn, the values of index 5 indicate that Argentina seems to converge faster to a stable distribution among the states, while Australia and especially the World, are slower to converge to such distribution. This can be understood as that the latter two are more prone to be subject to unforeseen contingencies while in Argentina the instability seems inherent to the system. Finally, index 8 shows that Australia has a high probability of staying in its high state, while Argentina has nearly the same chances of staying in its higher state or going down to its lower one. In the case of the World, even with its instability, has a slighty higher probability of staying in its higher state than Argentina.

The results of this exercise are consistent with studies comparing Argentina and Australia (Gerchunoff and Fajgelbaum (2005) Esposto and Tohmé (2009)). The case of the World economy is better analyzed in the context of the comparison with the behavior of the entire sample, of which it is the aggregate.

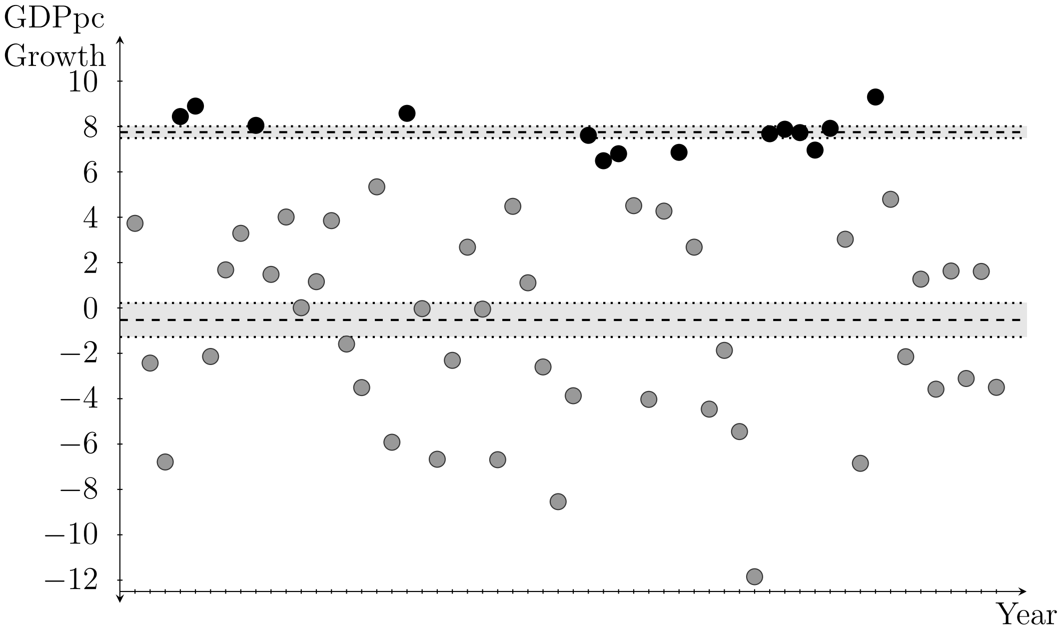



Figure 3: Growth of per capita GDP in *Argentina* and its estimated regimes

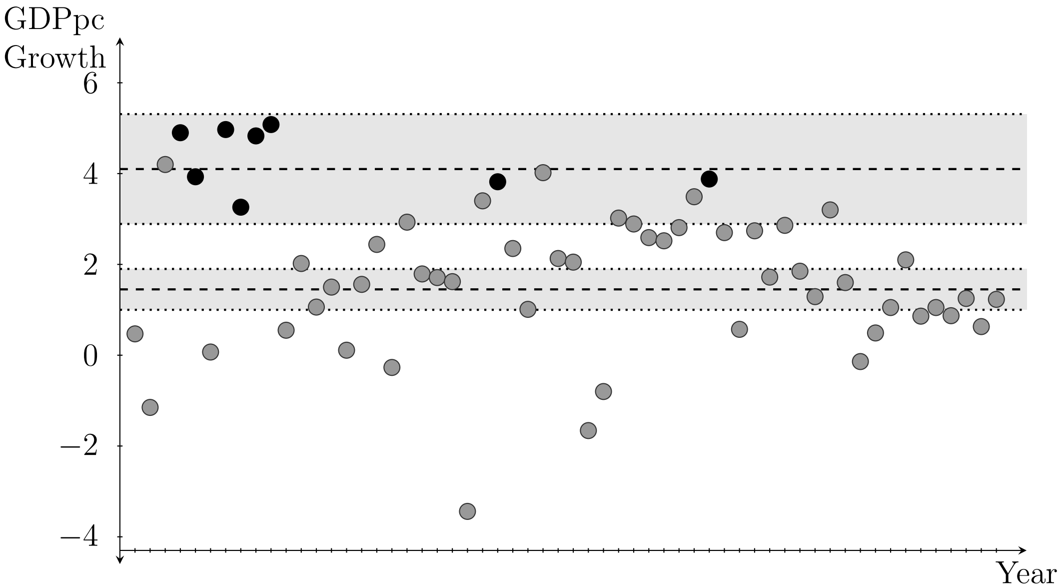



Figure 4: Growth of per capita GDP in *Australia* and its estimated regimes

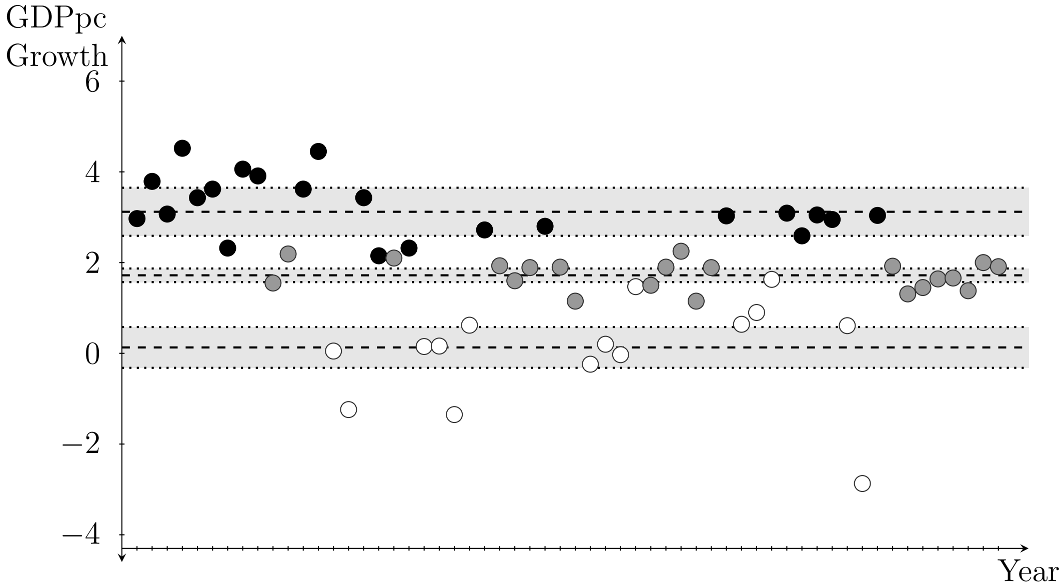



Figure 5: Growth of per capita GDP in *World* and its estimated regimes

Table 5: Indexes of ARG and AUS

| Index | Argentina | Australia | World |
|---|---|---|---|
| Index 1 (Trace) | 0.594 | 0.486 | 0.913 |
| Index 2 (Power) | 0.228 | 0.171 | 0.281 |
| Index 3 (Square Root) | 0.478 | 0.413 | 0.53 |
| Index 4 (Polynomial) | 0.353 | 0.236 | 0.833 |
| Index 5 (Eigenvalue) | 0.406 | 0.514 | 0.645 |
| Index 6 (Eigenvalue*) | - | - | 0.165 |
| Index 7 (1st Eigenvector) | 0.543 | 0.668 | 0.443 |
| Index 8 (State Probability) | 0.542 | 0.919 | 0.654 |
| Index 9 (Cross Probability) | 0.458 | 0.081 | 0.061 |

# 4 The impact of the indexes on average growth

The final step of our analysis involves seeing how the properties of the transition matrix impact on the average growth rate of an economy. The workflow of our entire analysis is captured in Algorithm 1.

**Algorithm 1** Workflow of the data analysis

1: Obtain the Markov Switching model for each country using the Expectation Maximization Algorithm (EM), Dempster et al. (1977), implemented in the `R` package `MSwM` (Sanchez-Espigares and Lopez-Moreno (2018)).
2: Select the specification with the lower AIC.
3: Given the transition matrix of each country, compute the indexes described in Table 4.
4: Compute the average per capita GDP growth for each country (AvePCGrowth).
5: Run a standard OLS regression of the form:
   - $\text{AvePCGrowth}_i = \beta_0 + \beta_1 \text{Index}_i + \epsilon_i$

The results of running the regression on the 87 series are shown in table 8. Notice that the p-values of the Breusch-Pagan test, all above 0.1, indicate that we cannot reject the null hypothesis of homoskedasticity. We can also see that from our nine indexes, seven are significant. Five of them at 1% (Trace, Power, Polynomial, Square Root, and State Probability). Eigenvalue and $p_{11}$ are significant at 5%. Finally, the third eigenvalue $\lambda_3$ (in the case of 3-state processes) and the index derived from the first eigenvector are not significant.

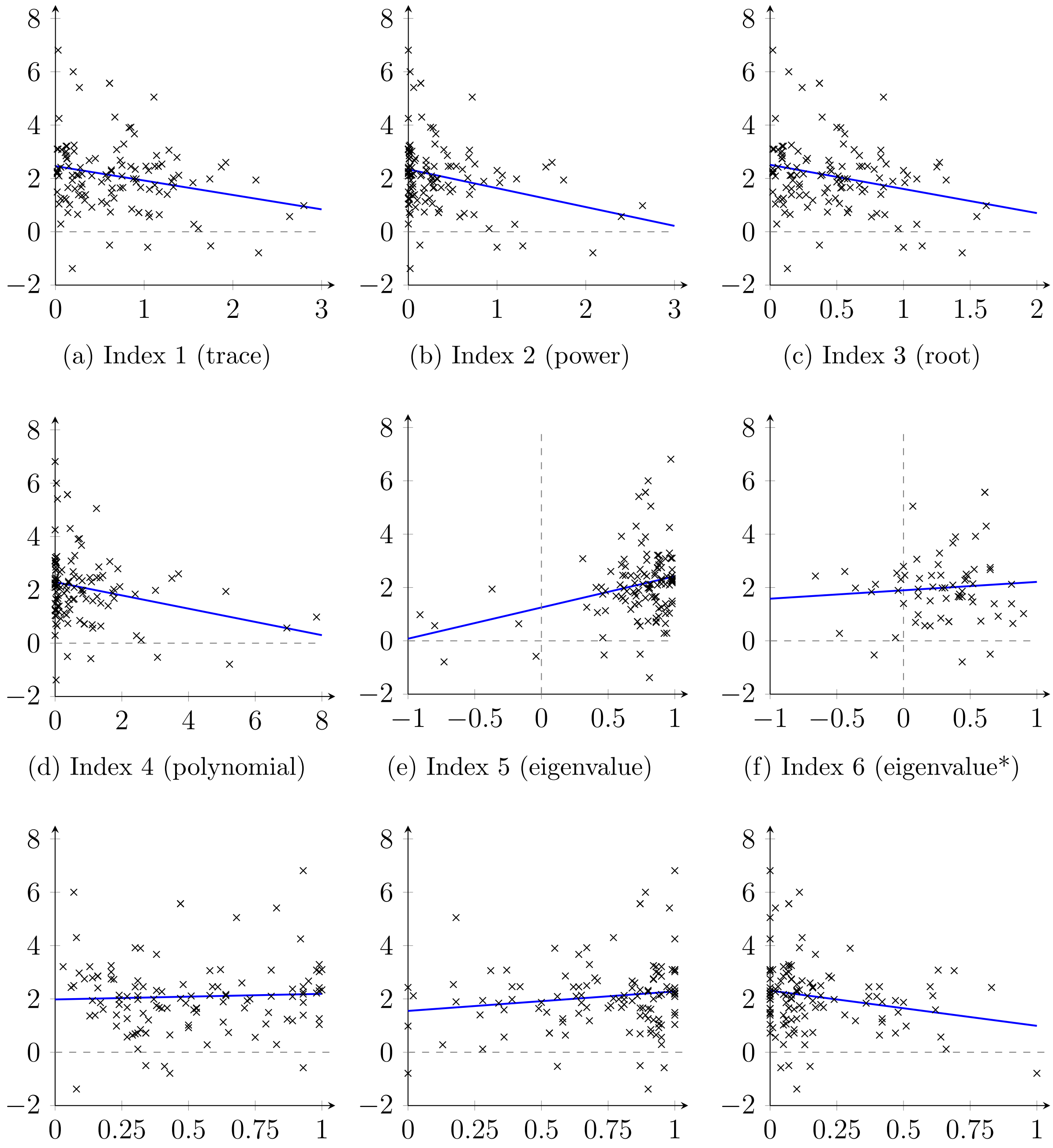
The interpretation of these results is that the average growth rate is affected by the degree in which states are absorbing, represented also by the probabilities of staying at the highest state. It is also impacted by the speed of convergence to a steady distribution of states. Notice that while these results are rather unsurprising, they convey an important insight: the systemic behavior of the growth rate, *independently of other relevant variables*, has by itself an impact on its average value.

The evidence (graphically represented in figure 6) indicates clearly that frequent changes of state are associated to lower average growth rates (the impact of indexes 1 to 4). On the other hand, a fast convergence towards a steady distribution (index 5) and a high probability of staying at the highest state are related to higher average growth rates.

## 4.1 Robustness

Traditional studies of economic growth processes emphasize on the impact of variables like investment, exports, etc. (see, for instance, Sala-i Martin (1994), Sala-i Martin (1997), Levine and Renelt (1992), Berg et al. (2018) and Caraballo et al. (2017)). A question that can be raised is whether our previous results are an artifact of the omision of such variables, leading to rather tautological results.

To check out this possibility we add as explanatory variables in our regressions both traditional and non traditional determinants of growth, like investment, exports (both measured as percentage of GDP), human capital (as percentage of enrollment in secondary



(a) Index 1 (trace) (b) Index 2 (power) (c) Index 3 (root)

(d) Index 4 (polynomial) (e) Index 5 (eigenvalue) (f) Index 6 (eigenvalue*)

(g) Index 7 (1st eigenvector) (h) Index 8 (state probability) (i) Index 9 (cross probability)

Figure 6: Economic Growth vs. Index values

Note: The $x$-axis represents the value of the corresponding index while the $y$-axis represents the average economic growth of the per capita GDP. Each country observation is represented by an $\times$. The dashed lines stannd for the 0 in each axis. The thick line represents the OLS regression fitted model.

schools), inequality (using the Gini index), per-head income in 1960 and inflation. All the data is drawn from the World Bank statistics for the same period of time. Table 6 in the Appendix summarizes this information, indicating the number of observations, the values of the mean, standard deviation, minimum and maximum of the sample, as well as the 25% and 75% percentiles.

A first piece of evidence is obtained by considering the correlogram in figure 7. We can see that these new variables are rather uncorrelated with the indexes obtained above. This suggests that the information in the indexes is *not* the same as in the more traditional determinants of growth.

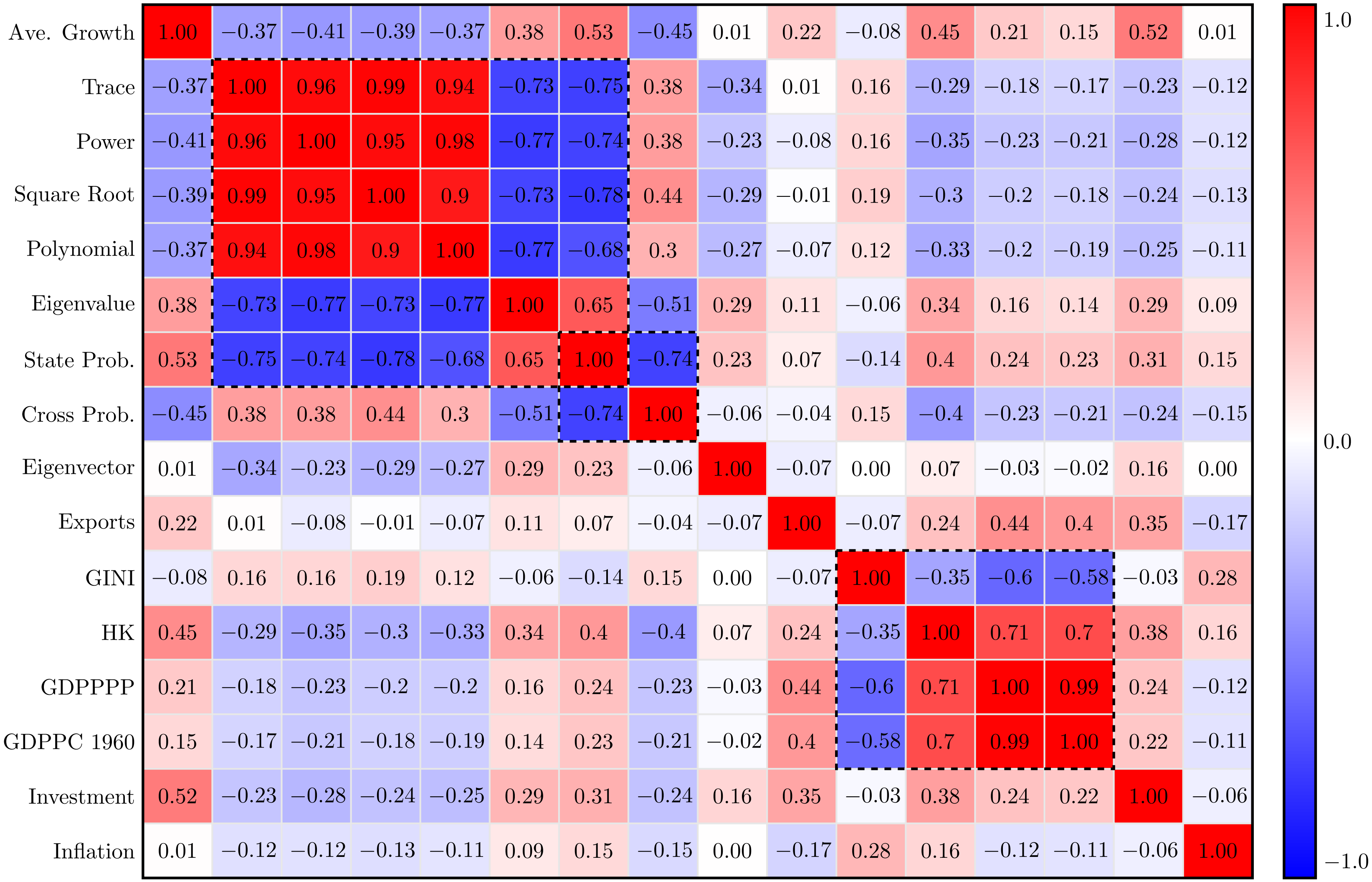



Figure 7: Correlations between indexes

The results of running the regression adding the new variables can be seen in table 9. Notice that the initial conditions in 1960 are significant in combination with all the indexes, same as investment (except in one case) and human capital, having all of them the expected signs.

With respect to the indexes, the ones reflecting the frequency of switchings (indexes 1 to 4) and the probability of staying in state 1 once there (index 7) are still highly significant. On the other hand, the second eigenvalue losses significance, meaning that the speed of convergence to a stable distribution losses explanatory power once investment and human capital enter in the picture.

One interesting case is that of the regression in which the third eigenvalue is an explanatory variable. Neither this index nor investment are significant while exports becomes significant. This could indicate that in economies with three regimes (the only ones in which $\lambda_3$ can be defined) international trade may be a relevant factor in defining the average growth rate. But the data does not seem to provide much support for this conjecture. In effect, as shown by the correlogram corresponding to index 6 (figure 9 in the Appendix), it is not correlated with exports or any other of the determinants of growth included in this robustness check.

In any case, the takeaway of this exercise is that the results obtained by regressing average growth on the indexes are not artifacts nor hide the impacts of far more relevant variables. On the contrary, they indicate that the properties of $P$, reflecting an implicit rule guiding the growth process, have an impact on the long-run behavior of economies.

# 5 Conclusion

In this paper, we have explore the behavior of economic growth processes as generated by Markov dynamics in which states are associated to *regimes*. The evidence shows that the following are properties of growth processes:

- The series are usually stationary.
- They have few (two or three) regimes.
- The Markov processes are ergodic.
- One of the regimes dominates the other. Each series stays much longer on the dominant regime.

This means that growth processes are quite stable, in general. Thus, the high dispersion in regimes and their average values seems to indicate that there is little evidence of a convergence process, as already hinted in Bickenbach and Bode (2003). Thus, economies seem to respond to the traditional determinants of growth as well as to their intrinsic dominant regime of growth.

The main conclusion of this work is that in order to understand any particular growth process it is relevant to focus on its dominant regime. Futher work is needed to understand the fundamentals that explain why each economy has a particular structure of regimes and which one is the dominant one.

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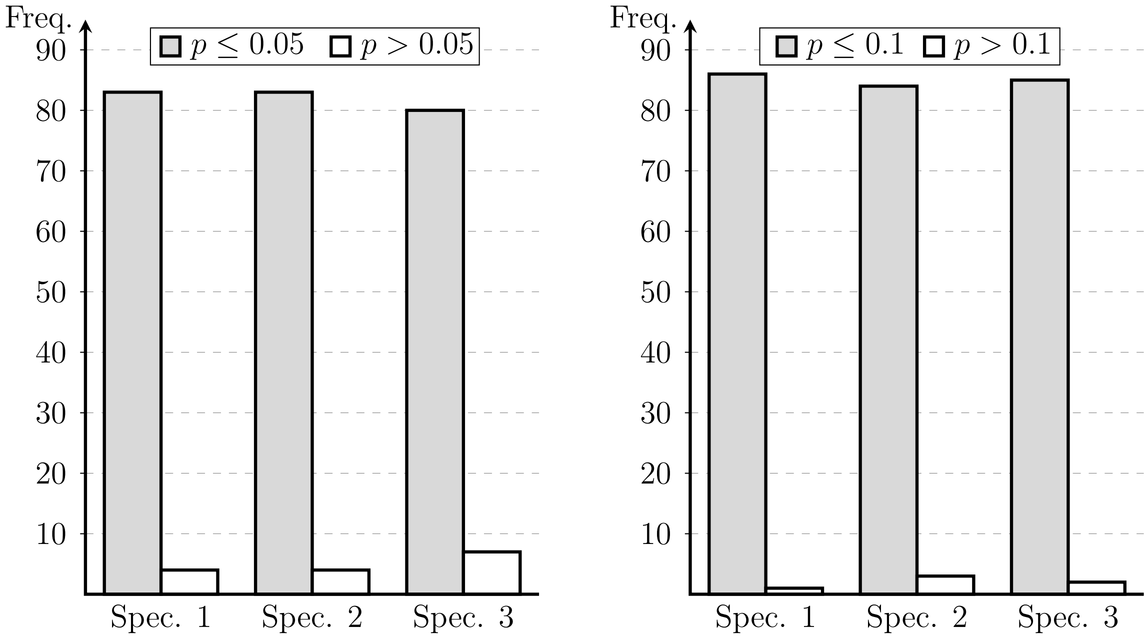
# 6 Appendix

Table 6: Descriptive Statistics

| Statistic | N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|---|
| Ave. Growth | 87 | 1.994 | 1.357 | −1.382 | 1.262 | 2.521 | 6.814 |
| Trace | 87 | 0.744 | 0.643 | 0.020 | 0.200 | 1.118 | 2.800 |
| Power | 87 | 0.414 | 0.560 | 0.0004 | 0.025 | 0.622 | 2.640 |
| Square Root | 87 | 0.503 | 0.403 | 0.020 | 0.159 | 0.788 | 1.625 |
| Polynomial | 87 | 0.962 | 1.470 | 0.0004 | 0.040 | 1.250 | 7.839 |
| Eigenvalue | 87 | 0.654 | 0.373 | −0.912 | 0.569 | 0.874 | 0.980 |
| Eigenvalue* | 47 | 0.263 | 0.382 | −0.722 | 0.001 | 0.550 | 0.902 |
| State Prob. | 87 | 0.728 | 0.278 | 0.000 | 0.626 | 0.928 | 1.000 |
| Cross Prob. | 87 | 0.173 | 0.234 | 0.000 | 0.030 | 0.194 | 1.000 |
| Eigenvector | 87 | 0.507 | 0.289 | 0.028 | 0.266 | 0.724 | 0.999 |
| Exports | 81 | 31.180 | 24.095 | 7.170 | 17.706 | 39.018 | 168.261 |
| GINI | 80 | 42.001 | 8.766 | 26.405 | 34.791 | 48.306 | 61.714 |
| HK | 77 | 57.516 | 25.927 | 7.454 | 34.080 | 78.754 | 99.658 |
| GDPPPP | 83 | 10,458.760 | 14,694.290 | 254.652 | 1,091.698 | 12,762.650 | 66,938.730 |
| GDPPC 1960 | 87 | 4,419.994 | 6,189.039 | 0.000 | 620.478 | 4,834.917 | 27,867.780 |
| Investment | 81 | 23.026 | 5.880 | 10.935 | 19.420 | 25.969 | 43.409 |
| Inflation | 80 | 19.820 | 48.958 | 1.073 | 4.646 | 11.615 | 309.122 |

**Note**: HK: Secondary Enrollment (%); Exports and Investment are presented as % of the GDP.




(a) p-value $\leq 0.05$ (b) p-value $\leq 0.1$

Figure 8: ADF test p-values for multiple specifications

The hight of the bars indicates the number of series satisfying the conditions.
**Note**: (Spec. 1) is a linear model with no drift and linear trend, (Spec. 2) is a linear model with drift but no linear trend and (Spec. 3) is a linear model with both drift and linear trend.

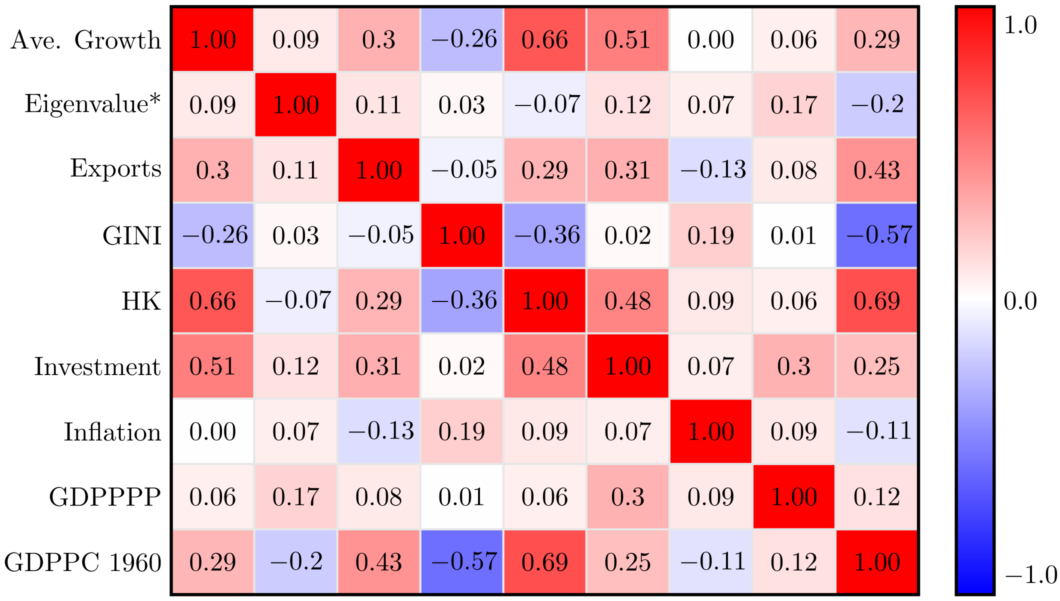



Figure 9: Correlations with three states

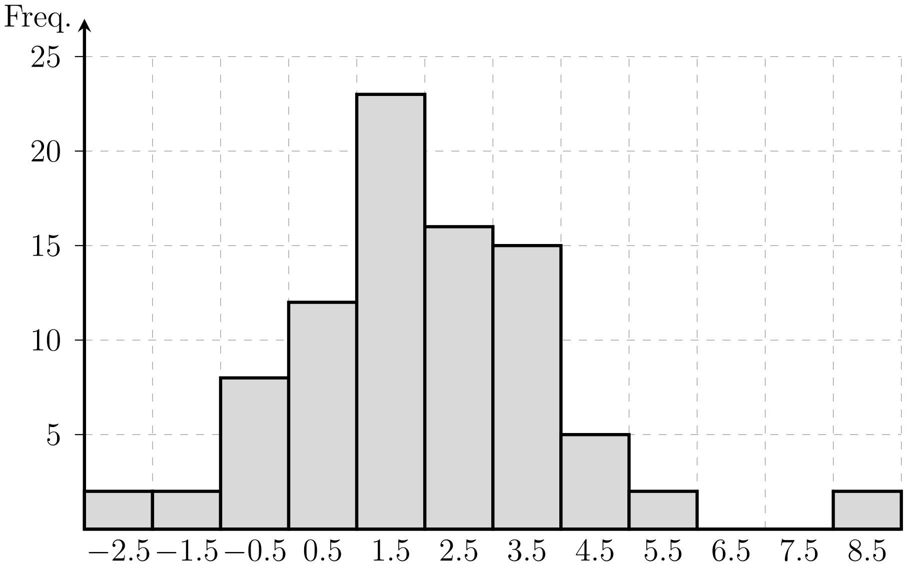



Figure 10: Histogram of means of the dominant regimes

Note: Shapiro-Wilk normality test: W = 0.96419, p-value = 0.01652

# Table 7: Regimes

| Country (Spec) | Regime 1 | Regime 2 | Regime 3 |
|---|---|---|---|
| ARG (1) | 7.75 (0.26) , 24.1% | −0.53 (0.75) , 75.9% | — |
| AUS (1) | 1.45 (0.45) , 86.2% | 4.1 (1.21) , 13.8% | — |
| AUT (2) | 4.03 (0.27) , 39.7% | 1.67 (0.27) , 60.3% | — |
| BDI (4) | 1.33 (0.32) , 36.2% | −2.58 (0.39) , 24.1% | 0.48 (1.46) , 39.7% |
| BEL (2) | 4.4 (0.24) , 34.5% | 1.53 (0.24) , 65.5% | — |
| BEN (4) | −1.05 (1.77) , 17.2% | 2.17 (0.62) , 32.8% | 1.23 (0.58) , 50% |
| BFA (2) | 0.28 (1.01) , 51.7% | 3.85 (1.01) , 48.3% | — |
| BGD (1) | −0.1 (1.53) , 50% | 3.15 (0.38) , 50% | — 0 |
| BLZ (5) | 2.38 (0.42) , 36.2% | 8.6 (0.42) , 22.4% | −0.73 (0.42) , 41.4% |
| BOL (2) | 2.54 (0.63) , 84.5% | −3.49 (0.63) , 15.5% | — |
| BRA (1) | 1.08 (0.49) , 84.5% | 7.61 (1.17) , 15.5% | — |
| BWA (1) | 4.18 (0.5) , 86.2% | 10.71 (3.29) , 13.8% | — |
| CAF (5) | −3.62 (3.43) , 1.7% | 1.83 (3.43) , 74.1% | −2.21 (3.43) , 24.1% |
| CHL (5) | 3.89 (0.61) , 87.9% | −9.97 (0.61) , 6.9% | 1.45 (0.61) , 5.2% |
| CHN (1) | 8.43 (0.41) , 62.1% | 2.74 (2.63) , 37.9% | — |
| CIV (2) | −2.28 (0.82) , 56.9% | 4.52 (0.82) , 43.1% | — |
| CMR (4) | −8.21 (1.06) , 12.1% | 1.38 (0.19) , 63.8% | 5.04 (1.89) , 24.1% |
| COD (1) | 1.43 (0.63) , 53.4% | −4.78 (1.86) , 46.6% | — |
| COG (1) | 0.31 (0.58) , 93.1% | 7.01 (7.69) , 6.9% | — |
| COL (5) | 1.08 (1.23) , 56.9% | −5.76 (1.23) , 1.7% | 3.74 (1.23) , 41.4% |
| CRI (5) | 3.28 (0.76) , 77.6% | −6.89 (0.76) , 3.4% | 0.92 (0.76) , 19% |
| DNK (1) | 2.21 (0.39) , 82.8% | 0.95 (0.22) , 17.2% | — |
| DOM (3) | 3.21 (0.52) , 36.2% | 3.21 (0.52) , 63.8% | — |
| DZA (6) | 1.39 (0.27) , 17.2% | 1.39 (0.27) , 51.7% | 1.39 (0.27) , 31% |
| ECU (1) | 1.09 (0.33) , 94.8% | 8.89 (0.84) , 5.2% | — |
| EGY (4) | 2.39 (0.28) , 48.3% | 0.28 (0.85) , 25.9% | 5.81 (0.85) , 25.9% |
| ESP (4) | −0.31 (0.73) , 25.9% | 2.79 (0.37) , 44.8% | 6.06 (0.69) , 29.3% |
| FIN (4) | 4.58 (0.89) , 41.4% | −1.21 (1.24) , 17.2% | 2.3 (0.19) , 41.4% |
| FJI (4) | 5.43 (1.41) , 5.2% | 2.25 (0.52) , 87.9% | −6.35 (0.77) , 6.9% |
| FRA (2) | 0.95 (0.27) , 58.6% | 3.93 (0.27) , 41.4% | — |
| GAB (1) | 0.58 (0.7) , 77.6% | 5.09 (4.8) , 22.4% | — |
| GBR (2) | 2.58 (0.73) , 86.2% | −1.64 (0.73) , 13.8% | — |
| GHA (4) | 1.6 (0.18) , 43.1% | −1.62 (1.12) , 41.4% | 3.97 (0.73) , 15.5% |
| GRC (5) | 7.32 (1.25) , 22.4% | 1.67 (1.25) , 69% | −5.9 (1.25) , 8.6% |
| GTM (6) | 1.36 (0.14) , 5.2% | 1.36 (0.14) , 67.2% | 1.36 (0.14) , 27.6% |
| GUY (4) | −10.46 (5.76) , 5.2% | 0.19 (2.5) , 50% | 5.46 (2.97) , 44.8% |
| HND (1) | 0.43 (0.65) , 53.4% | 2.59 (0.31) , 46.6% | — |
| HTI (4) | −0.05 (0.55) , 51.7% | 0.66 (0.33) , 22.4% | −2.47 (0.5) , 25.9% |
| IDN (1) | 0.8 (1.43) , 29.3% | 4.37 (0.19) , 70.7% | — |
| IND (1) | 1.98 (0.58) , 77.6% | 5.04 (0.38) , 22.4% | — |
| ISL (5) | −0.72 (2.5) , 43.1% | 4.72 (2.5) , 19% | 4.9 (2.5) , 37.9% |
| ISR (5) | −0.18 (0.81) , 8.6% | 1.71 (0.81) , 75.9% | 7.85 (0.81) , 15.5% |
| ITA (5) | 1.87 (4.13) , 32.8% | 4.29 (4.13) , 53.4% | −0.68 (4.13) , 13.8% |
| JPN (2) | 9.11 (0.33) , 22.4% | 1.99 (0.33) , 77.6% | — |
| KEN (4) | 3.07 (0.26) , 29.3% | −0.86 (0.21) , 34.5% | 3.2 (1.84) , 36.2% |
| KOR (1) | 3.2 (0.84) , 44.8% | 8.42 (0.46) , 55.2% | — |
| LSO (3) | 3.1 (0.4) , 53.4% | 3.1 (0.4) , 46.6% | — |
| LUX (4) | 1.34 (0.33) , 58.6% | 5.56 (0.71) , 32.8% | −0.63 (0.92) , 8.6% |
| MDG (4) | −0.82 (0.85) , 27.6% | −5.28 (2.22) , 6.9% | 0.33 (0.48) , 65.5% |
| MEX (4) | 1.8 (0.31) , 60.3% | −2.29 (1.89) , 17.2% | 5.65 (0.59) , 22.4% |
| MMR (1) | 8.27 (0.61) , 32.8% | 1.04 (0.97) , 67.2% | — |
| MRT (5) | 0.47 (2.31) , 8.6% | −0.47 (2.31) , 84.5% | 15.5 (2.31) , 6.9% |
| MWI (4) | 1.23 (0.21) , 36.2% | −4.01 (1.12) , 8.6% | 2.75 (0.79) , 55.2% |
| MYS (4) | 3.94 (0.21) , 56.9% | 1.84 (1.38) , 20.7% | 6.65 (0.21) , 22.4% |
| NER (1) | −19.25 (2.4) , 3.4% | 0.09 (2.4) , 96.6% | — |
| NGA (4) | 0.37 (2.66) , 27.6% | −0.06 (0.84) , 39.7% | 3.69 (0.47) , 32.8% |
| NIC (1) | −5.08 (2.95) , 20.7% | 2.43 (0.61) , 79.3% | — |
| NLD (4) | 3.77 (0.5) , 39.7% | 0.08 (0.53) , 25.9% | 1.88 (0.16) , 34.5% |
| NOR (2) | 3.77 (0.27) , 60.3% | 0.82 (0.27) , 39.7% | — |
| NPL (5) | 3.01 (0.69) , 84.5% | −2.19 (0.69) , 15.5% | 2.54 (0.69) , 0% |
| PAK (1) | 3.56 (0.42) , 56.9% | 0.67 (0.58) , 43.1% | — |
| PAN (5) | 5.1 (1.66) , 67.2% | 0.04 (1.66) , 31% | −15.19 (1.66) , 1.7% |
| PER (4) | 0.35 (0.73) , 44.8% | −11.15 (1.54) , 6.9% | 4.65 (0.51) , 48.3% |
| PHL (2) | −9.79 (0.27) , 3.4% | 2.29 (0.27) , 96.6% | — |
| PNG (4) | 4.89 (1.32) , 41.4% | −2.05 (0.63) , 46.6% | 4.42 (1.17) , 12.1% |
| PRI (5) | 4.04 (0.31) , 34.5% | 5.92 (0.31) , 25.9% | −0.52 (0.31) , 39.7% |
| PRT (4) | −0.04 (0.69) , 41.4% | 6.86 (0.49) , 32.8% | 3.26 (0.29) , 25.9% |
| PRY (5) | 2.68 (0.71) , 72.4% | 8.18 (0.71) , 12.1% | −2.98 (0.71) , 15.5% |
| RWA (1) | 2.93 (0.64) , 86.2% | −2.82 (8.29) , 13.8% | — |
| SDN (4) | −1.57 (1.12) , 56.9% | 11.36 (1.48) , 5.2% | 3.43 (0.7) , 37.9% |
| SEN (4) | 2.01 (0.43) , 55.2% | 3.69 (0.12) , 3.4% | −2.1 (0.91) , 41.4% |
| SGP (1) | −4.21 (0.53) , 8.6% | 5.89 (0.44) , 91.4% | — |
| SLE (1) | −1.09 (2.51) , 24.1% | 1.53 (0.39) , 75.9% | — |
| SUR (4) | −1.49 (0.97) , 39.7% | 9.58 (1.97) , 12.1% | 2.57 (0.58) , 48.3% |
| SWE (4) | −1.39 (1.19) , 15.5% | 1.31 (0.25) , 24.1% | 3.22 (0.33) , 60.3% |
| SYC (3) | 3.02 (0.27) , 5.2% | 3.02 (0.27) , 94.8% | — |
| TCD (4) | −0.8 (0.63) , 44.8% | 1.35 (1.18) , 46.6% | 2.24 (1.04) , 8.6% |
| TGO (1) | 0.91 (0.88) , 87.9% | 2.92 (0.21) , 12.1% | — |
| THA (4) | 3.57 (0.08) , 39.7% | 2.49 (1.22) , 29.3% | 7.36 (0.02) , 31% |
| TTO (2) | −2.68 (0.67) , 24.1% | 4.69 (0.67) , 75.9% | — |
| TUR (4) | −0.2 (1.87) , 32.8% | 2.76 (0.49) , 34.5% | 6.2 (0.47) , 32.8% |
| URY (1) | −1.79 (1.71) , 31% | 3.58 (0.69) , 69% | — |
| USA (3) | 1.99 (0.54) , 81% | 1.99 (0.54) , 19% | — |
| VCT (6) | 2.54 (0.45) , 12.1% | 2.54 (0.45) , 74.1% | 2.54 (0.45) , 13.8% |
| ZAF (5) | −3.01 (0.44) , 17.2% | 0.9 (0.44) , 50% | 3.09 (0.44) , 32.8% |
| ZMB (1) | 3.08 (0.65) , 25.9% | −1.1 (0.78) , 74.1% | — |
| ZWE (4) | 12.75 (1.5) , 12.1% | 0.98 (0.64) , 70.7% | −7.65 (2.39) , 17.2% |
| WLD (4) | 0.13 (0.45) , 25.9% | 1.72 (0.15) , 36.2% | 3.12 (0.53) , 37.9% |

Table 8: OLS results

| | *Dependent variable:* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | AveGrowth | | | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Trace | −0.601*** | | | | | | | | |
| | (0.218) | | | | | | | | |
| Power | | −0.702*** | | | | | | | |
| | | (0.251) | | | | | | | |
| Polynomial | | | −0.262*** | | | | | | |
| | | | (0.096) | | | | | | |
| Square Root | | | | −0.958*** | | | | | |
| | | | | (0.349) | | | | | |
| Eigenvalue | | | | | 0.967** | | | | |
| | | | | | (0.381) | | | | |
| Eigenvector | | | | | | 0.398 | | | |
| | | | | | | (0.505) | | | |
| StateProb | | | | | | | 1.587*** | | |
| | | | | | | | (0.500) | | |
| CrossProb | | | | | | | | −1.433** | |
| | | | | | | | | (0.615) | |
| Eigenvalue* | | | | | | | | | 0.262 |
| | | | | | | | | | (0.409) |
| Constant | 2.447*** | 2.289*** | 2.252*** | 2.480*** | 1.360*** | 1.794*** | 0.839** | 2.235*** | 1.692*** |
| | (0.216) | (0.175) | (0.169) | (0.226) | (0.287) | (0.293) | (0.389) | (0.176) | (0.186) |
| Observations | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 48 |
| $R^2$ | 0.082 | 0.084 | 0.081 | 0.082 | 0.071 | 0.007 | 0.106 | 0.060 | 0.009 |
| Adjusted $R^2$ | 0.071 | 0.074 | 0.070 | 0.071 | 0.060 | −0.004 | 0.096 | 0.049 | −0.013 |
| Residual Std. Error | 1.307 | 1.306 | 1.308 | 1.308 | 1.316 | 1.360 | 1.290 | 1.323 | 1.063 |
| F Statistic | 7.620*** | 7.837*** | 7.486*** | 7.542*** | 6.456** | 0.619 | 10.082*** | 5.435** | 0.411 |
| AIC | 297.51 | 297.31 | 297.64 | 297.59 | 298.61 | 304.35 | 295.23 | 299.59 | 146.07 |
| BP Test | 0.16 | 0.38 | 0.30 | 0.24 | 0.88 | 0.93 | 0.75 | 0.60 | 0.51 |

*Note:* $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 9: OLS results (cont):

| | *Dependent variable:* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | AveGrowth | | | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Exports | 0.008 | 0.007 | 0.007 | 0.007 | 0.006 | 0.006 | 0.007 | 0.007 | 0.019** |
| | (0.007) | (0.007) | (0.007) | (0.007) | (0.007) | (0.007) | (0.006) | (0.007) | (0.007) |
| GINI | −0.007 | −0.007 | −0.009 | −0.005 | −0.010 | −0.011 | −0.007 | −0.007 | −0.026 |
| | (0.015) | (0.015) | (0.015) | (0.015) | (0.016) | (0.016) | (0.015) | (0.015) | (0.017) |
| Inflation | −0.002 | −0.002 | −0.002 | −0.002 | −0.001 | −0.001 | −0.002 | −0.002 | −0.002 |
| | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.003) |
| Investment | 0.063*** | 0.063*** | 0.065*** | 0.062*** | 0.067*** | 0.076*** | 0.056** | 0.064*** | 0.019 |
| | (0.023) | (0.023) | (0.023) | (0.023) | (0.023) | (0.024) | (0.022) | (0.022) | (0.022) |
| HK | 0.022*** | 0.022*** | 0.022*** | 0.022*** | 0.022*** | 0.025*** | 0.019*** | 0.020*** | 0.037*** |
| | (0.006) | (0.007) | (0.007) | (0.006) | (0.007) | (0.007) | (0.006) | (0.006) | (0.006) |
| GDPPC 1960 | −0.0001** | −0.0001** | −0.0001** | −0.0001** | −0.0001** | −0.0001** | −0.0001** | −0.0001** | −0.0001*** |
| | (0.00003) | (0.00003) | (0.00003) | (0.00003) | (0.00003) | (0.00003) | (0.00003) | (0.00003) | (0.00003) |
| Trace | −0.361** | | | | | | | | |
| | (0.167) | | | | | | | | |
| Power | | −0.408** | | | | | | | |
| | | (0.193) | | | | | | | |
| Polynomial | | | −0.131* | | | | | | |
| | | | (0.072) | | | | | | |
| Square Root | | | | −0.640** | | | | | |
| | | | | (0.271) | | | | | |
| Eigenvalue | | | | | 0.439 | | | | |
| | | | | | (0.290) | | | | |
| Eigenvector | | | | | | −0.366 | | | |
| | | | | | | (0.389) | | | |
| StateProb | | | | | | | 1.359*** | | |
| | | | | | | | (0.396) | | |
| CrossProb | | | | | | | | −1.257*** | |
| | | | | | | | | (0.470) | |
| Eigenvalue* | | | | | | | | | −0.120 |
| | | | | | | | | | (0.305) |
| Constant | −0.006 | −0.030 | −0.063 | 0.010 | −0.444 | −0.269 | −0.954 | 0.062 | 0.343 |
| | (0.787) | (0.787) | (0.795) | (0.781) | (0.782) | (0.800) | (0.748) | (0.772) | (0.821) |
| Observations | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 40 |
| $R^2$ | 0.448 | 0.446 | 0.437 | 0.455 | 0.427 | 0.414 | 0.501 | 0.467 | 0.669 |
| Adjusted $R^2$ | 0.385 | 0.383 | 0.373 | 0.393 | 0.362 | 0.348 | 0.444 | 0.407 | 0.597 |
| Residual Std. Error | 0.862 | 0.863 | 0.870 | 0.856 | 0.877 | 0.887 | 0.819 | 0.846 | 0.669 |
| F Statistic | 7.182*** | 7.126*** | 6.864*** | 7.392*** | 6.605*** | 6.267*** | 8.887*** | 7.774*** | 9.258*** |
| AIC | 187.29 | 187.53 | 188.68 | 186.37 | 189.85 | 191.39 | 180.21 | 184.75 | 90.45 |

*Note:* *p<0.1; **p<0.05; ***p<0.01. Robust standard errors in parenthesis