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Cooperation and Retaliation in Legislative Bargaining*

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Abstract

We study a legislative-bargaining divide-the-pie game in which some legislators have the ability to affect the amount of resources to be distributed (positively or negatively). If included in the winning coalition, these legislators cooperate and increase the size of the pie. If excluded, they retaliate and decrease it. Cooperation and retaliation produce significant changes in the equilibrium allocation relative to Baron and Ferejohn (1989): The bargaining position of cooperating and retaliating legislators improves, and thus they are more likely to be included in the winning coalition (which may be larger-than-minimum). Some of these legislators may be excluded from the winning coalition, creating inefficient output losses. Moreover, output losses increase with legislators' patience.

JEL Classification: D72, D74, D61.

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1 Introduction

Distributive policies generally determine winners and losers. What if unhappy losers retaliate, causing a cost to society? From strikes and demonstrations to government shutdowns and filibusters, retaliation can reduce the amount of resources to be distributed. Conversely, happy winners may cooperate, producing positive externalities. For example, consider tax collection and environmental or trade treaties. Benefiting individuals or countries may increase their tax-collection effort, reduce their carbon emissions, or foster trade. A similar logic applies to political negotiations. Madison (1787), in The Federalist Papers (# 10) states this clearly: "If a faction consists of less than a majority, relief is supplied by the republican principle, which enables the majority...by regular vote." While a minority faction would not decide policies, it "may clog the administration, convulse the society, [and] it will be unable to execute and masks its violence under the forms of the Constitution."

In this paper, we focus on the allocation of resources through legislative bargaining (Baron and Ferejohn, 1989). We modify the setting by adding a fixed number of "active" legislators who – through *cooperation* or *retaliation*– have the ability to affect output. Active districts not included in the winning coalition cause a loss in output (retaliators would decrease resources and cooperators would not contribute to increase them). Hence, the resources to be divided are endogenous, and depend on the number of active districts that are excluded from the winning coalition. The agenda setter anticipates the externalities caused by the composition of the winning coalition. The nuances of our model implications paint a more cynical version than Madison's: Those players who can cooperate or retaliate are more likely to get a larger share of resources than the others.

Intuitively, the potentiality to "convulse the society" allows factions (cooperators and retalitors in the model) to gain something out of it, even if they are a minority: They are more likely to be included in a winning coalition, even if they are not pivotal in the legislative process. This rationale may hold even when the agenda setter already has a sufficient number of votes to pass her policies, resulting in larger-than-minimal winning coalitions.[1] In spite of the additional resources that they may bring (or not destroy), some of the active legislators may be left out of the winning coalitions. Thus, in some equilibria there are inefficient output losses.

[1]These results are in line with the empirical literature, in which larger-than-minimal winning coalitions are the norm. See Knight (2008) and references therein.

Contrary to results in other dynamic games of legislative decision making, e.g. Piguillem and Riboni (2015), in our setting an increase in patience leads to *more* inefficient outcomes: With sufficiently low patience, all active districts are included in the coalition and there are no output losses. Since continuation values increase with patience, active districts eventually stop being called into the winning coalition with certainty and there are output losses. Efficiency also depends on the voting rule, with an increase in the required supermajority reducing output losses. That an increase in the supermajority, or a reduction in patience, increases efficiency suggests that procedural rules should be made contingent on the number of cooperating or retaliating legislators.

We endogeneize the decision to become active and show that all legislators that have the option to become active choose to do so.[2] Legislators are agents of their constituencies, and in some circumstances the decision to become active is the result of grassroots movements. For example, the Great Recession, and the slow recovery from it, produced an outburst of protests in established democracies around the world. Occupy Wall Street in the United States, "indignados" in Spain, the anti-austerity movement in Greece are examples of demonstrations that can have an impact on economic activity, and may have affected legislators' actions.[3]

An example of bargaining spillovers that affect legislative outcomes is provided by the 1990 Clean Air Act Amendment in the U.S. This arrangement established the first environmental program to rely on tradable emission permits to control acid rain pollution by reducing sulfur dioxide (SO_2) and nitrogen oxide (NO_x) emissions from coal-powered electric generating plants. At the time, mid-west states were high emitters of SO_2 and NO_x, and delayed their support for the new regulation until they received sufficient compensation. This was achieved by giving polluting utilities free emission permits, which amounted to expected transfers of approximately two billion dollars per year in 2019 dollars.[4] Thus, polluting states used their higher bargaining power to obtain a benefit from the new legislation.

Our results do not only apply to bargaining in formal legislatures. Environmental negotiations that take place in the international arena are subject to

[2]The presence of output losses for some equilibria render this decision non trivial.

[3]Recent cases were the protests seen in 2019 and 2020 in Iran, Lebanon, Hong Kong, Colombia, Chile, Gilet Jaunes in France, Black Lives Matter in the U.S., etc.

[4]At the time of the regulation market prices were expected to be roughly $200/ton and permits for more than five million tons were initially allocated. The value of free permits was approximately 0.5% of total federal grants to state and local governments in 1990. For Ohio and Indiana, two of the most polluting states, the expected value of free permits was equivalent to 2.8% and 4.7% of federal grants in 1990. See Cooper et al. (2010) for a detailed analysis of the political negotiations that resulted in the free allocation of pollution permits.

different rules of engagement. A polluting country which does not support the outcome of an international environmental agreement may threaten to sustain pollution (imposing a negative externality on all other countries) unless it obtains a better deal. Conversely, countries may allocate more effort in reducing pollution if they perceive a benefit from cooperation. An example of how cooperation and retaliation forces might shape international agreements is the clean development mechanism set up in the aftermath of the 1997 Kyoto Protocol.[5]

Literature review

Since Baron and Ferejohn's (1989) seminal paper on multilateral bargaining in legislatures, there have been multiple and diverse contributions to the field.[6] Banks and Duggan (2000) generalizes bargaining to multidimensional policies, while Eraslan (2002) does the same regarding heterogeneous recognition probabilities and discounting. The effect of endogenous status quo, or persistence of agenda-setting power, has also been studied (Baron, 1996; Riboni and Ruge-Murcia, 2008; Diermeier and Fong, 2011). Snyder et al. (2005) look at voting power and recognition probabilities when legislators voting weights depend on parties' vote share. We differentiate from this literature by allowing for (positive and negative) externalities, an idea informally discussed in Calvert and Dietz (2006).

As in Baron and Ferejohn (1989), our model has an exogenous status quo. A notable feature of this type of models is that a proposal is passed with the minimum amount of votes required, i.e., with minimum winning coalitions.[7] It has been shown that larger than minimial winning coalitions are possible when Baron and Ferejohn's (1989) assumptions are relaxed: Banks (2000) and Groseclose and Snyder (2000) use sequential voting, Dal Bó (2007) relies on "pivotal bribing" in

[5]The clean development mechanism allows countries to implement part of their committed emission abatement targets through projects in countries that have ratified the Kyoto protocol but are not subject to such targets. This gives incentives to ratify the protocol both to countries that have to reduce emissions, as they can do so at a lower cost, and to countries that do not have to reduce emissions, as they will be recipients of foreign investment. See Beccherle and Tirole (2011).

[6]Examples of early works include Austen-Smith and Banks (1988), Baron (1991), Romer and Rosenthal (1978), and Romer and Rosenthal (1979).

[7]In his classical work, Riker (1962) poses that bargaining games with zero sum games must only feature minimum winning coalitions in equilibrium. Although this has been disputed since Shepsle (1974), and does not hold empirically, there are few papers that can account for larger-than-minimum winning coalitions. In single shot games, these larger winning coalitions can be explained with "open bargaining rules" that allow for amendments (for instance, Fréchette et al., 2003), while in dynamic games, unanimity can sometimes be achieved in steady state (e.g. Baron and Bowen, 2018). Similarly with endogenous status quo, like Anesi and Seidmann (2015).

committees, and Hummel (2009) allows for lobbying. Under these changes it turns out that it is possible to assemble cheaper coalitions than minimum winning ones.

Our work is also related to the theory of political failure by which politically determined policy choices lead to an inefficient allocation of resources (Acemoglu, 2003). The source of the inefficiency in our setting is that resources to be distributed is endogenous to the legislators who approved the proposed distribution, i.e. there are externalities. This type of endogeneity is present in the coalitional bargaining literature, where coalitional surplus depends on the coalition's members (see Stole and Zwiebel (1996); Manzini (1999); Dasgupta and Maskin (2007); Ray and Vohra (1999)).[8] In contrast to the protocols considered in this literature, and summarized in Ray and Vohra (2015), we find that –due to the constraints of Baron and Ferejohn's (1989) protocol and the nature of externalities– inefficiency is more likely the more patient legislators are.[9]

To our knowledge, ours is the first paper to study legislative bargaining, under Baron and Ferejohn's (1989) protocol, in which the size of rents depends on the composition of the winning coalition. Eraslan and Merlo (2017) consider a model in which players are heterogeneous with respect to the potential surplus they bring to the bargaining table, but the size of the pie depends on the (random) identity of the agenda setter. The paper closest to ours is Baranski (2019). In its setting players make costly contributions to a common surplus after having bargained over its division, thus the size of rents is also endogenous to the agenda setters' equilibrium proposals. Differently to our model, in Baranski (2019) players receive an endowment and are identical when negotiations begin. Output is the result of joint production, and bargaining is over equity shares instead of quantities. While some results are qualitatively similar to ours (e.g. winning coalitions are typically formed by two types of players), we delve deeper in two directions: our equilibria might feature non-minimal winning coalitions, and we allow for agents that can decrease rents. Additionally, we characterize efficiency and show that output losses are increasing in patience.

In most papers, policy making takes place exclusively within formal institutions, disregarding informal channels of influence. An exception is Scartascini and

[8]In our model, like in the coalitional bargaining literature, the efficient outcome is not achieved because the agenda setter "seeks to maximize his own payoff...but in doing so it will generally need to enlist partners who have to be suitable compensated. The nature and amount of that compensation depends crucially on the protocol." (page 60, Ray and Vohra (2015)).

[9]The reason for this divergence is that in our setup it is not true that average surplus is maximized for the grand coalition due to the introduction of retaliation and cooperation (i.e. in their notation, it is not true that $v(S)/S$ is maximal for $S = N$, where N is the total number of legislators and $v(S)$ is the surplus of a coaltion of S legislators).

Tommasi (2012), where political actors can choose to play in the legislative arena, or outside of it. If they stay outside parliament, they become active in the informal arena and they channel their demands through mobilizations, riots, strikes, etc. Protests are placated with transfers from the formal institution. The authors focus on the long run determinants of institutionalization of policy making, understood as the fraction of actors choosing the formal arena. Contrary to Scartascini and Tommasi (2012), in our paper all demands are channeled inside the parliament, the size of the pie depends on the winning coalition, and the legislative game has more than one round, being repeated until there is an agreement. Also, we allow for positive and negative actions, which can take place simultaneously.

Other studies on political actions outside the parliament focus on the causes of protests, broadly defined. Ray and Esteban (2017) discuss how excluded factions (e.g. ethnic groups) can cause conflict and retaliation. Moreover, they link conflict with inequality, lower economic activity and development. In terms of our setup, the exclusion of an ethnic group from the winning coalition can backlash into conflict. Edmond (2013) is a recent example of theoretical work on the coordination aspects of protesting, emphasizing (weak) institutional quality as a catalyst for protesting. Battaglini (2017) focuses on whether protests, or petitions, influence policy makers' actions by aggregating information through the "wisdom of the crowds".

The rest of the paper is organized as follows. Section 2 describes the environment, and defines the equilibrium concept. Section 3 characterizes equilibria, and section 4 presents the main results and provides some comparative statics. Section 5 solves for the decision to commit to become active and section 6 considers the effect of relaxing some of our modeling assumptions. Section 7 concludes, and an appendix collects all proofs.

2 Model

We consider an economy with n districts represented by the set of legislators N, with $|N| = n$, who have to decide how to divide aggregate resources, $\tilde{Y}$. Following Baron and Ferejohn (1989), legislators bargain over the distribution of resources using closed rules (i.e., no amendments) with equal probabilities of recognition and discounting. From N, a legislator is randomly chosen to make a proposal $x \in X \subset R^n$, where X is the set of all proposals that satisfy the budget constraint. That is, a proposal assigns $x_j \geq 0$ to each district $j \in N$, represented by a legislator who is also indexed with j, such that $\sum_j x_j \leq \tilde{Y}$. Let the voting rule

q be such that if a (super) majority of $n/2 < q \leq n$ votes to approve the proposal, resources are distributed and the game is over. If the proposal is not approved, a legislator is drawn to make a proposal and a new round of bargaining begins. There can be an infinite number of rounds or sessions. We assume $u_j(x) = x_j$ for all j, and all players discount the future with $0 < \delta < 1$.

Before the legislature convenes, districts' types are drawn, and remain fixed throughout the game. The main departure from Baron and Ferejohn (1989) is the following. Districts are either "active", if they can affect the pie $\tilde{Y}$, or "passive" otherwise. There are r active districts, out of which r^+ are "productive", if they may increase rents, and r^- are "destructive", if they may decrease them. In terms of the model primitives, if and only if a legislator from a productive district votes in favor of the proposal, then aggregate resources increase by η. If and only if a legislator from a destructive district votes against the proposal, then there is a reduction of aggregate resources by η.[10] We interpret these changes in aggregate output as cooperation and retaliation.

All legislators who support the proposal are considered to be in the winning coalitions. Thus, productive districts in the winning coalition cooperate, while destructive ones outside the winning coalition retaliate.[11] In Section 5 we endogeneize districts' types by giving them a choice to commit to their types before the legislature convenes, and show that those districts that have this option will excercise it.

Total rents to be distributed, $\tilde{Y}$, are given by an exogenous pie Y plus or minus the production or destruction of the active legislators (feasibility requires that $n\eta < Y$). The presence of cooperation and retaliation introduces two innovations with respect to Baron and Ferejohn (1989): first, the resources to be distributed, $\tilde{Y}$, are endogenous and depend on the profile of districts' votes. Second, ex ante payoffs are not necessarily the same across districts' types, even if they have the same probability of being agenda setters.

[10] Restricting output changes to be of the same magnitude for productive and destructive districts simplifies the characterization of equilibrium, see lemma 3. After characterizing equilibria we show that asymmetries, such as e.g. $\eta^- > \eta^+$, would result in retaliating districts' continuation value being greater, and them being more likely to be called in the winning coalition (see section 6.2).

[11] The relationship between transfers and actions that affect output is documented in different strands of the political economy literature (as bribing, ear-marked spending, etc.) and it is specially salient in the papers on conflict, even in developed countries. For instance, see Gillezeau (2015) on the effect of anti-poverty spending on the abatement of the 1960s riots in the U.S.

2.1 Strategies and Equilibrium concept

Let $t = 0, 1, \ldots$ index the legislative rounds and denote by $h^t \in H^t$ a history of the legislative game up to round t. History includes, for each session, who was the agenda setter, what proposal was made, and how members voted. Legislators are of different types, with types being constant across rounds, and all legislators have full information about the history of the game.. Let $s^t_j(h^t)$ be a proposal strategy for an agenda setter from district j given history h^t. A pure strategy for the agenda setter (i.e. the randomly recognized legislator) specifies how much to offer to each legislator for any history h^t, i.e., $s^t_j(h^t) : H^t \to X$.

Similarly, let $a^t_j(h^t)$ be the voting strategy for each legislator $j \in N$. Given a proposal by the agenda setter and a history of the game, each legislator must decide whether to accept it or not, i.e., $a^t_j(h^t, s^t_j(h^t)) : H^t \times X \to \{\texttt{yes}, \texttt{no}\}$. A randomized strategy $\sigma^t_j(h^t)$ for legislator j at history h^t is a probability distribution over the strategies $(s^t_j(h^t), a^t_j(h^t))$. Following Baron and Ferejohn (1989) we assume that floor legislators who are indifferent between accepting or rejecting a proposal vote in favor.

Each possible history h^t of the legislative game up to round t defines a subgame of the repeated game beginning at that round. We denote $x(\sigma^t_j(h^t)) = x(\sigma^t_j(h^t), \sigma^t_{-j}(h^t))$ the expected proposal associated with strategy σ_j. A strategy profile $\sigma^* = (\sigma^*_1, ..., \sigma^*_n)$ is a subgame-perfect equilibrium if for all histories of the game $h^t \in H^t$, for all $j \in N$, and for all feasible $\sigma^t_j(h^t)$,

$$u_j(x(\sigma^{*t}_j(h^t), \sigma^{*t}_{-j}(h^t)) \geq u_j(x(\sigma^t_j(h^t), \sigma^{*t}_{-j}(h^t))).$$

We restrict our attention to stationary subgame-perfect equilibria in stage undominated strategies, as it is customary in this literature. Thus, we look at history independent equilibria: at every node t the available actions and strategies must be the same, up to the agenda setter's type. Furthermore, all voting decisions are made as if the legislator was pivotal (Baron and Kalai, 1993). Hence, we drop dependence to h^t from all notation that follows. A proposal in mixed strategies is characterized by a probability distribution over feasible pure strategy proposals, $\pi_j(s_j)$, such that $\pi_j \geq 0$, and $\int_{s_j \in X} \pi_j(s_j) = 1$ for all j.

Definition 1 (Stationary Subgame-Perfect Equilibria)**.** The strategy profile $\sigma^* = (\sigma^*_1, \ldots, \sigma^*_n)$ is a stationary subgame-perfect equilibria if, for all $j \in N$, and for all

$\sigma_j \in X$,

$$u_j(x(\sigma_j^*, \sigma_{-j}^*)) \geq u_j(x(\sigma_j, \sigma_{-j}^*)),$$

Note that this equilibrium concept is analogous to a Markov Perfect Equilibrium in which the only state variable is the agenda setter's type (which is independent of the previous round or period's state), where weakly dominated strategies are never played. Stage undomination ensures identical payoffs of all the legislators of the same type included in the winning coalition.[12] In terms of notation, this allows us to replace the j indexes with an index for the legislators' types. Let i index the agenda setter's type and k the floor-legislators' types. Thus, i and k refer to whether a legislator (agenda setter or floor legislator) comes from a passive, retaliating or cooperating district, i.e. $i, k \in \{0, -, +\}$ respectively. In particular, all districts of type k that are offered a positive payoff should receive the same amount, $x^k(i)$. Therefore, we can characterize pure strategy proposals, s_i, by how much to offer ($x^k(i)$ for all i and k), to how many legislators ($m^k(i)$ for all i and k), such that $\sum_k m^k(i)x^k(i) \leq \tilde{Y}$. More generally, with mixed strategy proposals, $m^k(i)$ denotes the *expected* number of legislators of type k to whom an agenda setter of type i offers a positive payoff.[13]

A direct implication of stationarity is that any agenda setter, following any history of the game, would solve the same optimization problem. That is, an agenda setter of type i chooses a proposal π_i that maximizes her objective function $\tilde{Y}(\pi_i, a_i) - \sum_k m^k(i)x^k(i)$ subject to participation constraints, feasibility constraints and the resource constraint.

To simplify the presentation of our main results we henceforth assume that active districts are only productive, and denote them by $i = 1$ (instead of $i = +$). Lemma 3 shows that results extend to the general case with both productive and destructive districts.

The participation constraints require that the proposal made by the agenda setter $i \in \{0, 1\}$ induces at least $q - 1$ floor legislators to vote `yes`. In other words, these floor legislators obtain at least the value of waiting, which is the continuation value δv^k, i.e., $x^k(i) \geq \delta v^k$, where v^k is the ex ante value of legislators of type k. The feasibility constraints require that (i) no more than the available active districts are offered to be in the coalition and (ii) at least the minimum amount of required active districts are offered to be in the coalition. Suppose the agenda

[12] In section 6.1 we consider the effects of lifting the restriction to stage undominated strategies, such that districtics of the same type might be offered different transfers.

[13] For simplicity we omit the argument π of the underlying proposal in mixed strategy.

setter comes from an active district, then the former constraint for floor legislators from active districts is $m^1(1) \leq r-1$ and the latter is $m^1(1) = \max\{0, q+r-n-1\}$. The resource constraint captures how the profile of expected votes determines the pie $\tilde{Y}$. Given that the agenda setter's utility is decreasing in $x^k(i)$, constraints for $x^k(i)$ are always binding, and $x^k(i) = \delta v^k$, for all i and k. Since the agenda setter takes as given continuation values, her strategy is then reduced to choosing $m^1(i)$ and $m^0(i)$, i.e. the composition of her coalition. Any solution to the following maximization problem (for agenda setter of type $i = 0, 1$), and the continuation values, is then a stationary equilibrium.

$$\max_{m^1(i), m^0(i)} \quad \tilde{Y}(m^1(i)+i) - m^1(i)\delta v^1 - m^0(i)\delta v^0 \tag{1}$$

$$\begin{aligned} \text{s.t.} \quad & \tilde{Y}(m^1(i)+i) = Y + (m^1(i)+i)\eta, \\ & m^1(i) \leq r - i, \\ & m^1(i) \geq \max\{0, q+r-n-i\} \qquad (2) \\ & m^0(i) \leq n - r - i, \\ & m^0(i) \geq \max\{0, q-r-i\}. \end{aligned}$$

3 Analysis

We begin the analysis with the characterization of the set of winning coalitions that must be considered in equilibrium. A minimum winning coalition is one in which $m^1(i) + m^0(i) = q - 1$, i.e. exactly $q - 1$ legislators plus the agenda setter vote `yes`. Larger-than-minimal winning coalitions might arise in equilibrium if the benefit of adding a district to the coalition outweighs its cost. Note that no agenda setter will consider winning coalitions in which "additional legislators" (i.e., beyond q) come from passive districts when they have positive continuation values. Doing so would suppose a cost for the agenda setter with no gain. Thus, we are led to the following lemma:

Lemma 1. For all i:
If $\sum_{k\in\{0,1\}} m^k(i) > q - 1$, and $v^0 > 0$, then $m^0(i) = 0$. That is, when an agenda setter considers larger-than-minimal winning coalitions, all members, except perhaps the agenda setter, come from active districts.
If $m^1(i) \leq q - 1$, then the support of mixed strategies is in $[0, q-1]$. That is, no mixed strategy contemplates calling more than $q - 1$ active districts and so all equilibria have minimum winning coalitions.

Proof. All proofs are in the appendix. □

Note that if $v^0 = 0$, there is a large number of "trivial" larger-than-minimal winning coalitions in which passive districts are offered 0 and vote `yes`. We conjecture, and later verify, that in equilibrium $v^0 > 0$. Under the conjecture, lemma 1 reduces the set of larger-than-minimum winning coalitions to be considered in equilibrium. Lemma 1 also shows that minimum winning coalitions do not consider calling more than $q-1$ active districts. Thus, there will be no mixing across types of winning coalitions: no larger than minimum winning coalition would include "redundant" passive districts, and no minimum winning coalition would include "extra" active districts. This simplifies the maximization problem. In particular, the expected number of passive districts called into a minimal coalition is $q-1-m^1(i)$ for all i. More generally, irrespective of the coalition, the expected number of passive legislators is $\max\{0, q-1-m^1(i)\}$. Henceforth we denote by $m(i)$ the expected number of active districts called into the winning coalition.

Thus, the problem of an agenda setter of type $i = 0, 1$ is simplified to

$$\begin{aligned} \max_{m(i)} \quad & \tilde{Y}(m(i)+i) - m(i)\delta v^1 - \max\{0, q-1-m(i)\}\delta v^0 \qquad (3) \\ \text{s.t.} \quad & \tilde{Y}(m(i)+i) = Y + (m(i)+i)\eta, \\ & m(i) \leq r - i, \\ & m(i) \geq \max\{0, q+r-n-i\}. \end{aligned}$$

The corresponding Lagrangian is given by

$$\begin{aligned} \mathcal{L}^i(m(i)) \quad = \quad & Y + (m(i)+i)\eta - m(i)\delta v^1 - \max\{0, q-1-m(i)\}\delta v^0 \\ & + \overline{\lambda}(i)[r-i-m(i)] - \underline{\lambda}(i)[\max\{0, q+r-n-i\} - m(i)], \end{aligned}$$

where $\overline{\lambda}(i)$ and $\underline{\lambda}(i)$ are, respectively, the multipliers on the upper and lower bounds of $m(i)$. It is important to note that the problem is non-linear due to the presence of the max operator. Despite this, lemma 1 implies that all mixing strategies either satisfy $m(i) > q-1$, or all satisfy $m(i) \leq q-1$. Thus, we can treat the optimization as being effectively linear, and consider the effects of a marginal increase in the expected number of active districts willing to vote in favor of the proposal. The first order condition of the agenda setter's problem, is then given by

$$\eta - \delta v^1 + \mathbb{1}_{m(i)<q-1}\delta v^0 - \overline{\lambda}(i) + \underline{\lambda}(i) = 0. \qquad (4)$$

The indicator function shows that for minimum winning coalitions the agenda setter contemplates reducing the expected number of passive legislators increasing one-for-one the expected number of active legislators. A marginal increase in $m(i)$ has three effects on the agenda setter's payoffs: they increase by η as the increase in the expected number of active districts increases rents, are reduced by δv^1 reflecting the cost of added active districts, and they increase by δv^0 from cost savings from less passive districts. Thus, the first order condition for an interior equilibrium with a minimum winning coalition is characterized by $\eta - \delta v^1 + \delta v^0 = 0$.

For larger-than-minimal coalitions, the indicator function shows that the decision is on enlarging the coalition with new members from active districts. There are thus two changes in the agenda setter's payoff from a marginal increase in $m(i)$: they increase by η as the increase in the expected number of active districts increases rents, and are reduced by δv^1 reflecting the cost of these added active districts. Thus, the first order condition for an interior equilibrium with a larger-than-minimum winning coalition is characterized by $\eta - \delta v^1 = 0$.

We denote "corner-equilibria" those equilibria in which either $\underline{\lambda}(i) > 0$, $\bar{\lambda}(i) > 0$, or $m(i) = q - 1$, and "interior equilibria" those equilibria in which at least one type of agenda setter' choice is unconstrained, i.e., one for which $\underline{\lambda}(i) = \bar{\lambda}(i) = 0$, and $m(i) \neq q - 1$. Since in interior equilibria $m(i)$ generically will not be an integer, we are led to the following characterization of equilibria.[14]

Remark 1. Generically, interior equilibria are mixed-strategy equilibria, and corner equilibria are pure strategy equilibria.

Note that the second order conditions trivially hold due to the effective linearity of the objective function and the constraints. For the same reason, uniqueness is generally not warranted. In the case of corner equilibria, the constraints are solved for a unique value of $m(i)$, which guarantees uniqueness. In the case of interior equilibria, where multiplicity arises, equilibria are payoff equivalent: v^0, v^1, and the expected number of active districts in a winning coalition, $E(m) \equiv \frac{n-r}{n} m(0) + \frac{r}{n}(m(1) + 1)$, do not depend on the particular mix that leads to $m(i)$ for $i = 0, 1$.[15]

For any v^0 and v^1, let the expected cost of forming a coalition of $m(i)$ active districts be: $e(m^i) = m(i)\delta v^1 + \max\{0, q - 1 - m(i)\}\delta v^0$. Let ρ^i be the probability of a type i legislator being called into a coalition by the agenda setter. Then,

[14] Our distinction between pure and mixed-strategy equilibria relates to whether strategies call for an integer number of legislator of each type, or if there is randomization between different integers. In legislative bargaining, due to anonymity, strategies are usually mixing in the sense that there is randomization between legislators of a given type.

[15] Payoff equivalence for mixing strategies is shown in the proof of propositions 5 and 6.

taking into account that the probability of recognition as an agenda setter is the same for all types, stationarity implies that, for $i = 0, 1$, we can write valuations as follows:

$$v^i = \frac{1}{n}(\tilde{Y} - e(m^i)) + \frac{n-1}{n}\rho^i \delta v^i. \tag{5}$$

From the equations above we can solve for v^i as a function of ρ^i, which depends on the coalitions proposed by legislators of type i, summarized in $m(i)$. That is, we need to calculate $\rho^i(m(0), m(1))$.

Given a pair of strategies $m(0)$ and $m(1)$, the construction of ρ^i for $i = 0, 1$ is mechanical. For instance, in the case of a passive legislator, we construct the probability that he is called into a coalition, ρ^0, as follows. With probability $r/(n-1)$, the agenda setter comes from an active district, hence, the probability that a passive legislator is called in the coalition depends on how many passive districts the active agenda setter needs to call, $\max\{0, q - m(1) - 1\}$, divided by the total number of available passive districts $(n - r)$. With probability $\frac{n-r-1}{n-1}$ the agenda setter is from a passive district, and the probability that a passive legislator is called in the coalition depends on how many passive districts the passive agenda setter needs to call, $\max\{0, q - m(0) - 1\}$, divided the total number of available passive districts $(n - r - 1)$. Similarly for the case of a legislator from an active district that is not the agenda setter. Hence,

$$\rho^0(m(0), m(1)) = \frac{r}{n-1}\frac{\max(0, q - m(1) - 1)}{n-r} + \frac{n-r-1}{n-1}\frac{\max(0, q - m(0) - 1)}{n-r-1}, \tag{6}$$

$$\rho^1(m(0), m(1)) = \frac{r-1}{n-1}\frac{m(1)}{r-1} + \frac{n-r}{n-1}\frac{m(0)}{r}. \tag{7}$$

Any stationary subgame-perfect equilibrium must solve the system of equations (4), and (5) for all $i \in \{0, 1\}$, with ρ^i given by (6) and (7). Existence of stationary subgame-perfect equilibria is shown in the next proposition.

Proposition 1. There exists a stationary subgame perfect equilibrium outcome.

In the lemma below we show that $v^1 > v^0$, i.e. it is more costly to include active districts in a winning coalition than passive ones. After establishing this result, we provide an example that clarifies the intuition for mixing strategies and output losses.

Lemma 2. For all $1 \leq r < n$, it is always the case that $v^1 > v^0$.

Example 1. Consider the case in which there is only one productive district, i.e., $r = 1$. For simplicity, let's also assume that legislators are fully patient, i.e., $\delta \to 1$. The only non-trivial choice is that of a passive agenda setter who must choose $m(0) = \rho^1$. Suppose that in equilibrium the active district is always included, i.e. $m(0) = 1$. Plugging $m(0) = 1$ in equations (5), we obtain $v^1 \to Y + \eta$ and $v^0 \to 0$.[16] The first order condition (4) is met if

$$\delta(v^1 - v^0) \leq \eta,$$

but with the strategy above $\delta(v^1 - v^0) \to Y + \eta$, which is greater than η. Then, it is not optimal to have $m(0) = 1$, and instead $m(0) < 1$. Since there is a positive probability that the active district will be left out of the winning coalition, i.e. $1 - \rho^1 > 0$, there are expected output losses in equilibrium.

Similarly, suppose the passive agenda setter never includes the active district in the winning coalition, i.e. $m(0) = 0$. Following the same procedure as above, after some algebra, and using feasibility, $Y > n\eta$, we find

$$v^1 - v^0 = -\frac{1}{n}\left(\eta - \frac{q-1}{n}Y\right) < 0.$$

which contradicts lemma 2. Thus, $m(0) = 0$ is not optimal and the equilibrium is in mixed strategies, i.e. $0 < m(0) < 1$.

The example shows that, when patience is high, active districts must be left out of the winning coalition with positive probability, such that their continuation values decrease enough for the agenda setter to find it profitable to include them in a winning coalition. This creates inefficient output losses in equilibrium.

4 Results

In what follows we restrict the analysis to q-supermajorities that exclude the unanimity rule. Since legislators must receive their continuation value to approve a proposal, with $q = n$ they all have to be included in a winning coalition. Hence, they all have the same continuation value and the distinction between types disappears. In that case, equilibrium is the same as in Baron and Ferejohn (1989) with unanimity rule.[17]

Remark 2. With $q = n$ the equilibrium in this game is identical to Baron and Ferejohn (1989), and the expected payoff of all legislators is $\frac{1}{n}(Y + r\eta)$.

[16] This result is not surprising, as $m(0) = 1$ gives the active district veto power, and thus can extract all the surplus as players become perfectly patient.

[17] Equal ex ante values are also the outcome in the limit as $\delta \to 0$.

In light of lemma 1, all winning coalitions that include more than q members are composed of active districts, except perhaps for the agenda setter. As a consequence, if there are less active districts than $q-1+i$, it must be the case that coalitions are minimal. Therefore, larger-than-minimal winning coalitions are a potential phenomena only when there is a relatively large number of active districts. Taking these issues into account, in proposition 2 we first determine conditions for larger-than-minimal winning coalitions to be an equilibrium. Then in proposition 3 we analyze equilibrium characteristics for a relative low number of active districts, and finally we study the case of a large number of active districts in proposition 4.

Proposition 2 provides our first result. It establishes that, when allowing for retaliation and cooperation that may change the size of rents, larger-than-minimum winning coalitions are possible in equilibrium.

Proposition 2. $\exists!\ \delta_q \in [0,1)$ such that winning coalitions are minimal if and only if $\delta > \delta_q$. If $r \leq q-1+i$, $\delta_q = 0$, for $i = 0, 1$.[18]

Relative to the voting rule, q, the number of active districts, r, and the potential change in output, η, the discount factor determines how costly it is to get a legislator's support. When the discount factor is large enough, only minimum winning coalitions can be sustained in equilibrium. Indeed, for high δ, $\delta > \delta_q$, since legislators give a relatively large weight to the future, their continuation values are high. Thus, the cost of adding a non-necessary legislator into the winning coalition is high as well. In this case, the agenda setter does not want to form a larger-than-minimal winning coalition. Conversely, for low δ, $\delta \leq \delta_q$, the legislators' continuation values are small, and the cost of including an extra active district in the coalition might be lower than the output loss if excluded. Since the agenda setter acts as a residual claimant, she is willing to add a non-necessary active district, even though the cost of the coalition increases by δv^1, because rents increase by η, and so her utility increases by $\eta - \delta v^1$.[19]

[18] Note that δ_q only depends on i when $r = q$. In this case when $i = 0$, $\delta_q > 0$, but when $i = 1$, $\delta_q = 0$. To avoid cumbersome notation we have decided to drop i as a determinant of δ_q.

[19] Notice that the description of the equilibrium assures that the budget constraint is always satisfied in equilibrium. We can study feasibility under individual deviations in which an active legislator is offered his continuation value, and the proposal is approved without his vote (because initially there was a larger-than-minimal coalition). It can be shown that the budget constraint would still be always satisfied, with the agenda setter absorbing the reduction in resources. Moreover, a sensible alternative assumption rules out these deviations: that the agenda setter distributes x_j to district j if an only if (i) the proposal is approved, and (ii) legislator j voted `yes`.

This formalization of larger-than-minimal winning coalitions provides a bridge between the theoretical prediction of minimum winning coalitions with evidence that larger-than-minimum coalitions are frequently observed.[20] Proposition 2 provides a rationale for larger than-minimum winning coalitions in the Baron and Ferejohn (1989) setting: they are an equilibrium if and only if for the agenda setter the cost of additional legislators is lower than the increase in rents from including them in the coalition.

The following proposition characterizes equilibria when there are so few active districts that there are no incentives to have larger-than-minimum winning coalitions.

Proposition 3. For $i = 0, 1$ and $r \leq q - 1 + i$, $\exists!\ \bar{\delta} \in (0, 1)$ such that $m(i) = r - i$ if and only if $\delta \leq \bar{\delta}$.

From proposition 2, $\delta_q = 0$ and so all coalitions are minimal. For $\delta \leq \bar{\delta}$, there exists a unique corner solution in which all active districts are offered their continuation value, they all vote `yes` and there is no output loss. When legislators are more patient, including them in the coalition is more costly. In this case, even though the agenda setter has the technology to avoid inefficiencies, she does not use it. She reduces the probability of calling active districts to lower their continuation value, up to the point in which she is indifferent between active and passive districts, i.e. $\delta(v^1 - v^0) = \eta$.

Thus, in equilibrium, some active districts might be left out of the coalition, as shown in figure 1. Therefore, proposition 3 presents our second result, that it is possible for output to be inefficient in equilibrium (which happens whenever $m(i) + i < r$). This result reflects the fact that in models of legislative bargaining with linear utility, the agenda setter's actions can be interpreted as if she only cared about the welfare of the winning coalition. Thus, if the cost of replacing a passive legislator by an active one is higher than the output gain, not all active districts will be called into the coalition. In contrast, a social planner that cared for aggregate social welfare would never exclude active districts, as this implies an inefficient loss of output.

The following proposition characterizes equilibria when the presence of a large number of active districts raises the possibility of having larger-than-minimum winning coalitions.

Proposition 4. For $i = 0, 1$ and $r > q - 1 + i$
(i) $\exists!\ \bar{\delta}$ such that $m(i) = q - 1$ if and only if $\delta \in [\delta_q, \bar{\delta}]$. Otherwise, for $\delta > \bar{\delta}$ there

[20] See Riker (1962), and Knight (2008) and references therein.

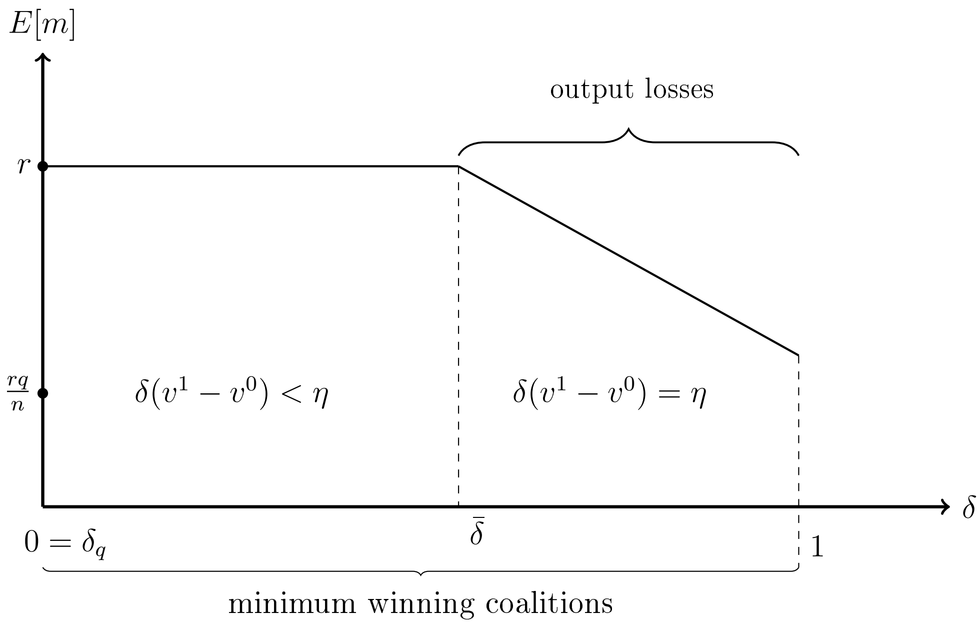

Figure 1: Equilibria when $r \leq q - 1 + i$

exist interior equilibria with $m(i) < q - i$.
(ii) $\exists!$ $\underline{\delta}$ such that $m(i) = r - i$ if and only if $\delta \in [0, \underline{\delta}]$. Otherwise, for $\delta \in (\underline{\delta}, \delta_q)$, there only exists interior equilibria with $q - i < m(i) < r - i$.

In (i) equilibria are similar as those in proposition 3, as the agenda setter only proposes minimum winning coalitions, and for high δ these imply output losses. In (ii), when $m(i) < r - 1$, some active districts are left out of the winning coalition and the equilibrium is also inefficient. When $m(i) > q-1$, the first order condition for an interior equilibrium is

$$\delta v^1 = \eta.$$

Similarly to proposition 2, for δ high enough, the benefits to include active districts beyond the minimum-winning coalition must be in balance with the costs. Therefore, for $\underline{\delta} < \delta < \delta_q$, there are mixed strategy equilibria with larger-than-minimum winning coalitions. If δ becomes so small that the benefits of including active districts beyond $q-1$ is always higher than the costs, then there is a unique pure strategy equilibrium in which all active districts are called into the coalition. Figure 2 describes equilibria for (i) and (ii).

Equilibria in which not all active districts are a part of the winning coalition are inefficient, and there are two types of inefficiency. In the first, there are larger than minimum winning coalitions, and thus no passive districts. Active districts are included up to the point that their private cost to the agenda setter equals

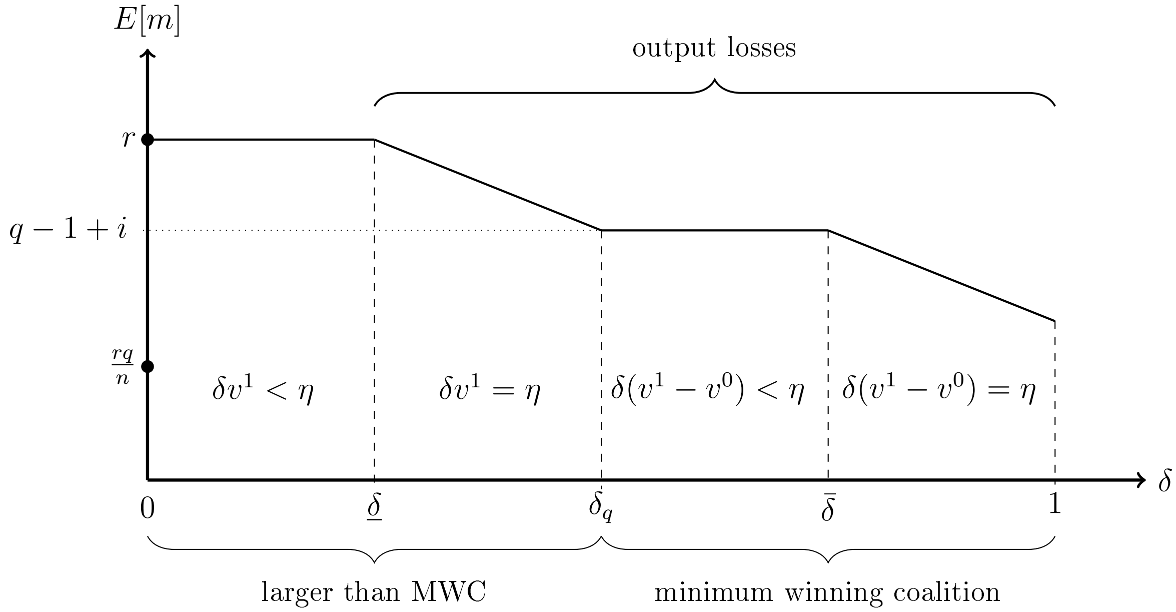

Figure 2: Equilibria when $r > q - 1 + i$

their social benefit. In the second, the winning coalition is minimal and the agenda setter acts strategically by calling, or threatening to call, some passive districts to reduce active districts' continuation values. In this case, the social benefit of active districts is strictly higher than the private cost to the agenda setter.

Corollary 1. (i) Legislators from active districts have a higher probability of being in the winning coalition. (ii) For all $1 \leq r < n$, it is always the case that $v^0 > 0$.

Corollary 1 presents our final results. It shows that active districts are more likely to be called into a winning coalition. In fact, as shown in the proof, it is precisely their higher probability of being in the winning coalition that leads them to have higher ex ante payoffs. We also verify our conjecture that $v^0 > 0$. Thus, we can rule out trivial larger-than-minimum coalitions formed by calling into them legislators with zero continuation values.

We now consider the case in which some active districts can reduce output, instead of only being able to increase it as we have assumed so far, and the agenda setter must consider how many legislators of each type to include in her winning coalition. The following lemma shows that, when the effect on output is the same for rioters and cooperators, the agenda setter is indifferent about the composition of active districts in her coalition, i.e. she only cares about $m(i) = m^+(i) + m^-(i)$. Thus, lemma 3 extends all results derived under the restriction that active districts were only of the productive type.

Lemma 3. For all $1 \leq r^+$ and $1 \leq r^-$, it is always the case that $v^+ = v^-$.

4.1 Comparative Statics

The next propositions summarize some comparative static results. In particular, we are interested in the effect of parameter changes on the likelihood of having minimum winning coalitions and on output losses, defined as $(r - E(m))\eta$, where recall the definition of the expected number of active districts in a winning coalition, $E(m) = \frac{n-r}{n}m(0) + \frac{r}{n}(m(1) + 1)$. As expected, these results depend on the effect of parameter changes on the continuation values of active and passive districts.

Proposition 5. (i) An increase in the required supermajority q, increases the range of parameters for which minimum winning coalitions are an equilibrium outcome, and reduces the expected output losses. Additionally, active districts' payoffs increase, except when $r \leq q - 1 + i$ and $\delta < \bar{\delta}$, for $i = 0, 1$. (ii) Output losses (weakly) increase with δ.

An increase in the needed supermajority (weakly) raises the number of both types of legislators in the winning coalition. The increase in the expected number of active districts reduces output losses in equilibrium. The mechanism by which active legislators are (weakly) more likely to be part of the winning coalition depends on whether the number of active legislators is higher or lower than q.

First, consider the case of a large number of active legislators ($r > q - 1 + i$), depicted in figure 3. An increase in q has a direct effect on the region of δ for which there are minimum winning coalitions because more legislators are needed in these coalitions. This mechanical effect implies that both corresponding thresholds, δ_q and $\bar{\delta}$, decrease. On the contrary, q has no effect on $\underline{\delta}$, nor on $m(i)$ for $\delta \in (\underline{\delta}, \delta_q)$. Thus, the supermajority does not affect the equilibrium, in particular output losses, for $\delta \in [0, \delta_q)$. According to lemma 1, in the region of minimum winning coalitions, $\delta \in [\delta_q, \bar{\delta}]$, additional legislators needed to achieve the new supermajority come from active districts. Thus, in this region, an increase in q reduces output losses. Finally, for $\delta > \bar{\delta}$, $E(m)$ increases with q (since otherwise v^0 would increase more than v^1), thus also reducing output losses.

If there is a small number of legislators ($r \leq q-1+i$), the agenda setter's initial response to an increase in q is to call more passive districts into the minimum winning coalition (if $\delta \in [0, \bar{\delta})$, there is no other course of action as all active districts are already in the coalition). This increases passive districts' continuation

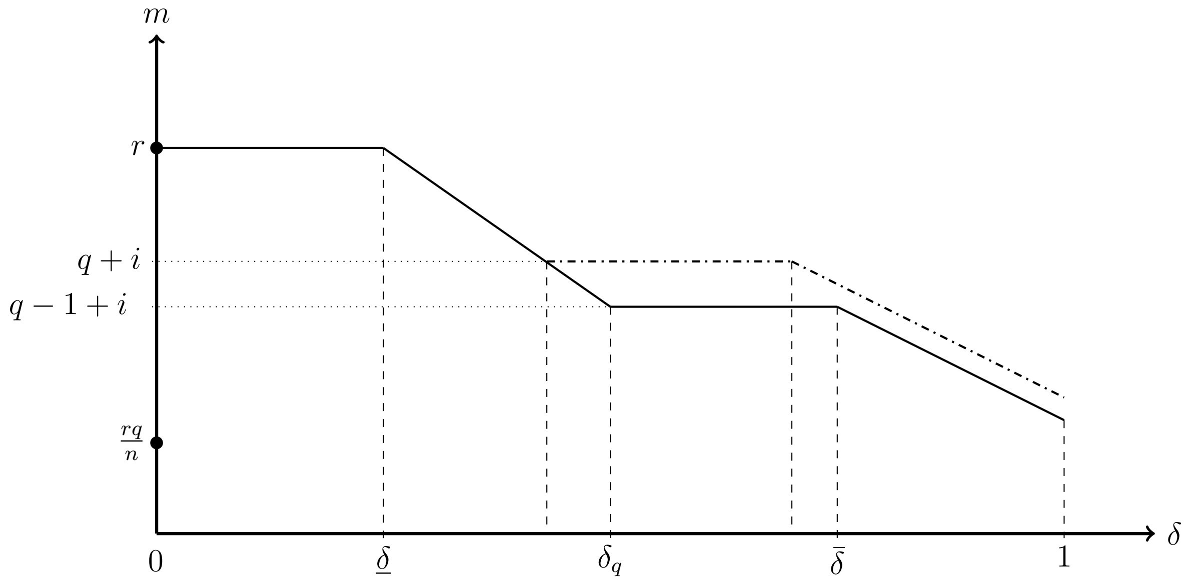

Figure 3: Comparative statics in q: $r > q + i$

values, giving the agenda setter incentives to increase the probability of calling active districts when using a mixing strategy. As a result, $\bar{\delta}$ increases with q, as does $E(m)$ for $\delta > \bar{\delta}$. Thus, an increase in q reduces output losses.

The different mechanisms that explain the decrease in output losses with greater supermajorities are then consistent with a non-monotonic effect of q on the continuation values of active districts. For $r > q-1+i$ some active districts are left out from the minimum winning coalition, thus increasing the needed supermajority increases the probability that they are called into it, rising their continuation value. For $r \leq q-1+i$, when $\delta < \bar{\delta}$, the effect of an increase in the supermajority reverses, as this now increases the probability that passive districts are called into the coalition. The increase in passive districts continuation values must be met, due to feasibility, by a decrease in active players' continuation values. Since when $r \leq q-1+i$, $\bar{\delta}$ is increasing in q, there is always a supermajority above which the continuation values of active districts is decreasing in q.[21]

Finally, note that contrary to results in other dynamic games (Piguillem and Riboni, 2015), more patience leads to more inefficient outcomes. Higher values of δ increase the continuation value of active districts inducing the agenda setter to call them less often into the winning coalition. This increases output losses.

Proposition 6. An increase in the potential damage η, or in the number of active districts, decreases the range of parameters for which minimum winning coalitions

[21] Formally, this threshold supermajority corresponds to q such that $\bar{\delta} = 1$. See (13) in the appendix.

are an equilibrium outcome. The effect on output losses is ambiguous.

An increase in η increases the agenda setter's incentives to include active districts in the winning coalition. In particular, the thresholds for larger-than-minimum winning coalitions including all active districts, $\underline{\delta}$, and for minimum winning coalitions, δ_q, increase. On the one hand, a larger η increases the expected number of active districts in the coalition ($E(m)$), on the other hand, it increases the damage of active districts left out from it. Hence, the effect on output losses, $(r - E(m))\eta$, is generally ambiguous. For example, with a large number of active districts ($r > q - 1 + i$), when mixed strategies are an equilibrium with larger-than-minimum winning coalitions, an increase in η reduces output losses. With minimum winning coalitions that do not include all active districts, an increase in η increases output losses.[22]

Alternatively, this exercise could have been performed over the ratio $\frac{\eta}{Y}$, with similar results. That is, an increase in η can also be interpreted as a reduction of Y.[23] Thus, changes in η can be interpreted as comparing different economies in a cross-section, or the same economy over the business cycle (for the latter an increase in η reflects a fall in Y). Then, since in a recession (boom) more (less) active districts are included in the winning coalition, endogenous rents dampen output shocks, i.e. $\frac{d\tilde{Y}}{dY} < 1$.

Consider now an increase in the number of active districts. There are two effects. First, it gives the agenda setter incentives to increase the number of districts called into the winning coalition. Second, it reduces the probability of a given active district to be called into the winning coalition. These two effects have opposite effects on the continuation value of active districts. When $r > q - 1 + i$, $\underline{\delta}$, $E(m)$ for $\delta \in (\underline{\delta}, \delta_q)$, and δ_q increase with r^- or r^+. From the latter, minimum winning coalitions are part of the equilibrium for a smaller set of δ. While an increase in r^+ reduces output losses for $\delta \in (\underline{\delta}, \delta_q)$, the effect of r^- is ambiguous (the higher is r^- the more likely output losses increase). For $\delta \in [\delta_q, \bar{\delta}]$, an increase in r^- or r^+ increases output losses, and for $\delta > \bar{\delta}$ the effect is ambiguous (it can be shown that output losses increase with r^- or r^+ when $\delta \approx 1$).

[22]When $\delta > \bar{\delta}$, the effect is ambiguous. It can be shown that when $\delta \approx 1$, output losses increase with η.

[23]If both Y and η were to increase proportionally, it would result in proportional increases of v^k. Thus, there would not be an effect on thresholds or optimal strategies.

5 Choice of Becoming Active

In the setup of the game, productive districts cooperate at no cost if included in the winning coalition and destructive ones retaliate if they are not included. In this extension, having characterized the equilibria for a given number of active districts, we endogeneize districts' commitment to behave in that way.

We assume that by default prohibitively large collective action costs prevent districts from committing to cooperate or retaliate as above. Each district can solve its collective action problem with an exogenous probability, and we assume this stochastic process to be i.i.d. across districts. An interpretation is that shocks can spur mobilization or agreement in societies (for instance, in Acemoglu and Robinson (2001) a shock may cause revolutions). We say that the collective action problem is solved with probability $\beta^+ + \beta^-$ and it is not solved with the remaining probability $1 - \beta^+ - \beta^-$. In particular, with probability β^+, district j has the option of committing to the productive strategy. Similarly, with probability β^- district j has the option of committing to the destructive strategy. Finally, with the remaining probability district j commits to take no action, regardless of the outcome of bargaining.[24] We will now show that all districts for which the collective action problem is solved, will choose to commit to their types.

With a bit of an abuse in notation, let's assume that r districts have the option to either become productive or destructive, and denote by $v^i(\cdot)$ ex ante payoffs as a function of the number of active districts. Without loss of generality, we consider the decision problem in one of these districts, that takes as given that the other $r-1$ districts will become active. Thus, this district is in effect comparing payoffs $v^1(r)$ and $v^0(r-1)$. Given that becoming active is assumed to be costless, it will be in the districts interest to do so whenever $v^1(r) > v^0(r-1)$. Note that the presence of output losses for some equilibria renders this condition non trivial.

Proposition 7. For all $1 \leq r \leq n$, it is always the case that $v^1(r) > v^0(r-1)$.

We thus verify that all districts that have an option to become active will do so. The assumption that becoming active is costless allows to characterize this decision without having to find explicit expressions for $v^1(r)$ and $v^0(r)$. If instead we assume that the action is costly, then each district, upon observing how many districts managed to solve their collective action problems, would have to compare the expected gain from becoming active with the cost. Furthermore,

[24] Thus, parameters β^- and β^+ can be seen as measures of institutional quality, or as measures of the degree of discretion that districts have to shield regional output from national taxation, or to promote growth opportunities with spillovers.

if information is imperfect, such that each district only observes if they can solve their collective action problem, the expected gain, $E[v^1(r) - v^0(r-1)]$, depends on the distribution of r (which depends on parameters β^+ and β^-). Thus, the decision on becoming active requires knowing $v^i(r)$ for all i and r.[25]

Denote by z the cost of becoming active. For small z, e.g. $z < \min_r[v^1(r) - v^0(r-1)]$, proposition 7 continues to hold, and all districts that have the option to become active will do so. Propositions 2, 3, 4, 5, and 6, and corollary 1 would hold as well.

6 Further Discussion

6.1 Reputational Concerns

The assumption that cooperators and retaliators are committed to cooperate when they are in the winning coalition and committed to riot when they are not was made for expositional clarity and serves to highlight the model's results. A rationale for making this assumption would be that legislators interact over time and have reputational concerns. Thus, a legislator would riot, even if this is a costly action with no immediate payoff, because not doing so would signal that his future threats to do so are vacuous. Our model assumptions capture these concerns without the need to introduce repeated game elements that would complicate the analysis.

It was for similar reasons that we restricted ourselves to stage undominated stationary equilibria. If we lift this assumption then we should consider strategies that discriminate among active districts, giving them a full payoff with some probability, and a smaller payoff (probably infinitesimal) in exchange for cooperating or not rioting. As long as these strategies satisfy (4) together with equations (5) and (6) and (7) they would constitute a stationary equilibria with no output losses.[26] If legislators interact repeatedly, they might not be willing to cooperate for "free" if they expect this would undermine their bargaining power in subsequent

[25] A microfoundation for actions with imperfect information is to have citizens (or a subgroup of them, such as public servants or scientists) in district i observe a noisy signal of the realization of a variable θ_i that summarizes institutional quality or growth opportunities in their district and decide non cooperatively whether to engage in destructive/productive action or not. If the mass of citizens choosing to act is larger than θ_i then the action is successful and we say that the district is active. See Edmond (2013) for a detailed analysis in an application to street protests.

[26] Generically it would still be the case that $m(i) < r$ if m is the number of active districts that are paid their continuation value. But the remaining $r - m(i) - i$ legislators would also be in the winning coalition in this case.

interactions.

6.2 Asymmetric Effects

We can conjecture how our equilibria would be affected if we lift some of our model's assumptions. If one of the types of active districts has a larger effect on rents, e.g. $\eta^- > \eta^+$, then retaliators will have a higher probability than cooperators to be called into the winning coalition and thus higher ex ante values (corollary 1). In fact, for an agenda setter to be indifferent on what type of active district to call into the winning coalition it must be the case that $\delta(v^- - v^+) = \eta^- - \eta^+$.[27]

Finally, we conjecture what would happen if one of the types of districts is more likely to be selected as agenda setter. This would have no direct effect on expected output if we assume that $m(0) = m(1) + 1$.[28] If passive districts are more likely to be agenda setters this would (weakly) increase their continuation value (as in Eraslan (2002)). This would reduce the probability that they are called into a winning coalition, and thus unambiguously would increase expected output. The converse happens when active districts are more likely to be agenda setters. Since their continuation value increases, they are less likely to be called into the winning coaltion and this reduces expected output.

7 Conclusions

We introduce a simple, and arguably natural, assumption in Baron and Ferejohn's (1989) canonical model of legislative bargaining: Some legislators have the ability to either "grease" or "sand" the wheels of policy-making. These legislators, if satisfied with the outcome of the bargaining, cooperate to increase output, rents, or resources available for taxation. Conversely, if unsatisfied, they may retaliate reducing output, rents, or the tax base. With this assumption, the pie to be distributed in the legislative bargaining game becomes endogenous, and determined by the composition of the winning coalition.

Given their ability to affect the level of aggregate resources, active districts are more likely to be called into a winning coalition than passive districts, thus the cost to include them in a winning coaltion is higher. When the agenda setter

[27] This guarantees that in the mixing equilibria with minimum winning coalitions $\delta(v^k - v^0) = \eta^k$, $k = +, -$; and for larger than minimum winning coalitions $\delta v^k = \eta^k$, $k = +, -$.

[28] In this case expected output depends on $E[m] = \frac{\alpha(n-r)}{n} m(0) + \frac{r}{n}\frac{n-(n-r)\alpha}{r}(m(1)+1)$, where α measures the relative likelihood that a passive legislator is the agenda setter (in the baseline $\alpha = 1$). It is clear that $\frac{dE[m]}{d\alpha} = 0$ when $m(0) = m(1) + 1$.

is choosing the composition of her winning coalition, she trades off the higher cost of active districts against the increase in output they produce. Therefore, as patience increases, active districts eventually stop being called into the winning coalition with certainty. This produces output losses, as either the gains of including cooperating legislators are not realized, or retaliation takes place.

When there is a relatively large number of active districts, larger-than-minimum winning coalitions are possible in equilibrium. This feature of our model resonates with the large empirical evidence on larger-than-minimum winning coalitions, and fills a gap in theoretical models of legislative bargaining under Baron and Ferejohn's (1989) protocol, where only minimum winning coalitions are possible. In our model, larger-than-minimum winning coalitions lessens the trade-off between expropriation of minorities and decision-making costs (Buchanan and Tullock (1962); Harstad (2005)). With impatient agents and a large number of active districts, larger-than-minimum coalitions only include active districts and exclude the passive minority while increasing the size of rents.

Our model allows us to conjecture, as discussed in section 6.2, what would happen in a different institutional setting in which not all active districts have the same effect on rents. For example, it is claimed that transitions to democratization in Western Europe and Latin America were fostered by negative economic shocks that allowed for organizing revolutions (as in Acemoglu and Robinson (2001)). If we interpret that negative shocks have a larger effect on resources than positive shocks, then those who have the larger potential to affect resources, retaliators, would be more likely to be called into government.

The mechanisms highlighted in our model may be informative of policymaking more generally. For example, one may interpret retaliation as produced by ethnic conflict, and patience as inversely related to mandate duration. Hence, a dictator can reduce conflict by co-opting opposing ethnic groups that are not necessarily needed to govern, if these perceive that ascension to power is unlikely and thus have few demands. In transition to democracy, as expected mandates shorten, more conflict arises (see Esteban et al. (2012)).

Our finding that an increase in the supermajority, or a reduction in the discount factor, increases efficiency has normative implications. Our model suggests that procedural rules should be made contingent such that voting on legislation is delayed (by e.g. requiring that more committees evaluate a proposal), or the effective supermajority be increased (e.g. by increasing the number of navette rounds in a bicameral legislature before a conference committee is convened), when there are a large number of active districts. This would reduce expected output losses in the presence of retaliators or cooperators, while having no effect on legislative rules when rents are exogenous.

Districts that have the opportunity to commit to the retaliating/cooperating strategies will do so in our setup. Such opportunistic behavior links our results to the literature on institutional strength (Scartascini and Tommasi (2012); Levitsky and Murillo (2009)). Districts only cooperate if they get transfers, incentivizing only conditional cooperation. A weak institutional setting, with large potential damage, many active districts, and/or impatient agents sustains an equilibrium with systematic transfers to active districts. In turn, this leads to greater incentives to become an active member, weakening the institutional framework even further.

Our work provides the foundations for a dynamic game, in which a share of available resources can be used to invest in strengthening institutions, e.g. by reducing the probability that districts can engage in retaliating activities in the following legislative session. Legislative bargaining can thus introduce persistence to output shocks. Similarly, if damages have permanent effects, a dynamic extension can be employed to study climate negotiations. We leave the analysis of such extensions for future work.

References

Acemoglu, D. (2003): "Why not a political Coase theorem? Social conflict, commitment, and politics," Journal of Comparative Economics, 31(4), 620–652.

Acemoglu, D. and J. A. Robinson (2001): "A theory of political transitions," American Economic Review, 91, 938–963.

Anesi, V. and D. J. Seidmann (2015): "Bargaining in standing committees with an endogenous default," Review of Economic Studies, 82, 825–867.

Austen-Smith, D. and J. Banks (1988): "Elections, Coalitions, and Legislative Outcomes," American Political Science Review, 82(2), 405–422.

Banks, J. and J. Duggan (2000): "A bargaining model of collective choice," American Political Science Review, 94(1), 73–88.

Banks, J. S. (2000): "Buying Supermajorities in Finite Legislatures," American Political Science Review, 94(3), 677–681.

Baranski, A. (2019): "Pre-distribution: Bargaining over Property Rights with Endogenous Production," Ssrn.

Baron, D. and R. Bowen (2018): "Dynamic Coalitions," SSRN (Working paper), 3757.

Baron, D. and E. Kalai (1993): "The simplest equilibrium of a majority-rule division game," .

Baron, D. P. (1991): "A Spatial Bargaining Theory of Government Formation in Parliamentary Systems," American Political Science Review, 85(1), 137–164.

——— (1996): "A Dynamic Theory of Collective Goods Programs," American Political Science Review, 90, 316–330.

Baron, D. P. and J. Ferejohn (1989): "Bargaining in Legislatures," American Political Science Review, 83, 1181–1206.

Battaglini, M. (2017): "Public protests and policy making," Quarterly Journal of Economics, 132(1), 485–549.

Beccherle, J. and J. Tirole (2011): "Regional initiatives and the cost of delaying binding climate change agreements," Journal of Public Economics, 95(11-12), 1339–1348.

Buchanan, J. and G. Tullock (1962): The calculus of consent, University of Michigan Press, Ann Arbor.

Calvert, R. L. and N. Dietz (2006): "Legislative Coalitions in a Bargaining Model With Externalities," in Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks, vol. 32(1), 37–58.

Cooper, R. N., A. D. Ellerman, P. L. Joskow, R. Schmalensee, J.-P. Montero, and E. Bailey (2010): Markets for Clean Air: The U.S. Acid Rain Program, vol. 79, Cambridge University Press, Cambridge.

Dal Bó, E. (2007): "Bribing Voters." American Journal of Political Science, 51(4), 789–803.

Dasgupta, P. and E. S. Maskin (2007): "Bargaining and Destructive Power," Annals of Economics and Finance, 8, 217–227.

Diermeier, D. and P. Fong (2011): "Legislative bargaining with reconsideration," Quarterly Journal of Economics, 126(2), 947–985.

Edmond, C. (2013): "Information manipulation, coordination, and regime change," The Review of Economic Studies, 80(1), 1422–1458.

Eraslan, H. (2002): "Uniqueness of stationary equilibrium payoffs in the Baron–Ferejohn model," Journal of Economic Theory, 103(1), 11–30.

Eraslan, H. and A. Merlo (2017): "Some unpleasant bargaining arithmetic?" Journal of Economic Theory, 171, 293–315.

Esteban, J., L. Mayoral, and D. Ray (2012): "Ethnicity and conflict: An empirical study," American Economic Review, 102, 1310–1342.

Fréchette, G. R., J. H. Kagel, and S. F. Lehrer (2003): "Bargaining in legislatures: An experimental investigation of open versus closed amendment rules," American Political Science Review, 97, 221–232.

Gillezeau, R. (2015): "Did the War on Poverty Stop the 1960s Race Riots ?" Working paper, University of Michigan.

Groseclose, T. and J. M. Snyder (2000): "Vote Buying, Supermajorities, and Flooded Coalitions," American Political Science Review, 94(3), 683–684.

Harstad, B. (2005): "Majority Rules and Incentives," The Quarterly Journal of Economics, 120, 1535–1568.

Hummel, P. (2009): "Buying supermajorities in a stochastic environment," Public Choice, 141(3-4), 351–369.

Knight, B. (2008): "Legislative Representation, Bargaining Power and The Distribution of Federal Funds: Evidence From The US Congress," The Economic Journal, 118(532), 1785–1803.

Levitsky, S. and M. V. Murillo (2009): "Variation in Institutional Strength," Annual Review of Political Science, 12, 115–133.

Madison, J. (1787): "The federalist no. 10," November, 22, 1787–88.

Manzini, P. (1999): "Strategic bargaining with destructive power," Economics Letters, 65, 315–322.

Piguillem, F. and A. Riboni (2015): "Spending-biased legislators: Discipline through disagreement," Quarterly Journal of Economics, 130(2), 901–949.

Ray, D. and J. Esteban (2017): "Conflict and Development," Annual Review of Economics, 9(1), 263–293.

Ray, D. and R. Vohra (1999): "A Theory of Endogenous Coalition Structures," Games and Economic Behavior, 26, 286–336.

——— (2015): "Coalition formation," Handbook of Game Theory with Economic Applications.

Riboni, A. and F. J. Ruge-Murcia (2008): "The dynamic (in)efficiency of monetary policy by committee," Journal of Money, Credit and Banking, 40, 1001–1032.

Riker, W. H. (1962): The theory of political coalitions, Yale University Press, New Haven.

Romer, T. and H. Rosenthal (1978): "Political resource allocation, controlled agendas, and the status quo," Public choice, 33, 27–43.

——— (1979): "Bureaucrats Versus Voters: On the Political Economy of Resource Allocation by Direct Democracy," The Quarterly Journal of Economics, 93(4), 563–587.

Scartascini, C. and M. Tommasi (2012): "The making of policy: institutionalized or not?" American Journal of Political Science, 56(4), 787–801.

Shepsle, K. A. (1974): "On the Size of Winning Coalitions," American Political Science Review, 68(2), 505–518.

Snyder, J. M., M. M. Ting, and S. Ansolabehere (2005): "Legislative Bargaining under Weighted Voting," American Economic Review, 95(4), 981–1004.

Stole, L. and J. Zwiebel (1996): "Organizational Design and Technology Choice under Intrafirm Bargaining : Reply," American Economic Review, 86, 456–457.

8 Appendix

8.1 Proof of Lemma 1

Suppose that $m^1(i) + m^0(i) > q - 1$, such that there is a larger-than-minimum coalition. Consider the alternative strategy in which $m^0(i)$ is reduced by Δm. As long as $m^1(i)+m^0(i)-\Delta m \geq q-1$, and $m^0(i)-\Delta m \geq 0$, the proposal is feasible, will be approved and the agenda setter's payoff increases in $\Delta m \delta v^0 \geq 0$. Thus, it must be the case that, when $v^0 > 0$, all equilibria with larger-than-minimum coalitions feature $m^0(i) = 0$.

Now consider a minimum winning coalition with $m^1(i) < q - 1$. Suppose that as part of the mixing strategy, the agenda setter considers calling with positive probability $z \geq q$ active districts. Consider the alternative strategy of reducing by $\Delta\pi$ the probability of doing this and instead increase by $\Delta\pi$ the probability of including $q-1$ districts. The agenda setter's payoff increases by $\Delta\pi(z-q+1)\delta v^1 > 0$. Thus, no minimum winning coalition would consider including more than $q-1$ active districts.

8.2 Proof of Proposition 1

Every pair of $m(i)$, $i = 0, 1$, such that the first order conditions and the value functions are satisfied for all ρ^i (equations (4), (5), (6) and (7)) is an equilibrium. We show that such pair always exists. Without loss of generality, we classify strategies into two types: (a) $m(i) = \max[0, q + r - n - i]$, and (b) $m(i) > \max[0, q + r - n - i]$ and $m(1) = m(0) - 1$.[29] We will first show that $v^0 > v^1$ for all strategies of type (a) such that there is no equilibrium in these strategies. This intermediate result is useful to show that an equilibrium can only exist in strategies of type (b), and establish existence by continuity of v^i.

We begin with (a):

When $m(1) = m(0) = 0$, $\rho^1(0,0) = 0$, $v^1 = \frac{1}{n}[Y - (q-1)\delta v^0]$ and $v^0 = \frac{1}{n}[Y - (q-1)\delta v^0] + \rho^0(0,0)\delta v^0$. Since $\rho^0(0,0) > 0$, $v^0 > v^1$ and this violates (4). When $m(i) = q + r - n - i$, the algebra is more cumbersome, yielding $v^1 = [Y+(q+r-n)\eta][rn-r\delta(n-1)]/\Delta$, $v^0 = [Y+(q+r-n)\eta]n[r-\delta(q+r-n)]/\Delta$, resulting again, since $\Delta > 0$, in $v^0 > v^1$ and thus violating (4).[30]

We continue with (b):

[29] Note that $m(i) = \max[0, q + r - n - i]$ is the minimum amount of active districts that an agenda setter needs to call.

[30] The term $\Delta = \delta n(n + \delta)(n - q) + (\delta + n)[(1 - 2\delta)n + \delta q]r + \delta^2 r^2$.

We first note that there is a one-to-one relation between $\tilde{Y}$ and $m(0)$. Equations (6) and (7) imply a one-to-one relation between strategies and probabilities of being a floor legislator in a winning coalition, $\rho^i(m(0), m(0)-1)$. Given these probabilities and $m(0)$, equations (5) determine the corresponding value functions for passive and active legislators. It is straightforward from equations (5) that an increase in $m(0)$ (and $m(1) = m(0) - 1$) will increase v^1 and decrease v^0, relative to values found in (a), and that value functions are continuous in strategies.

We consider the case in which all active districts are called: $m(0) = r$. For $r \geq q$, equations (5) can be solved yielding

$$v^1 = \frac{(Y + r\eta)}{n - (n-r)\delta}, \quad v^1 - v^0 = \frac{\delta(Y + r\eta)}{n - (n-r)\delta}$$

- For $\delta \to 1$, $v^1 - v^0$ is larger than η and this violates (4). By continuity, there $\exists\ 0 < m(0) < r$ that satisfies (4), either as $v^1 - v^0 = \eta$ or $v^1 = \eta$, or $v^1 - v^0 < \eta$, $v^1 > \eta$ and $m(0) = q - 1$. Note that as $v^1 - v^0$ is increasing in $m(0)$ only one type of equilibrium will be attained.

- For $\delta < 1$ as long as $\delta(v^1 - v^0) > \eta$. For lower δ, such that $\delta(v^1 - v^0) < \eta$ when $m(0) = r$, either $\delta v^1 < \eta$ and $m(0) = r$ is an equilibrium, or there $\exists\ 0 < m(0) < r$ that satisfies (4) as established above.[31]

Next, for $r < q$ if $m(0) = r$ equations (5) can be solved yielding

$$v^1 - v^0 = \frac{\delta(Y + r\eta)(n-q)}{n(1-\delta)(n-r) - r(n-q)\delta}.$$

- When $\delta \to 1$, $v^1 - v^0$ is larger than η and this violates (4). By continuity, there $\exists\ 0 < m(0) < r$ that satisfies (4), in this case it must be an equilibrium with $(v^1 - v^0) = \eta$.

- For $0 < \delta < 1$ if $\delta(v^1 - v^0) > \eta$ when $m(0) = r$. Otherwise the equilibrium is in a corner with $m(0) = r$ and $\delta(v^1 - v^0) < \eta$.

8.3 Proof of Lemma 2

From first order condition (4) it is immediate that, if $v^0 \geq v^1$, an agenda setter would never choose to have a passive district in her coalition when an active one is available. If $r \geq q$, no passive is called into the winning coalition, so the value of a

[31] By continuity, (4) is satisfied, either as $\delta(v^1 - v^0) = \eta$ or $\delta v^1 = \eta$, or $\delta(v^1 - v^0) < \eta$, $\delta v^1 > \eta$ and $m(0) = q - 1$.

passive legislator is just the recognition probability, $\frac{1}{n}$, times the proposer's payoff of a passive agenda setter. But an active agenda setter would have a larger surplus output (since she comes from an active district the proposer's payoff, conditional on the same voting majority, is higher), the same recognition probability, and would be called into a winning coalition with higher, positive, probability. Thus, it must be the case that $v^1 > v^0$. Consider now the case that $r < q$. A passive district then has positive probability of being called into the winning coalition. But, this probability is 1 for active districts and thus higher than for passive districts (proposer's payoff, conditional on the same voting majority, is higher for an active agenda setter). Therefore, it is also the case that $v^1 > v^0$.

8.4 Proof of Proposition 2

The threshold $\delta_q(r)$ is zero when even including all active districts the winning coalition is minimal. This is always the case when $r \leq q-1$, and is also the case when $r = q$ and the agenda setter is from a active district.

When $r > q$ or $r = q$ and the agenda setter is from a passive district, the threshold $\delta_q(r)$ will be determined by solving the equilibrium under the assumption that $m(i) = q-1$, and verifying that the agenda setter does not prefer to increase the expected number of active districts into the coalition by Δm. For this case, from equations (5), values must satisfy

$$\begin{aligned} v^0 &= \frac{1}{n}\left[Y + (q-1)\eta - (q-1)\delta v^1\right], & (8)\\ v^1 &= \frac{1}{n}\left[Y + q\eta - (q-1)\delta v^1\right] + \frac{n-1}{n}\left[\frac{n-r}{n-1}\frac{q-1}{r} + \frac{r-1}{n-1}\frac{q-1}{r-1}\right]\delta v^1. & (9)\end{aligned}$$

From the second equation we can solve for v^1

$$v^1 = \frac{r\,(Y + q\eta)}{nr - \delta(n-r)(q-1)}. \tag{10}$$

Whenever $\delta v^1 > \eta$, the agenda setter will be unwilling to increase by $\Delta m > 0$ the expected number of active districts into the coalition, as this would reduce her payoff by $\Delta m(\delta v^1 - \eta) > 0$. Thus the agenda setter forms a minimum winning coalition calling $q-1$ active districts into it. Thus, δ_q is implicitly determined by $\delta_q v^1 = \eta$,

$$\eta = \delta_q \frac{r\,(Y + q\eta)}{nr - \delta_q(n-r)(q-1)}. \tag{11}$$

Since then RHS of the last equation is increasing in $\delta_q(r)$, the coalition will be minimal when $\delta \geq \delta_q$.

8.5 Proof of Proposition 3

To determine the threshold $\bar{\delta}$ we solve for a corner equilibrium with $m(i) = r - i$, and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition. To solve for the value functions, from equations (5),

$$\begin{aligned} v^0 &= \frac{1}{n}\left[Y + r\eta - r\delta v^1 - (q-r-1)\delta v^0\right] + \frac{n-1}{n}\left[\frac{n-r-1}{n-1}\frac{q-r-1}{n-r-1} + \frac{r}{n-1}\frac{q-r}{n-r}\right]\delta v^0, \\ v^1 &= \frac{1}{n}\left[Y + r\eta - (r-1)\delta v^1 - (q-r)\delta v^0\right] + \frac{n-1}{n}\delta v^1. \end{aligned}$$

Where the last equation shows that in this case all active districts are included in the winning coalition with probability one. Solving we find

$$\begin{aligned} v^0 &= \frac{(Y + r\eta)(1-\delta)(n-r)}{n(n-r)(1-\delta) + r\delta(n-q)}, \\ v^1 &= \frac{Y + r\eta - (q-r)\delta v^0}{n - (n-r)\delta}. \end{aligned} \tag{12}$$

Note that $\frac{dv^0}{d\delta} < 0$. Since expected output is independent of δ, and feasibility implies $rv^1(\delta) + (n-r)v^0(\delta) = Y + r\eta$, it must be the case that

$$r\frac{dv^1}{d\delta} + (n-r)\frac{dv^0}{d\delta} = 0.$$

Thus, $\frac{dv^1}{d\delta} > 0$ and $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$. To show that $0 < \bar{\delta} < 1$ we note that $v^0|_{\delta=0} = v^1|_{\delta=0} = \frac{Y+r\eta}{n}$, while $v^0|_{\delta=1} = 0$, and $v^1|_{\delta=1} = \frac{Y+r\eta}{r}$, implying $v^1|_{\delta=1} - v^0|_{\delta=1} > \eta$. Thus, $\bar{\delta}$ is determined by

$$\begin{aligned} \bar{\delta}\left(v^1|_{\bar{\delta}} - v^0|_{\bar{\delta}}\right) &= \eta, \\ \frac{\bar{\delta}(Y + r\eta)}{n(1-\bar{\delta}) + r\bar{\delta}}\left[1 - \frac{1 - \bar{\delta} + \bar{\delta}q/n}{1 + \frac{r\bar{\delta}(n-q)}{(1-\bar{\delta})(n-r)}}\right] &= \eta. \end{aligned} \tag{13}$$

and for $\delta > \bar{\delta}$, the expected number of active districts that the agenda setter would choose satisfies $m(i) < r - i$, as the cost or including all active districts in the coalition is higher than the resource cost of excluding some of them.

8.6 Proof of Proposition 4

(i) Since we assume $\delta \geq \delta_q$, from proposition 2 we are only considering minimum winning coalitions. To determine the threshold $\bar{\delta}$ we solve for a corner equilibrium with $m(i) = q-1$ and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition. Equations (8) and (9) characterize v^0 and v^1, from which we get

$$v^1 - v^0 = \frac{\eta}{n} + \frac{q-1}{n}\left[\frac{(n-r)}{r} + 1\right]\delta v^1$$

From (10) we have that $\frac{dv^1}{d\delta} > 0$ which implies $\frac{d\delta(v^1-v^0)}{d\delta} > 0$. The threshold $\bar{\delta}$ is characterized by

$$\eta = \bar{\delta}\left[\frac{\eta}{n} + \bar{\delta}\frac{(q-1)[Y+q\eta]}{nr-(n-r)(q-1)\bar{\delta}}\right]. \tag{14}$$

When $\bar{\delta} < \delta$, the agenda setter prefers to exclude some active districts from the minimum-winning coalition, and the equilibrium is interior.

(ii) To determine the threshold $\underline{\delta}$ we solve for a corner equilibrium with $m(i) = r-i$ and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition.

$$\begin{aligned} v^0 &= \frac{1}{n}\left[Y + r\eta - r\delta v^1\right], \\ v^1 &= \frac{1}{n}\left[Y + r\eta - (r-1)\delta v^1\right] + \frac{n-1}{n}\delta v^1. \end{aligned}$$

From the second equation we derive

$$v^1 = \frac{Y + r\eta}{n - \delta(n-r)}.$$

It is immediate that $\frac{dv^1}{d\delta} > 0$. An agenda setter will be willing to include all active districts in the coalition as long as the cost of doing so is lower than the damage they could produce on output. Thus, the threshold $\underline{\delta}$ is determined by $\underline{\delta}v^1 = \eta$,

$$\underline{\delta}\frac{Y + r\eta}{n-(n-r)\underline{\delta}} = \eta \implies \underline{\delta} = \frac{n\eta}{Y + n\eta}. \tag{15}$$

When $\underline{\delta} < \delta < \delta_q$ the agenda setter will form a coalition with $q-1 < m(i) < r-i$ active districts in expectation.

8.7 Proof of Corollary 1

It is straightforward that active districts have a higher probability of being in the winning coalition when $r > q-1$, and $\delta \in [0, \delta_q)$, as in this case passive districts are never called into a winning coalition. When $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q-1, r-i\}$, if $m(i) = q-1$, active districts have a positive probability of being in the winning coalition while passive districts are never called into it, while if $m(i) = r-i$ an active district's probability of being in the winning coalition is 1, thus higher than for a passive district.

We are thus left with the case $\delta \geq \bar{\delta}$. To prove that active districts must have a higher probability of being in the winning coalition we proceed by contradiction and assume that this probability is the same for every district (also see proof of lemma 2). If this were the case, the probability of being in the winning coalition must be $\frac{q-1}{n}$. This implies

$$m(0) = \frac{rq}{n}, \qquad m(1) = \frac{rq}{n} - 1.$$

We now use equations (5) to estimate v^0 and v^1:

$$\begin{aligned} v^0\left(1 - \delta\frac{q-1}{n}\right) &= \frac{1}{n}\left[Y + \frac{rq}{n}\eta - (\frac{rq}{n})\delta v^1 - (q - \frac{rq}{n} - 1)\delta v^0\right], \\ v^1\left(1 - \delta\frac{q-1}{n}\right) &= \frac{1}{n}\left[Y + \frac{rq}{n}\eta - (\frac{rq}{n} - 1)\delta v^1 - (q - \frac{rq}{n})\delta v^0\right]. \end{aligned}$$

These equations imply

$$(v^1 - v^0)\left(1 - \delta\frac{q-1}{n} - \frac{\delta}{n}\right) = 0. \tag{16}$$

But for an interior solution, as must be the case when $\delta \geq \bar{\delta}$, first order condition (4) implies

$$\delta(v^1 - v^0) = \eta. \tag{17}$$

Equation (16) is generically inconsistent with (17), and would imply that if districts have the same probability of being in the winning coalition they should have the same continuation values, i.e. $v^1 = v^0$. Thus, this tells us that the source of higher ex ante payoffs for active districts is precisely their higher probability of being in the winning coalition.

To prove that $v^0 > 0$ we start by assuming $v^0 = 0$, such that equations (5) give

$$\begin{aligned}
v^0 &= \frac{1}{n}[Y + m(0)\eta - m(0)\delta v^1] = 0, \\
v^1 &= \frac{1}{n}[Y + (m(1)+1)\eta] + \frac{n-r}{nr}m(0)\delta v^1,
\end{aligned}$$

From (4) we have that in equilibrium either $\delta v^1 < \eta$ and $m(0) = m(1) + 1 = r$, or $\delta v^1 = \eta$. The latter case can immediately be discarded as it would imply, from value function for v^0, that $Y = 0$ and by assumption $Y - n\eta > 0$. Let's consider then that $\delta v^1 < \eta$. The value function for v^1 gives $v^1 = \frac{Y+r\eta}{n-\delta(n-r)}$. Replacing this in the value function for v^0 gives

$$v^0 = \frac{1}{n}[Y + r\eta]\left(\frac{n(1-\delta)}{n-\delta(n-r)}\right).$$

But this is positive if $\delta < 1$. Thus, we prove that $v^0 > 0$.

8.8 Proof of Lemma 3

The proof proceeds by contradiction. Suppose $v^+ > v^-$. Then the agenda setter can increase her payoff by reducing m^+ by Δm, and increasing m^- by Δm, keeping $m^+ + m^-$ unaffected. This has no impact on resources to be distributed (excluded productive districts will not increase output by $\eta\Delta m$, but included destructive district will refrain from destroying resources by $\eta\Delta m$). And the change in the composition of the winning coalition increases the agenda setter's payoff by $\Delta m\delta(v^+ - v^-) > 0$. Thus, the agenda setter will try to replace productive by destructive districts as much as possible. If no agenda setter includes productive districts unless they are needed to reach a minimum winning coalition, that is, $m^+(i) = \max\{0, q + r^+ - n - \mathbb{1}_{i=+}\}$.

If $m^+(i) = 0$, then a destructive district as agenda setter would have the same surplus output as a productive one (at least when $m^-(i) \leq r^- - 1$, otherwise proof mirrors the case $m^+(i) = q + r^+ - n - \mathbb{1}_{i=+}$, see below), the same recognition probability, and would be called into a winning coalition with higher, positive, probability, $\frac{m^-(i)}{r^-} > 0 = \frac{m^+(i)}{r^+}$. Thus, it must be the case that $v^- \geq v^+$.

If $m^+(i) = q + r^+ - n - \mathbb{1}_{i=+}$, then a productive district as agenda setter would have a higher surplus output so we need to evaluate the value functions. These would be given by,

$$\begin{aligned}
v^{+} &= \frac{1}{n}\left[Y-(r^{+}-(q+r^{+}-n))\eta - r^{-}\delta v^{-} - m^{0}(+)\delta v^{0} - (q+r^{+}-n-1)\delta v^{+}\right] \\
&\quad + \frac{q+r^{+}-n-r^{+}/n}{r^{+}}\delta v^{+}, \\
v^{-} &= \frac{1}{n}\left[Y-(r^{+}-(q+r^{+}-n))\eta - (r^{-}-1)\delta v^{-} - m^{0}(-)\delta v^{0} - (q+r^{+}-n)\delta v^{+}\right] \\
&\quad + \frac{n-1}{n}\delta v^{-}.
\end{aligned}$$

Given that $m^0(+) = m^0(-)$ substracting we get

$$(v^{-}-v^{+})\left[1-\frac{\delta}{n}-\delta\frac{q+r^{+}-n-r^{+}/n}{r^{+}}\right] = \delta\left[\frac{n-1}{n}-\frac{q+r^{+}-n-r^{+}/n}{r^{+}}\right]v^{-}.$$

Since the terms in square brakes in the LHS and RHS are both positive, and $v^- \geq 0$, it must again be the case that $v^- \geq v^+$. Thus, we cannot have $v^+ > v^-$. Similar reasoning rules out $v^+ < v^-$, and we conclude that it must be the case that $v^+ = v^-$.

8.9 Proof of Propositions 5 and 6

We start by characterizing equilibria for the two types of interior equilibria: a) for minimum winning coalitions, $\delta > \bar{\delta}$, and b) for larger-than-minimum winning coalitions, $\delta \in [\underline{\delta}, \delta_q)$.

a) We expect to find multiple interior equilibria since we have a system of three equations, (5), and the indifference condition $\delta(v^1 - v^0) = \eta$, in four unknowns, v^0, v^1, $m(0)$, and $m(1)$. Using these three equations leads to a continuum of equilibria characterized by a relation between strategies, say $m(1) = f(m(0))$. This allows us to write $v^0(m(0))$ and $v^1(m(0))$, which from (5) are given by:

$$\begin{aligned}
v^{0}(m(0)) &= \frac{\frac{1}{n}\left[\tilde{Y}\left(m(0)\right)-e\left(C_{m}^{0}\right)\right]}{1-\frac{n-1}{n}\rho^{0}(m(0))\delta} \\
v^{1}(m(0)) &= \frac{\frac{1}{n}\left[\tilde{Y}\left(f(m(0)\right)-e\left(C_{m}^{1}\right)\right]}{1-\frac{n-1}{n}\rho^{1}(m(0))\delta}
\end{aligned} \tag{18}$$

Because strategies $m(0)$ and $f(m(0))$ satisfy $\delta(v^1 - v^0) = \eta$, for all feasible $m(0)$

we must have that

$$\frac{d\left[\tilde{Y}\left(m(0)\right)-e\left(C_m^0\right)\right]}{dm(0)}=\frac{d\left[\tilde{Y}\left(f(m(0))-e\left(C_m^1\right)\right]}{dm(0)}=0,$$

since agenda setters are indifferent with respect to the composition of their coalitions. We must also have that the total derivatives $\frac{dv^1(m(0))}{dm(0)}=\frac{dv^0(m(0))}{dm(0)}$ (to satisfy $\delta(v^1(m(0))-v^0(m(0)))=\eta$). Using expressions (18), after some algebra, this implies

$$\frac{dv^0(m(0))}{dm(0)}=\frac{n-1}{n}\delta\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}\frac{d\rho^0}{dm(0)}=\frac{n-1}{n}\delta\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}\frac{d\rho^1}{dm(0)}=\frac{dv^1(m(0))}{dm(0)}.$$

Taking total derivatives for the probabilities of being called into the winning coalition, (6), and (7), and replacing above we get

$$\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}\left(-\frac{r}{n-r}\frac{df(m(0))}{dm(0)}-1\right)=\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}\left(\frac{df(m(0))}{dm(0)}+\frac{n-r}{r}\right)$$

If $\frac{df(m(0))}{dm(0)}=-\frac{n-r}{r}$ then $\frac{dv^1(m(0))}{dm(0)}=\frac{dv^0(m(0))}{dm(0)}=0$. Otherwise we can eliminate from both sides the term $\left(\frac{df(m(0))}{dm(0)}+\frac{n-r}{r}\right)$ and

$$-\frac{n-r}{r}\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}=\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}.$$

But this is absurd since the LHS is negative and the RHS is positive. Thus the only possible solution is that $\frac{df(m(0))}{dm(0)}=-\frac{n-r}{r}$, and v^0 and v^1 are independent of $m(0)$. The intuition for this result comes from the fact that these strategies give legislators the same ex ante probability of being called into the winning coalition, and thus the same ex ante value since the probability of being agenda setter is always $\frac{1}{n}$.

Given that all solutions feature the same ex ante values we can apply a refinement to have a system of four equations in four unknowns. We choose that expected output be independent of the identity of the agenda setter:

$$Y-(r^{-}-m(0))\eta=Y-(r^{-}-m(1)-1)\eta.$$

Using the indifference condition $\delta(v^1-v^0)=\eta$ to replace v^1 as a function of v^0 in equation (5) for $i=0$, and the feasibility constraint (which can be used instead of (5) for $i = 1$) we get the following system of two equations in two unknowns

$$v^0 \left(1 - \frac{\delta}{n}\left(\frac{rq - nm(0)}{n-r}\right)\right)) = \frac{1}{n}\left[Y - r^{-}\eta\right] \tag{19}$$

$$nv^0 = Y - \left(\frac{r + \delta r^{-}}{\delta} - m(0)\right)\eta \tag{20}$$

b) The proof mirrors a), with the indifference condition now given by $\delta v^1 = \eta$. From (5) for $i = 1$ we can get the expression for v^1 as a function of $m(0)$ and $m(1)$. Imposing the condition $\delta v^1 = \eta$ for a mixed strategy equilibrium gives a continuum of equilibria characterized by a relation between strategies, $m(1) = f(m(0))$. This allows us to write $v^1(m(0))$, which from (7) is given by:

$$v^1(m(0)) = \frac{\frac{1}{n}\left[\tilde{Y}\left(f(m(0)) - e\left(C_m^1\right)\right]}{1 - \frac{n-1}{n}\rho^1(m(0))\delta}$$

A parallel reasoning as before tells us that both v^1, and the numerator of the expression above are invariant to changes in $m(0)$ as long as $\delta v^1(m(0)) = \eta$. Thus ρ^1 is independent of $m(0)$, which implies that, as before, $\frac{df(m(0))}{dm(0)} = -\frac{n-r}{r}$. As a corollary we have that v^0 is also independent of $m(0)$ ($\rho^0 = 0$ since passive districts are never called into a winning coalition when this is larger-than-minimum). We apply the same refinement that expected output be independent of the identity of the agenda setter.

Replacing the indifference condition, $v^1 = \frac{\eta}{\delta}$, into (5) for $i = 0$, and into the feasibility constraint, the latter results in

$$\begin{aligned}
(n-r)v^0 + r\frac{\eta}{\delta} &= Y - r^{-}\eta + m(0)\eta \\
(n-r)\frac{1}{n}\left[Y - r^{-}\eta\right] + r\frac{\eta}{\delta} &= Y - r^{-}\eta + m(0)\eta \\
-\left[Y - r^{-}\eta\right]\frac{r}{n} + r\frac{\eta}{\delta} &= m(0)\eta. \qquad (21)
\end{aligned}$$

We now continue the proof or our comparative static results with the following lemma, for which $E(m)$ is the expected number of active districts present in interior equilibria. Note that under our refinement, $E(m) \equiv m_0$.

Lemma 4. For the thresholds characterizing equilibrium types in proposition 4,

$$
\begin{array}{llllll}
\text{i) } r > q-1: & \dfrac{d\delta_q}{dq} < 0 \;, & \dfrac{d\delta_q}{d\eta} > 0 \;, & \dfrac{d\delta_q}{dr^-} > 0 \;, & \dfrac{d\delta_q}{dr^+} > 0, & \\
& \dfrac{d\underline{\delta}}{dq} = 0 \;, & \dfrac{d\underline{\delta}}{d\eta} > 0 \;, & \dfrac{d\underline{\delta}}{dr^-} > 0 \;, & \dfrac{d\underline{\delta}}{dr^+} = 0, & \\
& \dfrac{d\bar{\delta}}{dq} < 0 \;, & \dfrac{d\bar{\delta}}{d\eta} > 0 \;, & \dfrac{d\bar{\delta}}{dr^-} > 0 \;, & \dfrac{d\bar{\delta}}{dr^+} > 0, & (m(i) = q-1) \\
\text{ii) } r \leq q-1: & \dfrac{d\bar{\delta}}{dq} > 0 \;, & \dfrac{d\bar{\delta}}{d\eta} > 0 \;, & \dfrac{d\bar{\delta}}{dr^-} \lesseqgtr 0 \;, & \dfrac{d\bar{\delta}}{dr^+} \lesseqgtr 0. & (m(i) = r-i)
\end{array}
$$

For interior equilibria,

$$
\begin{array}{llllll}
\text{iii) } \delta > \bar{\delta}: & \dfrac{dE(m)}{d\delta} < 0 \,, & \dfrac{dE(m)}{dq} > 0 \,, & \dfrac{dE(m)}{d\eta} > 0 \,, & \dfrac{dE(m)}{dr^-} > 0 \,, & \dfrac{dE(m)}{dr^+} > 0, \\
\text{iv) } \delta \in [\underline{\delta}, \delta_q): & \dfrac{dE(m)}{d\delta} < 0 \,, & \dfrac{dE(m)}{dq} = 0 \,, & \dfrac{dE(m)}{d\eta} > 0 \,, & \dfrac{dE(m)}{dr^-} > 0 \,, & \dfrac{dE(m)}{dr^+} > 0.
\end{array}
$$

Note that i) is straightforward from (11), (14), and (15), and iv) is straightforward from (21). Note that (21) also allows to sign, when possible, the effect on output losses. The proof of iii) is a bit more complicated as there are two equations in the two unknowns, m_0 and v_0. Nevertheless, after some algebra to replace the derivatives of v_0 with respect to the different parameters we find the above results, which hold since $nm_0 > rq$ for all interior equilibria when $\delta > \bar{\delta}$ (otherwise it would not be the case that $v_1 > v_0$). For ii) the effect of q is straightforward from (13). For η this follows since we established that $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$ in the proof of proposition 4. For r^- and r^+ the effects are ambiguous.[32] The intuition is that an increase in the number of active districts has a negative effect on the continuation value of both active and passive districts. For the former due to the dilution of agenda setter rents, while for the latter due to lower probability of being in the winning coalition. Higher values of δ increase the continuation value of active districts inducing the agenda setter to call them less often into the winning coalition.

[32]For this we need to generalize (13) when active districts can be both productive and destructive. In this case it can be shown that

$$
\frac{\bar{\delta}(Y + r^+\eta)}{n(1-\bar{\delta}) + r\bar{\delta}} \left[1 - \frac{1 - \bar{\delta} + \bar{\delta}q/n}{1 + \frac{r\bar{\delta}(n-q)}{(1-\bar{\delta})(n-r)}} \right] = \eta.
$$

8.10 Proof of Proposition 7

We consider first the case with $r > q-1$ and $\delta \in [0, \underline{\delta})$, i.e. when all active districts are included in the winning coalition and this is larger-than-minimum. Since there are no output losses, the feasibility constraint implies that for all r

$$rv^1(r) + (n-r)v^0(r) = Y + r^+\eta. \tag{22}$$

Since, from lemma 2, $v^1(r) > v^0(r)$, equation (22) implies that $v^1(r) > \frac{Y+r^+\eta}{n} > v^0(r)$ for all r. Thus, it must be the case that $v^1(r) > v^0(r-1)$ for all r.

Next we consider the case $r > q-1$, and $\delta \in [\underline{\delta}, \delta_q)$, i.e. when there is a larger-than-minimum winning coalition but not all active districts are included in it. Since the agenda setter in these equilibria satisfies the first order condition (4) for an interior equilibrium, and $m(i) > q-1$, it must be the case that

$$\eta - \delta v^1(r) = 0.$$

Considering that the RHS of equation (22) now reflects an output loss, $Y - r^-\eta + m(r)\eta$, we infer that

$$v^1(r) = \frac{\eta}{\delta} > \frac{Y}{n} - \frac{r^- - m(r)}{n}\eta > v^0(r).$$

Thus, we find that $v^1(r) > v^0(r-1)$ for all r.

We now consider the case $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q-1, r-i\}$ active districts. If $m(i) = r-i$ then output is efficient and we can apply the logic of the case with $r > q-1$ and $\delta \in [0, \underline{\delta})$. Thus, we consider that $m(i) = q-1$ and there are output losses. The value functions $v^1(r)$ and $v^0(r)$ for this case must satisfy equations (8) and (9). Thus,[33]

$$\begin{aligned}
v^1(r) &= \frac{r\left(Y - r^-\eta + q\eta\right)}{nr - \delta(n-r)(q-1)}, \\
v^0(r-1) &= \frac{1}{n}\left[Y - (r^- - 1)\eta + (q-1)\eta - \frac{(q-1)\delta(r-1)\left(Y - (r^- - 1)\eta + q\eta\right)}{n(r-1) - \delta(n-r+1)(q-1)}\right], \\
&= \frac{\left(Y - r^-\eta + q\eta\right)\left[r - 1 - \delta(q-1)\right] - \delta(q-1)(r-1)\eta}{n(r-1) - \delta(n-r+1)(q-1)}.
\end{aligned}$$

[33] In what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.

Thus,

$$\begin{aligned} v^0(r-1) &= v^1(r)\frac{r-(1+\delta(q-1))}{r}\frac{nr-\delta(n-r)(q-1)}{n(r-1)-\delta(n-r+1)(q-1)} \\ &\quad -\frac{\delta(q-1)(r-1)\eta}{n(r-1)-\delta(n-r+1)(q-1)} \\ &< v^1(r)\frac{r-(1+\delta(q-1))}{r}\frac{nr-\delta(n-r)(q-1)}{n(r-1)-\delta(n-r+1)(q-1)}. \end{aligned}$$

Where the inequality in the last step follows since $r > 1$. Finally, the term multiplying $v^1(r)$ in the last expression is smaller than one (this follows since $r > q-1$). Thus, it is always the case that $v^1(r) > v^0(r-1)$ for all r.

We are left now with the last case, $\delta \geq \bar{\delta}$, i.e. interior equilibria that imply minimum winning coalitions. Since the agenda setter in these equilibria satisfies the first order condition (4) for an interior equilibrium, and $m(i) < q-1$, it must be the case that

$$\eta - \delta(v^1(r) - v^0(r)) = 0. \tag{23}$$

The RHS of the feasibility constraint, (22), now is given by $Y - r^-\eta + m(r)\eta$. Using equation (23) to write the LHS of the feasibility constraint in terms of $v^0(r)$ we have

$$nv^0(r) + r\frac{\eta}{\delta} = Y - r^-\eta + m(r)\eta.$$

Using this last equation for r and $r-1$ and equation (23) we get[34]

$$n[v^1(r) - v^0(r-1)] = \eta\left(\frac{n-1}{\delta} - 1\right) + \eta(m(r) - m(r-1)).$$

Since the first term in the RHS is positive, if $m(r) \geq m(r-1)$, then $v^1(r) > v^0(r-1)$. We prove this by contradiction. If $m(r) < m(r-1)$, we can show that there are strategies that result in higher values v^0 and v^1, implying that a choice of $m(r) < m(r-1)$ is suboptimal. For this we consider strategies that imply $m'(r) = m(r-1)$, which is a feasible option. We write feasibility constraints for $m(r)$ and $m'(r)$, using (23) to substitute $v^1(r)$ in terms of $v^0(r)$,

$$\begin{aligned} nv^0(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r)\eta, \\ nv^{0'}(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r-1)\eta \end{aligned}$$

[34] Again, in what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.

Subtracting these two equations we get

$$n[v^{0'}(r) - v^0(r)] = [m(r-1) - m(r)]\eta > 0.$$

Thus proving that $m(r) < m(r-1)$ is not optimal. This completes the proof that $v^1(r) > v^0(r-1)$ for all $r \geq 1$ and all δ.